Sound velocity in neutron stars: a new quality of dense matter

Toru Kojo

(Central China Normal University)

Refs)  
- Baym-Hatsuda-TK-Powell-Song-Takatsuka (2018): review
- TK (2021): mini-review, AAPPS Bull. 31 (2021) 1, 11
- TK (2021): PRD104 (2021)7, 074005
"heavy-ion collisions" ('80s-)

"multi-messenger astronomy" (2017-)

Neutron stars ('30s-)

lattice QCD

T

~ 155 MeV

QCD

"multi-messenger astronomy" (2017-)

NS

GWs

NS

~ 155 MeV

2/15
State of matter: overview

- few meson exchange
- nucleons only

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

- Baryons overlap
- Quark Fermi sea

most difficult

(d.o.f ??)

strongly correlated

(d.o.f : quasi-particles??)

steady progress

ab-initio nuclear cal.
laboratory experiments

Hints from NS

~ 2n₀

~ 1.4 Mₒ

~ 5n₀

~ 40n₀

~ 2 Mₒ

~ 0.16 fm⁻³

[Masuda+ '12; TK+ '14]

[pQCD]

[Freedman-McLerran, Kurkela+, Fujimoto+...]

~ 1.4 Mₒ

~ 2 Mₒ

~ ~ 0.16 fm⁻³

[3/15]
EoS & Neutron Star M-R relation

Einstein eq.: \[ G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

QCD (+EW) EoS

\[ \varepsilon \sim 5n_0 \]

gravity (source: \( \varepsilon_{QCD} \))

QCD pressure

\[ \frac{M}{M_\odot} \sim 2 \]

soft-to-stiff

\[ \varepsilon(1-2n_0) \]

\[ \varepsilon(\sim 5n_0) \]

1-to-1

soft-to-stiff

soft

stiff

soft

10-14 km

R
**Observations: summary**  
(2010-)  

*M/M_☉*  

**quarks?** (~ 3-5n₀)  

1<sub>st</sub> order P.T.  

**M = 2.08 ± 0.07 M_☉**  
**R<sub>2.08</sub> = 12.35 ± 0.75 km**  

NICER obs.  
J 0740+6620  

GW170817  

**R<sub>1.40</sub> = 12.45 ± 0.65 km**  

**R<sub>2.08</sub> ~ R<sub>1.40</sub> (!)**  

*nuclear*  

(n<sub>B</sub> < 1.5-2n₀)  

**11 – 13 km**  

[Miller+ '21]  

5/15
Soft to stiff is challenging: a new quality

Sound velocity: \( c_s^2 = \frac{\partial P}{\partial \epsilon} < 1 \) (causality)

- Nuclear & quark physics constrain each other

**Diagram:**
- \( P \) vs. \( \epsilon \)
- Soft to stiff regions
- \( M > 2M_\odot \)
- \( R_{1.4} \approx 12 \text{ km} \)
- \( \epsilon(1-2n_0) \) nuclear
- \( \epsilon(\sim 5n_0) \) quarks?
- \( c_s^2 \) forbidden
- \( c_s^2 > 1 \)
- \( 1/3 \)
- \( \sim 2n_0 \)
- \( \sim 5n_0 \)

**Baseline:** Quark-hadron continuity (QHC)
Crossovers & $c_s^2 = \frac{dP}{d\epsilon}$

- "non-relativistic" resonance gas
- pion gas
- QGP
- dip
- hadrons with $m >> T$ dominate (entropic effects)
- microphysics ?
- pure hadronic
- nuclear
- peak
- quark
- $\sim 2n_0$, $\sim 5n_0$
Quark-Hadron-Continuity EoS
Direct descriptions for $2-5n_0$?

confusing point:

- Switching from baryonic to quark bases

→ a source of confusions in hybrid models
  (e.g. normalization of energy)

Strategy

Follow quark states from nuclear to quark matter

(within a single model, e.g., percolation model, Fukushima-TK-Weise ‘20)
**Quarks in a baryon**

\( N_c (=3): \text{number of colors} \)

**Probability density:**

\[
Q_{in}(p, P_B) = \mathcal{N} e^{-\frac{1}{\Lambda^2} \left( p - \frac{P_B}{N_c} \right)^2}
\]

- **Mean:**
  \[
  \langle P_B \rangle = N_c \int p Q_{in}(p, P_B)
  \]

- **Variance:**
  \[
  \left\langle \left( p - \frac{P_B}{N_c} \right)^2 \right\rangle \sim \Lambda^2 \]

- **Energetic!**

**Localized:**

\( \Lambda \sim 200 \text{ MeV} \)

**Average energy (quark):**

\[
\langle E_q(p) \rangle_{P_B} = \mathcal{N} \int_p E_q(p) e^{-\frac{1}{\Lambda^2} \left( p - \frac{P_B}{N_c} \right)^2} \approx \langle E_q(p) \rangle_{P_B=0} + \frac{1}{6} \left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i} \right\rangle_{P_B=0} \left( \frac{P_B}{N_c} \right)^2 + \cdots
\]

\[ \times N_c \]

\( \sim N_c (M_q + \Lambda) \)

- **Baryon mass**

\[ \gg \]

\( \sim P_B^2 / (N_c E_q) \)

- **Baryon kin. energy**
A new unified model for QHC

density probability of quark state with $p$

density probability of baryon state with $P_B$

$$f_q(p; n_B) = \int_{P_B} \mathcal{B}(P_B; n_B) Q_{in}(p, P_B)$$

- e.g.) in ideal baryonic matter

\[ \sim \frac{n_B}{\Lambda^3} \]

\[ \sim \Lambda \]

\[ p \]

\[ l \]

\[ P_F \]

\[ P_B \]

\[ Q_{in} \]

\[ \sim \Lambda \]

\[ p \]

\[ p_1 \]

\[ p_2 \]

\[ p_3 \]

\[ P_B \]

\[ n_B = N_f \frac{P_F^3}{3\pi^2} \]

cf) [TK '21, TK-Suenaga '21]
Evolution of occ. probabilities

\[ f_q(p; n_B) = \int_{P_B} B(P_B; n_B)Q_{in}(p, P_B) \]

- baryon bases
- quark bases
- dual

| p | \( n_B < \sim n_0 \) | \( \Lambda \sim 200 \text{ MeV} \) | \( \sim \Lambda \) | \( \sim 1-3n_0 \) | \( \sim 5n_0 \) |

“quark saturation” constraint

→ relativistic baryons at low density, \( n_B \sim 1-3n_0 \)!

Peak in sound velocity

Ideal gas: patch work

\[ c_s^2 = \frac{dP}{d\varepsilon} \]

Quark forbidden by saturation

Baryon forbidden by confinement

"inevitable" stiffening

\[ f_q \]

\[ n_B/n_0 = 0.43, 0.64, 0.91, 1.25, 2.78, 5.39 \]

\[ \Lambda = 0.25 \text{ GeV} \]
Peak in sound velocity

more realistic picture

confinement

quark

baryon

nuclear int.

\[ c_s^2 = \frac{dP}{d\varepsilon} \]

(regulated by int.)

\[ n_B \]

\[ f_q \]

\[ n_B/n_0 = 0.43 \]

\[ 0.64 \]

\[ 0.91 \]

\[ 1.25 \]

\[ 2.78 \]

\[ 5.39 \]

\[ \Lambda = 0.25 \text{ GeV} \]
Summary

\[ R_{2.08} \sim R_{1.40} \ (!!) \rightarrow \text{strong 1}^{\text{st}} \text{ order P.T. unlikely for } n_B \sim 2-5n_0 \]

Quark-Hadron-Continuity: a good baseline

Peak in sound velocity \[ \rightarrow \] signature of quark matter formation (quark saturation effects)

Stiff quark matter EoS \[ \rightarrow \] bulk repulsion & Fermi surface attraction (disparity \[ \leftrightarrow \] channel dep. of gluon exchanges)

QCD knows the answers to key questions in the NS community
Back up
Quantum numbers?

quark quantum numbers; \( N_c, N_f, 2\)-spins (for a given spatial w.f.)

how many baryon species are needed to saturate quark states?

\[ \rightarrow \text{we need only } 2N_f = 6 \text{ species for } N_f = 3 \]

(full members of singlet, octet, decuplet are NOT necessary)

convenient *color-flavor-spin* bases

\[ \Delta_{s_z=\pm 3/2}^{++} = [u_R \uparrow u_G \uparrow u_B \uparrow], \quad [u_R \downarrow u_G \downarrow u_B \downarrow], \]

\[ \Delta_{s_z=\pm 3/2}^{-} = [d_R \uparrow d_G \uparrow d_B \uparrow], \quad [d_R \downarrow d_G \downarrow d_B \downarrow], \]

\[ \Omega_{s_z=\pm 3/2}^{-} = [s_R \uparrow s_G \uparrow s_B \uparrow], \quad [s_R \downarrow s_G \downarrow s_B \downarrow], \]
Stiff quark EoS? : a guide

\[ \varepsilon(n) = an^{4/3} + bn^\alpha \]

(\(n\): quark density)

\[ P = \frac{\varepsilon}{3} + b \left( \alpha - \frac{4}{3} \right) n^\alpha \]

For stiff EoS:

- for \(\alpha > 4/3\): \(b > 0\)
  (e.g. bulk repulsion, \(\sim + n_B^2/\Lambda^2\))
- for \(\alpha < 4/3\): \(b < 0\)
  (e.g. surface pairings, \(\sim - \Lambda^2 n_B^{2/3}\))

"Exotic" Fermi surface stiffens EoS!

Reminder: QCD int. are very channel dependent!
Quarks in ideal baryon gas: $f_q(p; n_B) = \frac{n_B}{n_B^c} e^{-p^2/\Lambda^2} + O(1/N_c^2)$

**LO:**
- the shape does not change
- the height grows linearly in $n_B$

**Energy density**
$$\varepsilon(n_B) = N_c \int_p E_q(p) \left( f_q^{LO}(p; n_B) + O(1/N_c^2) \right) = n_B M_B + O(1/N_c)$$

**Pressure**
$$\mathcal{P} = n_B^2 \frac{\partial (\varepsilon / n_B)}{\partial n_B} \sim 0 + O(1/N_c)$$

~ const. very soft
Quark matter formation: $f_q(p; n_B)$

A model after the saturation:

$$f_{q \text{ after}} = \theta(p_{\text{sat}} - p) + \theta(p - p_{\text{sat}}) f_q(p - p_{\text{sat}}; n_B^c)$$

$\varepsilon, n_B$ are continuous before and after the saturation.

any nontrivial consequences?
Quark matter formation: EoS

\[ P = n_B^2 \frac{\partial}{\partial n_B} \left( \frac{\varepsilon}{n_B} \right) \]

energy per particle

[cf] McLerran-Reddy (MR), PRL '19,...

\[ \varepsilon \sim N_c \Lambda_{QCD}^4 \]

\[ P \sim \Lambda_{QCD}^4/N_c \]

(!)
- location primarily determined by $\Lambda$ (or baryon size)
- width primarily determined by $p_w$ (or interactions)
- interactions: NOT driving forces, just temper the peak
quark energy; \hspace{1cm} \text{parameterization of MF}

\[ V_{CE}[f_q] = -C_E^A \times (1 - f_q^\beta) + C_E^S f_q^\beta \]

\begin{align*}
\text{for } f_q(p) &<< 1 \\
V_{CE}[f_q] &\simeq -C_E^A \\
\text{dilute in momentum space}
\end{align*}

\begin{align*}
\text{for } f_q(p) &\sim 1 \\
V_{CE}[f_q] &\simeq C_E^S \\
\text{color-sym. channels also enter for saturated levels}
\end{align*}

\[ \text{color-antisym. channels dominate} \]
\[ \rightarrow \text{the quark feels attractive correlations} \]
EoS with interactions

adjust $C_E^A$ (fit $M_B = 939$ MeV)

high density stiffening

peak in $c_s$
Step 2) A direct sketch for $\sim 2-5n_0$

A model of quark hopping

3D cubic lattice

how to describe explicitly?

on-site energy

hopping

for randomly generated baryon configurations

1. compute quark states for a given baryon configuration

2. take the ensemble average of the configurations

"quark-hadron continuity"
Comparisons with other scenarios

with 1\textsuperscript{st} order

Annala+ ('20)

Ours; Masuda+ ('12), McLerran+'19),...
Inversion problem: from $f_q$ to $B$

$$f_q(p; n_B) = \int_{P_B} B(P_B; n_B) Q_{in}(p, P_B)$$

How does *baryon occ. probability* look *after* the saturation?
**Inversion problem:** motivations to study $B$

- perhaps convenient to *use the baryonic bases* for low $E$ physics

\[ P(\mu_B) \big|_{\beta-eq} \rightarrow P(\mu_B, \mu_Q, T, \ldots) \]

- relations to the *McLerran-Reddy* (MR) model

*important* parameter

\[ \Delta = \frac{\Lambda^3}{k_{FB}^2} + \kappa \frac{\Lambda}{N_c^2} \]

*why this form?*

- phenomenological
  [McLerran-Reddy, PRL '19]
- derivation in excluded vol. model
  [Jeong-McLerran-Sen, '19]
A trial: *shell form*

\[ B^{sh}(P_B; P_{sh}) = h\theta(P_{sh} - P_B)\theta(P_B - P_{sh} - \Delta) \]

\[ f^s_{q}(p) \simeq h\Delta \frac{N_c^3}{\sqrt{\pi}} \frac{\tilde{P}_{sh}}{\tilde{p}} e^{-\tilde{p}^2 - \tilde{P}_{sh}^2} \left( e^{2\tilde{p}\tilde{P}_{sh}} - e^{-2\tilde{p}\tilde{P}_{sh}} \right) \]

\[ f^s_{q}(p) \sim h\Delta N_c^2 e^{-(\tilde{p} - \tilde{P}_{sh})^2} \]

\[ P_{sh} \sim N_c \Lambda \]

\[ P_{sh} \sim N_c p_{sat} \]

Matching of tails \( \sim h \Delta N_c^2 \)

Sat. model
Constraints from $f_q$ (for $P_{sh} \sim N_c \Lambda$)

$$f_q^{sh}(p) \sim h \Delta N_c^2 e^{-(\bar{p} - \bar{P}_{sh})^2}$$

constraint:  

$$f_q^{sh} < 1 \quad \Rightarrow \quad h \Delta < \Lambda / N_c^2$$

a possible scaling form: 

$$[h \Delta](P_{sh}) \sim c_0 \Lambda \left( \frac{\Lambda^2}{P_{sh}^2} + \frac{c_1}{N_c} \frac{\Lambda}{P_{sh}} + \frac{c_2}{N_c^2} \right)$$

MR-model (thin shell model)

$$h = 1 \quad \& \quad \Delta = \frac{\Lambda^3}{k_{FB}^2} + \kappa \frac{\Lambda}{N_c^2} \quad (c_1 = 0)$$
MR-model: \( EoS \)

\[
P_{sh} \sim N_c \Lambda \quad \text{baryon relativistic}
\]

but

\[
\begin{align*}
(n_{B}) \; \text{(shell)} & \; \approx \; \frac{\hbar}{\pi^2} \left( P_{sh}^3 - (P_{sh} - \Delta)^3 \right) \sim \hbar \Delta P_{sh}^2 \\
& \approx c_0 \Lambda^3 + c_1 \Lambda^2 \frac{P_{sh}}{N_c} + c_2 \Lambda \left( \frac{P_{sh}}{N_c} \right)^2
\end{align*}
\]

\[
(n_{B})^{(bulk)} \sim \Lambda^3
\]

\[
n_B \sim \Lambda^3 \; (!) \ll (N_c \Lambda)^3
\]

(kin.) energy density:

\[
\epsilon - m_B n_B \sim h \Delta \times \left[ E(P_{sh}) - m_B \right] \times 4\pi P_{sh}^2
\]

\[
\approx \Lambda / N_c^2 \times (N_c \Lambda)^2 / m_B \times (N_c \Lambda)^2 \sim N_c \Lambda^4
\]

consistent with quark's

relativistic pressure \( \sim N_c \Lambda^4 \) within \( n_B \sim \Lambda^3 \rightarrow \text{stiff} \; EoS \)
More on "$h \Delta$": another look

purely quark bases

$\delta n_B$  

$\delta p$  

$x N_c$

$\delta n_B \ (= \delta n_R = \delta n_G = \delta n_B)$

(need for consistency)

$p^2 \delta p$  

$(N_c p)^2 \times N_c \delta p$

naive

$x \ 1/ N_c^3$

occ. probability

phase space density is more dilute for composite particles
NS: brief history

1934) Zwicky & Baade (prediction) [1931: neutron discovered (Chadwick)]
   "supernova: ordinary star $\rightarrow$ NS"

1967) Bell & Hewish: discovery of a pulsar (NS) "existence of NS"

1974) Taylor & Hulse: a binary pulsar (double NSs)
   "indirect confirmation of gravitation waves (GWs)"
   "dawn of multi-messenger (GW, EM, neutrino) astronomy"

2010) Demorest+: discovery of 2M$_{-}$NS
   "NEW guide on high density equations of state"

2017) LIGO-Virgo: GWs from NS mergers (+ EM counterparts)
   "dawn of multi-messenger (GW, EM, neutrino) astronomy"