

Flavored hadron correlations and interactions from heavy ion collisions

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*The 8th Asian Triangle Heavy-Ion Conference (ATHIC 2021),
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- Introduction
- Basics in femtoscopic study of hadron-hadron interactions
- Recent & Future Femtoscopic Studies of HHI
- Summary

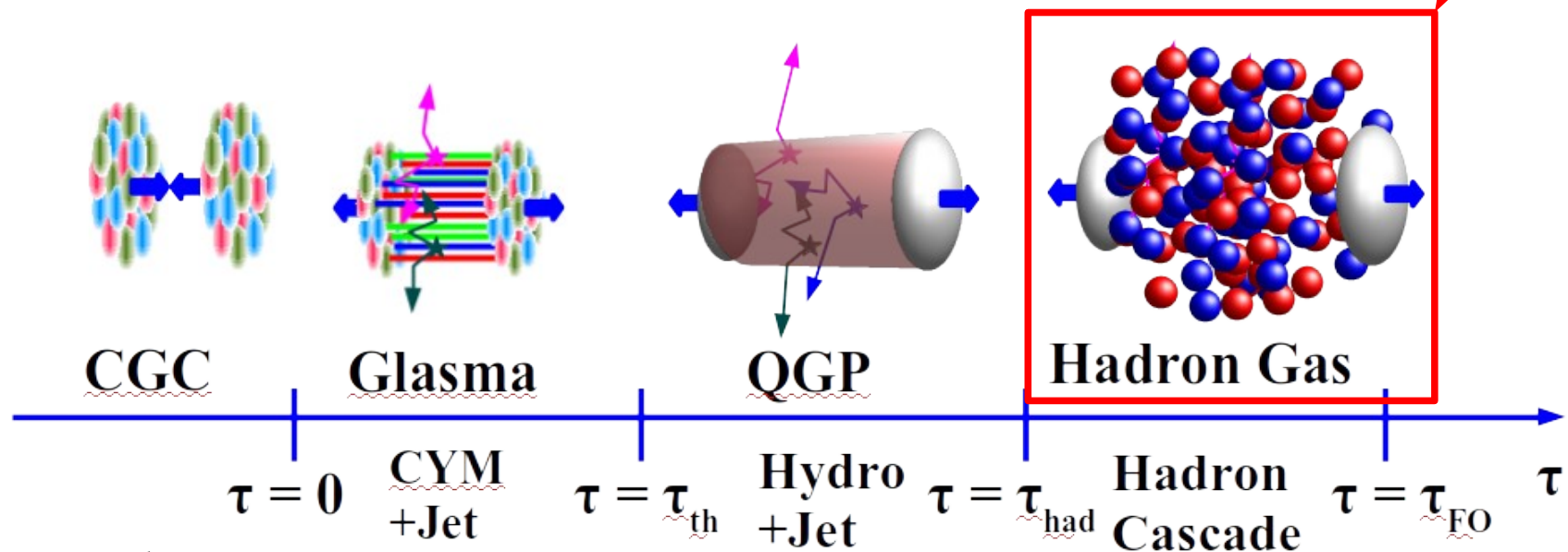
Hadron Physics using High-Energy HIC

High-Energy Heavy-Ion Collisions

- Too complex → Statistical → Simple and Clean !
- High T & Large volume → Abundant hadrons
- Nearly 4π detector / Vertex detector

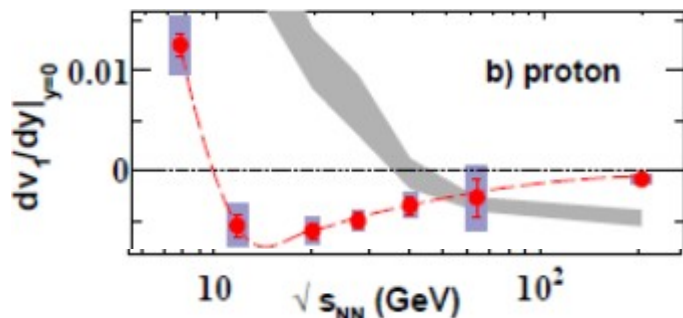
Let's study Hadron Physics

*(e.g. hadron-hadron interactions and hadronic matter EOS)
by using High-Energy Nuclear Collisions as Hadron Factories.*

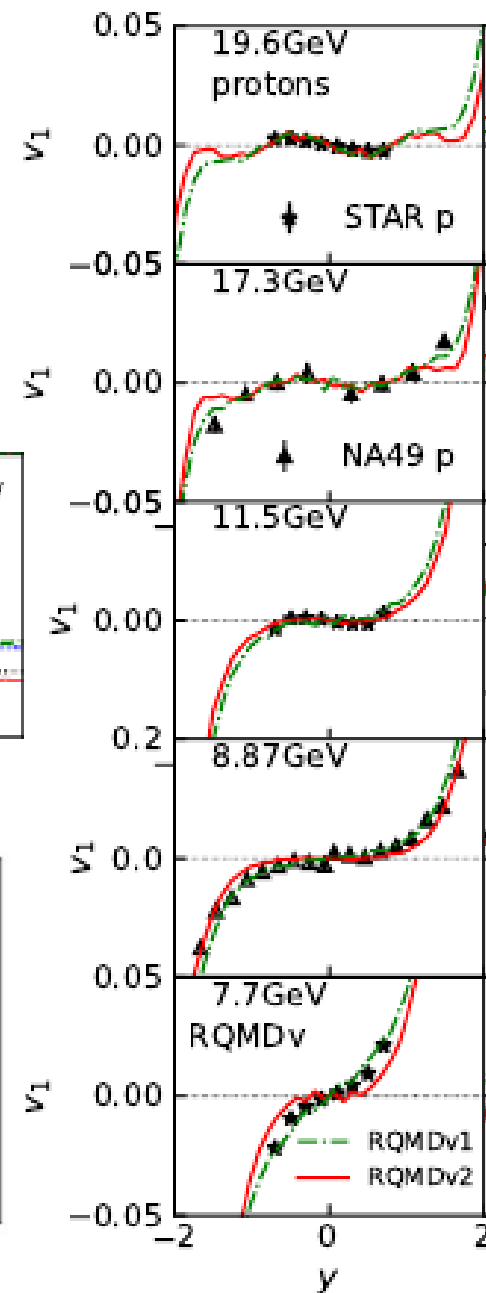
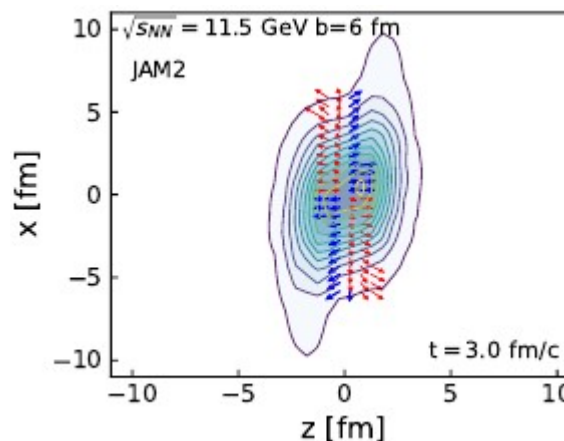
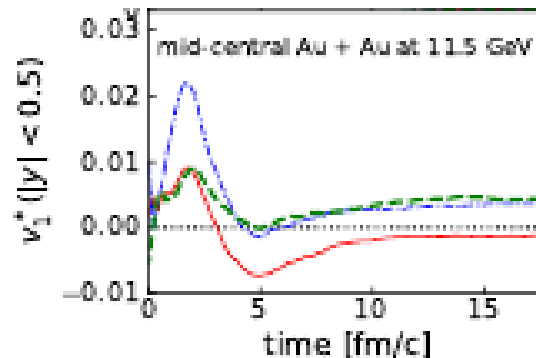


Revisiting dense hadronic matter EOS

- **Collective flow → Equation of State (EOS)**
 - Old but important subject in HIC
 - Non-monotonic energy dep. of proton v_1 slope = Unsolved puzzle [c.f. Chun Shen (Sat.)]
- **Recent discovery *Y. Nara, AO, arXiv:2109.07594***
 - JAM2+RQMDv (EOS effect)
 - Positive flow in compression, Negative flow in expansion due to tilted ellipsoid.
 - E-dep. of dv_1/dy is explained
 - *ATHIC abst. was not accepted.*



STAR, PRL112('14)162301



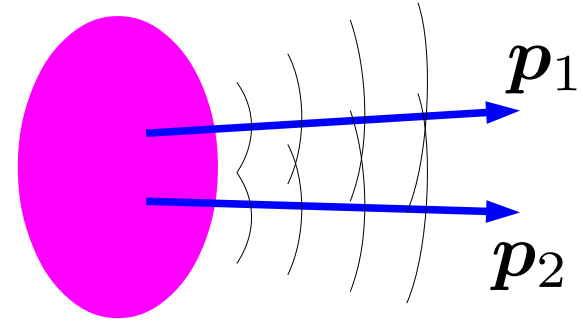
Femtoscopic study of hadron-hadron interaction

■ Correlation function (CF)

- One can access various hadron-hadron interactions using CFs (femtoscropy).

- CF=convolution of source fn. and |w.f.²

(Koonin-Pratt formula) *Koonin('77), Pratt('86), Lednicky+('82)*



$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N_{12}(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)} \simeq \int d\mathbf{r} \underbrace{S(\mathbf{r})}_{\text{source fn.}} \underbrace{|\varphi_{\mathbf{q}}(\mathbf{r})|^2}_{\text{relative w.f.}} \quad (\mathbf{q}=\text{rel. mom.})$$

■ Source size from quantum stat. + CF (HBT-GGLP effect)

Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)

■ Hadron-hadron interaction from source size + CF

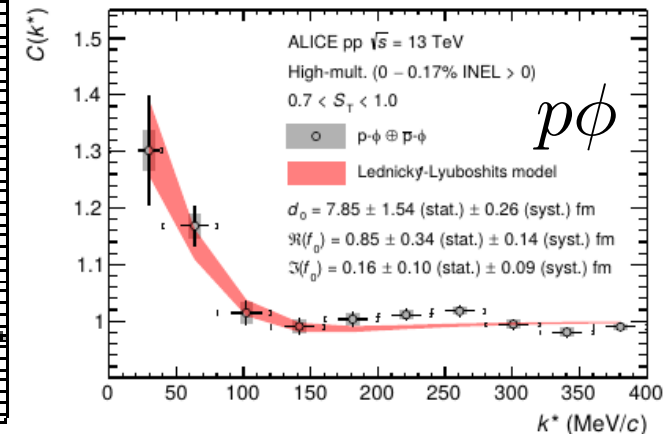
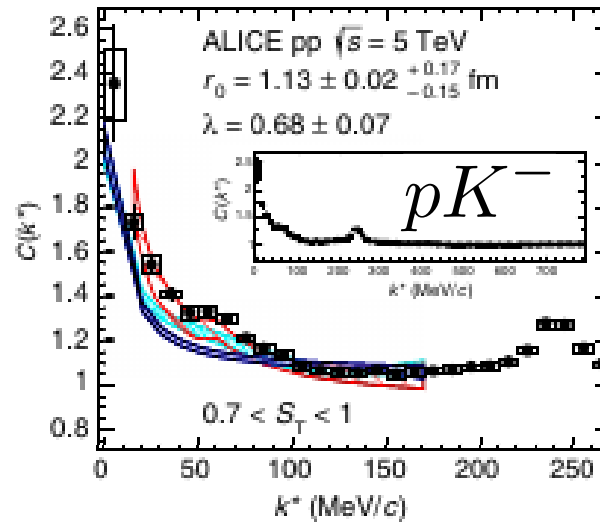
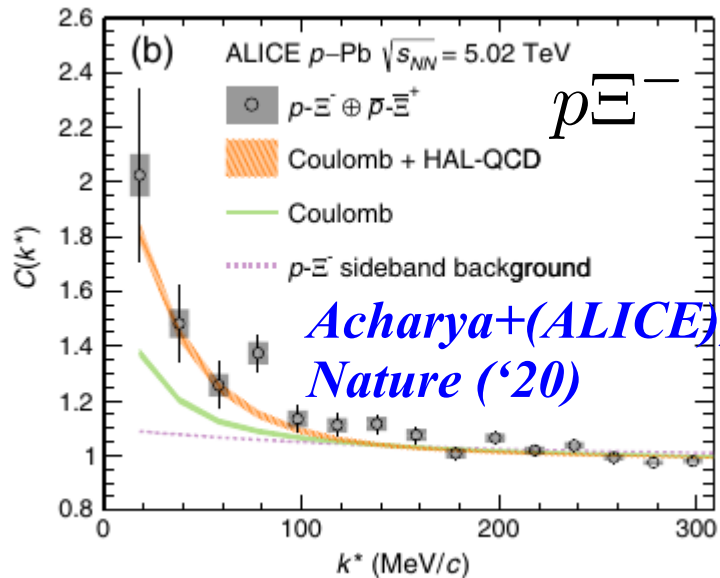
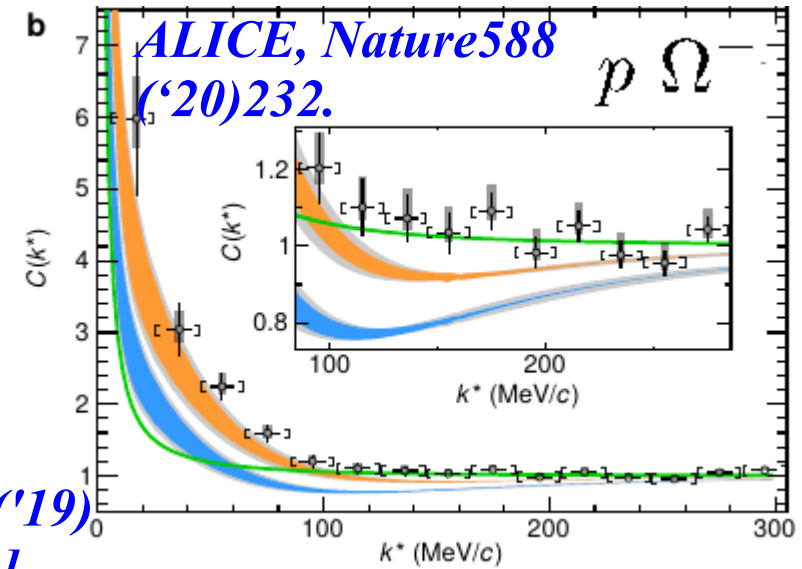
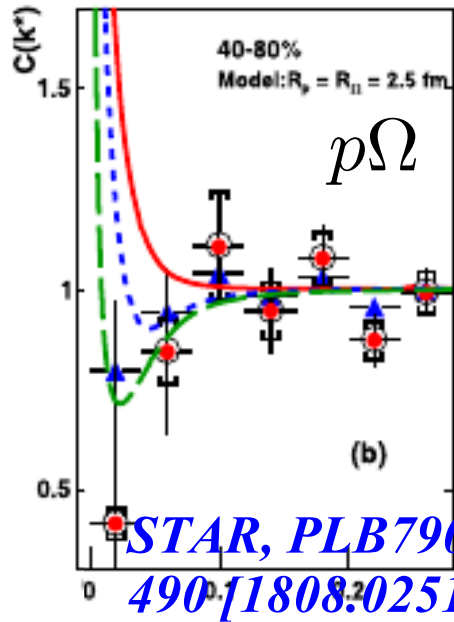
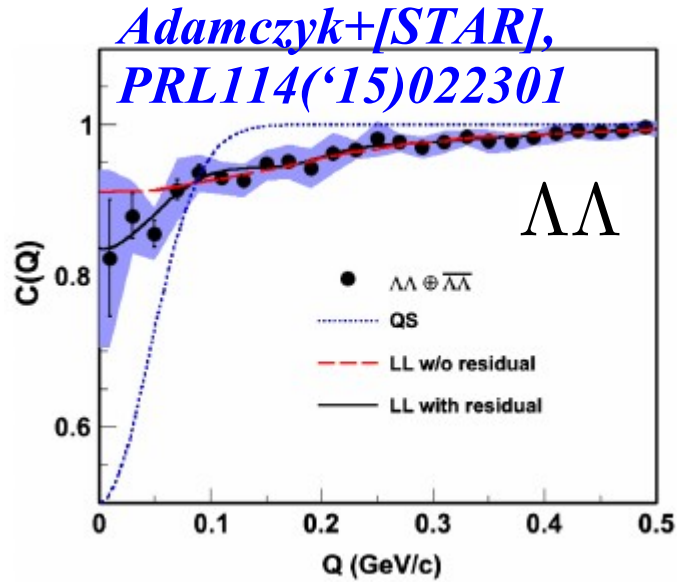
- CF of non-identical pair from static spherical source

Lednicky, Lyuboshits ('82); Morita, Furumoto, AO ('15)

$$C(\mathbf{q}) = 1 + \int d\mathbf{r} S(r) \{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \} \quad (\varphi_0 = \text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced $\rightarrow V_{hh}$ effects !

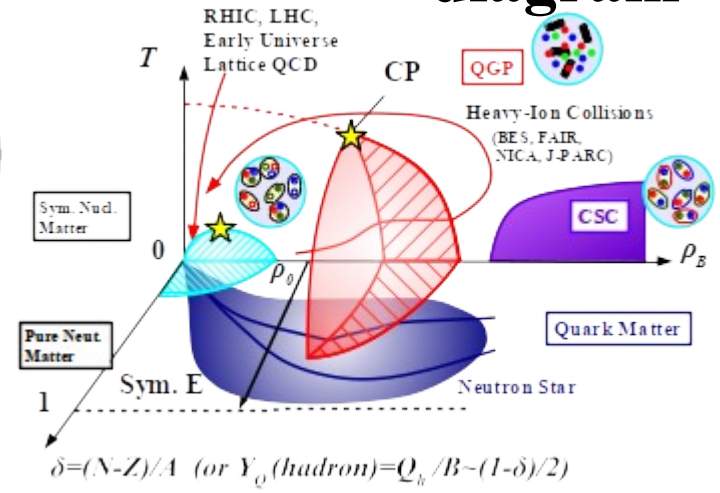
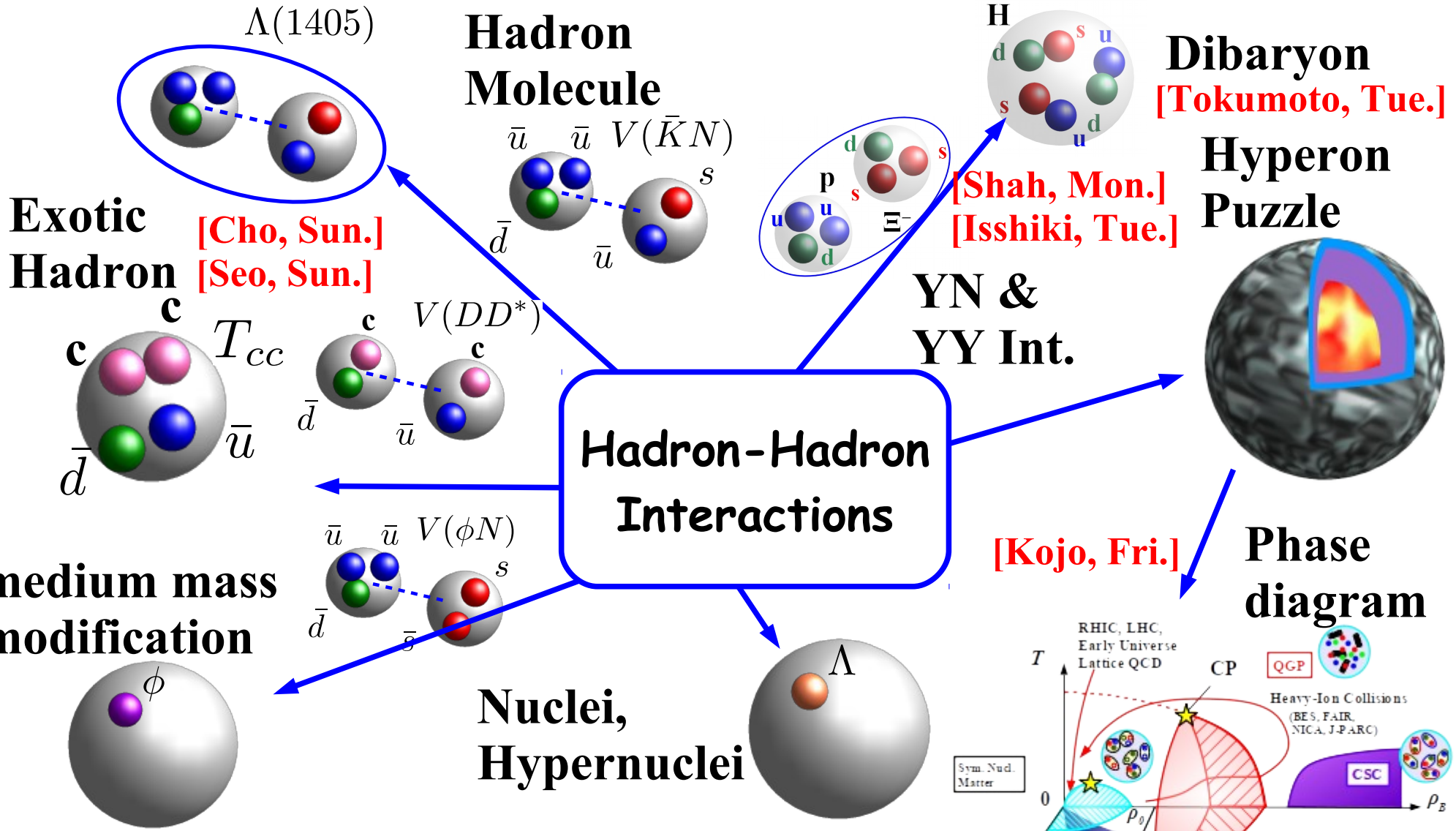
Measured Flavored Hadron CFs (examples)



*S. Acharya+[ALICE],
PRL124('20)092301*

ALICE, 2105.05578

Hadron-Hadron Interactions: Relevance



Att. or Repul., How strong, To be bound or not to be.



*Basic formulae
in femtoscopic study
of hadron-hadron interactions*

Lednicky-Lyuboshits (LL) model

- Lednicky-Lyuboshits analytic model
(Asymp. w.f.+eff. range corr.+Gaussian source)

Lednicky, Lyuboshits ('82)

- CF = a known function of $f(q)$, R , r_{eff} , and q .

$$\psi_0(r) \rightarrow \psi_{\text{asy}}(r) = \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \mathcal{S}^{-1} \left[\frac{\sin qr}{qr} + f(q) \frac{e^{iqr}}{r} \right]$$

$$C_{\text{LL}}(q) = 1 + \int dr S_{12}(r) (|\psi_{\text{asy}}(r)|^2 - |j_0(qr)|^2)$$
$$= 1 + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\text{eff}}}{R}\right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(2x) - \frac{\text{Im}f(q)}{R} F_2(2x)$$

($x = qR$, $R = \text{Gaussian size}$, F_1, F_2, F_3 : Known functions)

- Scattering amplitude at low energies

$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \mathcal{O}(q^4) \rightarrow f(q) = (q \cot \delta - iq)^{-1}$$

*From scattering length (a_0) and effective range (r_{eff}),
one can calculate the correlation function !*

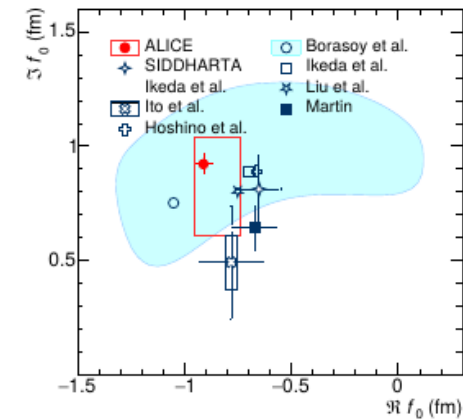
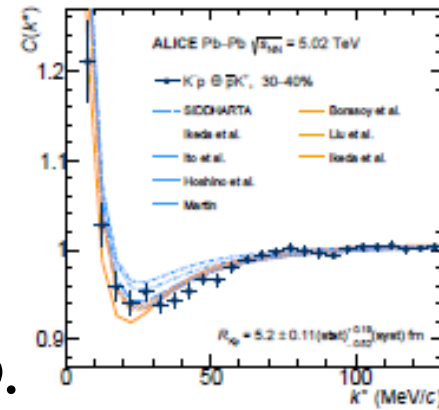
Scattering length from CFs

■ K^-p

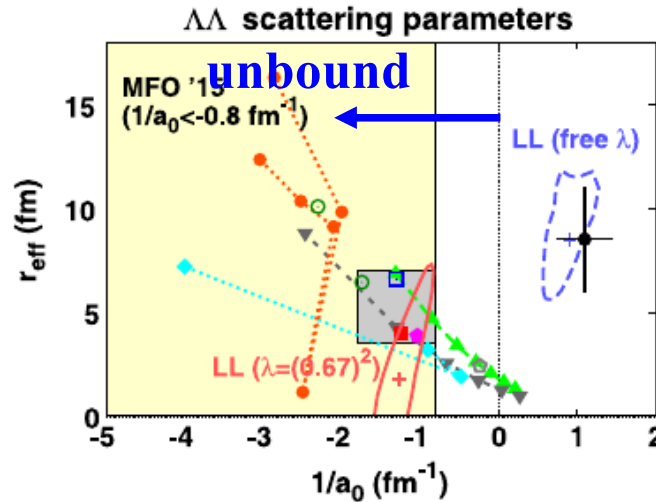
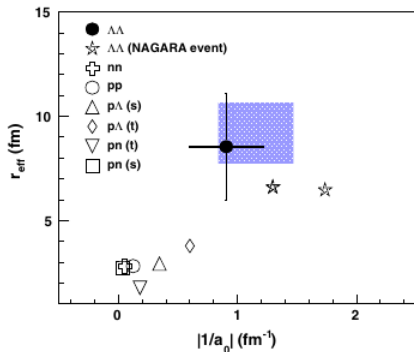
- LL (w/ Coulomb) gives a_0
- Comparable with kaonic atom data (SIDDHARTA).
- See also Siejka+[STAR], NPA982('19)359.

■ $\Lambda\Lambda$

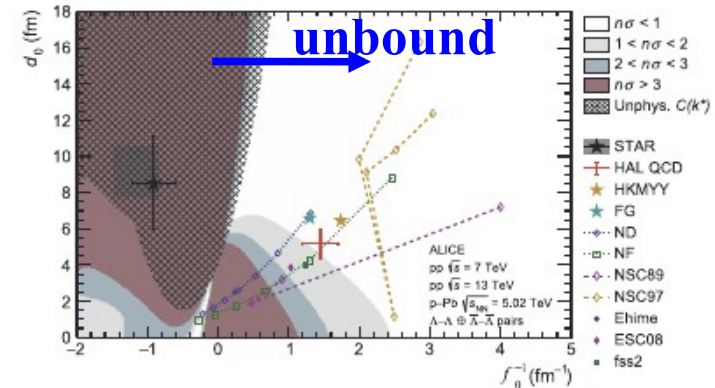
- Quantum statistics + LL
→ (a_0, r_{eff}) region



Acharya+[ALICE], PLB822('21),136708 [2105.05683] ($\delta \sim a_0 q$)



ND ▲
 NF ■
 NSC89 ◆
 NSC97 ●
 ESC08c □
 Ehime ◇
 fss2 ■
 FG □
 HKMY ○
 STAR ●



L. Adamczyk+[STAR], PRL114 ('15)022301

K.Morita, T.Furumoto, AO, PRC91 ('15) 024916; AO, Morita, Miyahara, Hyodo, NPA954('16)294 ($\delta \sim -a_0 q$)

S. Acharya [ALICE], PLB797 ('19) 134822 ($\delta \sim a_0 q$).

Femtoscopic diagnosis of bound state existence

- Lednicky-Lyuboshits model with zero effective range

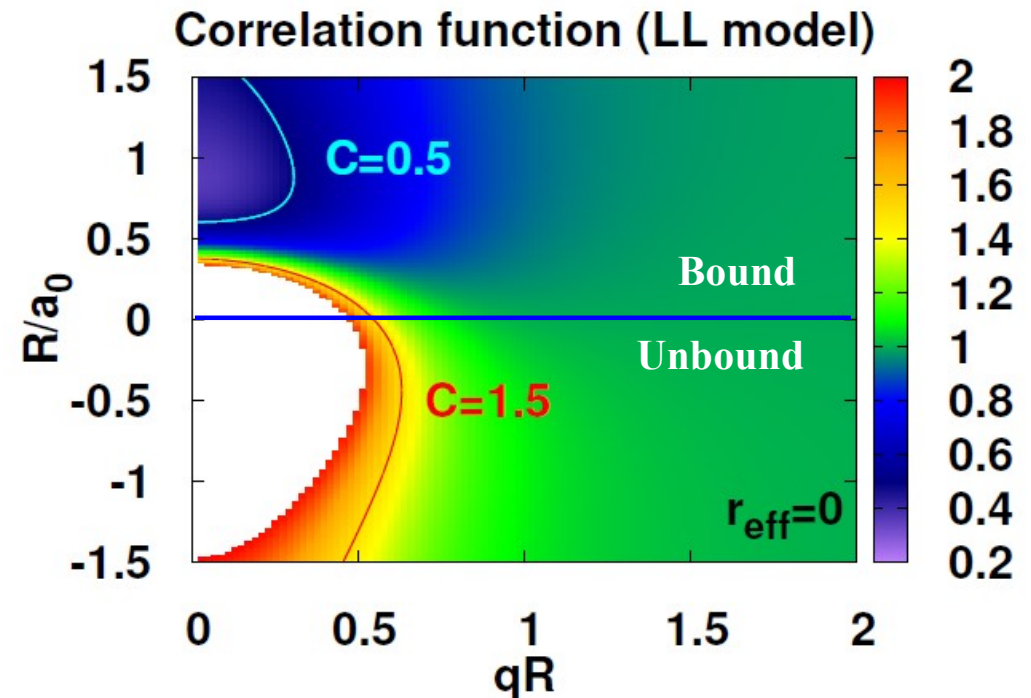
$$\rightarrow CF = C(qR, R/a_0)$$

$$r_{\text{eff}} = 0 \rightarrow q \cot \delta = -1/a_0 \rightarrow f(q) = (q \cot \delta - iq)^{-1} = -\frac{R}{R/a_0 + iqR}$$

$$C(x, y) = 1 + \frac{1}{x^2 + y^2} \left[\frac{1}{2} - \frac{2y}{\sqrt{\pi}} F_1(2x) - xF_2(2x) \right] \quad (x = qR, y = R/a_0)$$

$$= \frac{1}{2} \left(\frac{1}{y} - \frac{2}{\sqrt{\pi}} \right)^2 + 1 - \frac{2}{\pi} \quad (F_1 \rightarrow 1, F_2 \rightarrow 0 \text{ at } x \rightarrow 0)$$

- With a bound state, CF is suppressed at low q .
- Scattering w.f. needs to have a node to be orthogonal to b.s.



E.g. AO, Morita, Miyahara, Hyodo ('16)

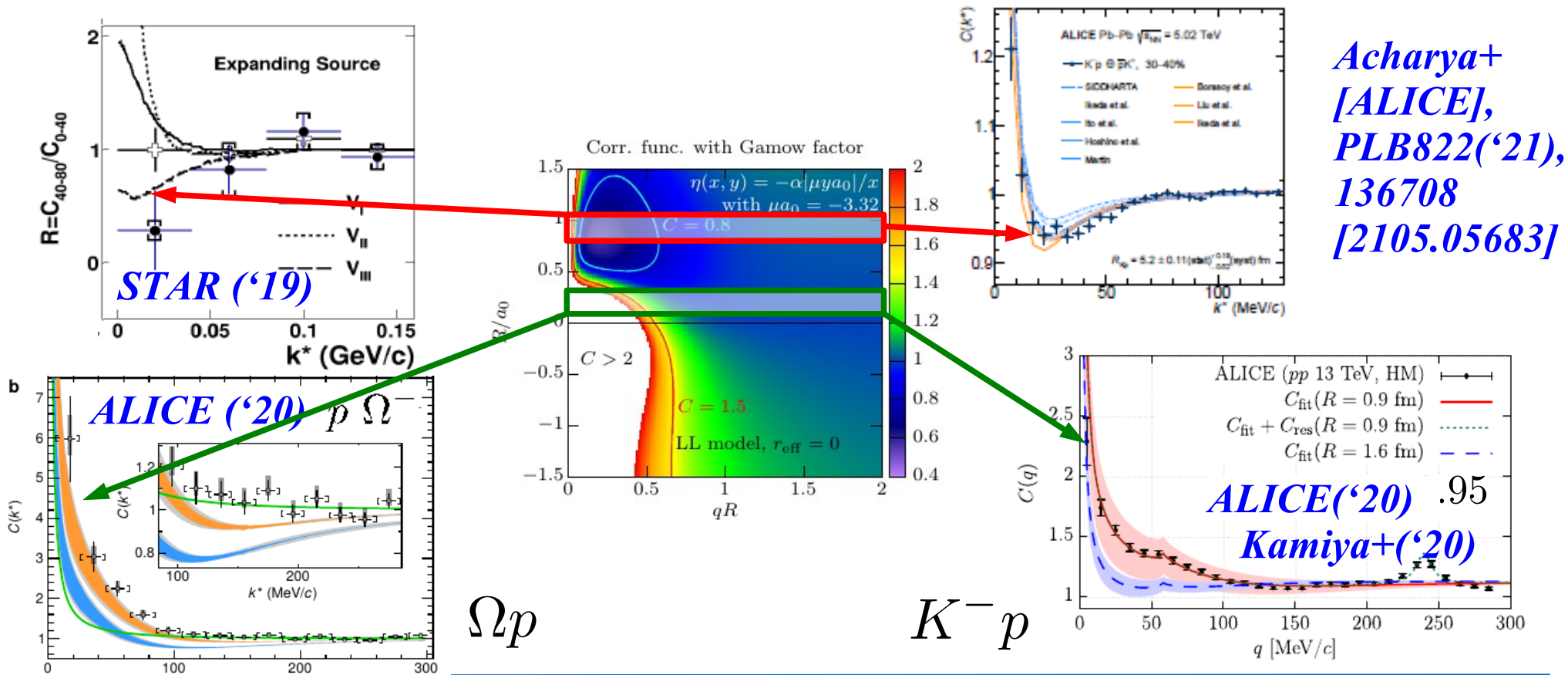
Hadronic molecules suggested by CFs

■ Ωp and $K^- p$

- Bound states are expected.

Dalitz, Tuan ('59); Akaishi, Yamazaki ('02); Goldman+('87); Oka ('88); Etminan+[HAL QCD] ('14); Iritani+[HAL QCD]('19).

- Dip is expected at $R \sim |a_0|$ *Morita+('16, '20); Kamiya+('20); Haidenbauer('18)*
- Data support the existence of a BS.



Acharya+ [ALICE], PLB822('21), 136708 [2105.05683]

ALICE ('20) .95 Kamiya+('20)

Ωp

$K^- p$



Coupled-Channel Effects

■ Correlation Function with Coupled-Channel Effects

- Koonin-Pratt-Lednicky-Lyuboshits-Lyuboshits (KPLLL) formula
Coupled-channel contributions with $\psi(-)$ boundary cond.

Lednicky, Lyuboshits, Lyuboshits, Phys. Atom. Nucl. 61 (1998), 2950;

J. Haudenbauer, NPA981('19) 1 [1808.05049];

Y. Kamiya, T.Hyodo, K.Morita, AO, W.Weise, PRL('20).

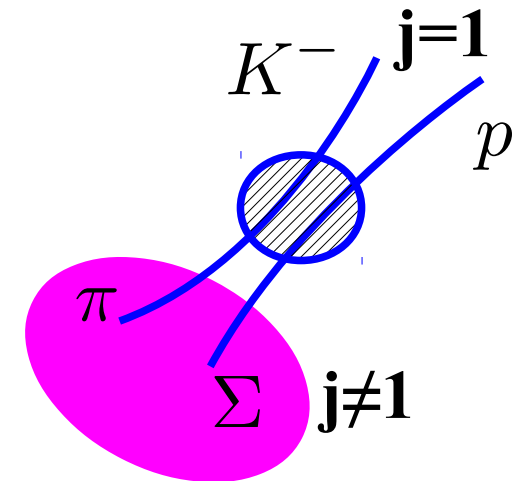
$$C(q) = 1 - \int dr S_1(r) |j_0(qr)|^2 + \int dr \sum_j \omega_j S_j(r) |\psi_j^{(-)}(q; r)|^2$$

$$\psi_{j=1}(r) \rightarrow [e^{iqr} + A_1(q)e^{-iqr}] / 2iqr \quad (\omega_1 = 1)$$

$$\psi_{j \neq 1}(r) \rightarrow A_j(q)e^{-iqr} / 2iqr \quad [\Psi^{(-)} \text{ boundary condition}]$$

(No Coulomb case)

- Wave functions of other channels also contribute to correlation functions.
- Source size R and weight ω_j ($j \neq 1$) are taken as the parameter.

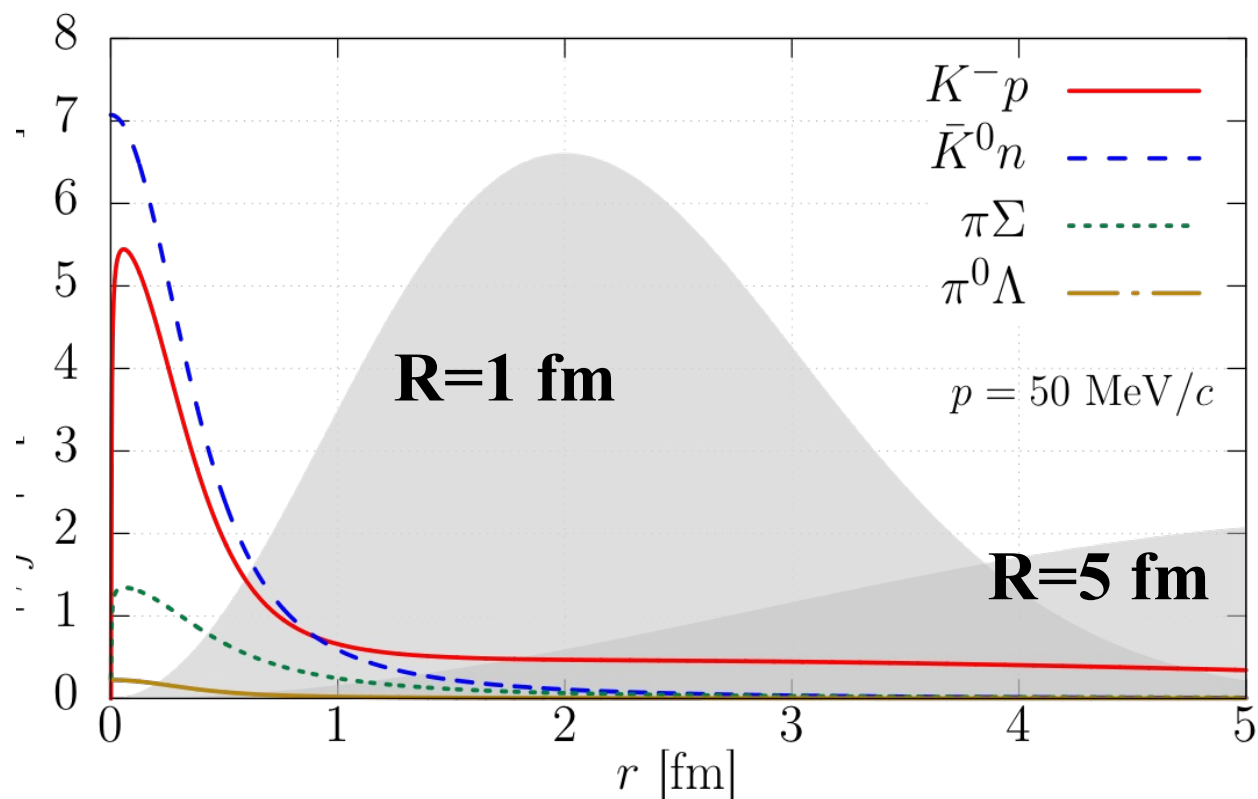


$$\omega_j S_j(r) |\psi_j^{(-)}(q; r)|^2$$

Source weight **Normalized Source fn.**

Discriminating Coupled-Channel Effects

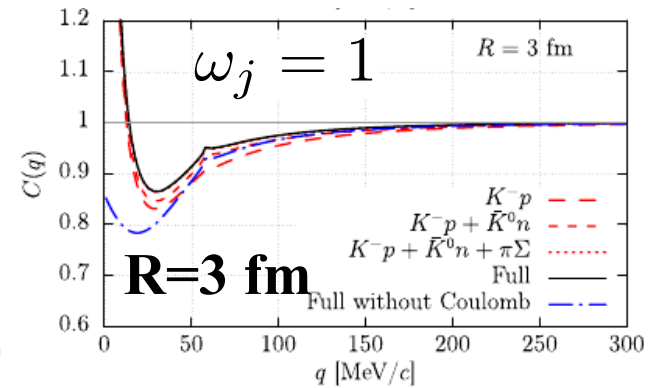
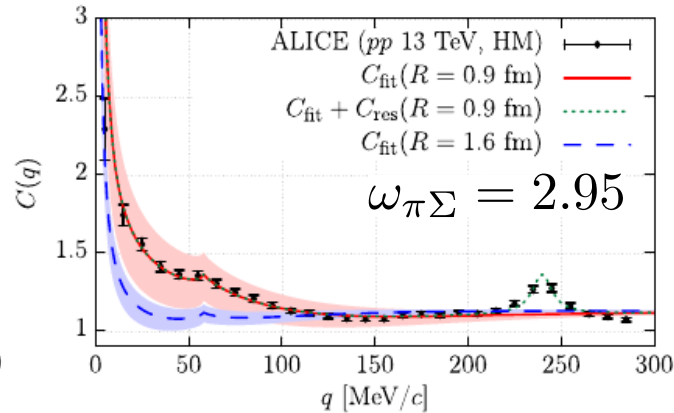
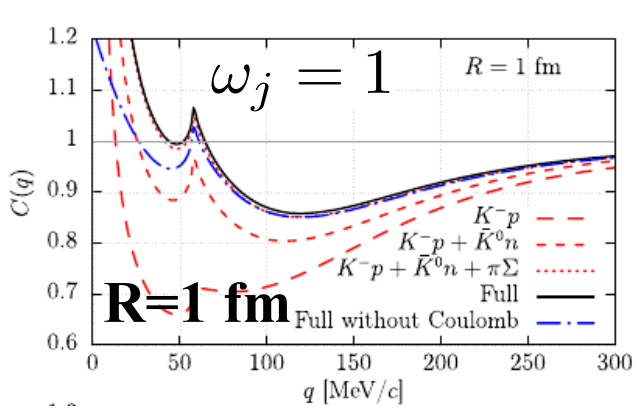
- Source size dependence again !
 - Unmeasured coupled-channel wave functions disappear soon.
→ CFs with large source is dominated by the measured channel wave function !
 - Scattering parameters from CFs with large source
Coupled-channel effects from CFs with small source.



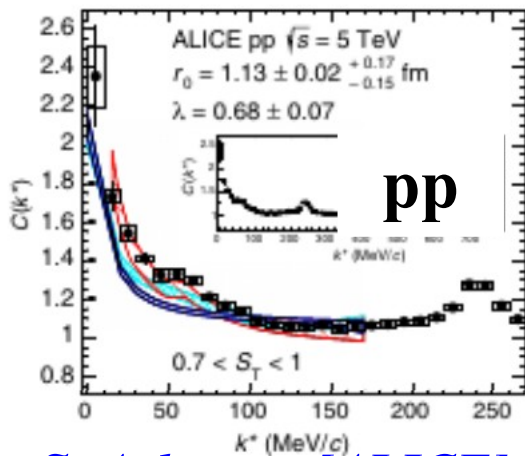
w.f. Kamiya+, arXiv:1911.01041v1

Source Size Dependence of $C(K^- p)$

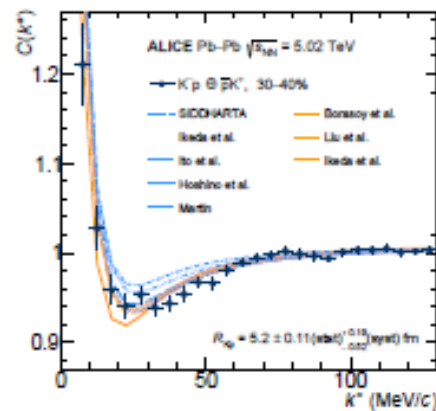
- Coupled-channel effects are suppressed when R is large, and “pure” $K^- p$ wave function may be observed in HIC.



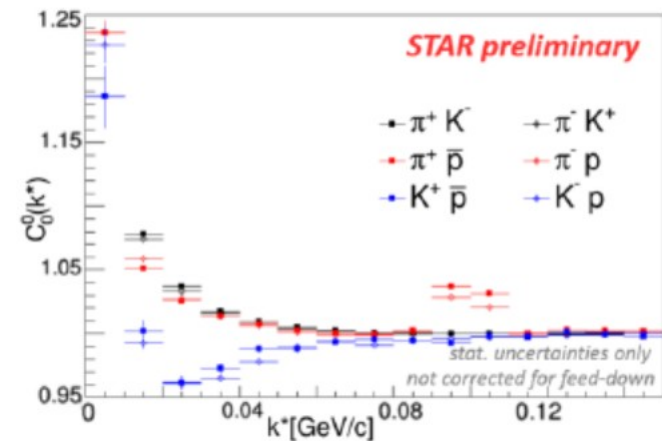
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.



S. Acharya+[ALICE], PRL124('20)092301



S. Acharya+[ALICE], PLB822('21)136708



Siejka+[STAR, preliminary], NPA982 ('19)359.

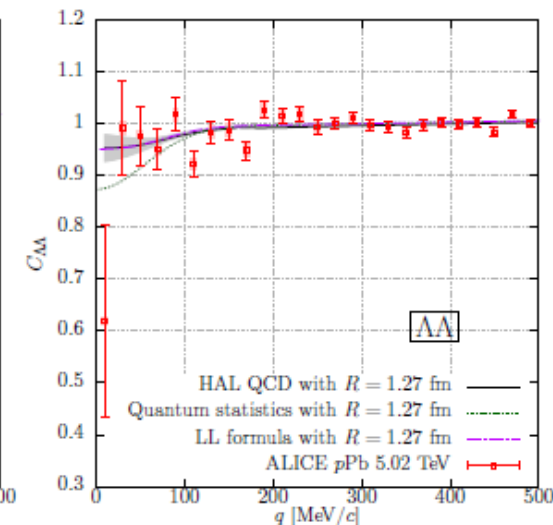
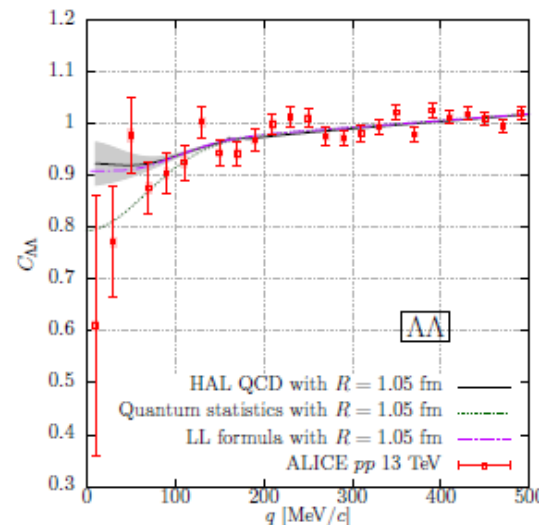
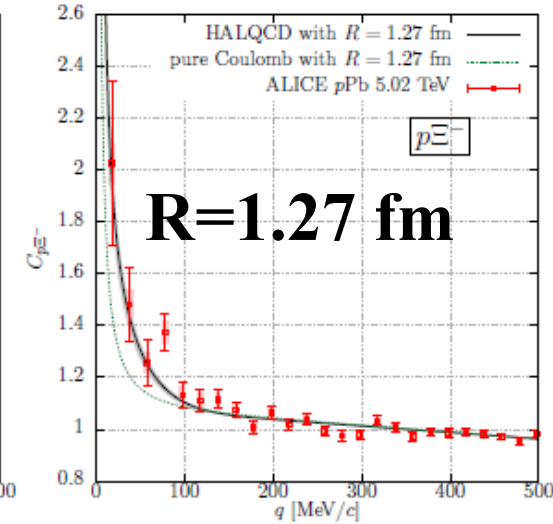
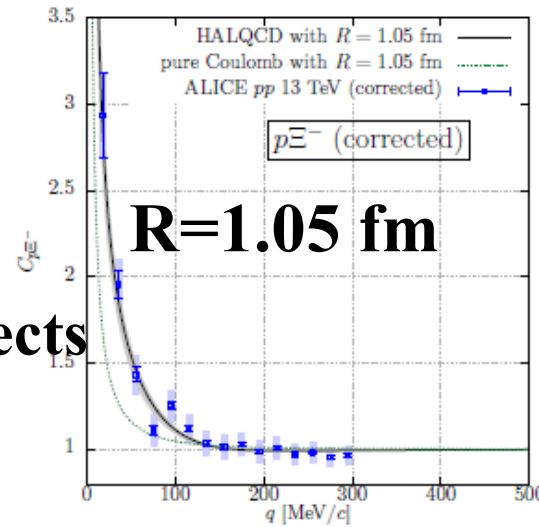
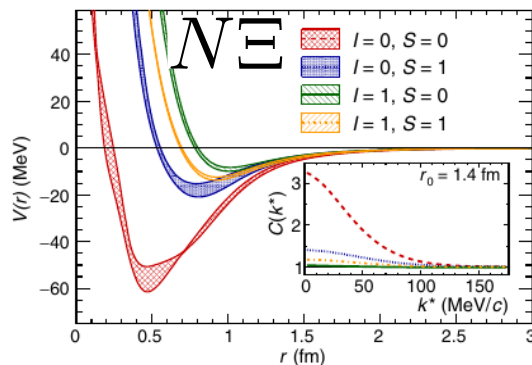
Recent & Future Femtoscopic Studies of HHI

Ξ^-p & $\Lambda\Lambda$ correlation functions (pp and pA)

Correlation function data from pp and pA collisions

S. Acharya et al. [ALICE], PLB 797('19)134822 ($\Lambda\Lambda$); PRL123('19)112002 (Ξ^-p from pA); Nature 588('20)232 (Ξ^-p from pp).

- CF(Ξ^-p) is enhanced at low q . \rightarrow Att. pot.
 - CF($\Lambda\Lambda$) is slightly enhanced from quantum stat. result. \rightarrow Weakly attractive pot.
 - CFs with coupled-channel effects using lattice QCD potential explains the data well.
- K. Sasaki [HAL QCD]('20);
Y.Kamiya+, arXiv:2108.09644.*



$\Xi^- p$ & $\Lambda \Lambda$ correlation functions (AA)

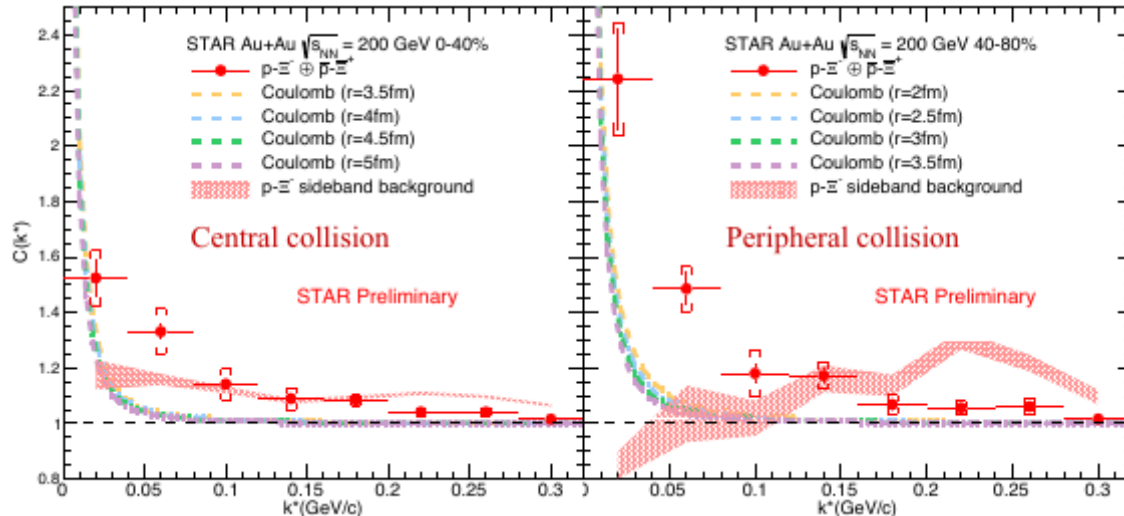
■ Correlation function data from AA collisions

[c.f. Shah, Mon., Isshiki, Tue.]

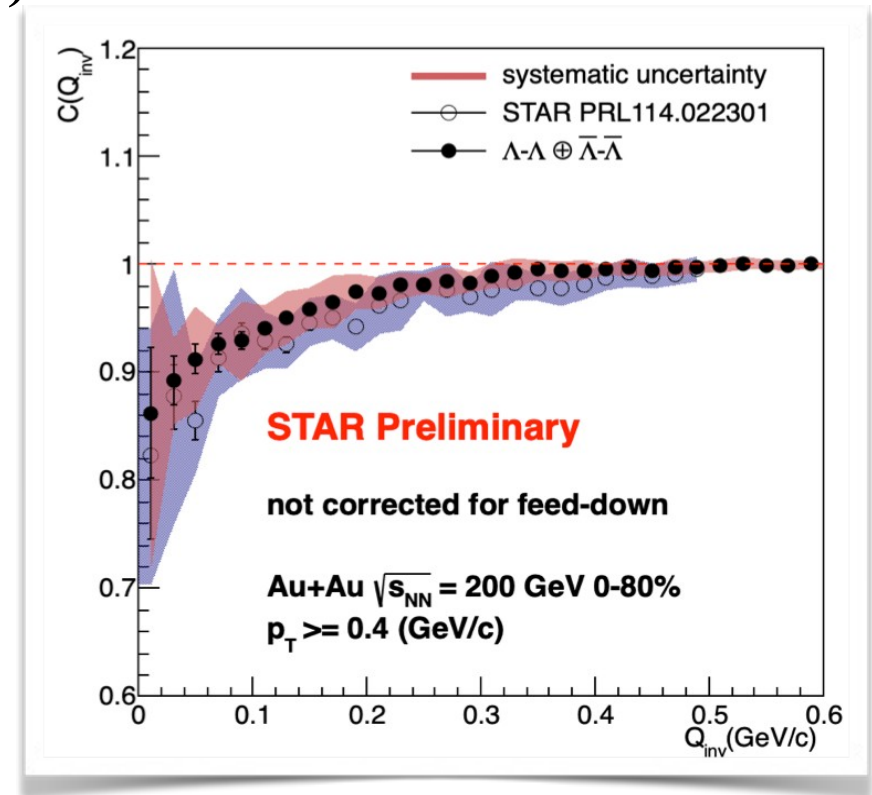
K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.

Moe Isshiki+ (STAR, preliminary), Strangeness physics workshop, 2021.

- We do not see a dip in $C(\Xi^- p)$ from Au+Au.
 - There will be no bound state of $\Xi^- p$.
- Much higher statistics data of $C(\Lambda \Lambda)$ from Au+Au are obtained.
 - LL formula fit will be possible.



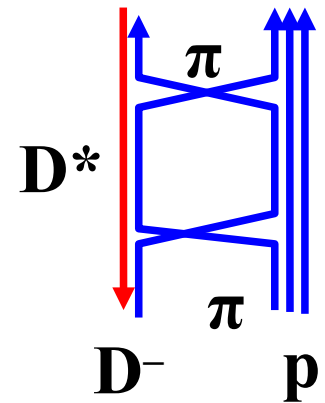
*K. Mi+(STAR, preliminary),
Au+Au 200 AGeV, APS2021.
(No Dip at larger R)*



Moe Isshiki+ (STAR, preliminary).

Charmed Hadron Interactions

- Charmed hadron interactions are extremely important in exotic hadron physics.
- Example: D^- p correlation
 - Proposed potentials generally predict weak or repulsive interaction. *Hofmann, Lutz ('05) (repulsive); Haidenbauer+('07) (repulsive); Yamaguchi+('11) (att., w/ bs); Fontoura+('13) (repulsive)*
 - Attraction from pion exchange *S. Yasui, K. Sudoh, PRD80('09)034008.*



Model	$a_{DN}^{I=0}$	$a_{DN}^{I=1}$	$a_{\bar{D}}$	
SU(4) contact [185]	-0.16	-0.26	-0.24	1. Hoffmann, Lutz ('05)
Meson exchange [194]	0.07	-0.45	-0.32	2. Haidenbauer+ ('07)
Pion exchange [192]	-4.38	-0.07	-1.15	3. Yamaguchi+('11)
Chiral quark model [219]	0.03-0.16	-(0.20-0.25)	-(0.14-0.15)	4. Fontoura+('13)

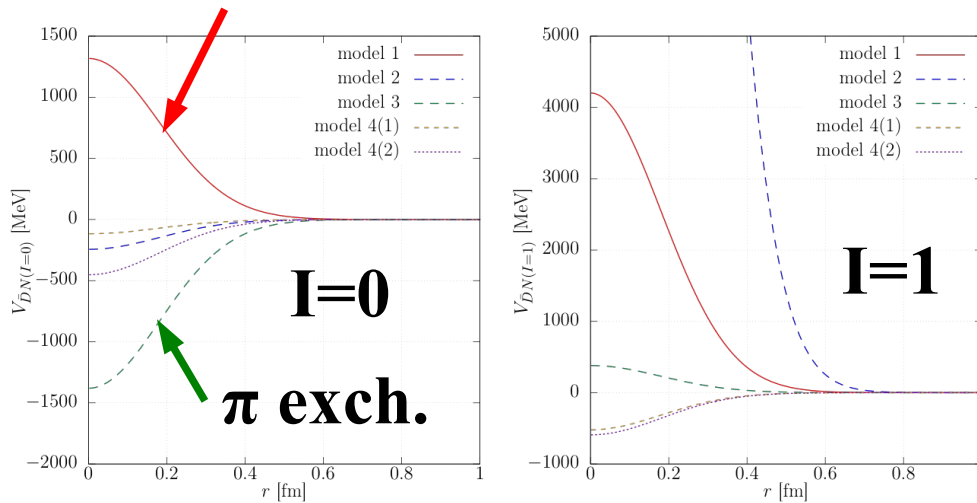
Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP96('17)88

Which picture is correct ?

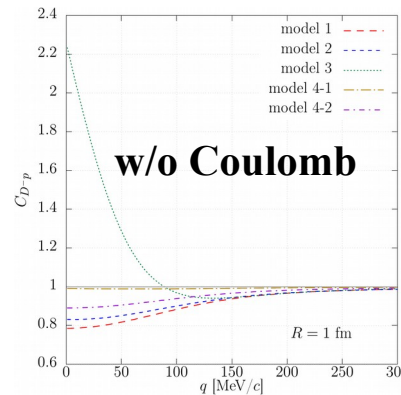
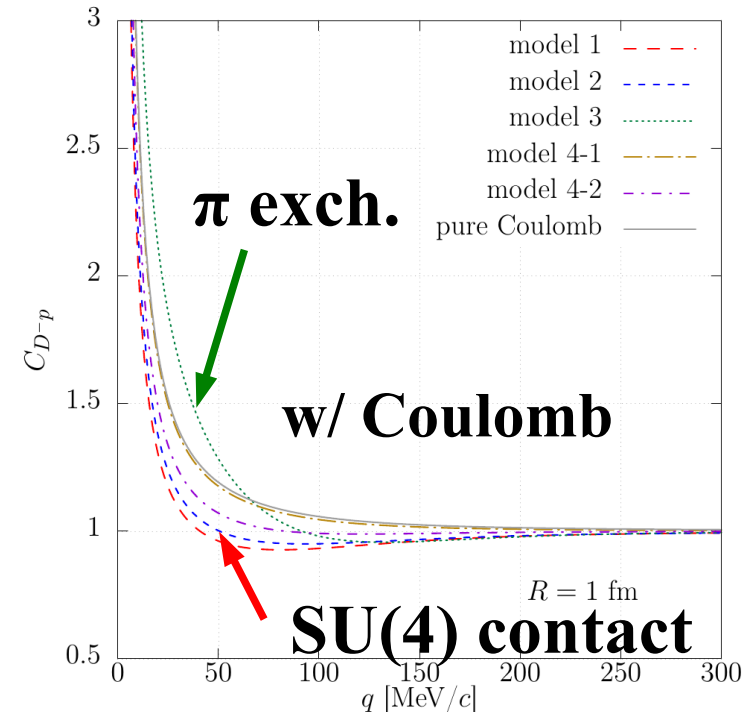
Charmed Hadron Interactions

- D^- p CFs from proposed scattering length *Kamiya, Hyodo, AO (in prog.)*
 - One-range Gaussian potential strength is fitted to proposed a_0 with the range of ρ meson exchange.
 - Measurable difference is found.

SU(4) Contact



1. Hoffmann, Lutz ('05)
2. Haidenbauer+ ('07)
3. Yamaguchi+ ('11)
4. Fontoura+ ('13)

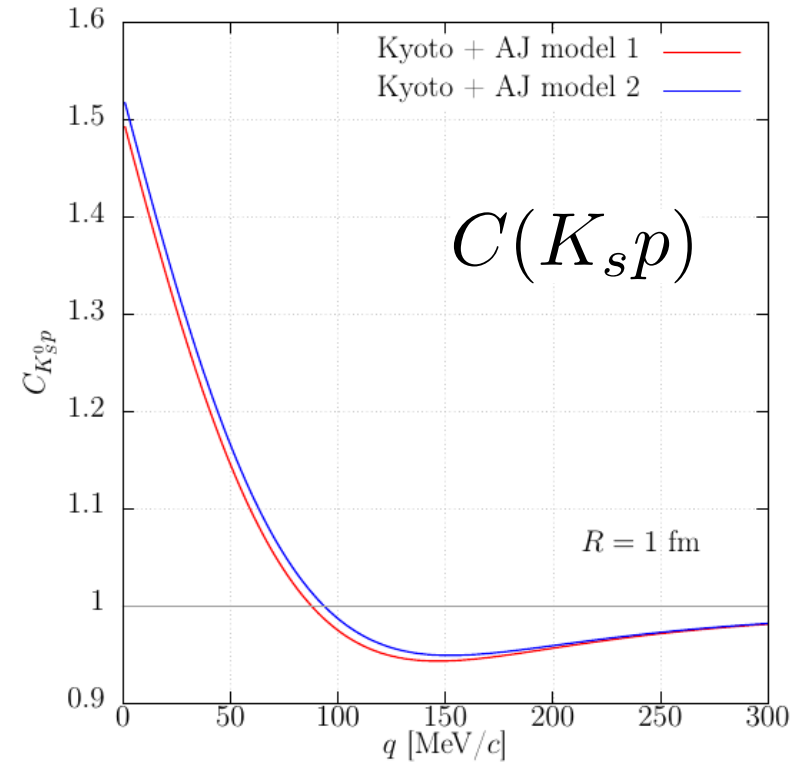
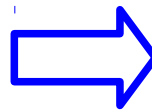
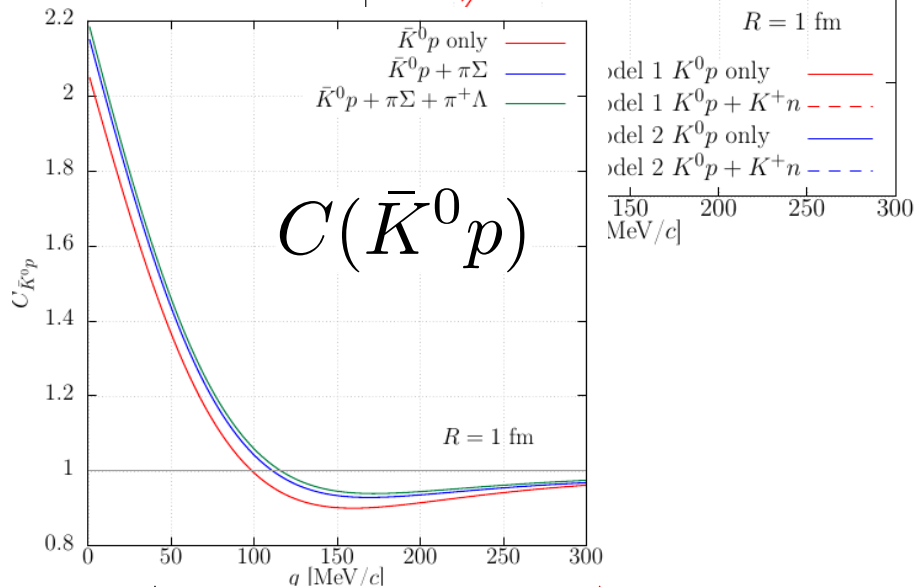
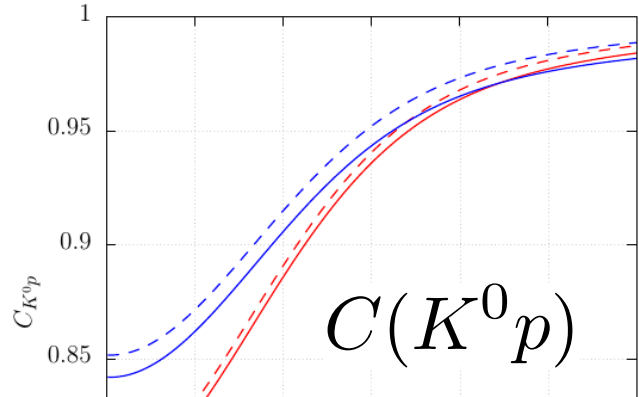


Data will discriminate these potentials !

Ks p correlation function

■ Ks wave function *Kamiya+(in prep.)*

$$|K_s\rangle = \frac{1}{\sqrt{2}} [|K^0\rangle + |\bar{K}^0\rangle] \rightarrow C(K_s p) = \frac{1}{2} [C(K^0 p) + C(\bar{K}^0 p)]$$



$K^- p, K^+ p, K_s p$, and $K^- n$ or $K^+ \bar{n}$ (missing)

to obtain KN and $\bar{K}N$ interactions with $I = 0, 1$

$V(\bar{K}N)$: Chiral SU(3) (Kyoto-Münich) model
(Miyahara, Hyodo, Weise ('18))

$V(KN)$: Chiral SU(3) based model
(Aoki, Jido ('19))

Other Correlation Functions

- $\bar{p}\bar{p}, p\bar{\Lambda}$ *E.g. A. Kisiel [ALICE], Acta Phys.Polon.Supp. 6 ('13)519*

- $K^\pm K_s^0$ *S.Acharya+ [ALICE], PLB774 ('17)64 [1705.04929]*
 - Slightly suppressed at low q
 - Tetraquark component of a_0 meson

- $p\phi$ *Acharya+[ALICE], PRL127('21)172301[2105.05578]*
 - Attractive potential which may be responsible to medium mass reduction

- $p\bar{\Lambda}, \Lambda\bar{\Lambda}$ [2105.05190],
 $p\Sigma^0$ ['20 [1910.14407]] [ALICE]

- deuteron-hadron CF

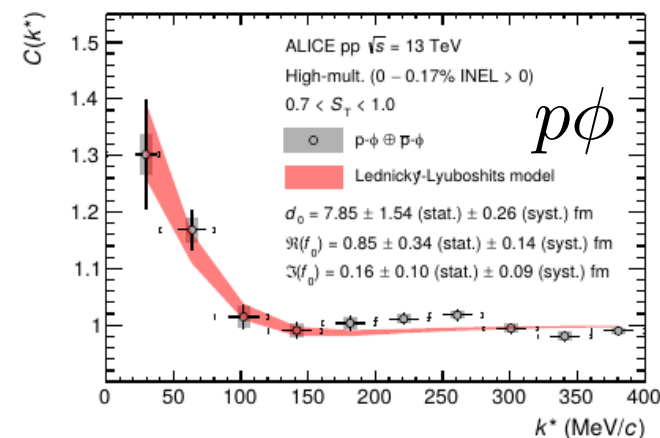
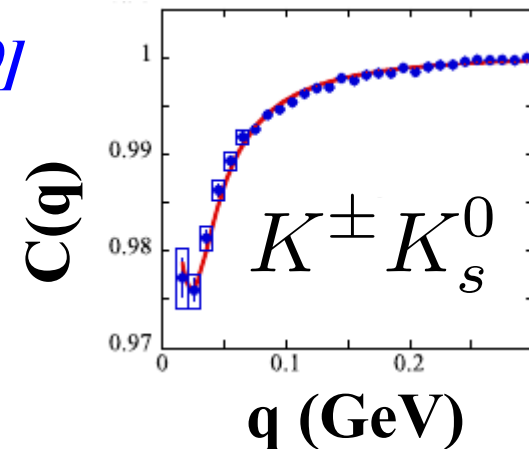
S. Mrówczyński and P. Słoń,

Acta Phys.Polon.B51('20)1739 [1904.08320];

F. Etminan, M. M. Firoozabadi, [1908.11484];

J. Haidenbauer, PRC102('20)034001 [2005.05012];

K.Ogata, T.Fukui, Y.Kamiya, AO [2103.00100].



*Acharya+[ALICE],
PRL127 ('21) 17,
172301[2105.05578]*

Summary

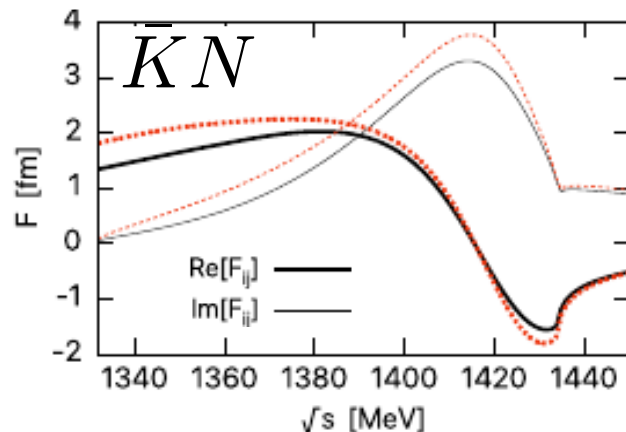
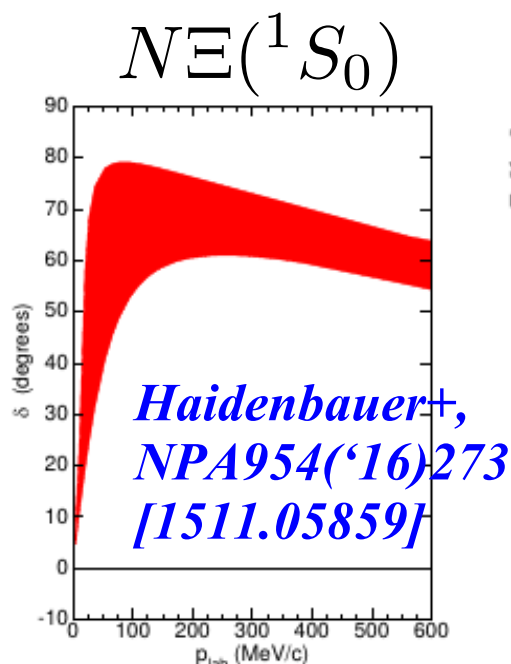
- **Correlation function from heavy-ion collisions (including high-energy pp and pA collisions) are useful to access hadron-hadron interactions.**
 - One can obtain the basic properties of HHIs such as Attractive or Repulsive, Strength, To be bound or not to be.
 - Source size dependence is useful in deducing the existence of a bound state and in discriminating the coupled-channel effects.
- **We are sorry for using a static Gaussian source functions.**
 - Flavored hadron CFs are measured in 1D (as a function of $|q|$), m_T dependence is not yet discussed, and errors are still large.
 - Effects of flow, elongated shape, and others are expected to be renormalized in changing the source size **at present**.
- **Many hadron physicists are waiting for your data.**

Thank you for your attention !

Theoretical Approaches to HHI

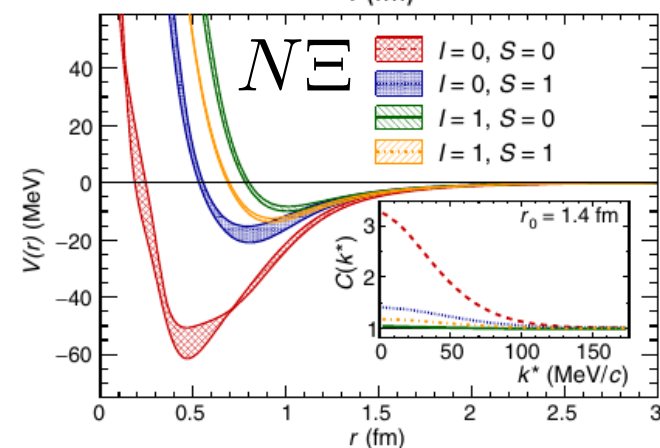
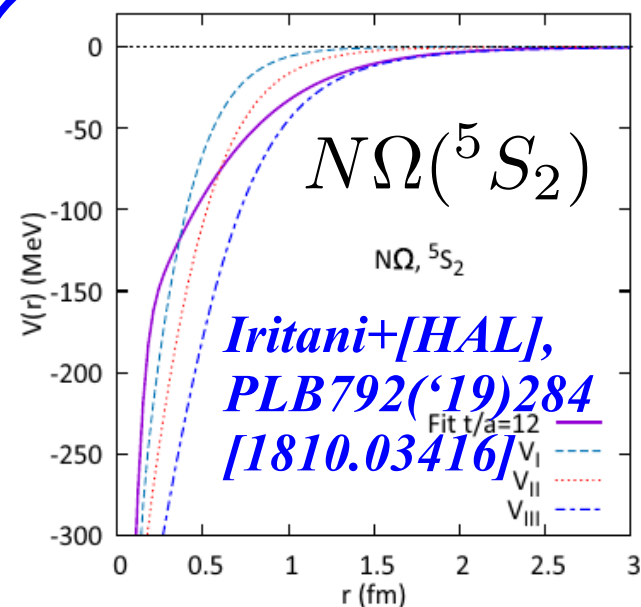
- Nuclear force models: meson exch., quark model, ... (need **data**)
- **Ab initio**: chiral EFT, lattice QCD (need **data** or **CPU resources**)

Chiral



*Miyahara, Hyodo,
Weise, PRC98('18),
025201 [1804.08269]
(Ikeda-Hyodo-Weise
amplitude)*

Lattice



*Sasaki+ [HAL], NPA998
(‘20)121737 [1912.08630]
(taken from ALICE(‘19))*

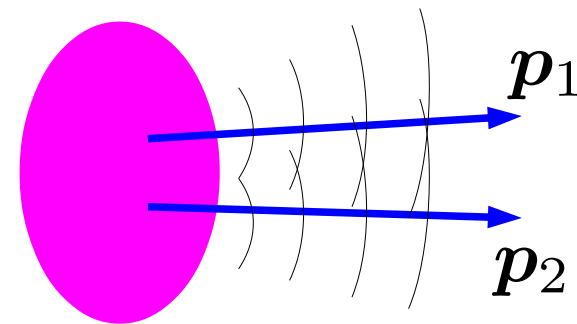
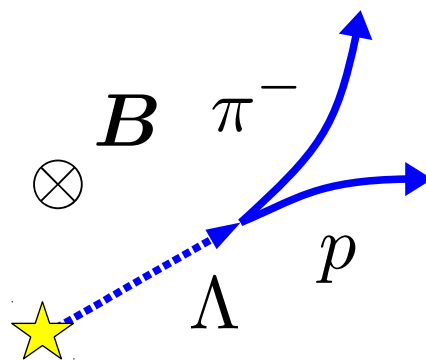
Experimental Approaches to HHI

■ Experimental approaches

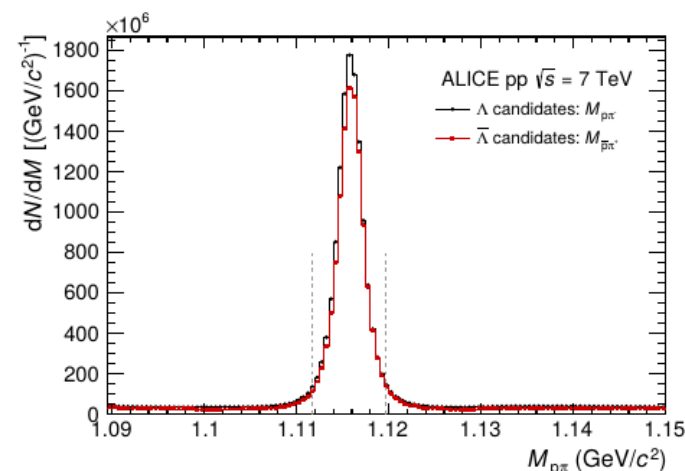
- hh scattering (NN, YN, π N, KN)
- Hadronic nuclei (normal nuclei, hypernuclei, kaonic nuclei)
- Hadronic atom (π^- , K^- , Σ^- , Ξ^- , ...)
- **Femtoscscopy**

■ Femtoscopic study of hh interactions

- Applicable to various hh pairs (NN, YN, KN, DN, YY, Yd, YNN ...)
- Valid when the source is chaotic
- Weakly decaying particles
→ Good pair purity
- Future measurements:
Charmed hadron, hNN, ...



$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N_{12}(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)}$$



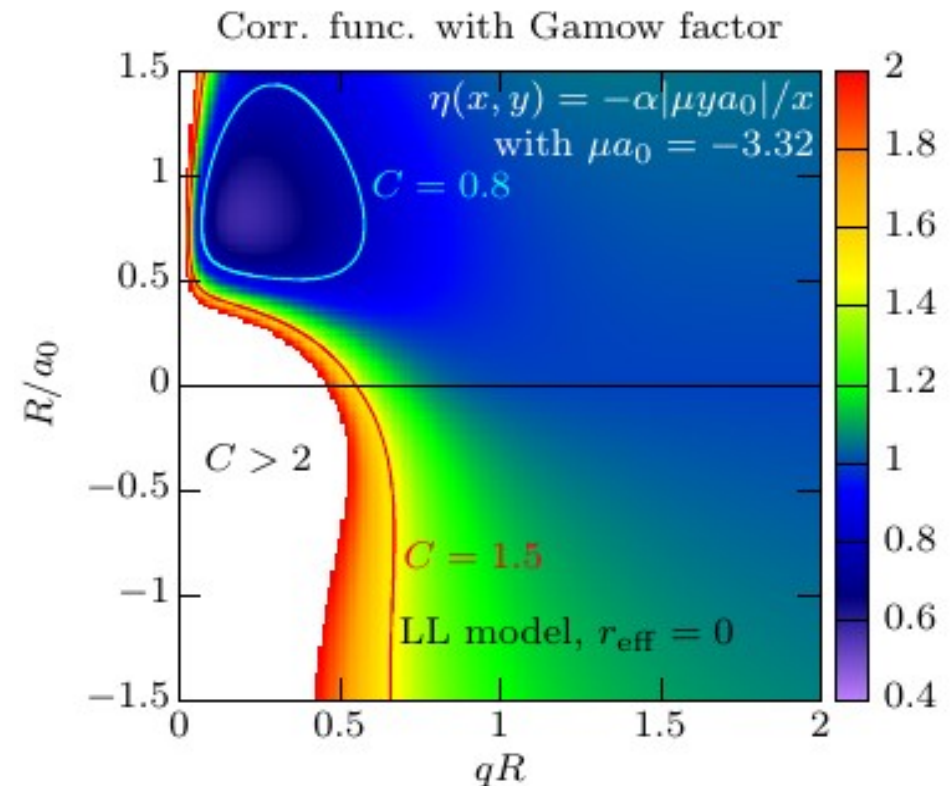
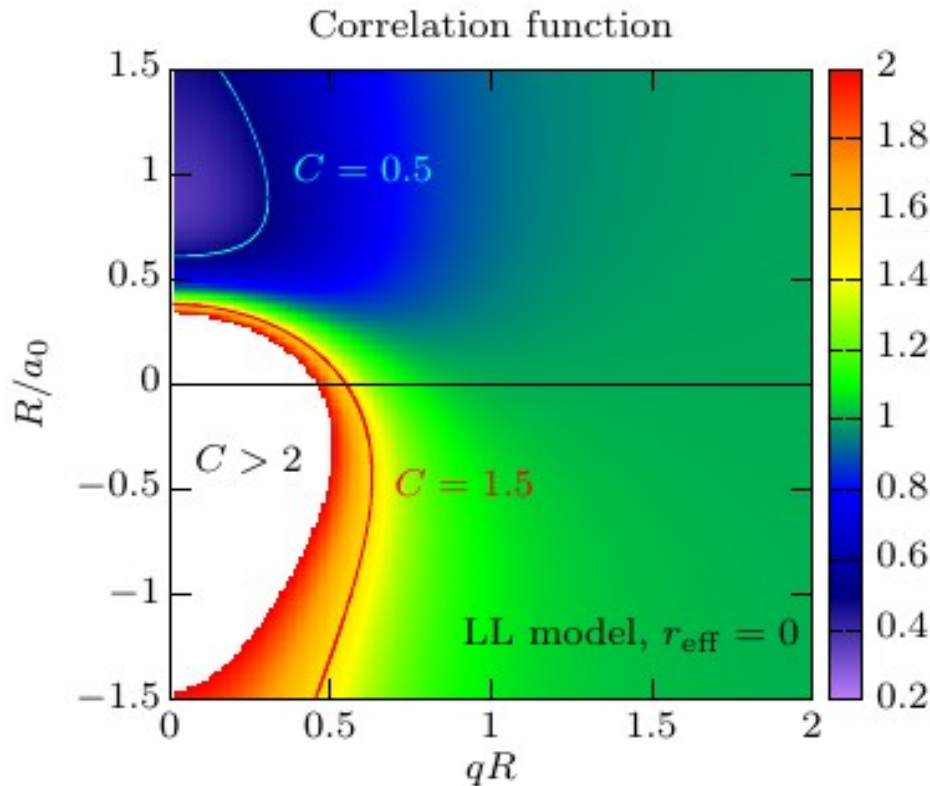
ALICE [1805.12455]

Femtoscopic Diagnosis of Bound State

- With Coulomb attraction, a bound state causes a dip.
 - E.g. LL formula and the Gamow factor

$$C(q) \rightarrow C_{\text{strong}}(q) \times A_{\text{Gamow}}(\eta)$$

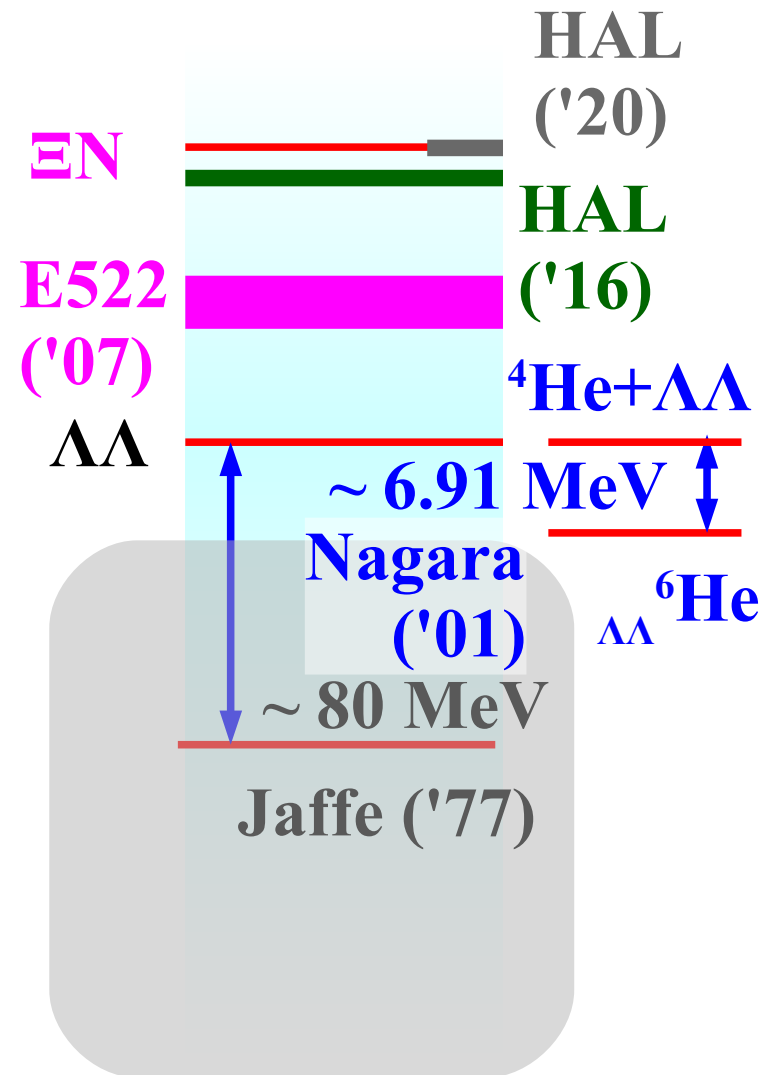
$$A_{\text{Gamow}}(\eta) = 2\eta / (\exp(2\eta) - 1) \quad (\eta = Z_1 Z_2 \mu |y a_0| / x)$$



Kamiya+('21), 2108.09644

H dibaryon state, to be bound or not to be bound ?

- H-dibaryon: 6-quark state (uuddss)
 - Prediction: *R.L.Jaffe, PRL38(1977)195*
 - Ruled-out by double Λ hypernucleus *Takahashi et al., PRL87('01) 212502*
 - Resonance or Bound “H” ? *Yoon et al.(KEK-E522)+AO ('07)*
- Lattice QCD results
 - Bound (below $\Lambda\Lambda$ threshold): *HALQCD('11), NPLQCD('11,'13), Mainz('19)* (heavier quark mass or SU(3) limit)
 - Resonance (Bound state of $N\Xi$): *HAL QCD ('16,18)* (HAL preliminary)
 - Virtual Pole (around $N\Xi$ threshold) *HAL QCD ('20)* (almost physical m_q)



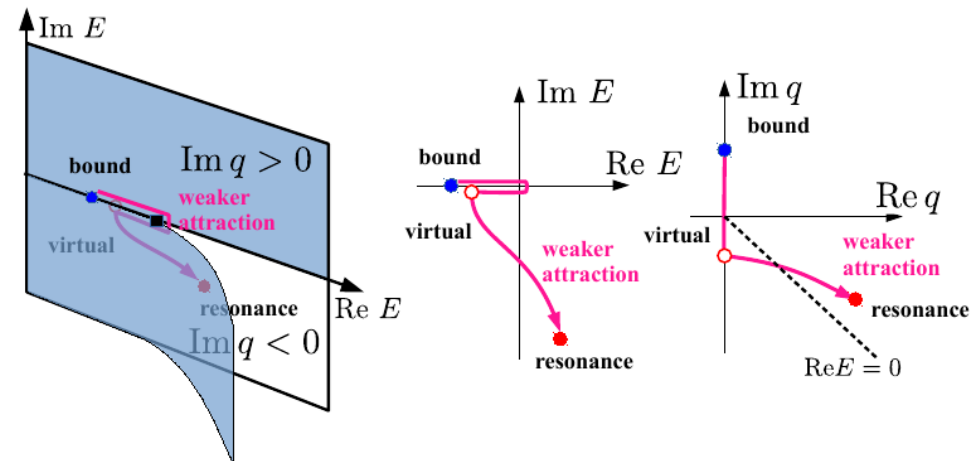
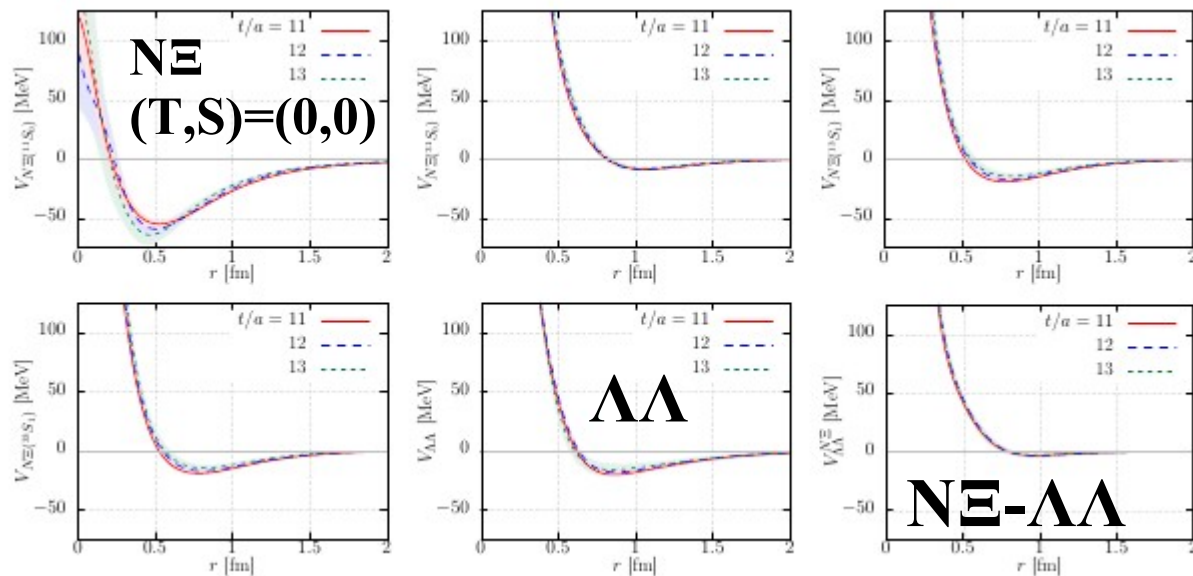
We examine LQCD $N\Xi$ - $\Lambda\Lambda$ potential and discuss H using CF !

$N\Xi-\Lambda$ potential from Lattice QCD

- $N\Xi-\Lambda\Lambda$ potential at almost physical quark mass ($m_\pi=146$ MeV) by HAL QCD Collaboration

K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

- Strong attraction in $(T,S)=(0,0)$ of $N\Xi$
- Weak attraction in $\Lambda\Lambda$ (Coupling with $N\Xi$ causes $\Lambda\Lambda$ attraction)
- **There is no bound state in $N\Xi-\Lambda\Lambda$ system (except for Ξ^- atom), but there is a virtual pole around the $N\Xi$ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, - , +) [relevant=(-,+,+)] (sign of Im (eigenmomentum))**

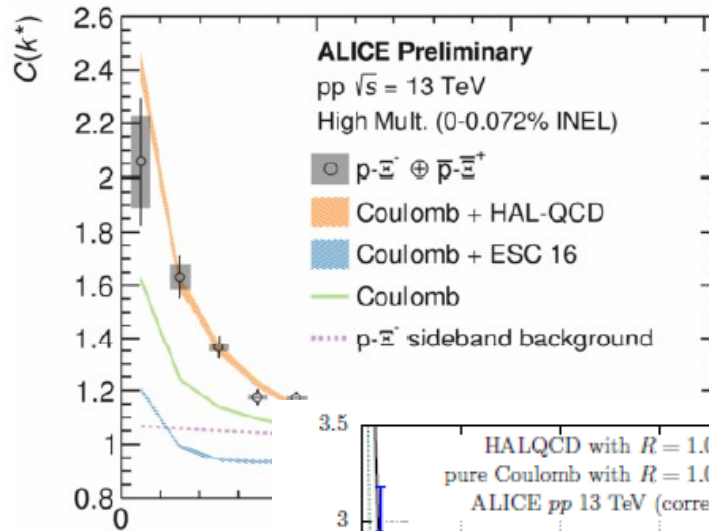
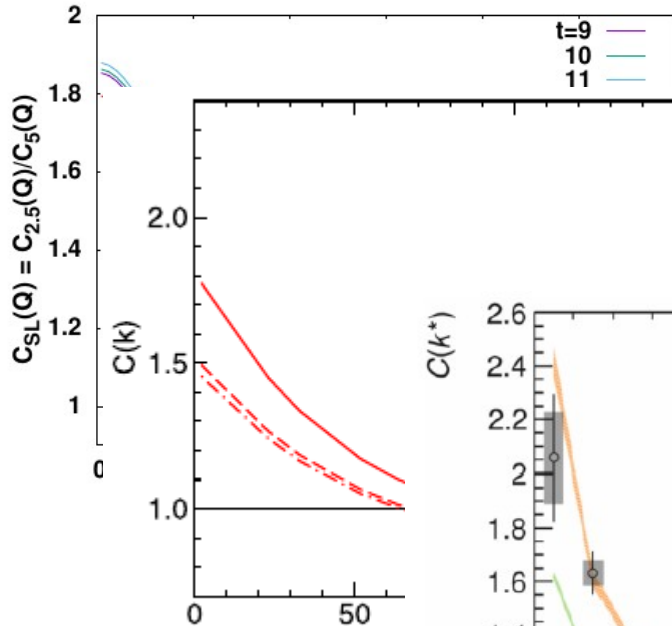


Ξ^- correlation function

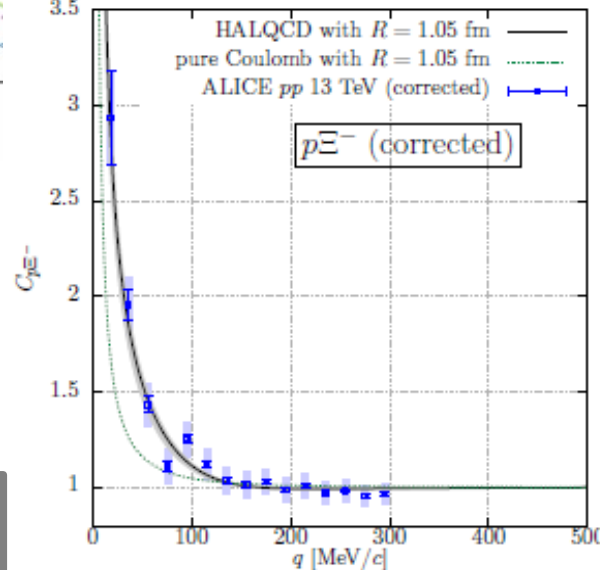
*T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856.
(heavier quark mass, $I=0$ only, w/o CC effects)*

*J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)*

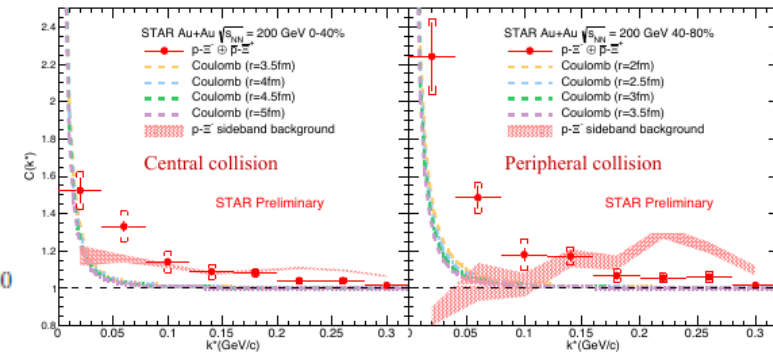
*D. L. Mihairov+[ALICE], NPA 1005 ('21)121760 (QM2019). (Nijmegen pot. does not explain the data. w/o CC)
Acharya+(ALICE), Nature ('20)*



ALI-PREL-315474



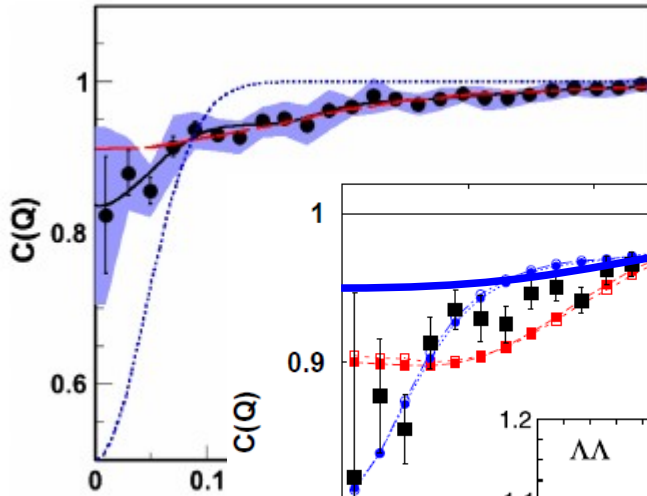
*K. Mi+(STAR, preliminary),
Au+Au 200 AGeV, APS2021.
(No Dip at larger R)*



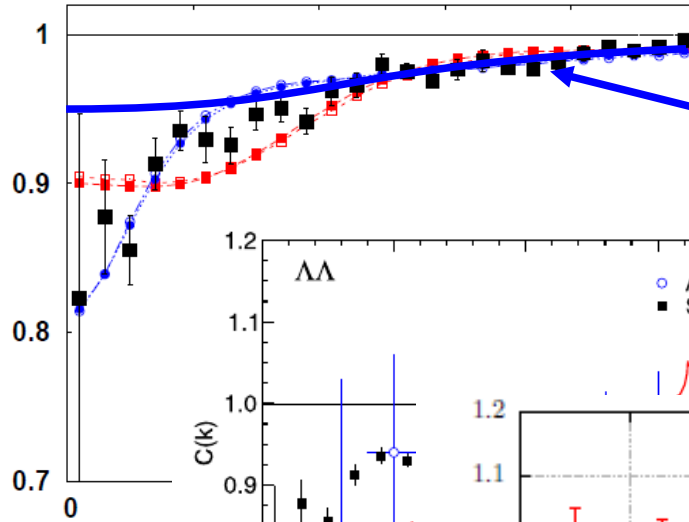
*Kamiya+, 2108.09644
w/ Lattice BB pot. at phys. m_q
CC effects with $\Lambda\Lambda$.*

**There is no signal
of bound state.**

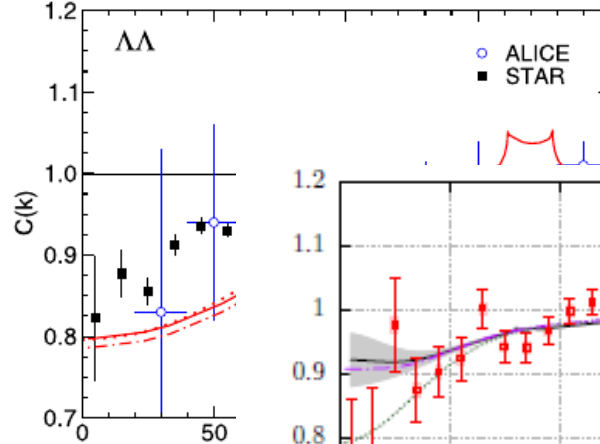
$\Lambda\Lambda$ correlation function



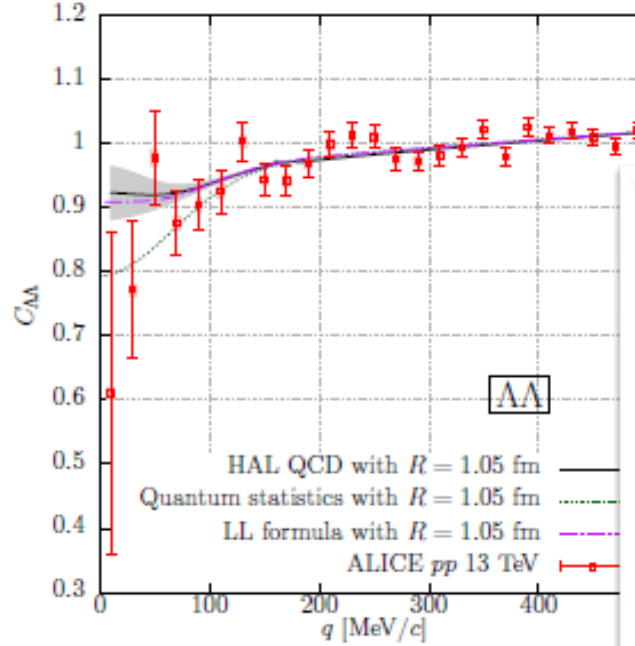
*Adamczyk+[STAR], PRL114('15)022301
(Residual source $R \sim 0.5$ fm was assumed.)*



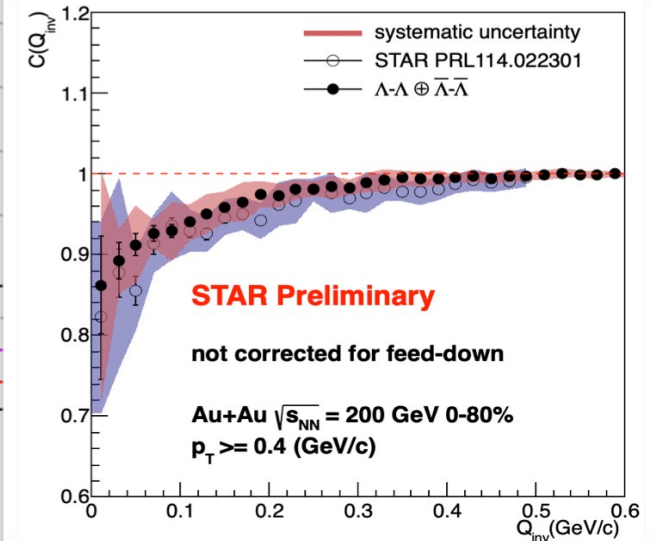
*Morita, Furumoto, AO, PRC91('15)
024916. (Res.Source + flow)*



*J. Haidenbauer, NPA981('19)1.
(NLO600)*



*Moe Isshiki+[STAR],
preliminary.*



*Kamiya+, 2108.09644
w/ Lattice BB pot. at phys. m_q
CC effects with $\Lambda\Lambda$.*

STAR Preliminary

not corrected for feed-down

Au+Au $\sqrt{s_{NN}} = 200$ GeV 0-80%
 $p_T \geq 0.4$ (GeV/c)

Scope of Femtoscopic study of HHI

pK^-
Chiral CC pot.
(examined)
Bound state
(favored)

$p\Xi^-$
Lattice QCD CC
pot. (examined)
Bound state
(disfavored)

$p\Omega$
Lattice QCD pot.
 $J=2$ (examined)
Bound state
(favored)

	n	p	K^-	K^+	π^-	π^+	Λ	Σ	Ξ^-	Ω^-	D^-	D^+	K_s	$+\alpha$
n														
p		0	0	0	Δ	Δ	0	0	0	0	0	0		
K^-		0	0	0	0	0							0	
K^+		0	0	0	0	0							0	
π^-		Δ	0	0	0	0								
π^+		Δ	0	0	0	0								
Λ		0					0							
Σ		0												
Ξ^-		0												
Ω^-		0												
D^-		0												
D^+		0												
K_s				0	0									
$+\alpha$														



pD^\pm
Chamed
hadron-
nucleon
interaction
(work in
prog.)

$K^\pm K_s^0$
Tetraquark
component
in a_0 meson

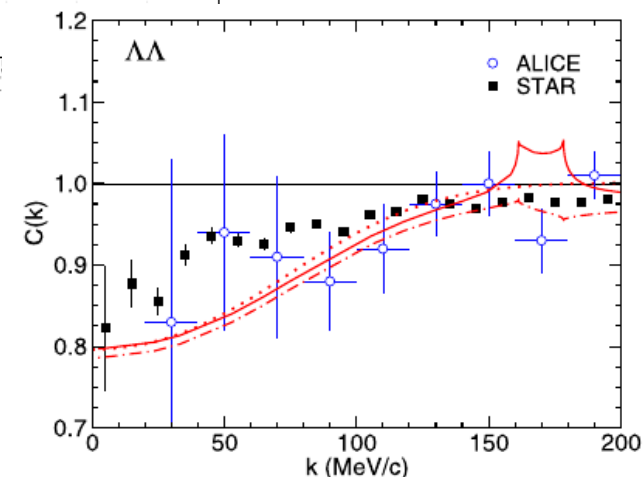
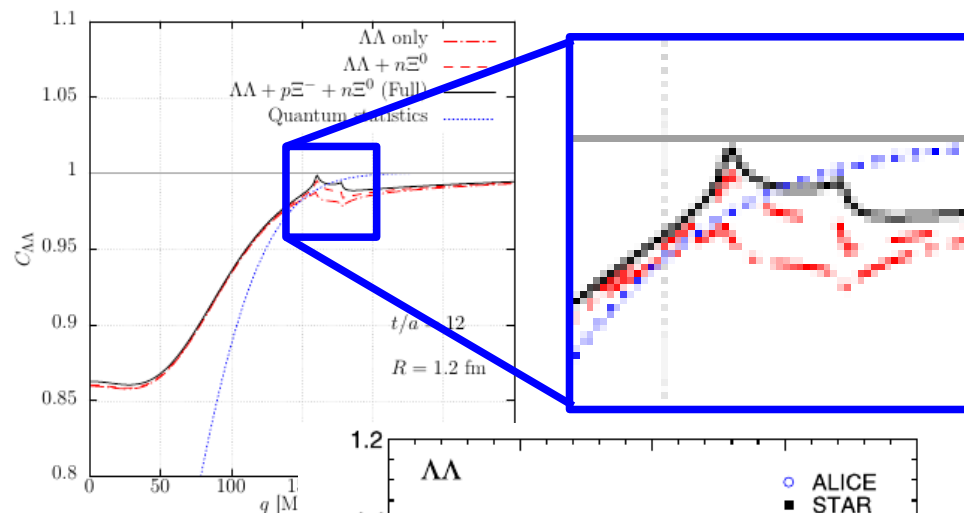
$\Lambda\Lambda$
Scattering pars. (a_0, r_{eff})
(constrained)
Bound state (disfavored)

Other bound states ?

■ $\Lambda\Lambda$ - $N\Xi$

- $C_{\Lambda\Lambda}(q)$ in AA(RHIC) and pp(LHC) are similar (No b.s. below $\Lambda\Lambda$).
- LQCD predicts a virtual pole near $N\Xi$ threshold, which can be detected as the cusp in $C_{\Lambda\Lambda}(q)$.
- NLO(600) potential predicts the same. (The fate of H particle)

K. Sasaki+[HAL QCD], NPA998('20)121737;
Y. Kamiya+, 2108.09644; Haidenbauer('19).



■ $K^{\text{bar}}N$

- $\Lambda(1405)$ is believed to be the bound state of $K^{\text{bar}}N$ and “dip” is expected at $R \sim a_0$.
- However, Coulomb and coupled-channel effects modify the dip-like behavior.

Kamiya+ ('20).

