



The 8th Asian Triangle Heavy-Ion Conference

ATHIC2021

5-9 November 2021

Inha University, Incheon, South Korea

Hydrodynamics: the best data machine of HIC

Long-Gang Pang (庞龙刚)

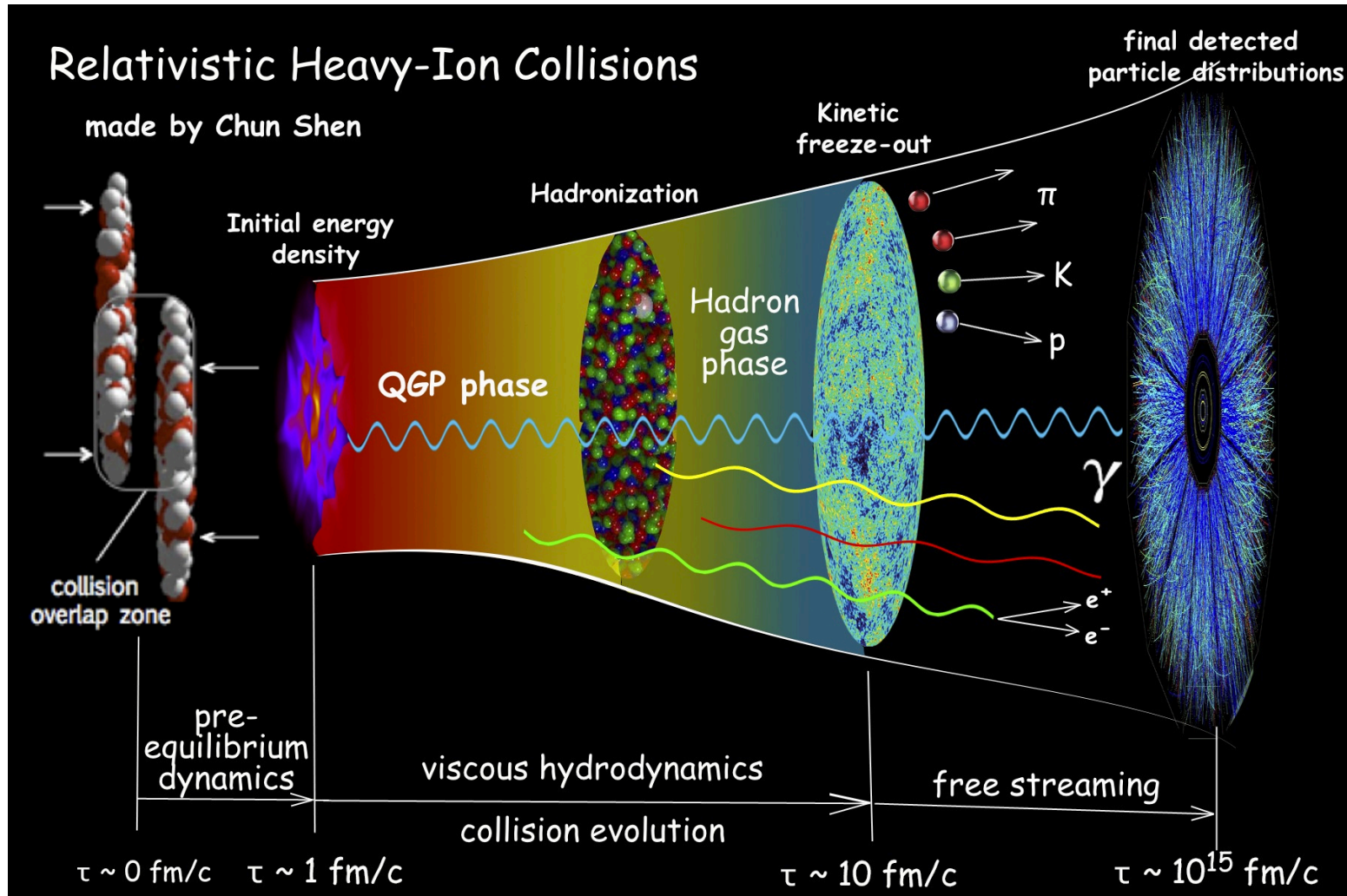
Central China Normal University

2021.11.5



The little bang

Credit: Chun Shen



Problem: what have been observed are the four-momenta of final state particles.
Not possible to access the initial state and quark gluon plasma directly.

The hottest fluid

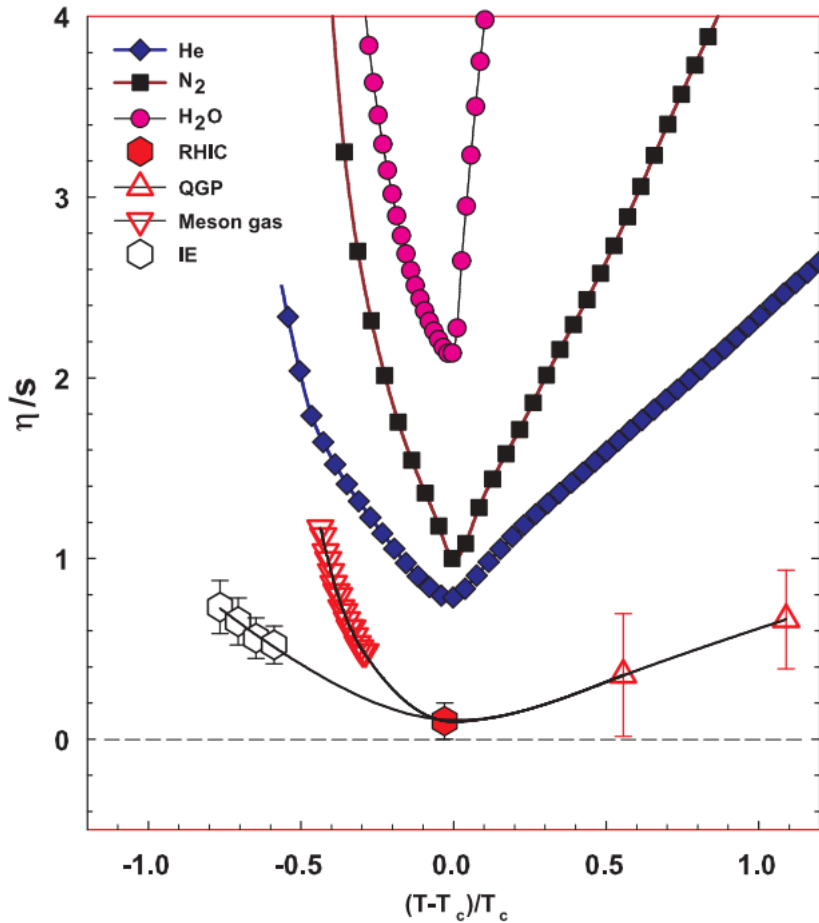
$$1 \text{ eV} = 11,604 \text{ Kelvin}$$

$$T_{\text{QGP}} \sim 350 \text{ MeV} = 4 \times 10^{12} \text{ K}$$

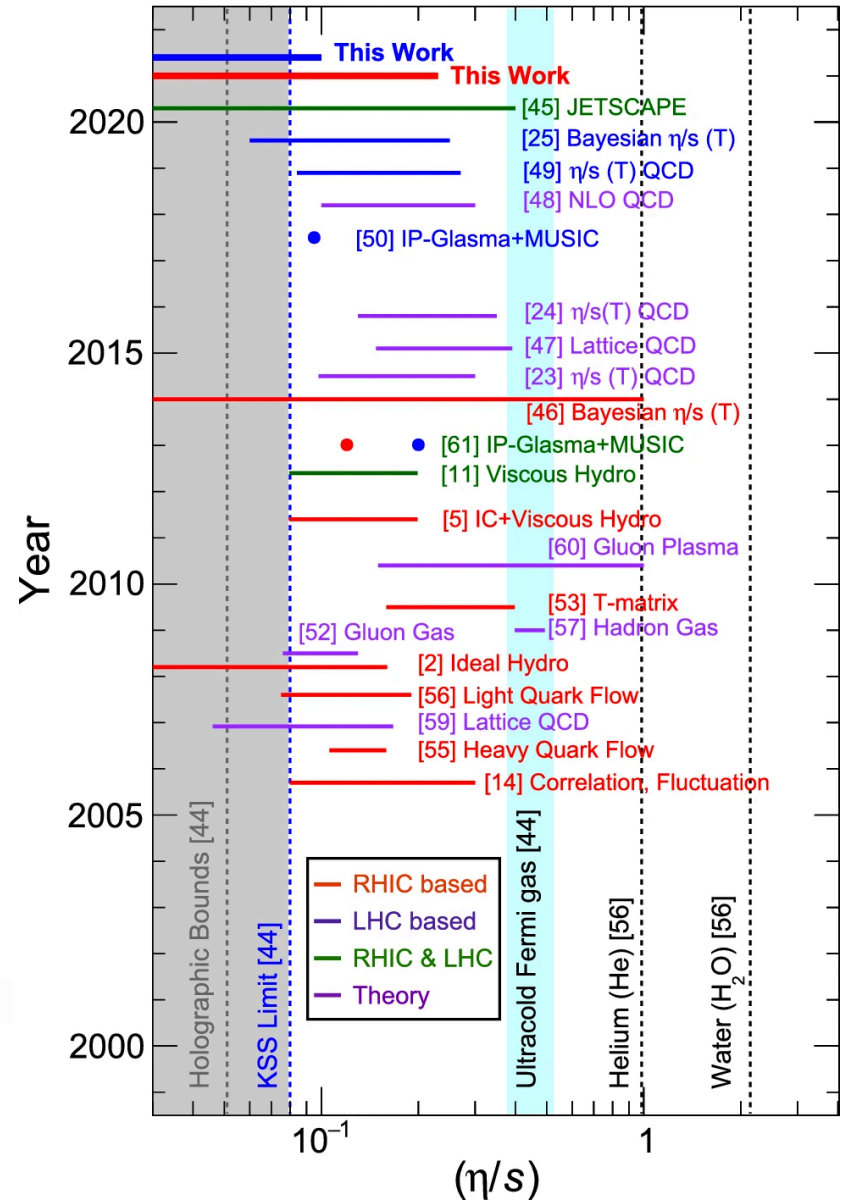
$$T_{\text{sun}\odot} \approx 15 \times 10^6 \text{ K}$$

$$T_{\text{QGP}} \approx 10^6 T_{\text{sun}\odot}$$

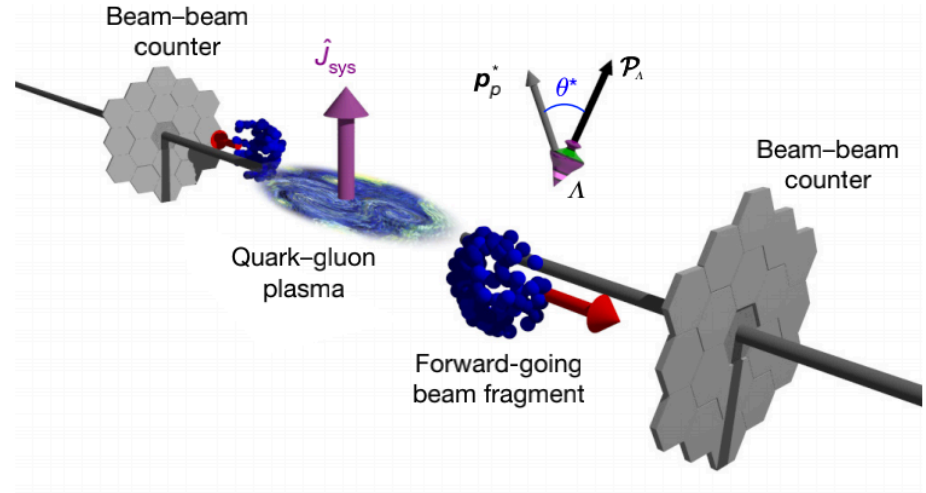
The most perfect fluid



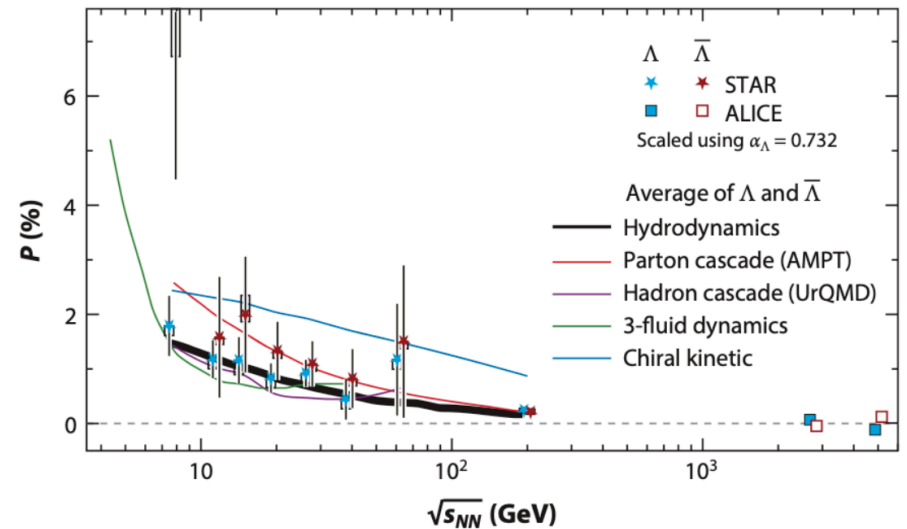
R. A. Lacey, N. N. Ajitanand, J. M. Alexander, et al. Phys. Rev. Lett. 98, 092301 (2007).
 Gonzalez, V., Basu, S., Marin, A. *et al.* Eur. Phys. J. C **81**, 465 (2021)



The most vortical fluid



F. Becattini and M. A. Lisa, *Ann. Rev. Nucl. Part. Sci.* 70, 395-423 (2020)



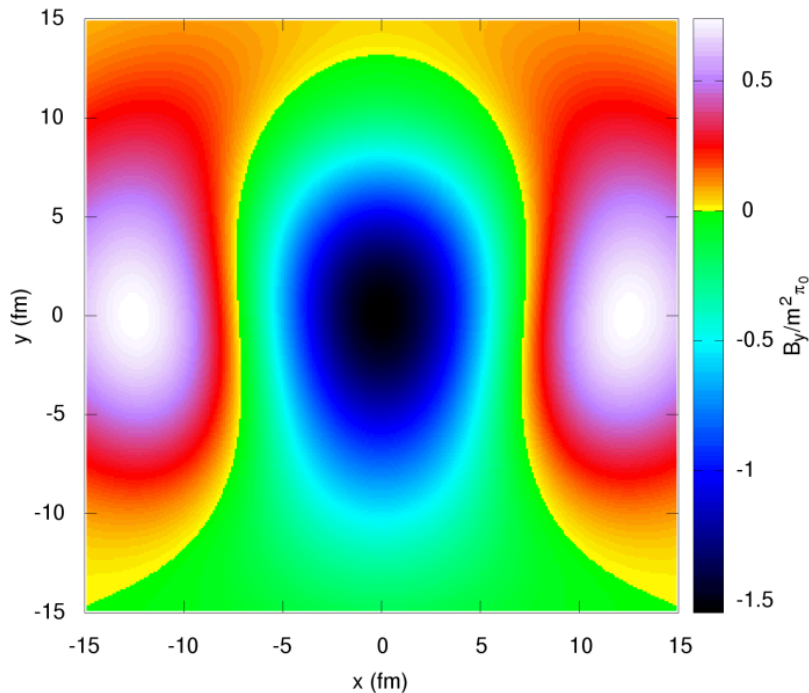
The STAR Collaboration. *Nature* 548, 62–65 (2017)

Liang, Wang, PRL 94, 102301 (2005)
Voloshin, nucl-th/0410089

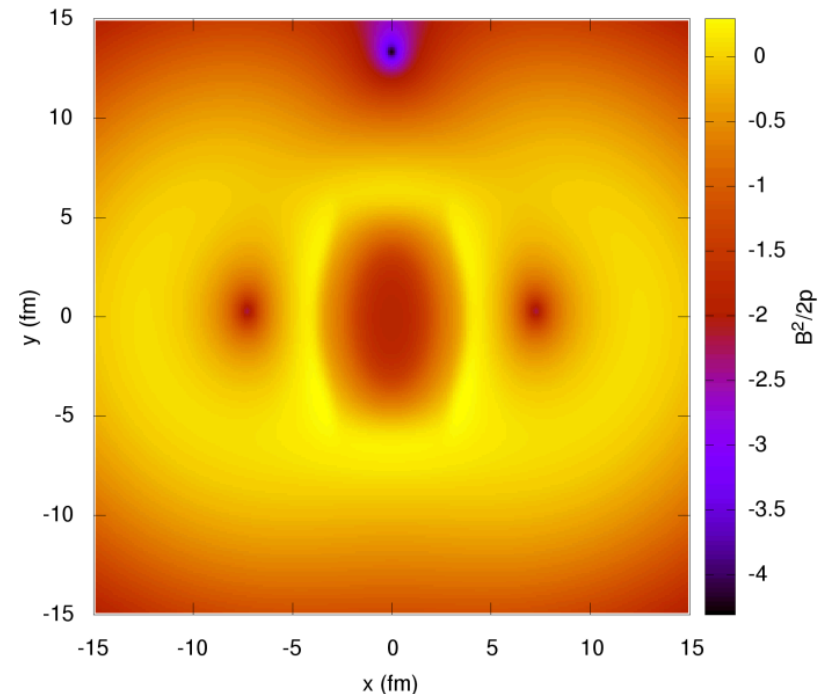
Strongest magnetic field at lab

3D+1 simulation of Au+Au collision at $\sqrt{s_{\text{NN}}} = 200$ AGeV
Conductivities of the medium ($\tau \leq \tau_0$): $\sigma = 5.8$ MeV, $\sigma_\chi = 1.5$ MeV

Gabriele Inghirami: : ideal 3D+1 MHD simulations with ECHO-QGP using Glauber i.c.



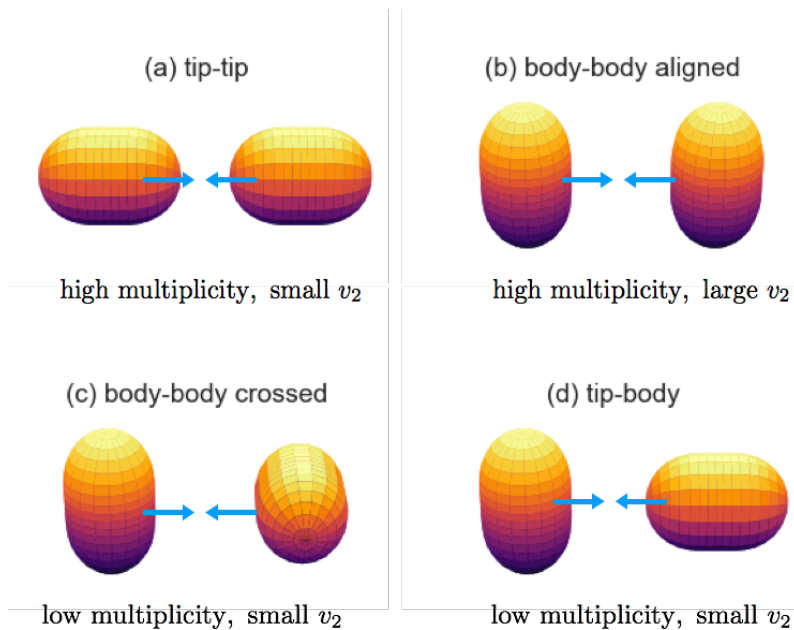
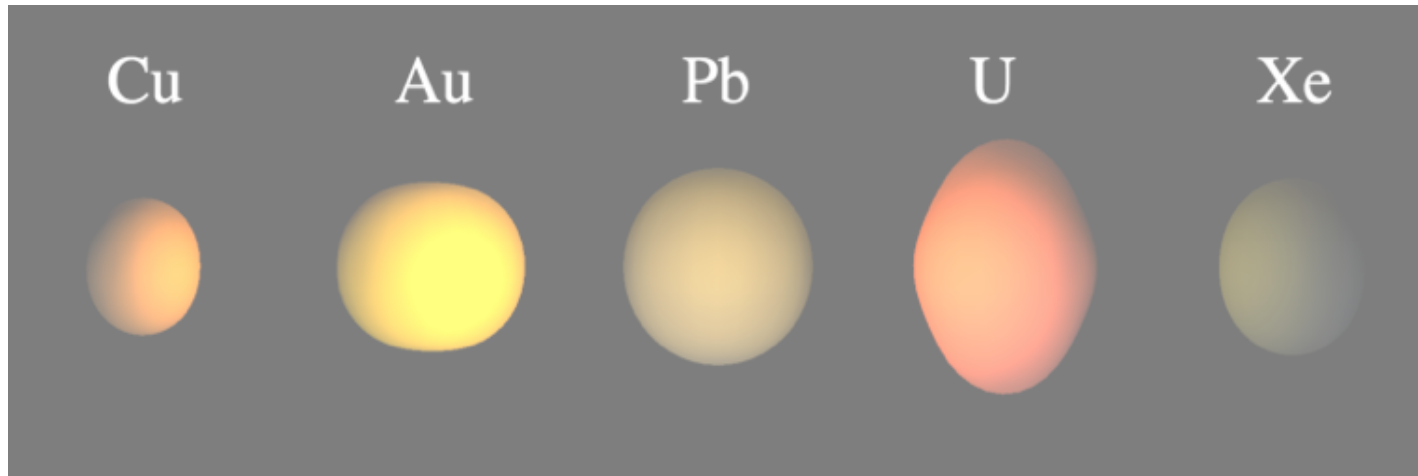
B_y at $\tau = 0.4$ fm/c and $\eta = 0$



Magnetic/thermal pressure ratio

Maximum magnetic field: $B_y \approx 1.5 m_\pi^2$

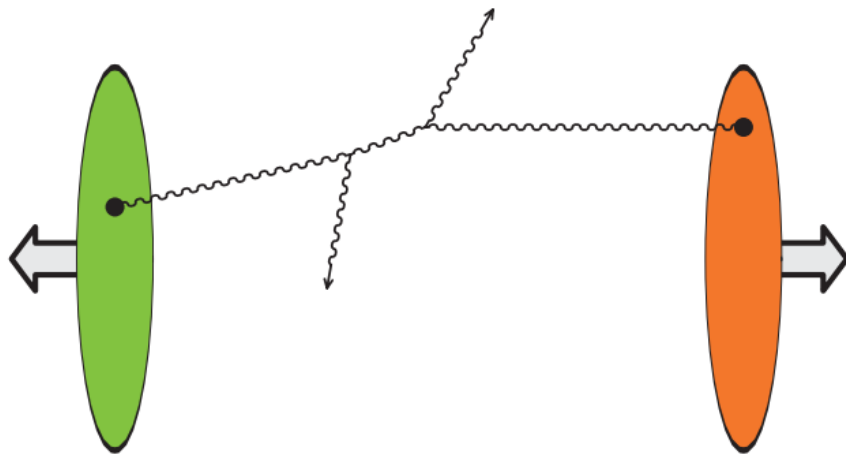
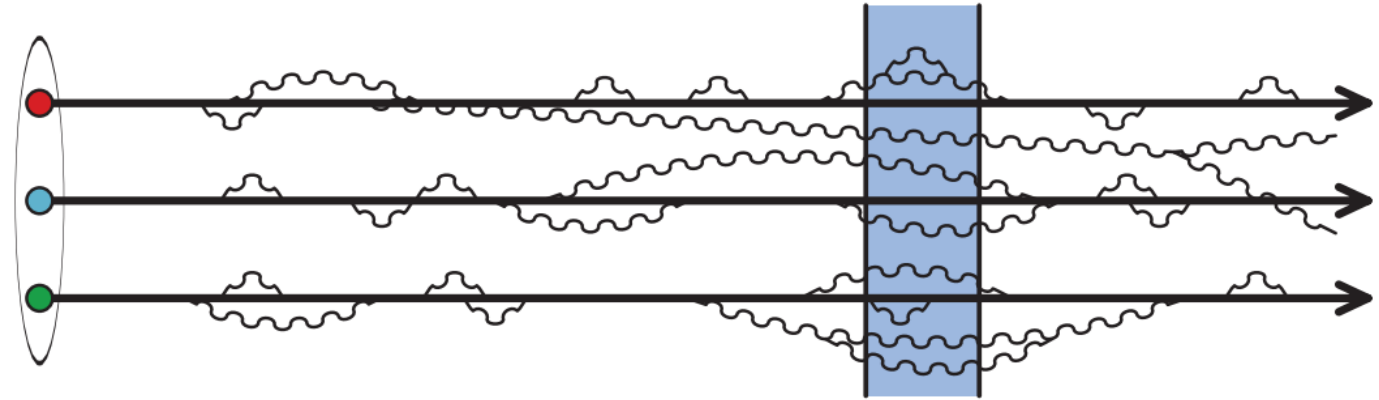
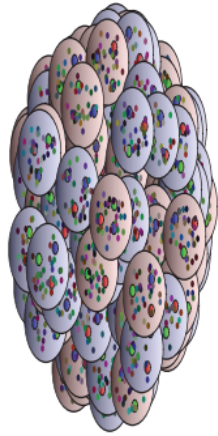
Effect of nuclear structure on HIC



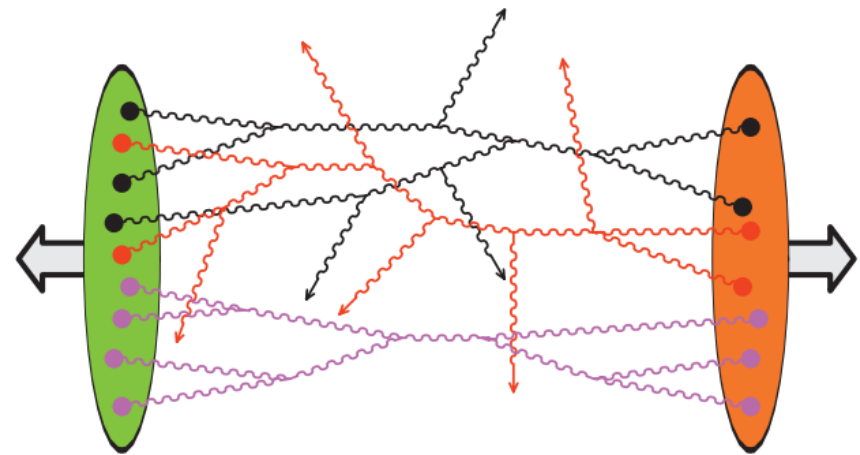
How nuclear structure affect HIC?

- Deformation
- Alpha cluster
- Neutron skin
- Two nucleon correlation
- ...

How CGC evolve to QGP?

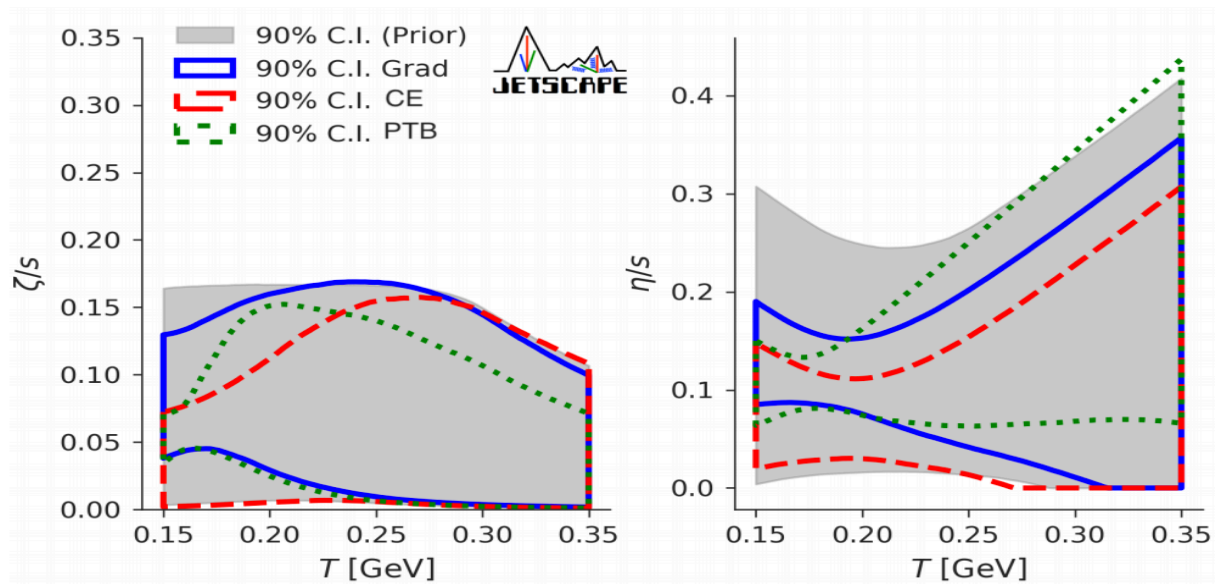
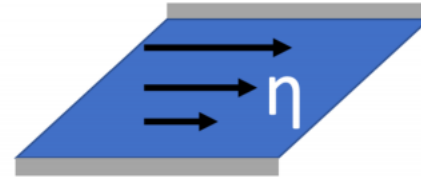
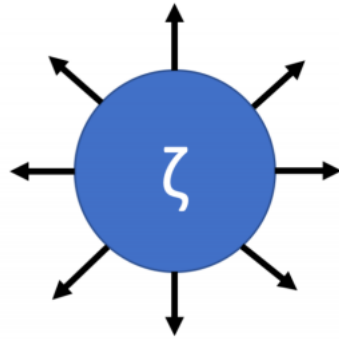


Dilute



Dense

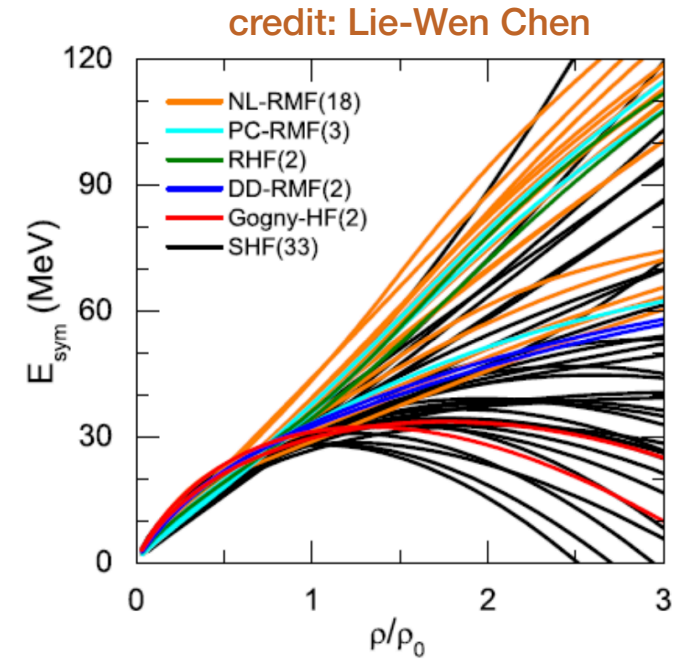
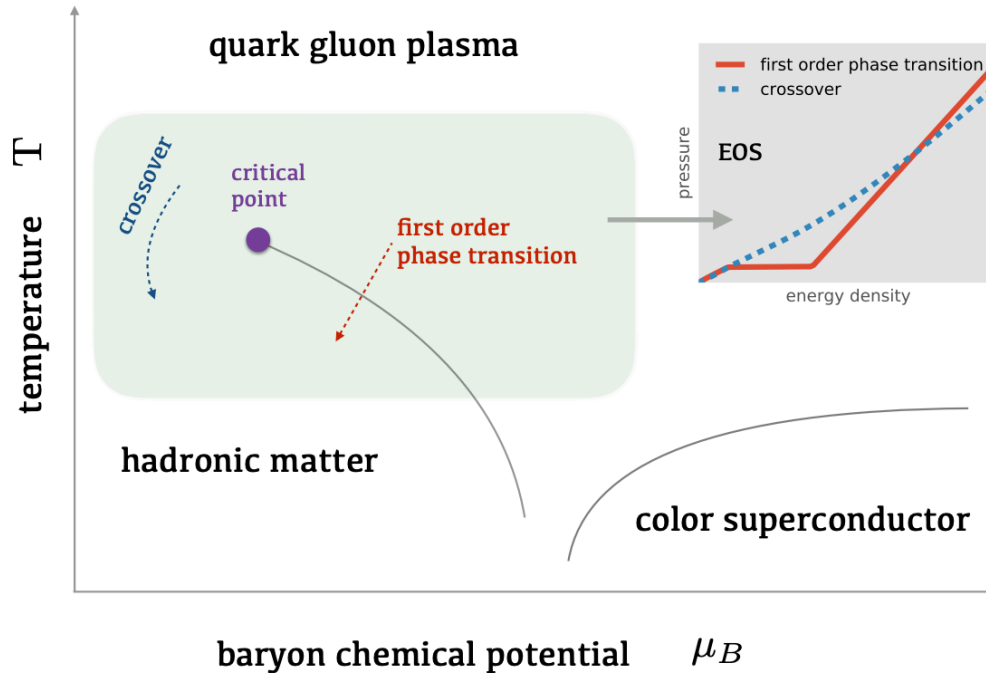
Uncertain viscosity of quark gluon plasma



The QCD phase transition and EoS

High temperature region: EoS

High density region: nuclear symmetry energy



List of questions in the field of HIC

- How the initial nuclear structure affect HIC?
- How energy and net baryons distribute in transverse plane and longitudinal direction
- How the saturated gluons evolve to locally thermalized QGP?
- What is the value of shear and bulk viscosity?
- Can we verify the lattice QCD EoS at 0 net baryon density?
- Can we determine the EoS at nonzero baryon chemical potential from experiment data?
- How long will the magnetic field survive in QGP?
- How OAM, vorticity, shear and temperature gradient, baryon chemical potential gradient couple to polarization of fermion spins?
- How quarks and gluons form hadrons on the freeze out surface?
- Is there local parity violation in strong interacting matter?
- ...

Each question corresponds to several parameters

Multiple parameters entangle with multiple observables

Model Parameter:

eqn. of state

shear viscosity

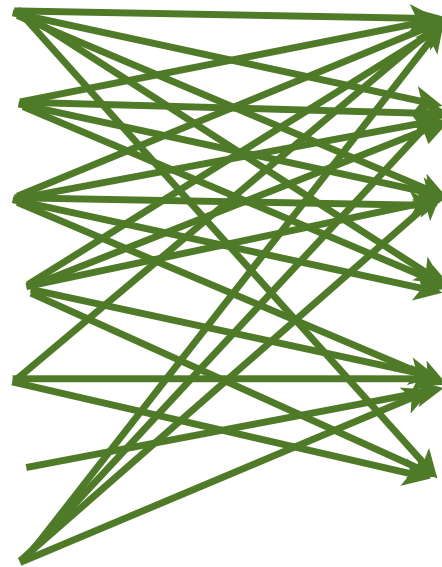
initial state

pre-equilibrium dynamics

thermalization time

quark/hadron chemistry

particlization/freeze-out



experimental data:

π /K/P spectra

yields vs. centrality & beam

elliptic flow

HBT

charge correlations & BFs

density correlations

Fig from S. Bass QM2017 (Bayesian method)

The observables entangle with model parameters

How to disentangle them

$$v_2 = f_1(\sqrt{s_{NN}}, \beta_2, \beta_4, b, \tau_0, \eta/s, \zeta/s, T_{\text{frz}}, \dots)$$

$$v_3 = f_2(\sqrt{s_{NN}}, \beta_2, \beta_4, b, \tau_0, \eta/s, \zeta/s, T_{\text{frz}}, \dots)$$

$$v_4 = f_3(\sqrt{s_{NN}}, \beta_2, \beta_4, b, \tau_0, \eta/s, \zeta/s, T_{\text{frz}}, \dots)$$

$$\langle p_T \rangle = f_4(\sqrt{s_{NN}}, \beta_2, \beta_4, b, \tau_0, \eta/s, \zeta/s, T_{\text{frz}}, \dots)$$

...

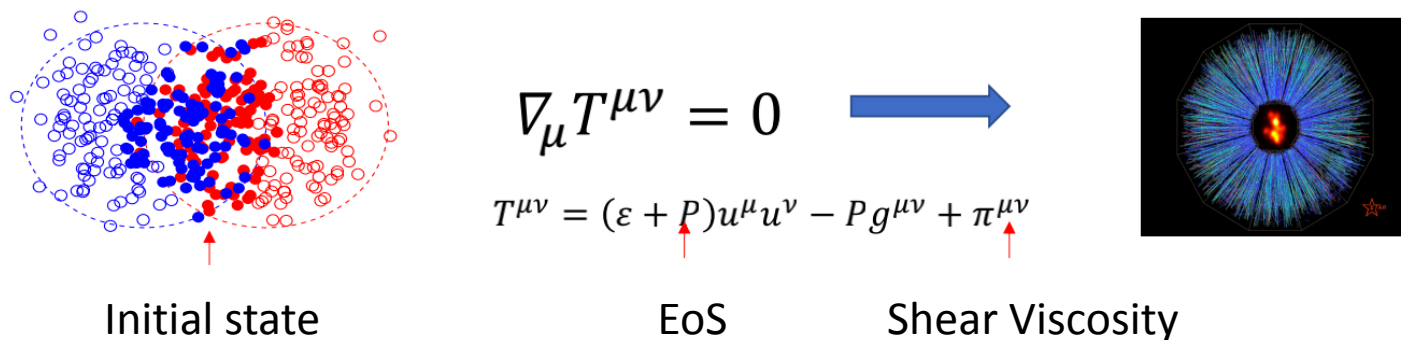
$$obs = f_n(\sqrt{s_{NN}}, \beta_2, \beta_4, b, \tau_0, \eta/s, \zeta/s, T_{\text{frz}}, \dots)$$

How to solve (disentangle) these equations inversely to determine properties of interests? E.g.,

$$\eta/s = g(v_2, v_3, v_4, \langle p_T \rangle, \dots)$$

Need to map initial to final state with adjustable parameters

CLVisc 1.0: Energy momentum conservation and evolution of shear stress tensor



$$\{p_i^{\mu}\} = f(\sqrt{s_{NN}}, \beta_2, \beta_4, b, \tau_0, \eta/s, \zeta/s, T_{\text{frz}}, \dots)$$

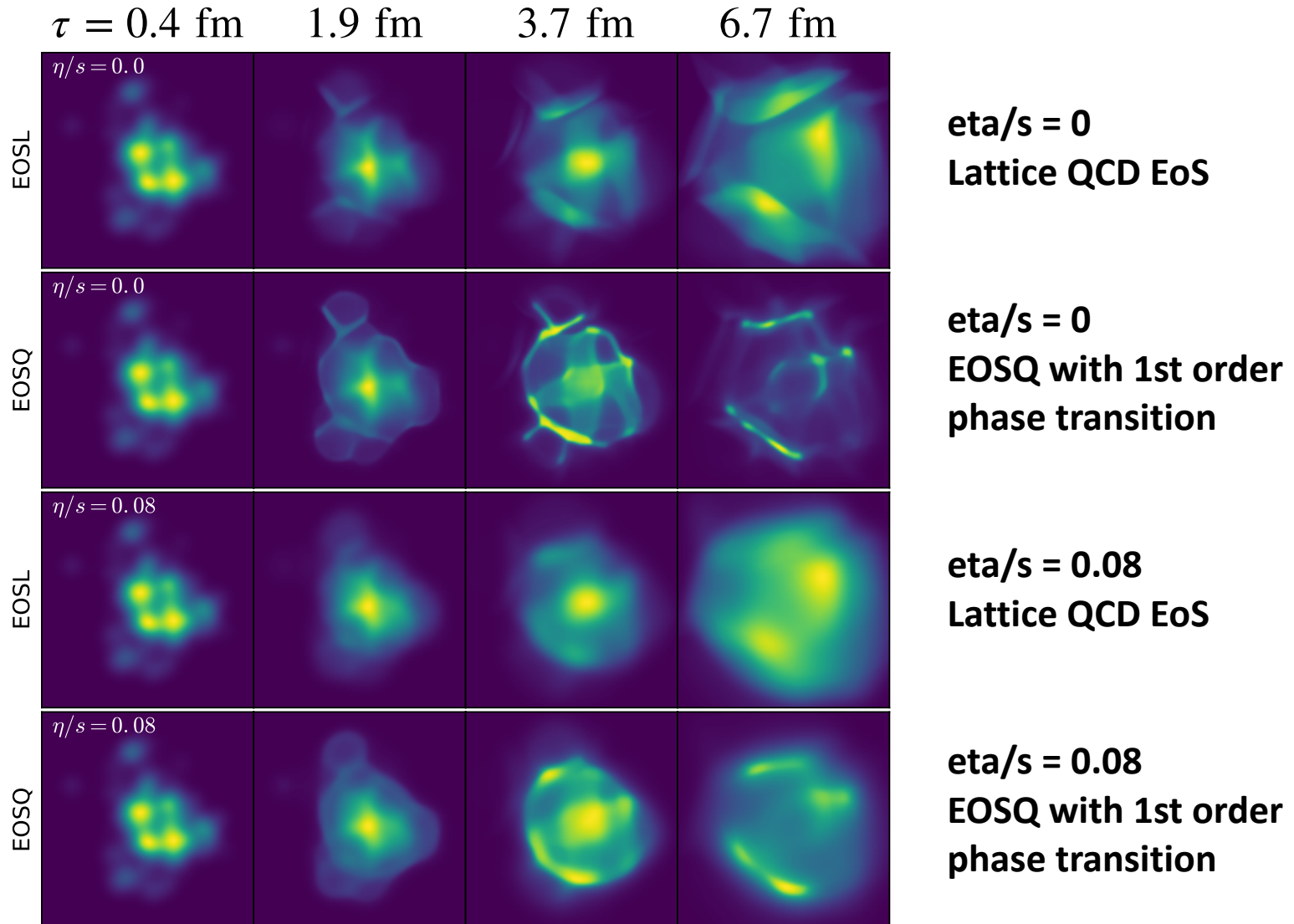
CLVisc: 3+1D viscous hydro on GPU using OpenCL

L.G. Pang, Q. Wang and X. N. Wang, PRC 86 (2012) 024911

L.G. Pang, B.W. Xiao, Y. Hatta, X.N.Wang, PRD 2015

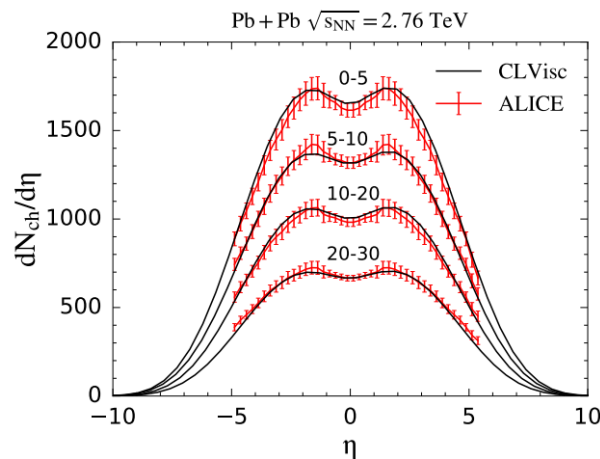
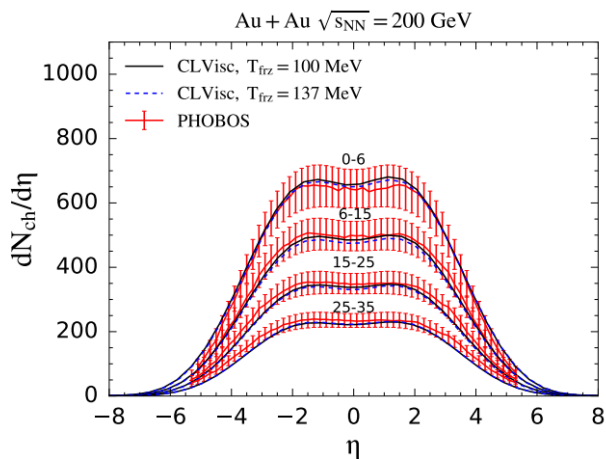
L.G. Pang, H.Petersen, XN Wang, PRC97(2018)no.6,064918

QGP evolution with adjustable parameters

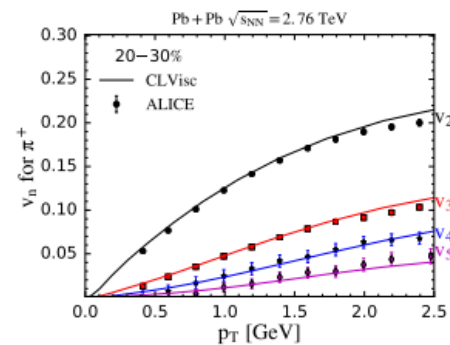
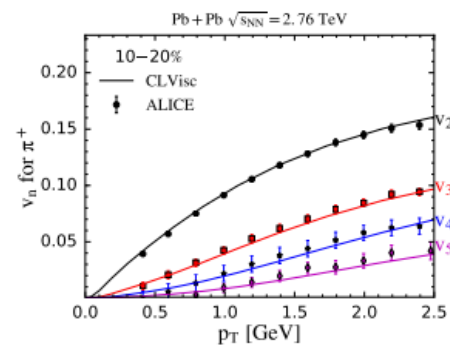
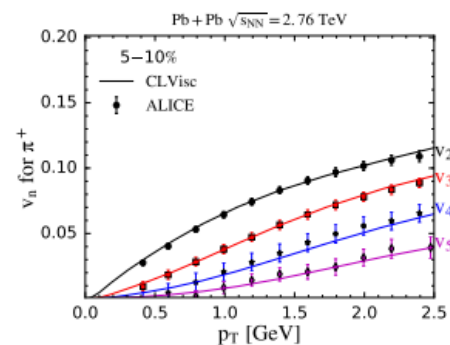


CLVisc simulation results

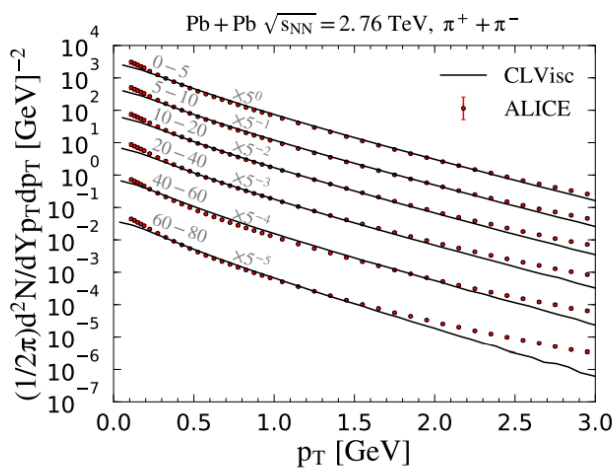
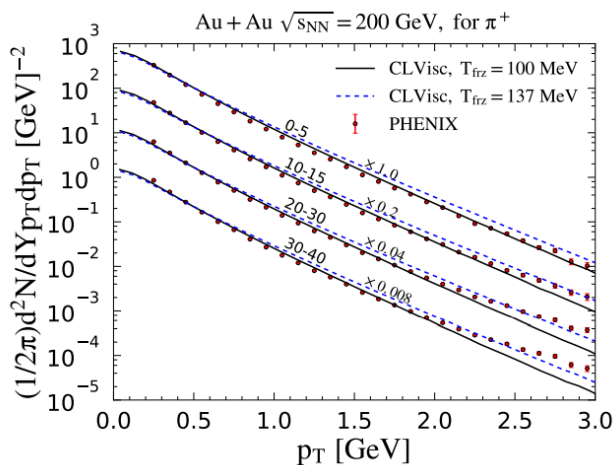
Longitudinal momentum distribution



Anisotropic flows



Transverse momentum distribution



CLVisc 2.0: for beam energy scan

$$\nabla_{\mu} T^{\mu\nu} = 0, \quad \nabla_{\mu} J^{\mu} = 0$$

where $T^{\mu\nu} = eU^{\mu}U^{\nu} - P\Delta^{\mu\nu} + \pi^{\mu\nu}$ is the energy-momentum tensor

and $J^{\mu} = nU^{\mu} + V^{\mu}$ is the net baryon current, nU^{μ} represent net baryons co-moving

with energy flow and V^{μ} the net baryon diffusion. $\pi^{\mu\nu}$ and V^{μ} evolve using IS equations,

$$\Delta_{\alpha\beta}^{\mu\nu} D\pi^{\alpha\beta} = -\frac{1}{\tau_{\pi}} (\pi^{\mu\nu} - \eta\sigma^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{5}{7}\pi^{\alpha<\mu}\sigma_{\alpha}^{\nu>} + \frac{9}{70}\frac{4}{e+P}\pi_{\alpha}^{<\mu}\pi^{\nu>\alpha}$$

$$\Delta^{\mu\nu} DV_{\nu} = -\frac{1}{\tau_V} \left(V^{\mu} - \kappa_B \nabla^{\mu} \frac{\mu_B}{T} \right) - V^{\mu}\theta - \frac{3}{10}V_{\nu}\sigma^{\mu\nu}$$

G. S. Denicol, C. Gale, S. Jeon, A. Monnai, B. Schenke, C. Shen, Phys. Rev. C98 (3) (2018) 034916.

X-Y Wu, G-Y Qin, L-G Pang, X-N Wang, [arXiv:2107.04949](https://arxiv.org/abs/2107.04949)

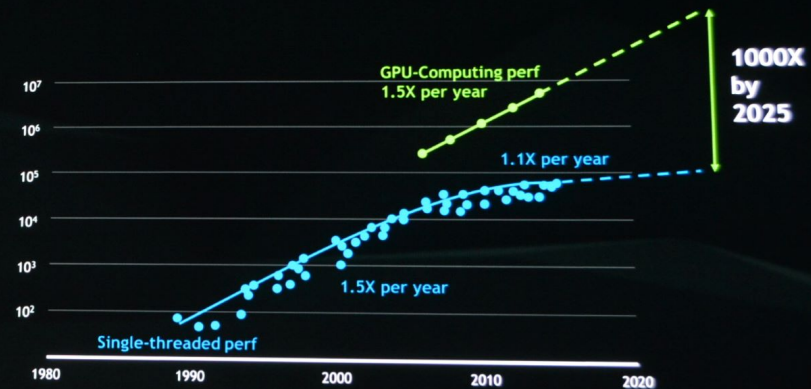
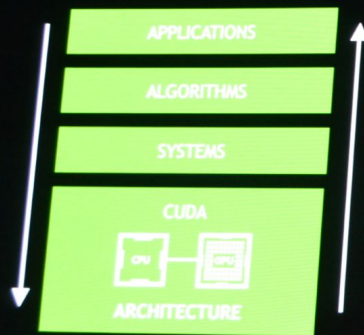
Not complete list of relativistic hydrodynamics

- OSU, PKU: BEShydro, iEBE-VISHNU
- McGill: MUSIC
- CCNU-LBNL: CLVisc
- T. Hirano's 3+1D hydro
- Frankfurt and Italy: vHLLE, ECHO-QGP
- CERN: EPOS
- Colorado: hydro using Lattice Boltzmann Method
- Instituto de Física: Smooth Particle Hydrodynamics
- ...

What makes CLVisc special: gpu parallel computing

CLVisc is 60~100 times faster on GPU than a single core CPU

RISE OF GPU COMPUTING



Original data up to the year 2010 collected and plotted by M. Horowitz, F. Labonte, O. Shacham, K. Olukotun, L. Hammond, and C. Batten New plot and data collected for 2010-2015 by K. Rupp

Bayesian analysis: scan 9-dim parameter space

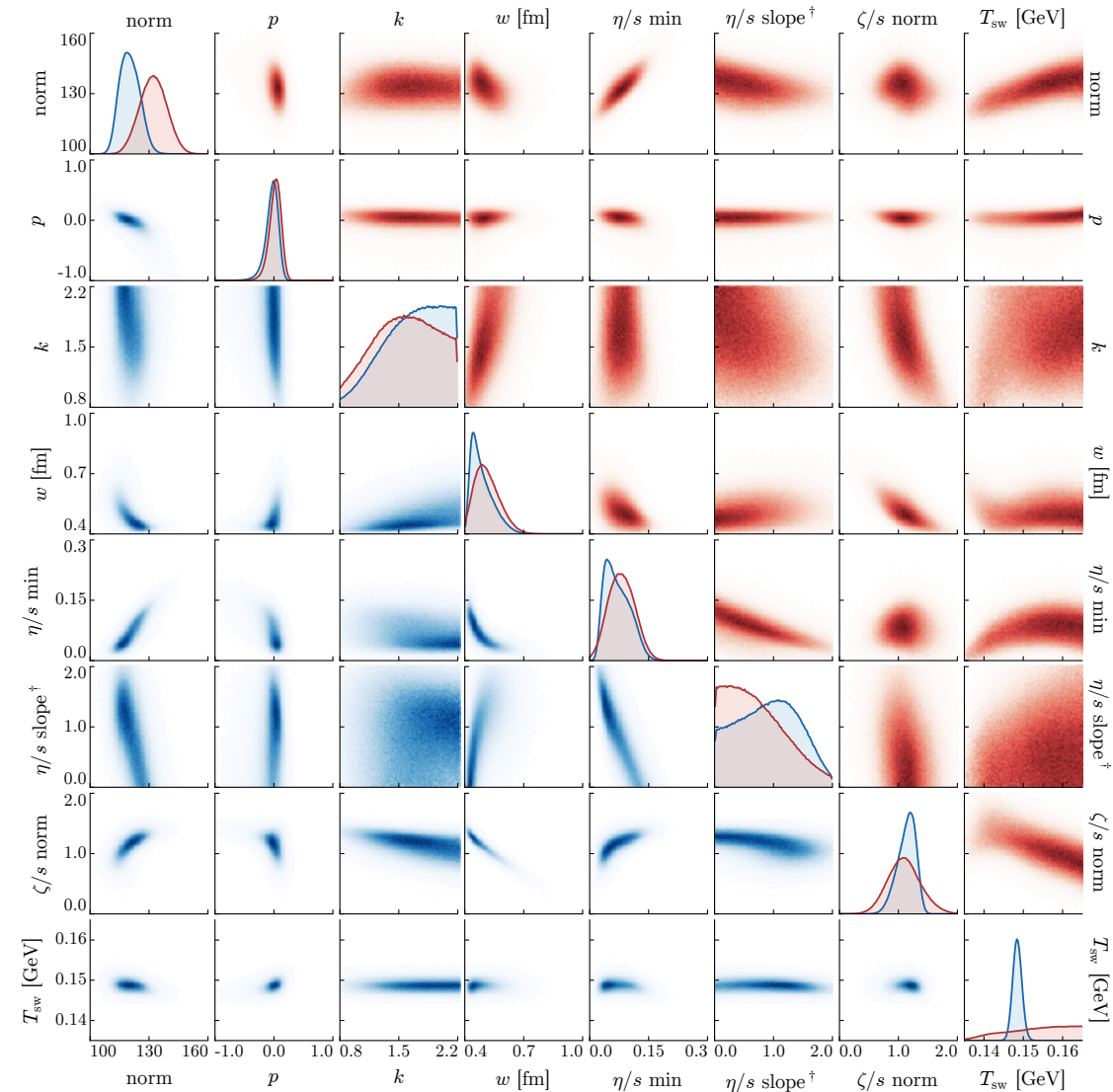


FIG. 7. Posterior distributions for the model parameters from calibrating to identified particles yields (blue, lower triangle) and charged particles yields (red, upper triangle). The diagonal has marginal distributions for each parameter, while the off-diagonal contains joint distributions showing correlations among pairs of parameters. [†]The units for η/s slope are $[\text{GeV}^{-1}]$.

TABLE I. Input parameter ranges for the initial condition and hydrodynamic models.

Parameter	Description	Range
Norm	Overall normalization	100–250
p	Entropy deposition parameter	−1 to +1
k	Multiplicity fluct. shape	0.8–2.2
w	Gaussian nucleon width	0.4–1.0 fm
η/s hrg	Const. shear viscosity, $T < T_c$	0.3–1.0
η/s min	Shear viscosity at T_c	0–0.3
η/s slope	Slope above T_c	0–2 GeV^{-1}
ζ/s norm	Prefactor for $(\zeta/s)(T)$	0–2
T_{switch}	Particlization temperature	135–165 MeV

Posterior Likelihood Prior

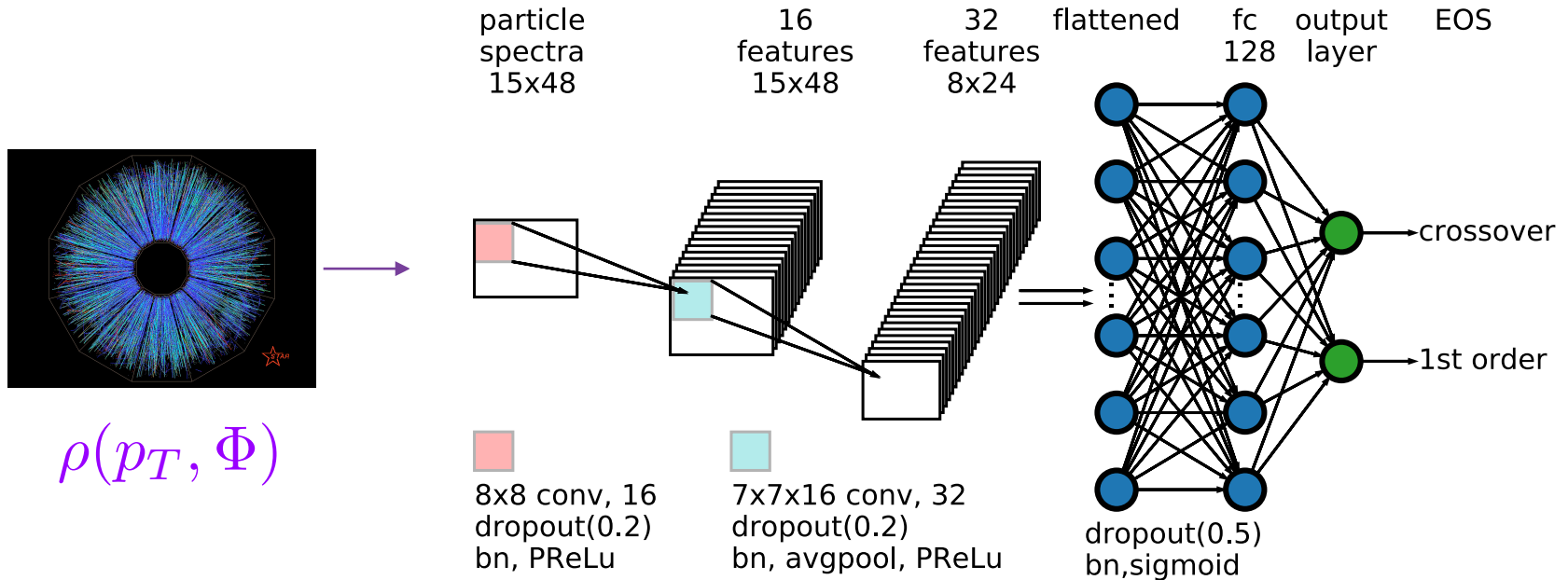
$$P(\mathbf{X} | \mathbf{Y}) = \frac{P(\mathbf{Y} | \mathbf{X}) P(\mathbf{X})}{P(\mathbf{Y})}$$

X: model — **Y**: data

PRC 94.024907, J.E.Bernhard. et.al.

PRL. 114, 202301, S. Pratt, et.el

Deep learning: Convolution Neural Network



L-G Pang, K Zhou, N Su, H Petersen, H Stocker, X-N Wang, Nature Commun 2018

Bayesian analysis: $\eta/s = g(v_2, v_3, v_4, \langle p_T \rangle, \dots)$

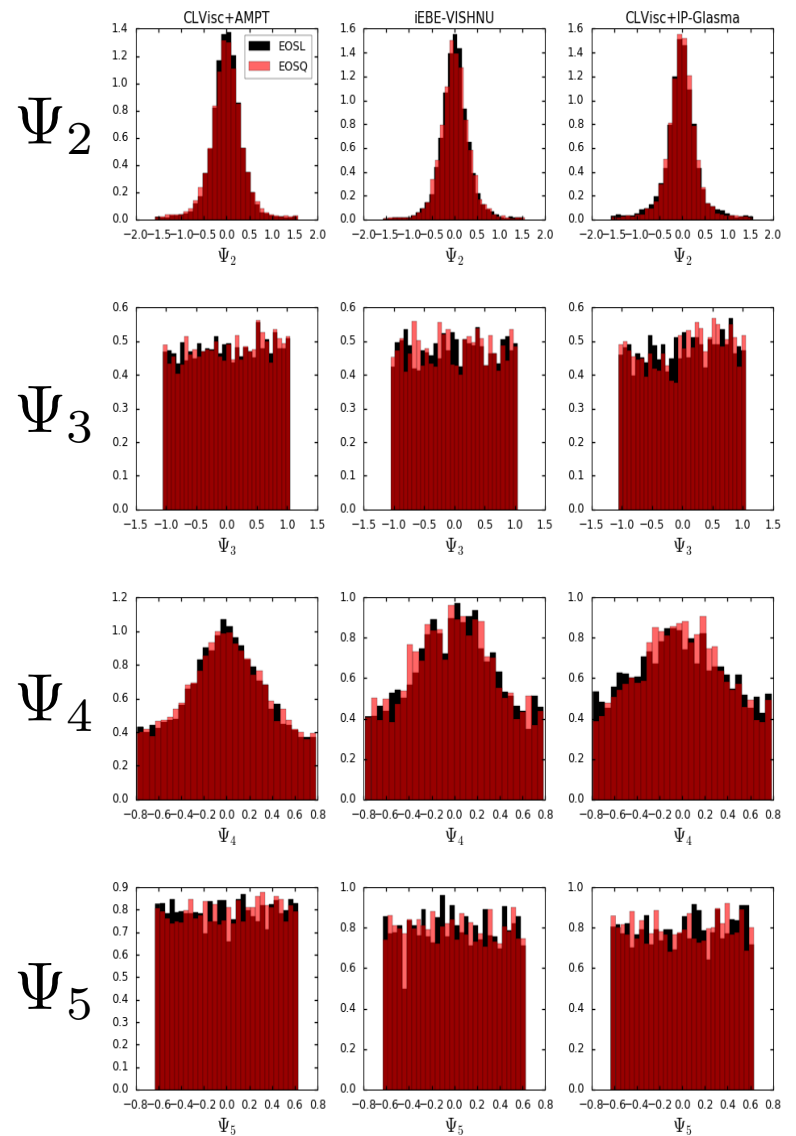
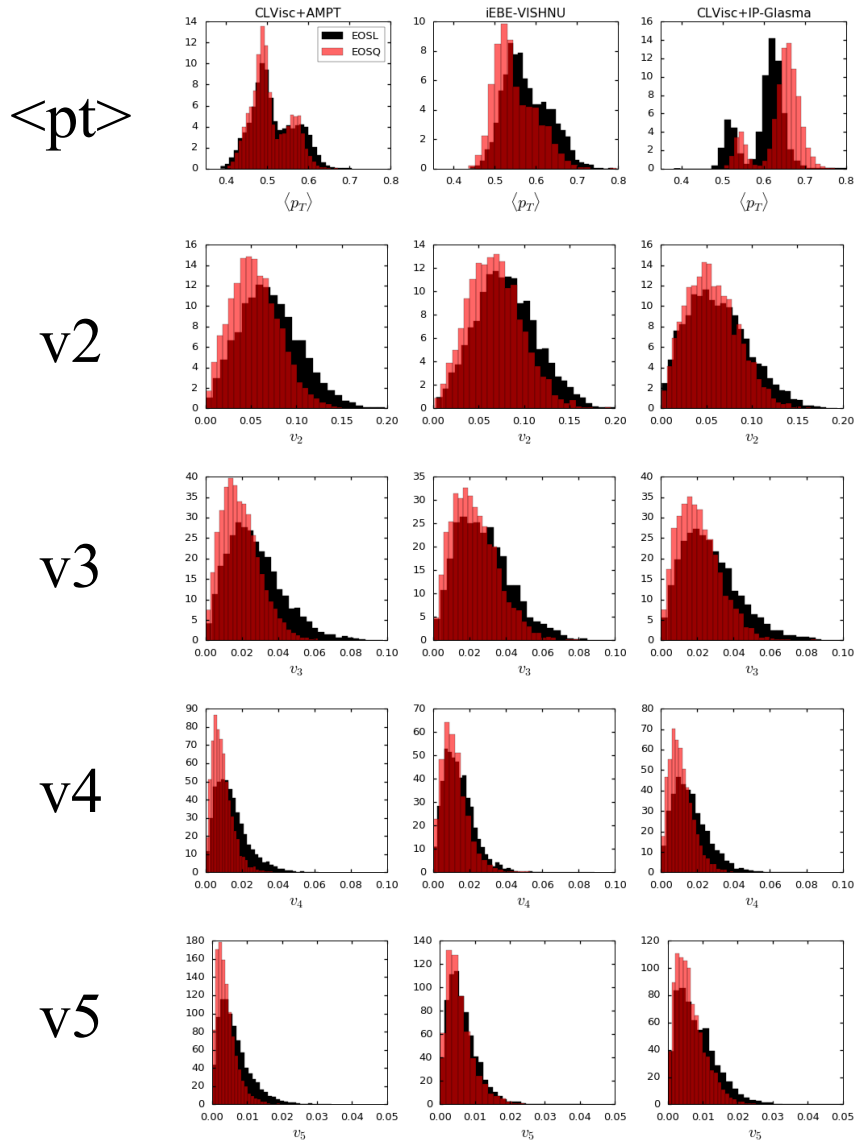
Deep CNN : $\eta/s = f(\rho(Y, p_T, \phi))$

In progress with Y-Q Luo, H-Chao Song, K Zhou

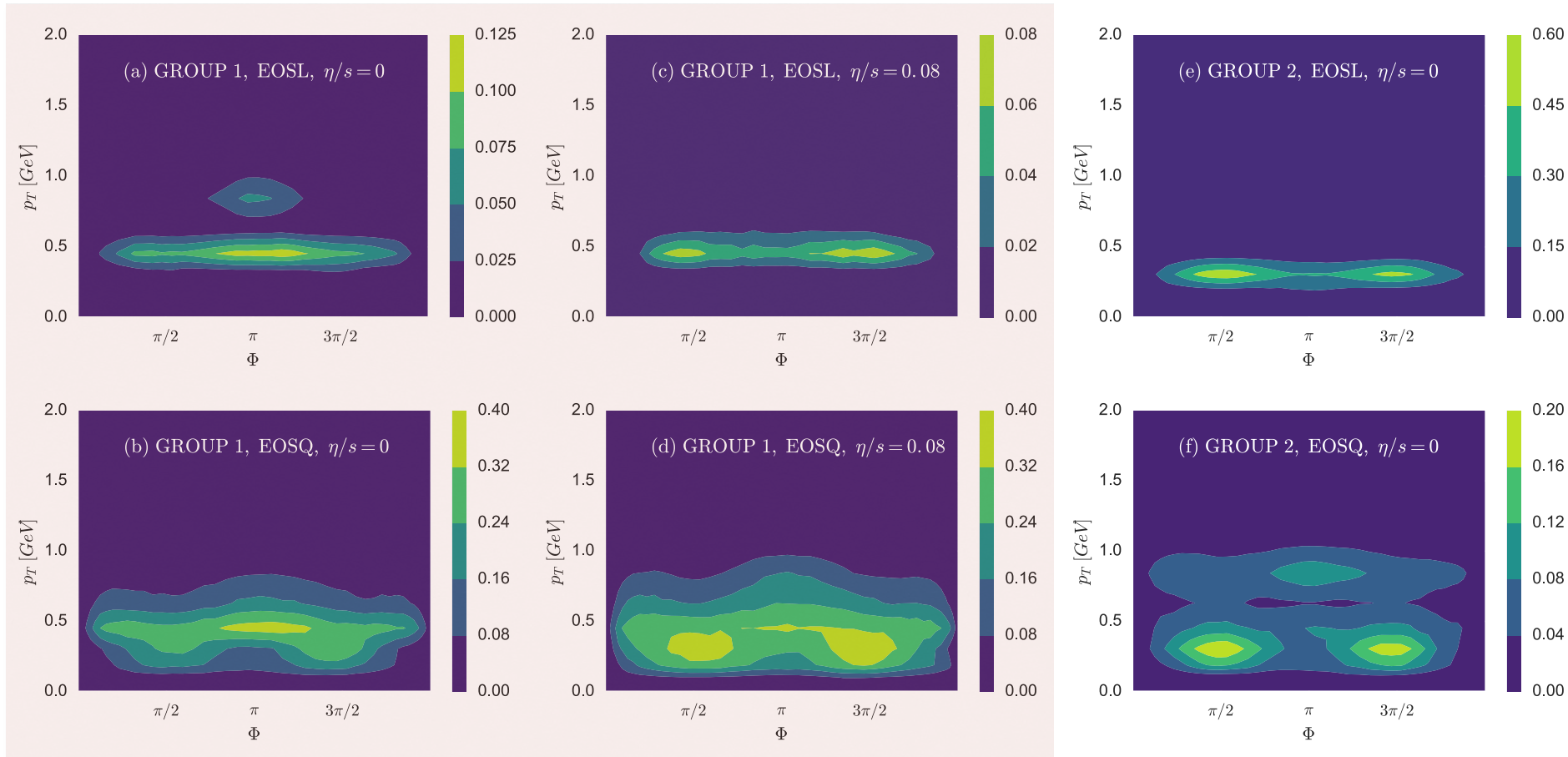
Key idea for deep learning study

Supervised learning using deep neural network with big amount of labeled training data (spectra, model parameter) from event-by-event relativistic hydrodynamics.

Key: diverge dataset to avoid over fitting



Interpretation: the attention of the network

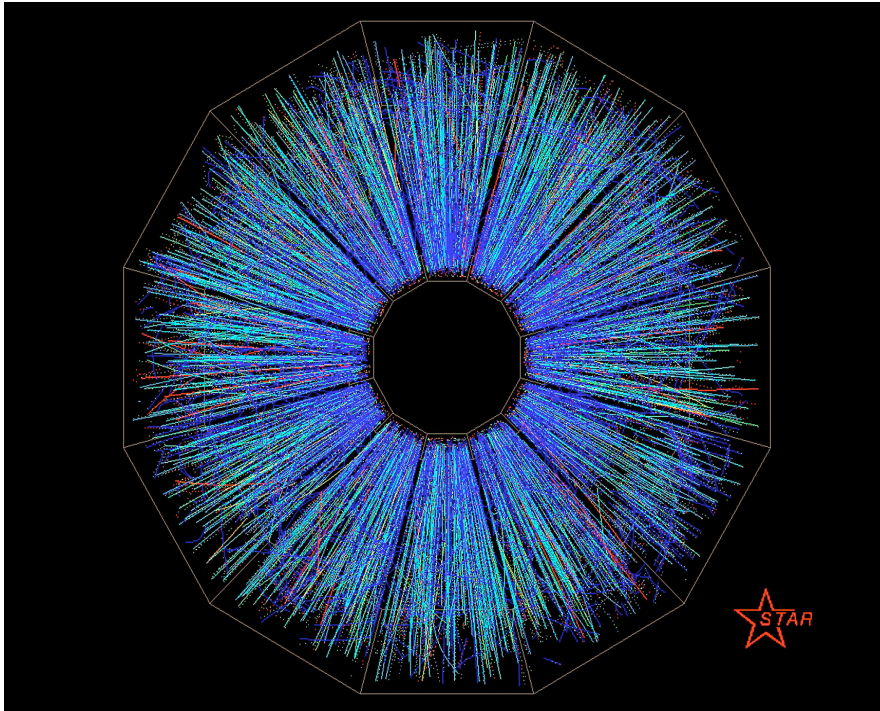


GROUP 1

GROUP 2

- Importance regions are different for different testing datasets
- η/s introduces a small difference

Better network architecture for particles



- Images: histograms
 - (px, py) or (pt, phi)
 - (px, py, pz)
 - (pt, phi, eta)
- Point cloud: particle list

E	Px	Py	Pz	pid
6.84	1.07	4.5	6.83	211
68.92	0.75	0.64	68.91	2212
40.4	0.06	0.54	40	321
...				

Deep CNN :

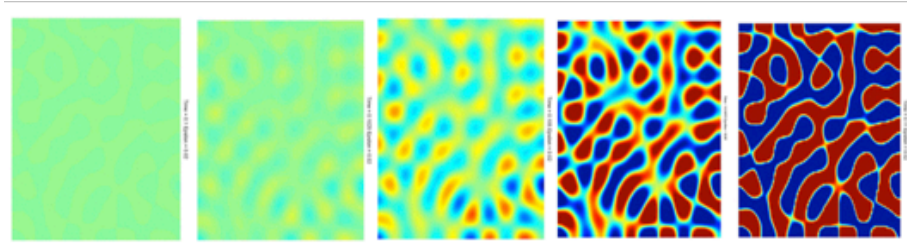
$$\eta/s = f(\rho(Y, p_T, \phi))$$

Point Cloud Network :

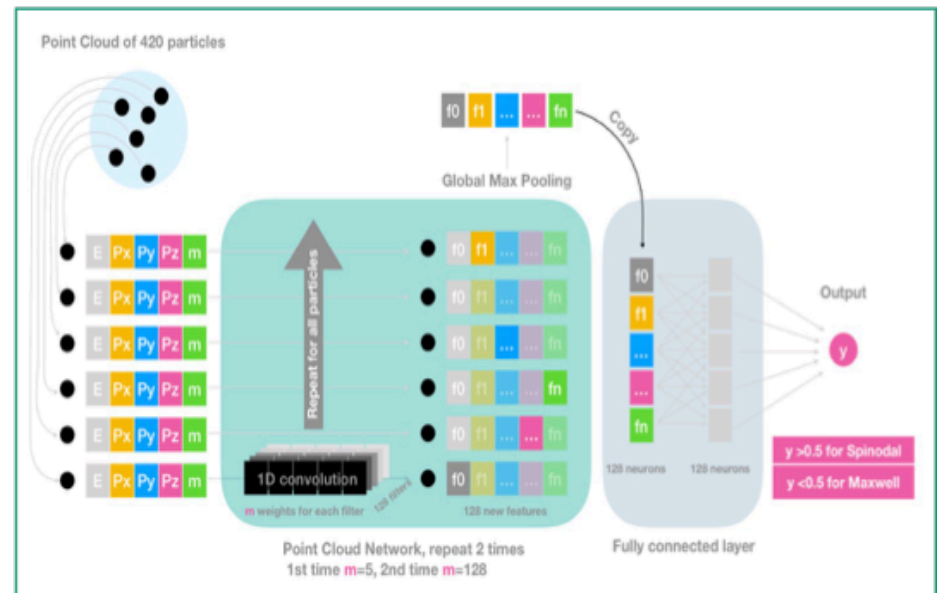
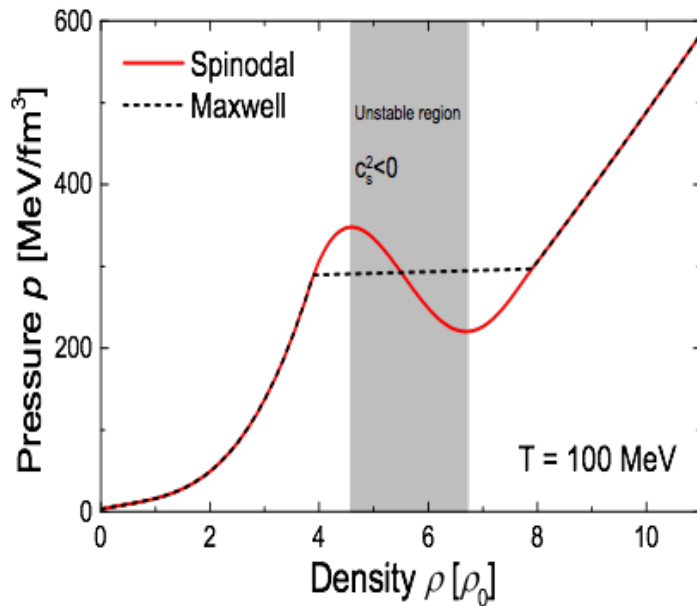
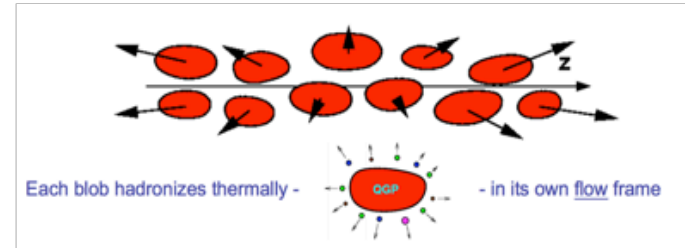
$$\eta/s = f(\{p_i^\mu, \text{pid}_i\})$$

Point cloud network for EoS classification

wiki

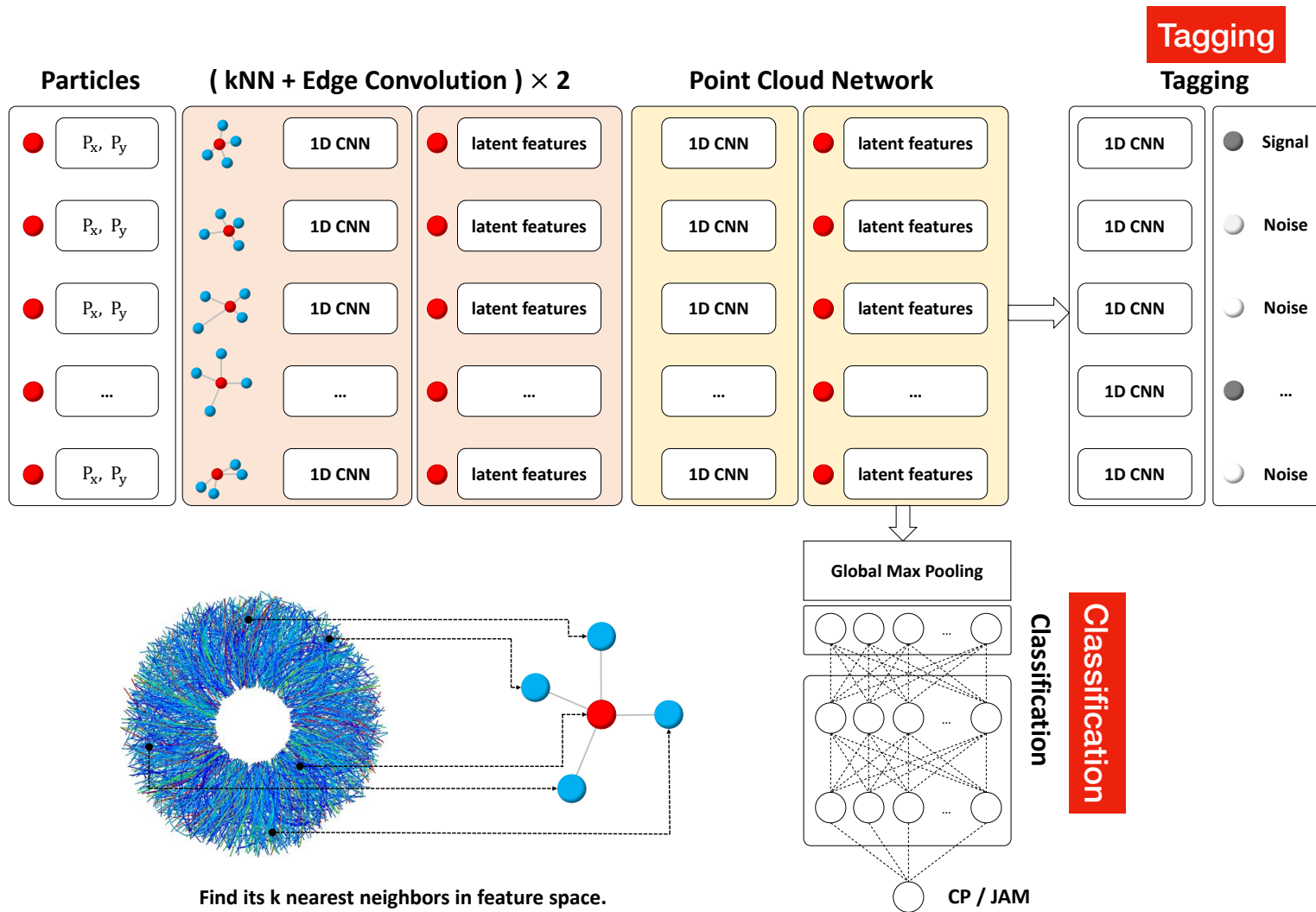


J. Randrup, INT 2009



J. Steinheimer, L.G. Pang, K. Zhou, V. Koch, H. Stoecker, J. Randrup, 2019, JHEP

Dynamical Edge convolution: more local correlation



Yi-Ge Huang, L.G. Pang, X.F. Luo, X.N. Wang, arxiv:2107.11828

The energy loss of energetic partons

$$k_a \cdot \partial f_a = \sum_{bcd} \prod_{i=b,c,d} \int \frac{d^3 k_i}{2\omega_i (2\pi)^3} (f_c f_d - f_a f_b) \\ \times |\mathcal{M}_{ab \rightarrow cd}|^2 \frac{\gamma_b}{2} (2\pi)^4 \delta^4(k_a + k_b - k_c - k_d), \quad (1)$$

$$\begin{array}{c} ij \rightarrow kl \\ \hline gg \rightarrow gg \end{array} \quad \begin{array}{c} |M|_{ij \rightarrow kl}^2 \\ \hline \frac{9}{2} g_s^4 \left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2} \right) \end{array} \quad (A-1)$$

$$gg \rightarrow q\bar{q} \quad \frac{3}{8} g_s^4 \left(\frac{4}{9} \frac{t^2+u^2}{tu} - \frac{t^2+u^2}{s^2} \right) \quad (A-2)$$

$$\begin{array}{c} gq \rightarrow gq \\ g\bar{q} \rightarrow g\bar{q} \end{array} \quad g_s^4 \left(\frac{s^2+u^2}{t^2} - \frac{4}{9} \frac{s^2+u^2}{su} \right) \quad (A-3)$$

$$\begin{array}{c} q_i q_j \rightarrow q_i q_j \\ q_i \bar{q}_j \rightarrow q_i \bar{q}_j \\ \bar{q}_i q_j \rightarrow \bar{q}_i q_j \\ \bar{q}_i \bar{q}_j \rightarrow \bar{q}_i \bar{q}_j \end{array} \quad \frac{4}{9} g_s^4 \frac{s^2+u^2}{t^2}, \quad i \neq j \quad (A-4)$$

$$\begin{array}{c} q_i q_i \rightarrow q_i q_i \\ \bar{q}_i \bar{q}_i \rightarrow \bar{q}_i \bar{q}_i \end{array} \quad \frac{4}{9} g_s^4 \left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} - \frac{2}{3} \frac{s^2}{tu} \right) \quad (A-5)$$

$$q_i \bar{q}_i \rightarrow q_j \bar{q}_j \quad \frac{4}{9} g_s^4 \frac{t^2+u^2}{s^2} \quad (A-6)$$

$$q_i \bar{q}_i \rightarrow q_i \bar{q}_i \quad \frac{4}{9} g_s^4 \left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} - \frac{2}{3} \frac{u^2}{st} \right) \quad (A-7)$$

$$q\bar{q} \rightarrow gg \quad \frac{8}{3} g_s^4 \left(\frac{4}{9} \frac{t^2+u^2}{tu} - \frac{t^2+u^2}{s^2} \right) \quad (A-8)$$

- $M_{ab \rightarrow cd}$ are the LO parton scattering amplitudes

- γ_b is the color-spin degeneracy for b

- f_a, f_d are Bose-Einstein or Fermi-Dirac distribution for gluons or quarks

Linear Boltzmann Transport for Jet Propagation in the Quark-Gluon Plasma I: Elastic Processes and Medium Recoil, PRC91, 054908, Ya-Yun He, Tan Luo, Xin-Nian Wang, Yan Zhu

The JETSCAPE framework, 1903.07706

Hydro as a background

Gradient Tomography of Jet Quenching in Heavy-Ion Collisions

Yayun He,¹ Long-Gang Pang,¹ and Xin-Nian Wang^{1,2,*}

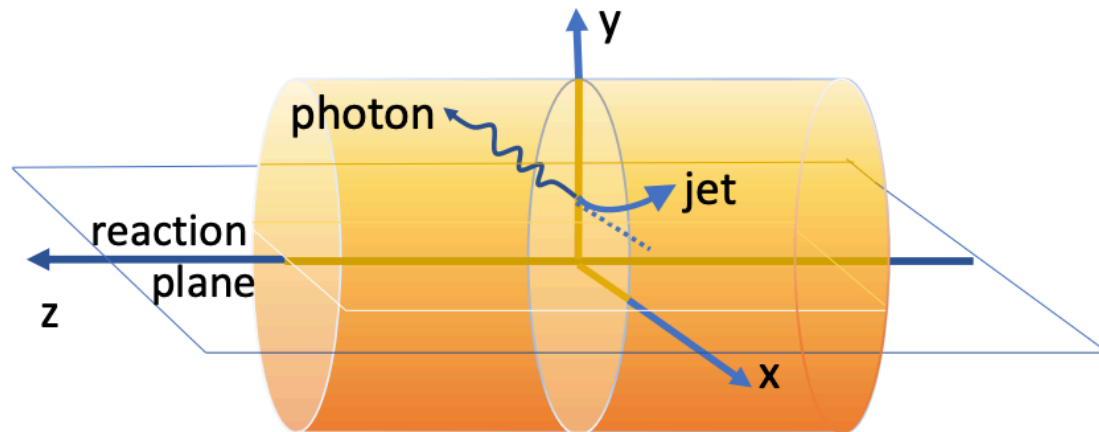
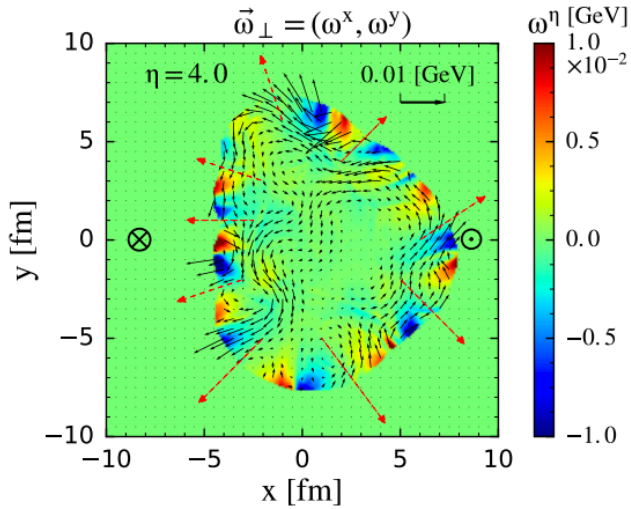


FIG. 1. (Color online) Illustration of the transverse geometry and a γ -triggered parton propagation in heavy-ion collisions.

Locating the transverse position using the gradient induced asymmetry.

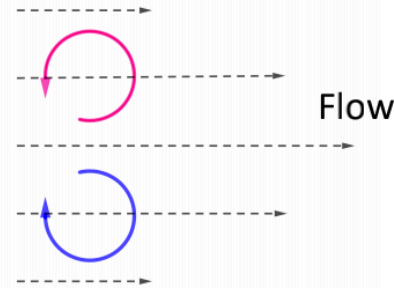
Complex local polarization in QGP



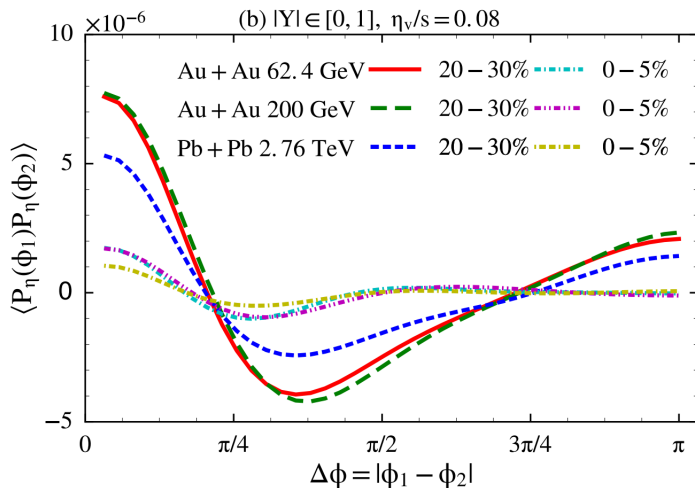
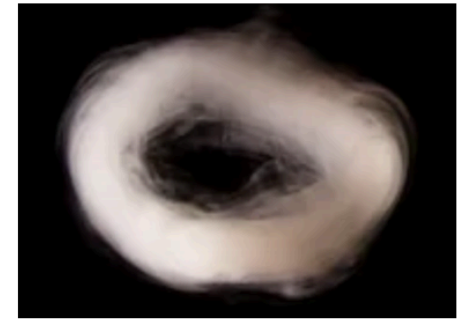
PRL. 117 (2016) no.19, 192301,
L-G Pang, H. Petersen, Q. Wang, X-N Wang

QGP fluctuation and angular momentum conserve

Vortical Pair



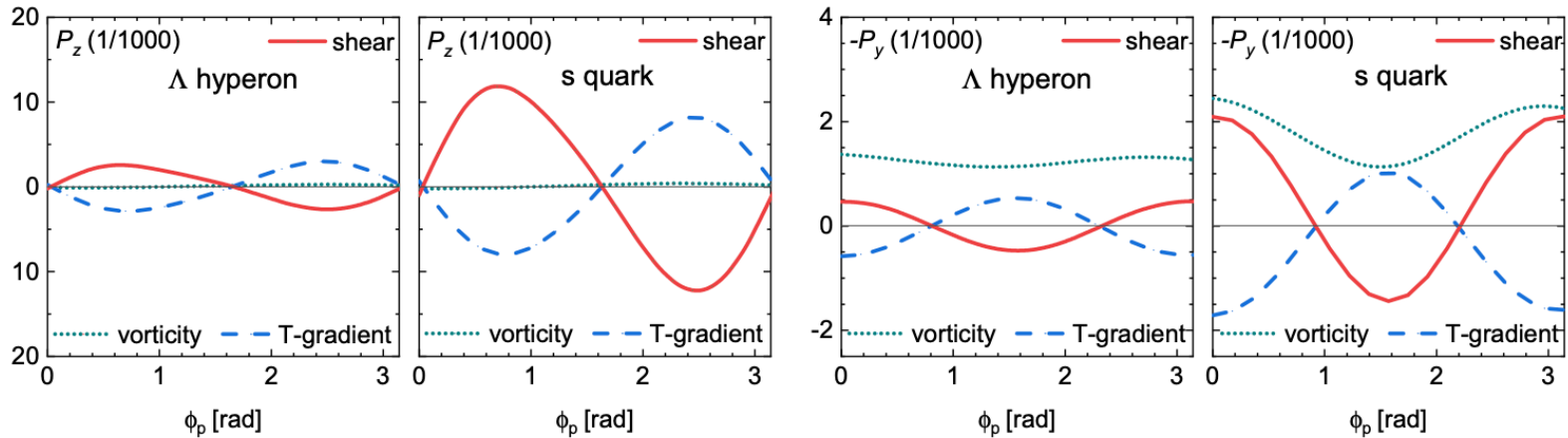
Vortical Ring



The azimuthal angle correlation of local polarization behaves like elliptic flow in semi-central and triangular flow in central collisions. Explained by the vortical pairs.

Shear induced polarization

$$\mathcal{A}_{\text{SIP}}^\mu = -\beta n_0(1 - n_0) \frac{p_\perp^2}{\varepsilon_0} \epsilon^{\mu\nu\alpha\rho} u_\nu Q_\alpha^\lambda \sigma_{\rho\lambda}$$



Local polarization has a “sign problem”, shear-induced polarization has the same sign toward solving the sign problem.

F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko, arXiv:2103.14621 [nucl-th]
 B. Fu, S. Y. F. Liu, L. Pang, H. Song and Y. Yin, Phys. Rev. Lett. 127, 142301 (2021)

Summary

- HIC is a typical inverse problem
- Relativistic hydrodynamics was used to investigate various interesting properties of QGP
- We need big data from models that are adjustable
- Bayesian analysis, Deep CNN, Point Cloud Net solves the inverse problem using different mapping functions
- Many extensions make hydro the best data machine (which I do not have time to introduce):
 - To include magnetic field
 - To include spin transport
 - To include critical point, hydrodynamics fluctuations
 - ...

Thanks