Phase Space Density Analysis
Transfer Matrices
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Introduction

- Transmission losses leads to a lower number of particles further downstream
- These may bias the downstream measurements when comparing them to the upstream measurements
- Going to try to determine the reason for the missingness
- Will try to find transfer matrix that describes the transport

Potential next step

- The missing data downstream is inaccessible, however the upstream sample which makes it downstream can be compared to the downstream sample
- The transport, M, of a covariance matrix from upstream to downstream can be given by:

$$\Sigma_{down} = \langle X_{down}\tilde{X}_{down}\rangle = \langle MX_{up}\tilde{M}\tilde{X}_{up}\rangle = M\langle X_{up}\tilde{X}_{up}\rangle\tilde{M} = M\Sigma_{up}\tilde{M}$$

- The determinant is given by:

$$|\Sigma_{down}| = \left|M\Sigma_{up}\tilde{M}\right| = |M|^2\left|\Sigma_{up}\right| = \left|\Sigma_{up}\right|$$

- The transfer matrix M has been previously investigated by Sophie Middleton and Chris Rogers
- A potential investigation would be to investigate the change in R for different fraction sizes of the beam. If stable it could be used to investigate the missing data downstream to see if it is due to scraping and magnet misalignment affects and nothing else

M Matrix

- The transfer Matrix components are then used to get the downstream distribution through:

$$
\begin{aligned}
x_d &= M_{00} + M_{01}x_u + M_{02}x'_u + M_{03}y_u + M_{04}y'_u \\
x'_d &= M_{10} + M_{11}x_u + M_{12}x'_u + M_{13}y_u + M_{14}y'_u \\
y_d &= M_{20} + M_{21}x_u + M_{22}x'_u + M_{23}y_u + M_{24}y'_u \\
y'_d &= M_{30} + M_{31}x_u + M_{32}x'_u + M_{33}y_u + M_{34}y'_u
\end{aligned}
$$

- Note this assumes the tracker stations are parallel with no rotations. In this case many of the Matrix components are simply zero. Corrections can be made for these if that is not the case.
- Corrections may also need to be added for any deviations in the magnetic field in x and y. Further corrections could also be added to include higher order effects. This also assumes the transverse only description is valid, but it may need to be extended to a full 6D description. The main problem may then be large errors, where it may become difficult to discern anything.
- If the error for the transfer of individual particles proves too large, another way too look at it would be through the transfer of moments

Higher Order

- Principle of transfer matrix can be extended to higher orders by considering squared terms, cubed terms, etc.
- Will also look at effect of including a offset term a0, i.e. the M_(i0) term
- Will also look at including pz term

First order

- Downstream x position from TKU S1 to TKD S1:
- Blue: $x_d = \boxed{a_{00}} + a_{01}x_u + a_{02}px_u + a_{03}y_u + a_{04}py_u + \boxed{a_{05}pz_u}$
- Orange: $x_d = a_{00}x_u + a_{01}px_u + a_{02}y_u + a_{03}py_u + \boxed{a_{04}pz_u}$
- Green: $x_d = a_{00}x_u + a_{01}px_u + a_{02}y_u + a_{03}py_u$
- Red: $x_d = \boxed{a_{00}} + a_{01}x_u + a_{02}px_u + a_{03}y_u + a_{04}py_u$

First Order

- Similar for downstream y, px, py and pz
- Write this for every point in sample, i.e. every downstream coordinate in terms of upstream coordinates
- Use linear least squares to solve (constants which minimize linear equations)
- Apply transfer matrix (constants) to upstream coordinates and compare to actual downstream data i.e. residuals

First Order Transfer Matrices

Blue

1.86982473e+01	7.91073650e-01	-6.70880696e-02	2.26740739e-01	1.77258700e+00	-1.44921274e-01
1.26265433e+00	-4.86970413e-02	6.20486835e-01	-7.16163409e-01	-2.31249175e-01	1.58834819e-02
-1.18198816e+00	-2.40262279e-01	-1.81033086e+00	7.81445555e-01	-1.07156657e-01	-1.47005147e-02
-8.65506330e-01	7.01521099e-01	2.53385814e-01	-6.50026350e-02	5.77103827e-01	2.56977824e-02
7.36893924e+01	1.38945059e-02	5.88241470e-03	3.05884176e-03	1.86652435e-02	4.34201028e-01

Orange

0.79067717	-0.06934735	0.2266646	1.77178437	-0.00602366
-0.04872381	0.62033427	-0.71616855	-0.23130338	0.02526295
-0.24023722	-1.81018804	0.78145037	-0.10710592	-0.02348077
0.70153945	0.25349039	-0.06499911	0.57714098	0.01926848
0.01233201	-0.00302135	0.00275876	0.01550209	0.98159356

Green

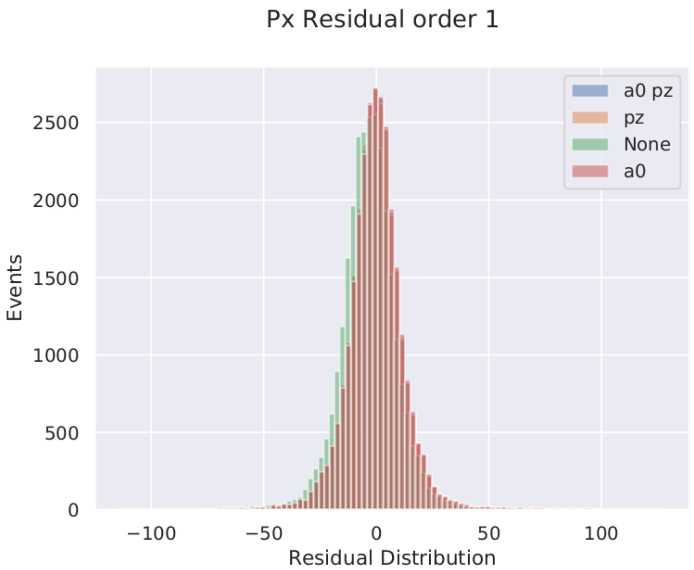
0.79378118	-0.06566514	0.22296971	1.77626513
-0.06174187	0.60489127	-0.70067237	-0.2500955
-0.22813752	-1.79583447	0.76704737	-0.08963949
0.69161036	0.24171176	-0.05317992	0.56280791

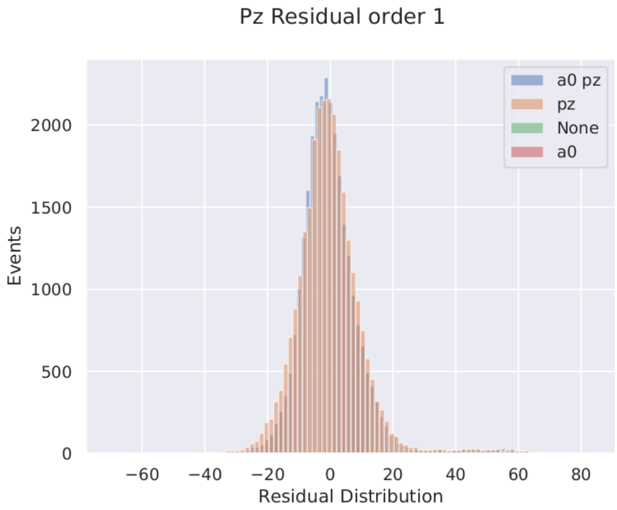
Red

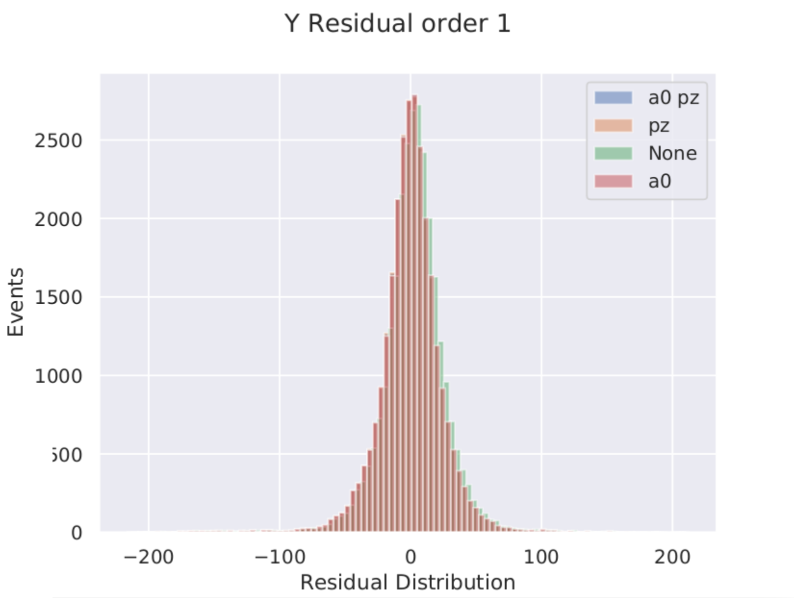
-0.76402214	0.79084041	-0.0692268	0.22644789	1.77201059
3.39573388	-0.04867148	0.62072124	-0.71613131	-0.231186
-3.15620058	-0.24028594	-1.81054781	0.78141585	-0.10721513
2.58558925	0.70156246	0.25376506	-0.06495071	0.57720604

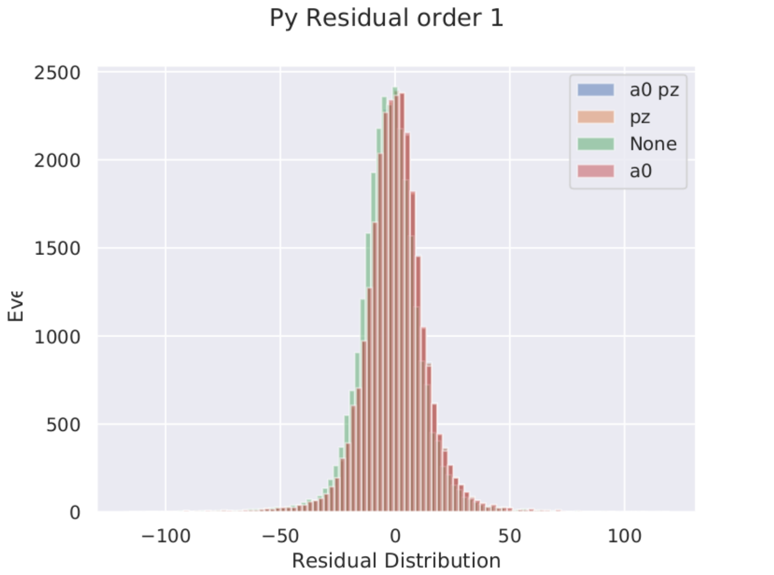
Residuals


X Residual order 1
a0 pz
pz
None
a0
Events
Residual Distribution


Px Residual order 1
a0 pz
pz
None
a0
Events
Residual Distribution

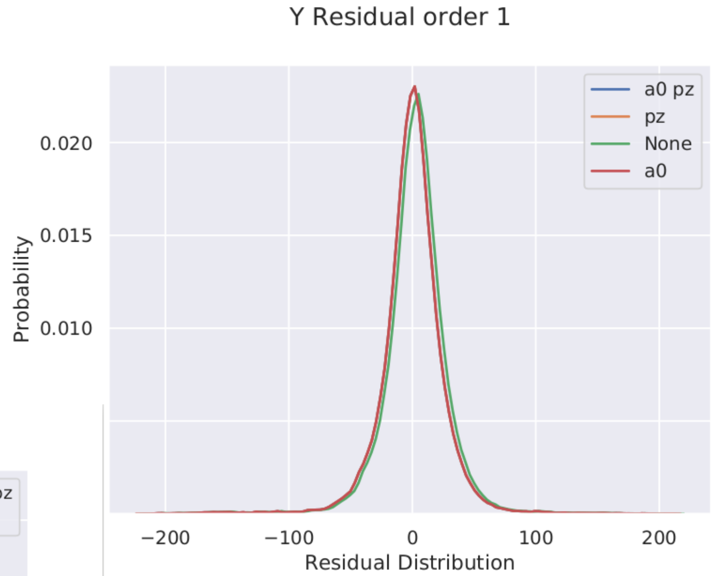

Pz Residual order 1
a0 pz
pz
None
a0
Events
Residual Distribution

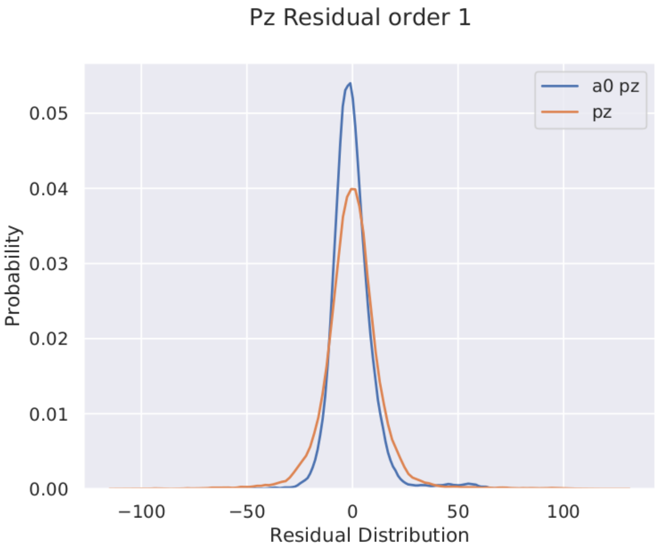

Y Residual order 1
a0 pz
pz
None
a0
Events
Residual Distribution

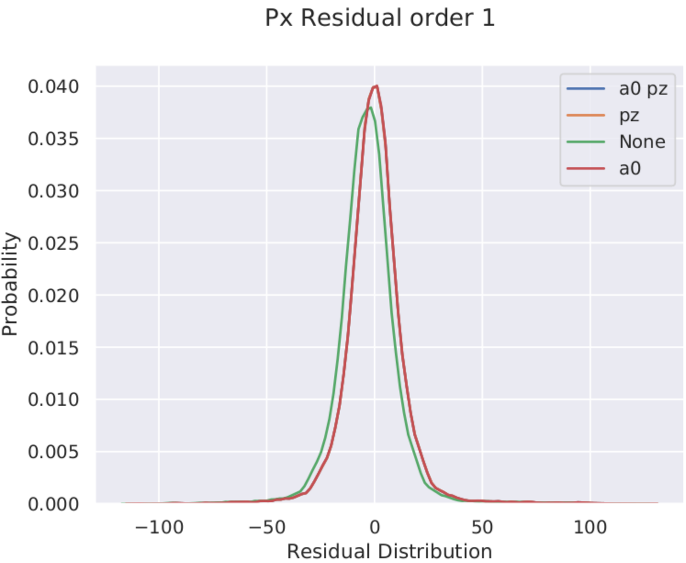

Py Residual order 1
a0 pz
pz
None
a0
Residual Distribution

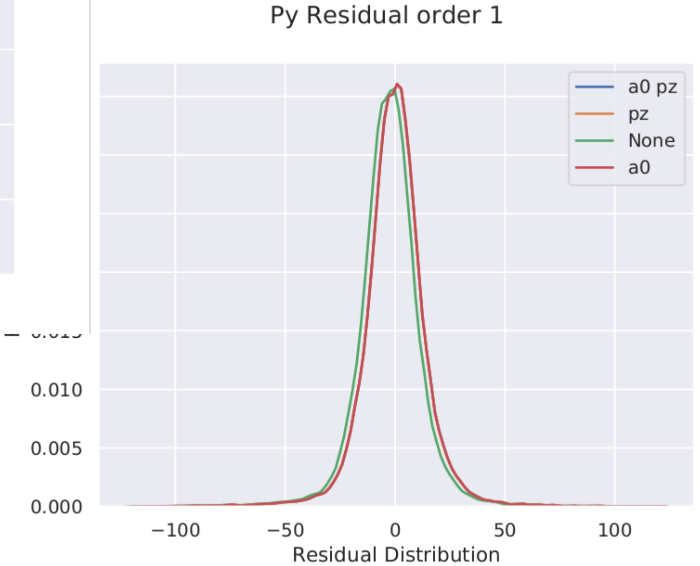
Residuals


X Residual order 1
Probability
Residual Distribution
a0 pz
pz
None
a0

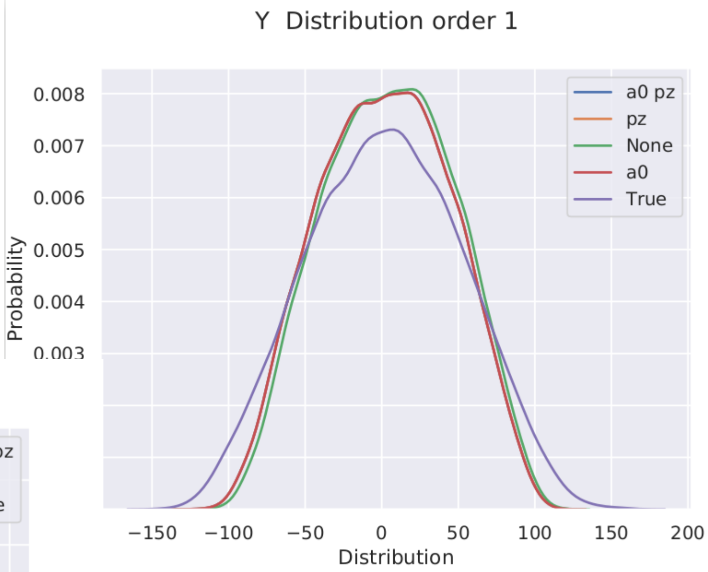

Y Residual order 1
Probability
Residual Distribution
a0 pz
pz
None
a0

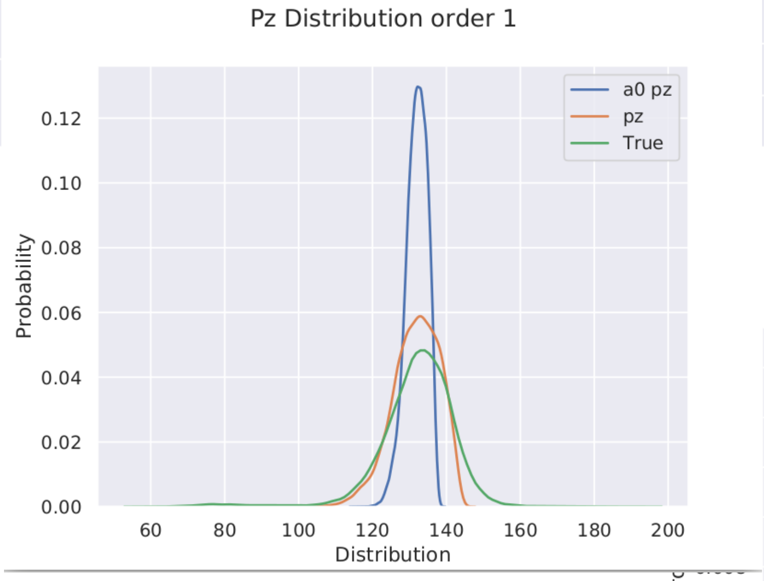

Pz Residual order 1
Probability
Residual Distribution
a0 pz
pz

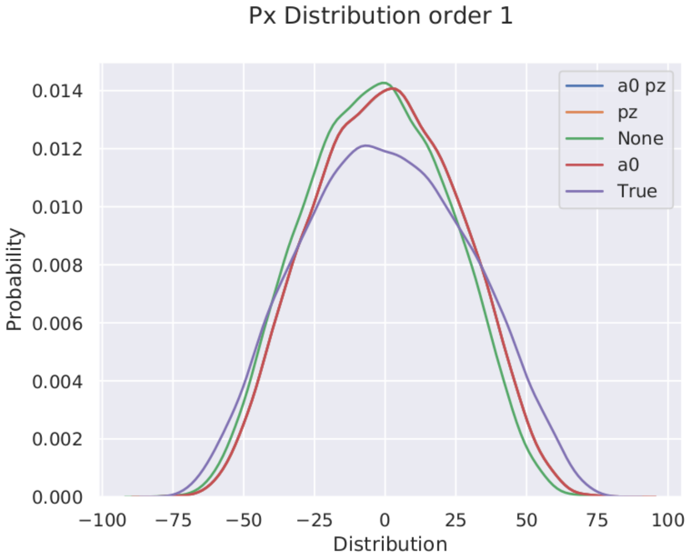

Px Residual order 1
Probability
Residual Distribution
a0 pz
pz
None
a0

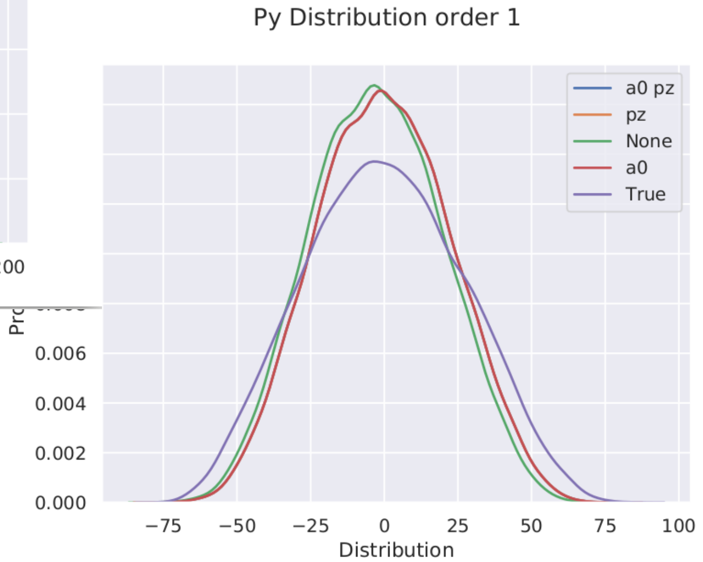

Py Residual order 1
Residual Distribution
a0 pz
pz
None
a0

Distributions





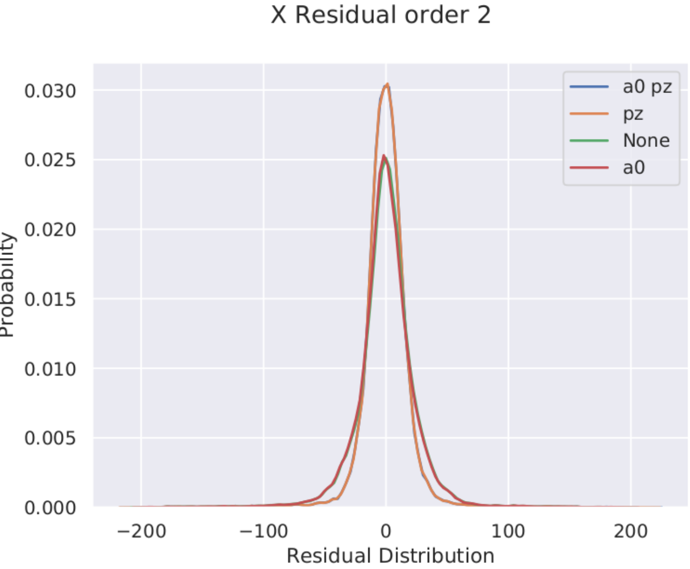


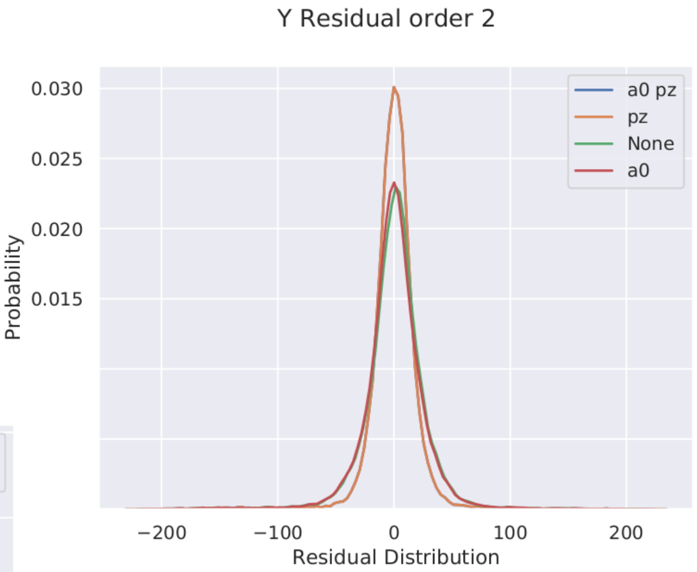


Higher Order

- Massive Residuals for 1st Order
- Always likely a higher order transfer matrix would be needed due to aberrations
- Higher order terms will be included now
- E.g. 2nd order has x^2, xpx, xy, etc.
- 3rd order has x^3, x^2px, xpxpy, etc.

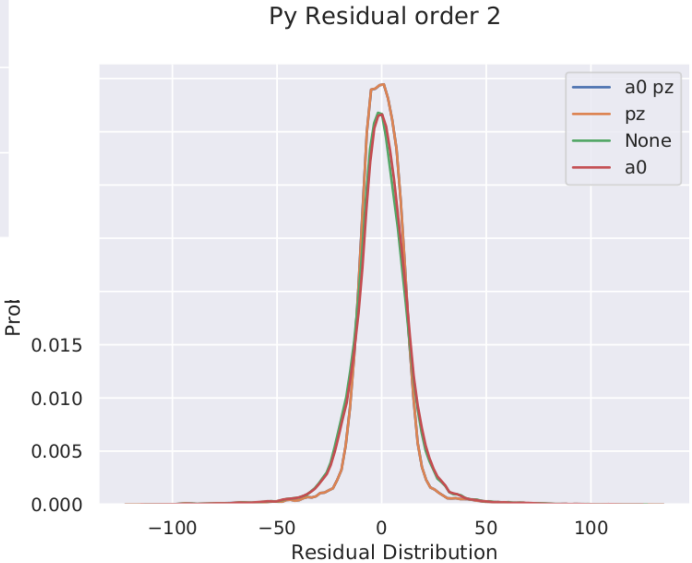
2^{nd} Order


X Residual order 2
a0 pz
pz
None
a0
Probability
Residual Distribution

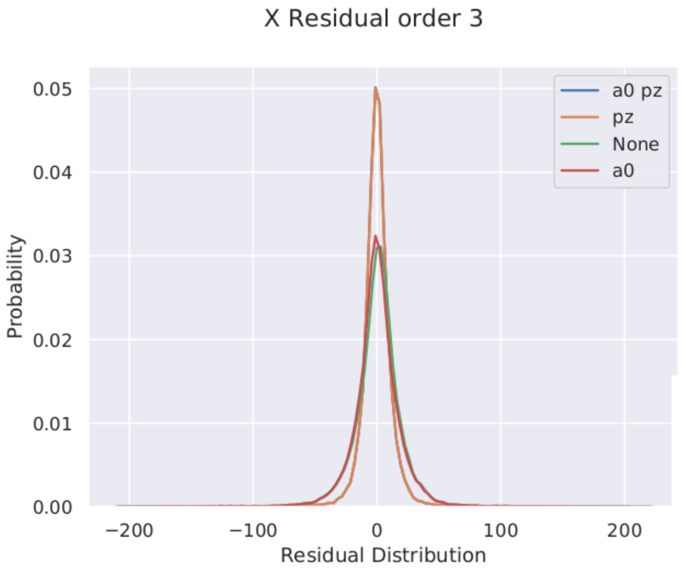

Y Residual order 2
a0 pz
pz
None
a0
Probability
Residual Distribution

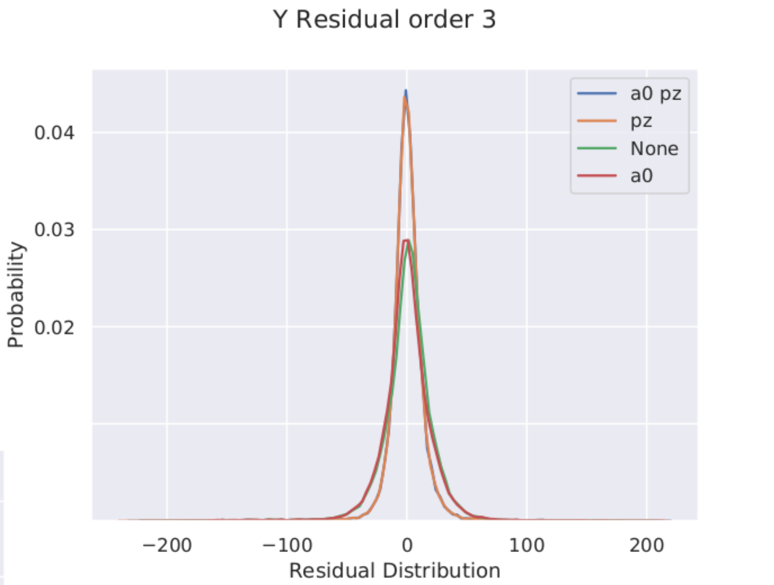

Pz Residual order 2
a0 pz
pz
Probability
Residual Distribution

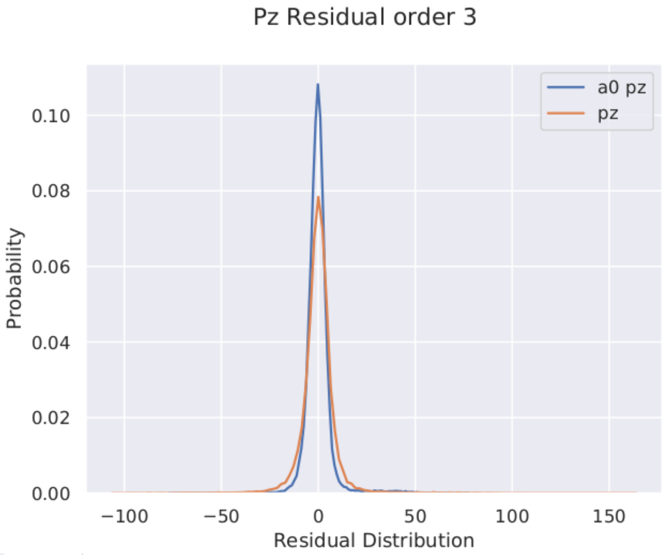

Px Residual order 2
a0 pz
pz
None
a0
Probability
Residual Distribution


Py Residual order 2
a0 pz
pz
None
a0
Residual Distribution

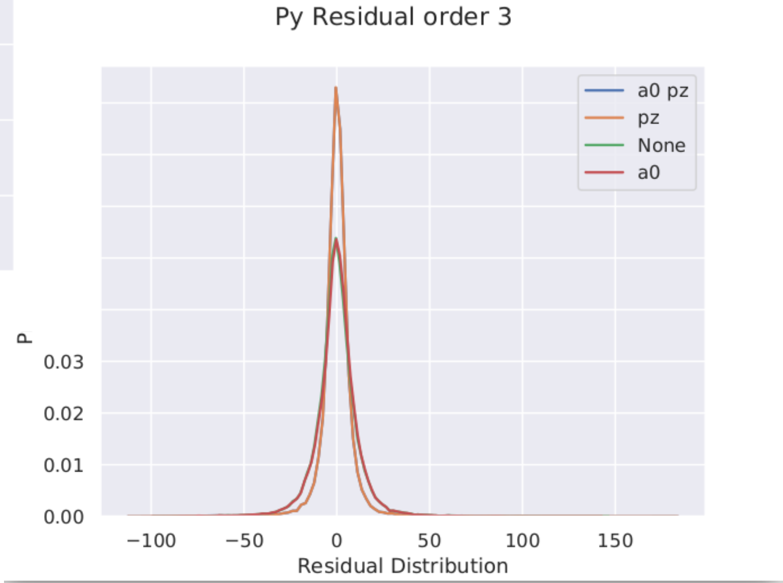
3^{rd} Order


X Residual order 3
Probability
Residual Distribution
a0 pz
pz
None
a0


Y Residual order 3
Probability
Residual Distribution
a0 pz
pz
None
a0

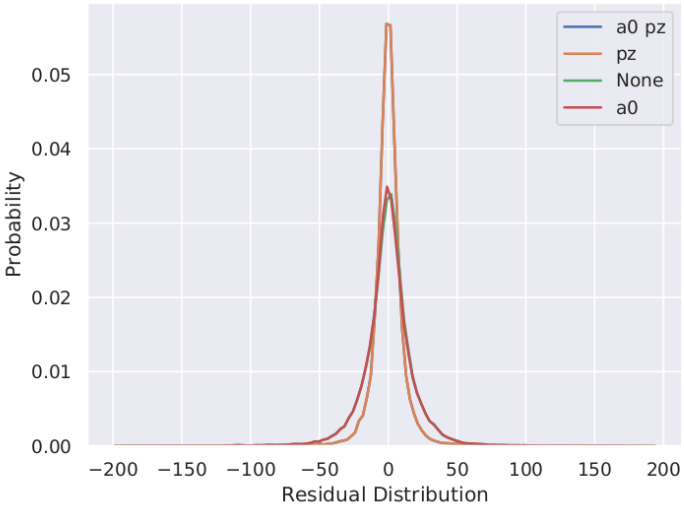

Pz Residual order 3
Probability
Residual Distribution
a0 pz
pz


Px Residual order 3
Probability
Residual Distribution
a0 pz
pz
None
a0

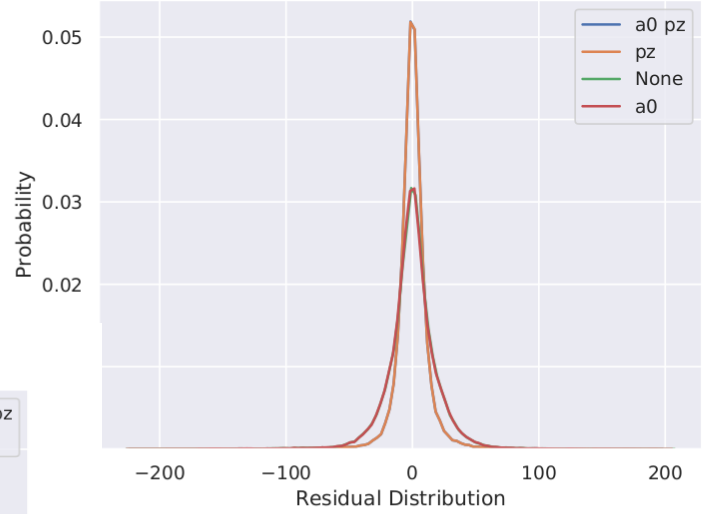

Py Residual order 3
P
Residual Distribution
a0 pz
pz
None
a0

6^{th} Order

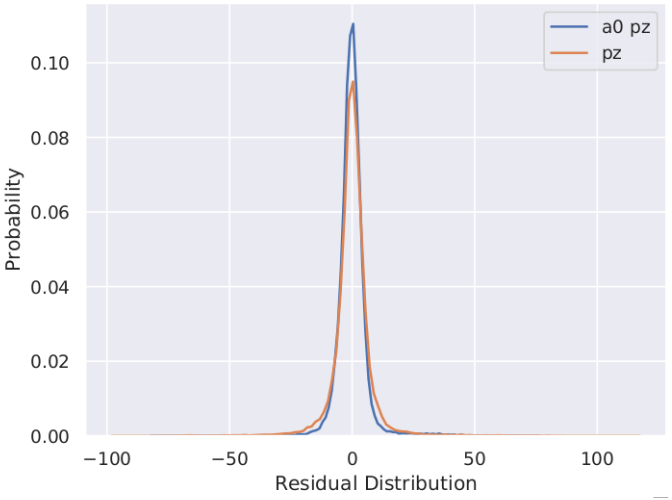
X Residual order 6



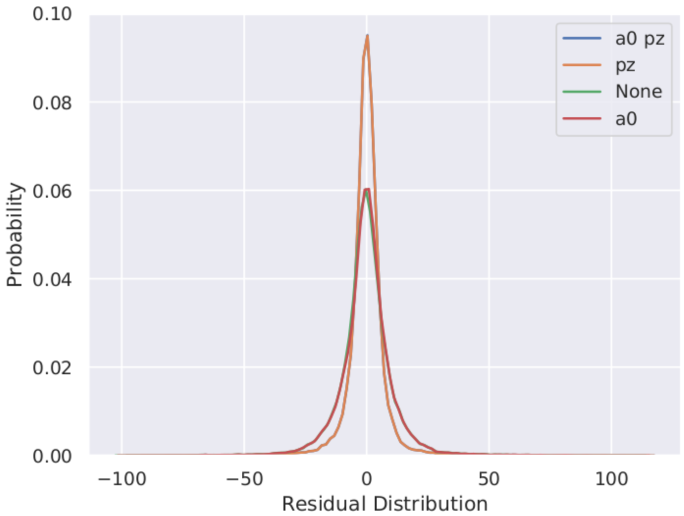
Y Residual order 6



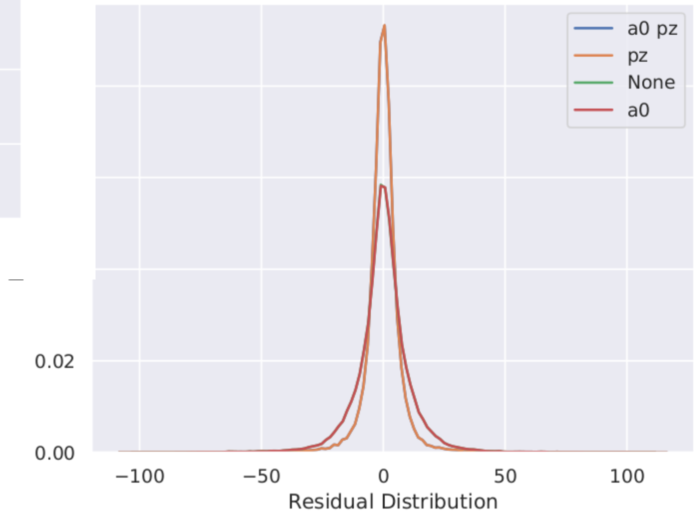
Pz Residual order 6



Px Residual order 6

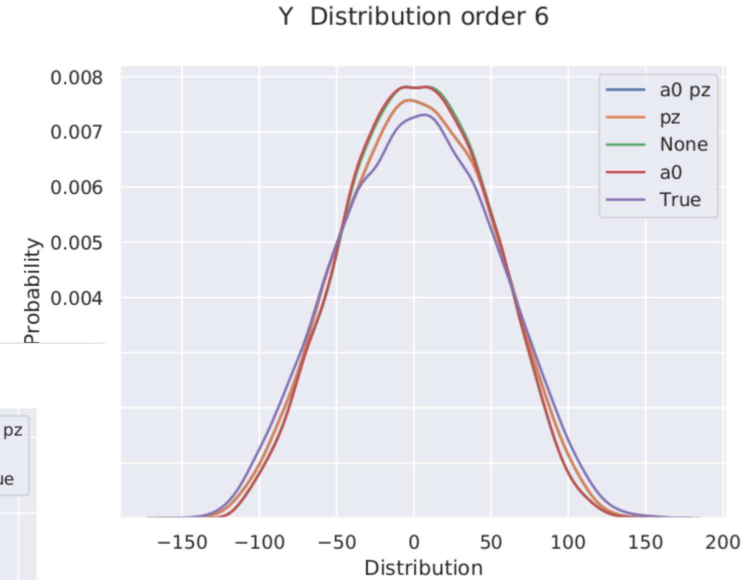


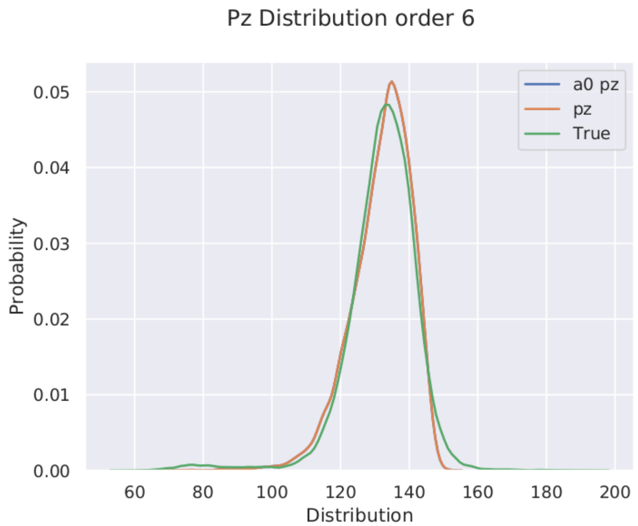
Py Residual order 6

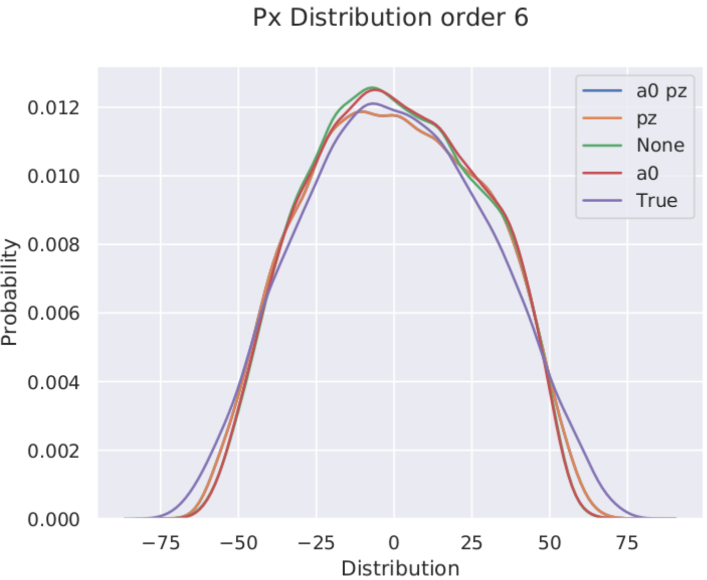


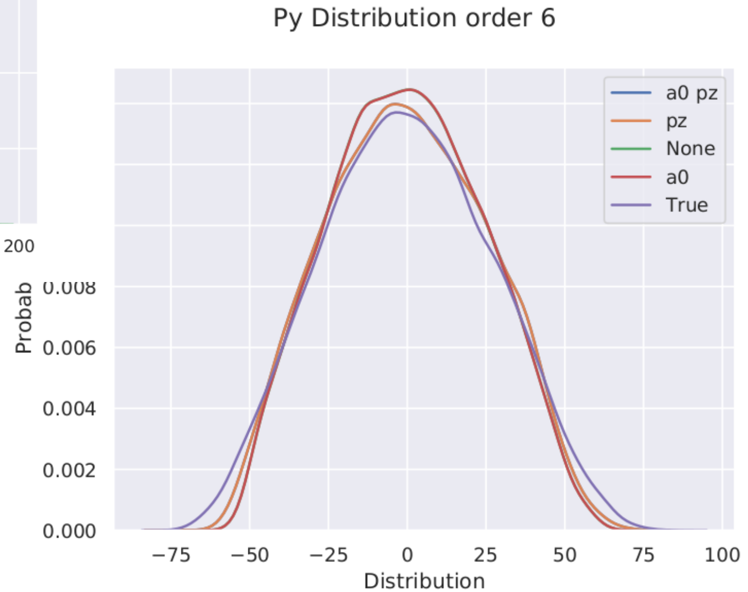
6^{th} Order


X Distribution order 6
a0 pz
pz
None
a0
True
Probability
Distribution


Y Distribution order 6
a0 pz
pz
None
a0
True
Probability
Distribution


Pz Distribution order 6
a0 pz
pz
True
Probability
Distribution


Px Distribution order 6
a0 pz
pz
None
a0
True
Probability
Distribution

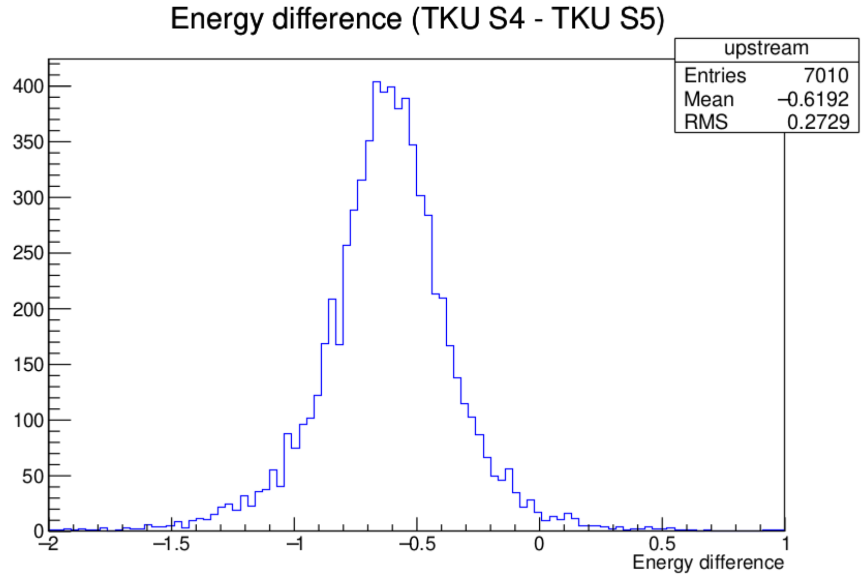

Py Distribution order 6
a0 pz
pz
None
a0
True
Distribution

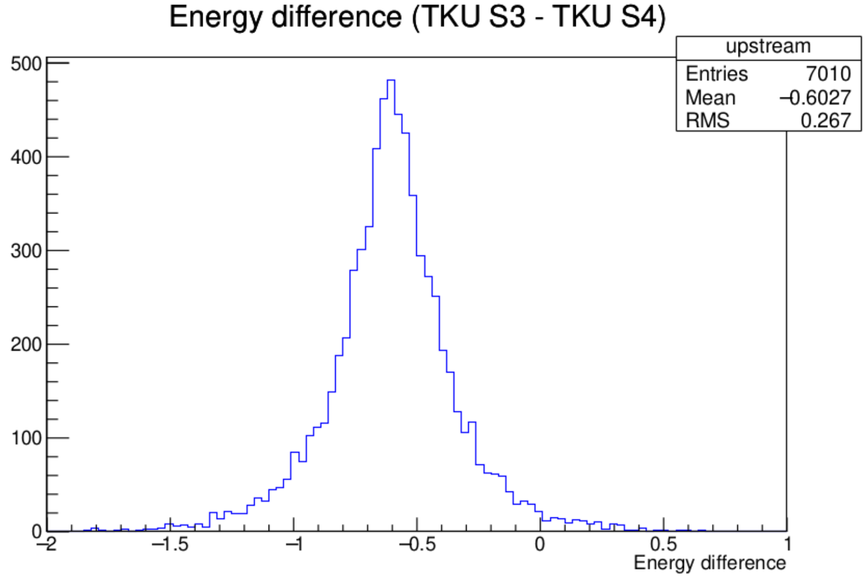
Overfitting

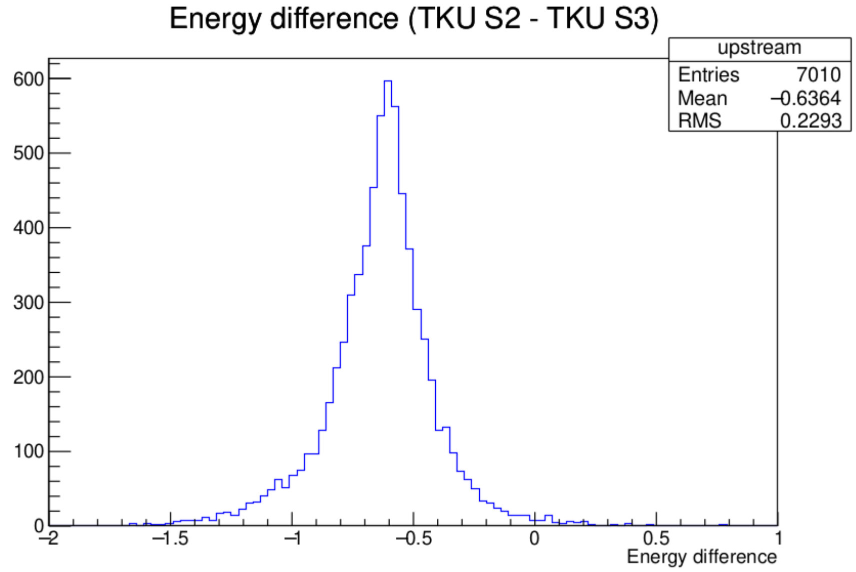
- Higher orders do improve residuals, but only minor improvements are seen after a while
- Improvements are due to free parameters to fit to, rather than actual physical meaning
- True downstream distribution is still very different than that obtained from transfer matrix approach

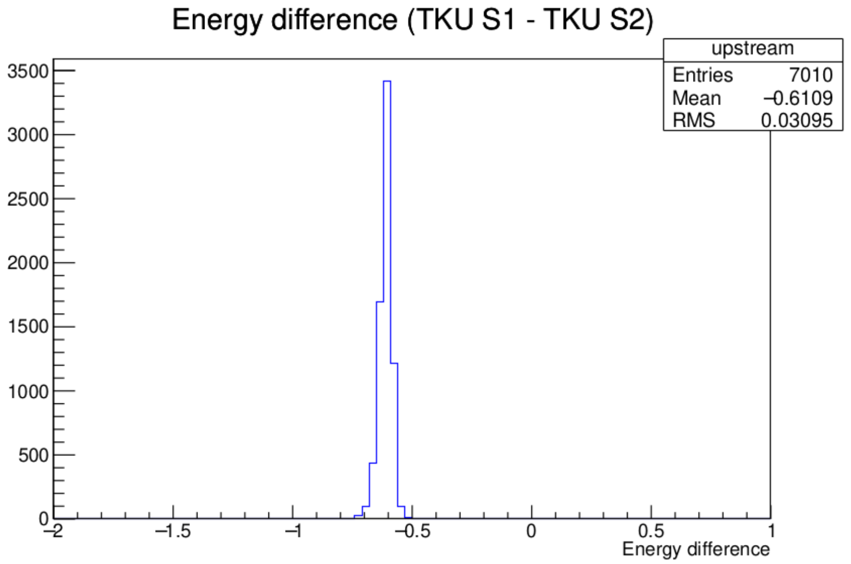
Skewness due to Scattering

- Transfer matrix assumes particles due not scatter, decay, etc.
- Including those particles will skew the linearised equations, and thus the constants and transfer matrices
- For simplicity will look between two stations where energy straggling is low, likely little scattering
- Will remove particle that deviates most and redo calculations
- Will repeat process a number of times
- Will apply transfer matrix to another no absorber run

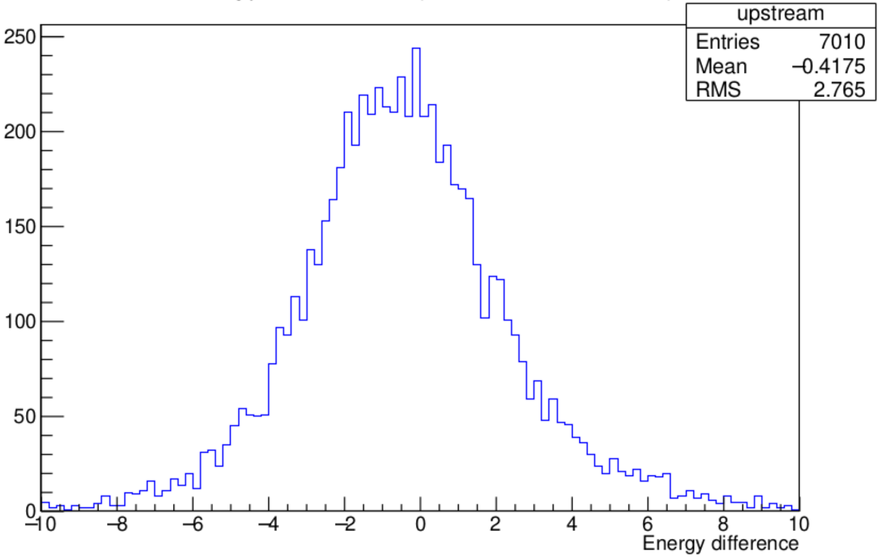

Energy difference (TKU S4 - TKU S5)
upstream
Entries 7010
Mean −0.6192
RMS 0.2729
Energy difference


Energy difference (TKU S3 - TKU S4)
upstream
Entries 7010
Mean −0.6027
RMS 0.267
Energy difference

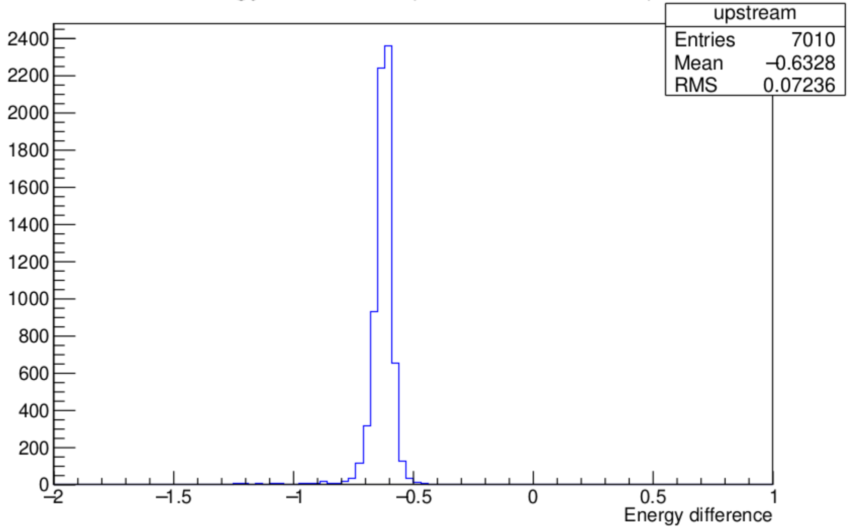

Energy difference (TKU S2 - TKU S3)
upstream
Entries 7010
Mean −0.6364
RMS 0.2293
Energy difference


Energy difference (TKU S1 - TKU S2)
upstream
Entries 7010
Mean −0.6109
RMS 0.03095
Energy difference

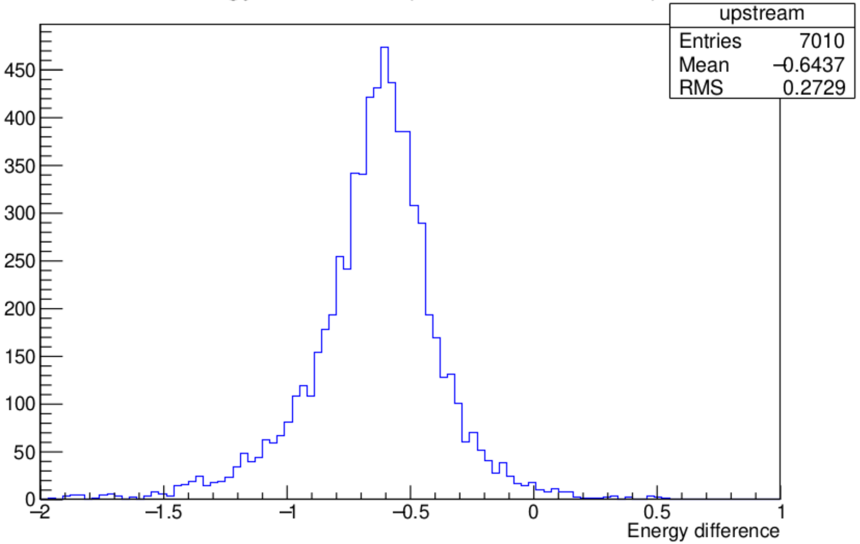
Energy difference (TKD S1 - TKU S1)



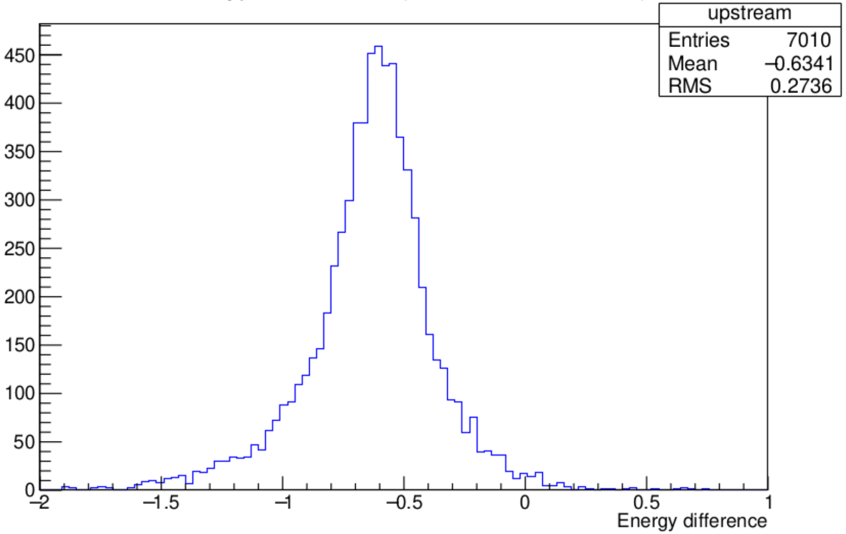
Energy difference (TKD S2 - TKD S1)



Energy difference (TKD S3 - TKD S2)



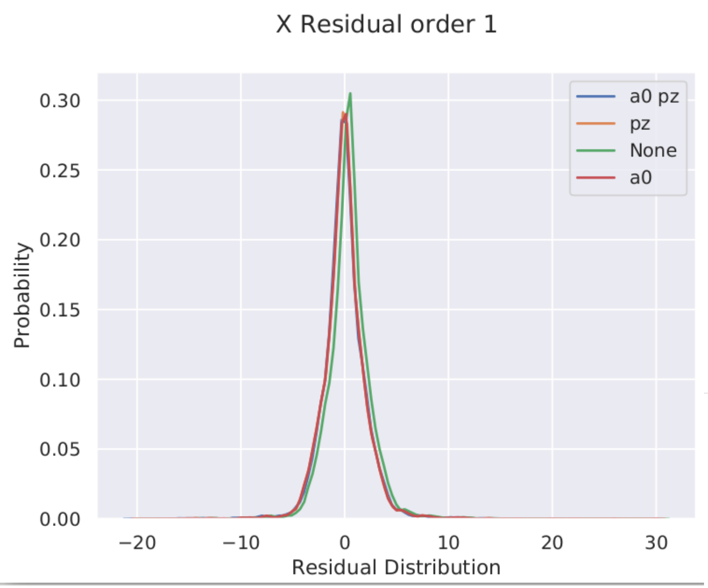
Energy difference (TKD S4 - TKD S3)

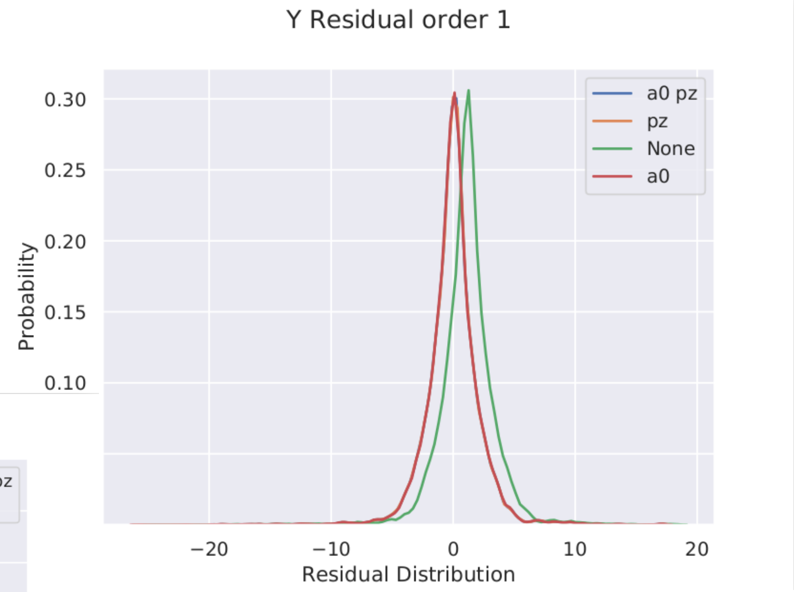


Energy Loss

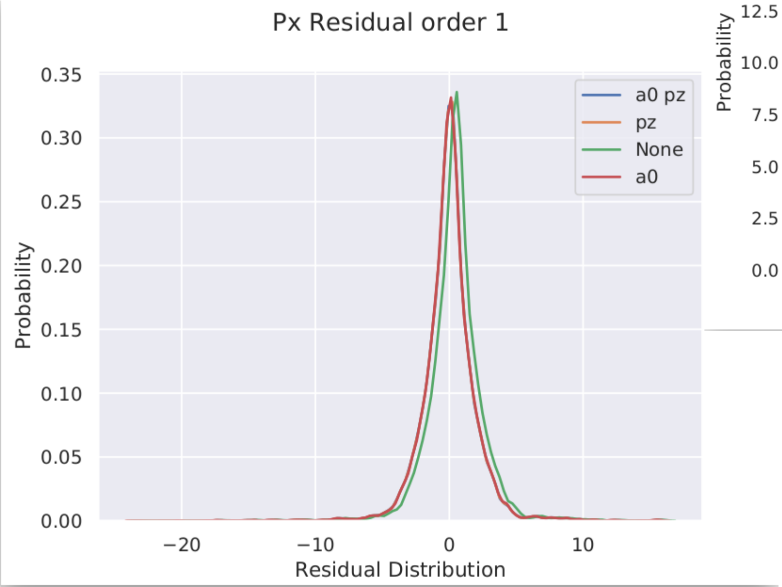
- The RMS Energy Loss is lowest between TKU S2 and TKU S1
- Initially have 7010 particles
- Will look at results after removing eight particles from consideration
- Those eight particles deviated significantly, and likely have scattered or scraped along the way

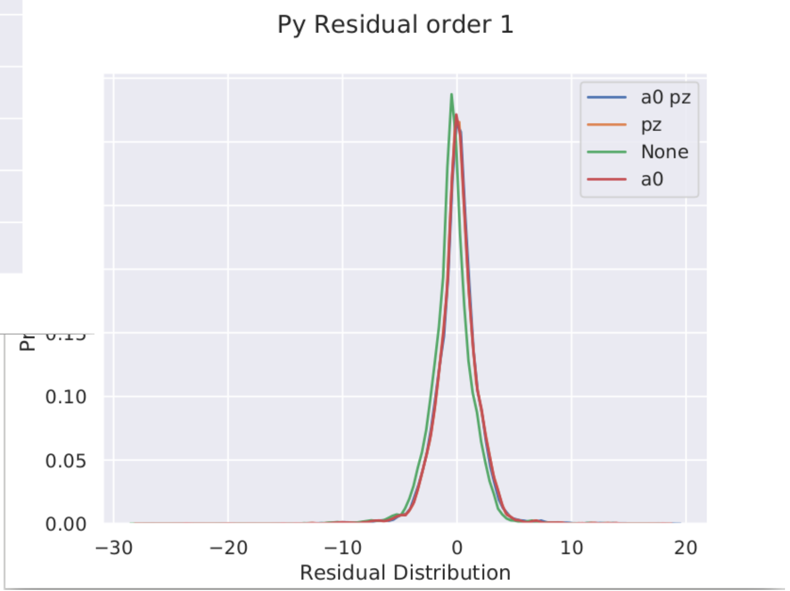
1^{st} Order


X Residual order 1
Probability
Residual Distribution
a0 pz
pz
None
a0


Y Residual order 1
Probability
Residual Distribution
a0 pz
pz
None
a0

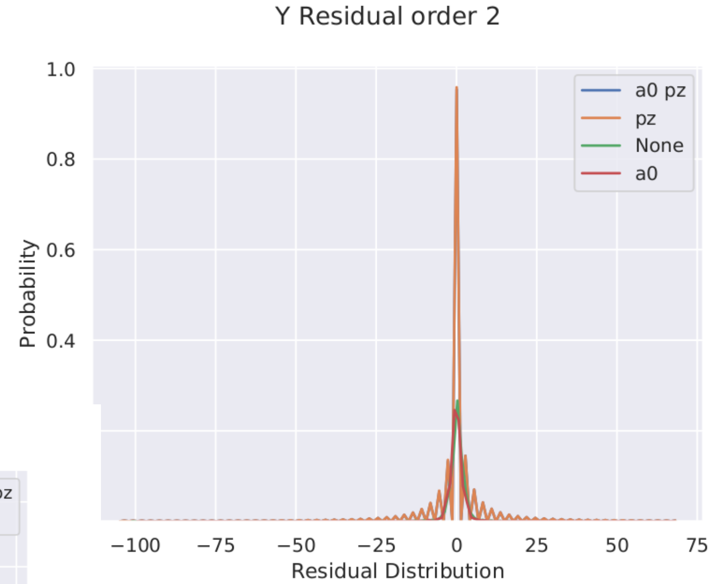

Pz Residual order 1
Probability
Residual Distribution
a0 pz
pz


Px Residual order 1
Probability
Residual Distribution
a0 pz
pz
None
a0

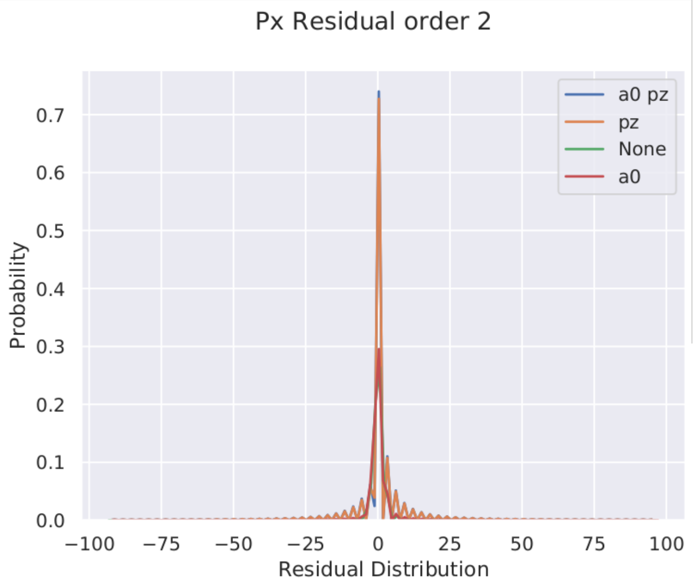

Py Residual order 1
Residual Distribution
a0 pz
pz
None
a0

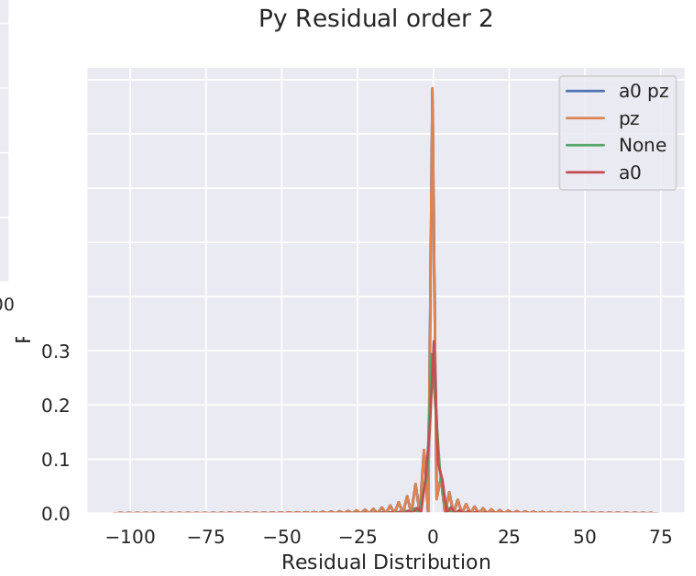
2nd Order


X Residual order 2
Probability
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
−75
−50
−25
0
25
50
75
100
Residual Distribution
a0 pz
pz
None
a0


Y Residual order 2
Probability
0.4
0.6
0.8
1.0
−100
−75
−50
−25
0
25
50
75
Residual Distribution
a0 pz
pz
None
a0

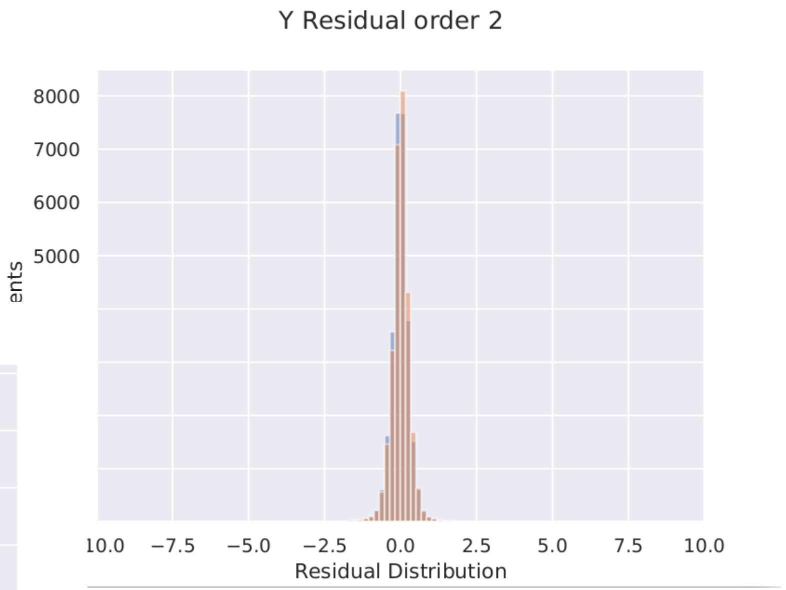

Pz Residual order 2
Probability
0
10
20
30
40
50
60
−100
−75
−50
−25
0
25
50
75
100
Residual Distribution
a0 pz
pz


Px Residual order 2
Probability
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
−100
−75
−50
−25
0
25
50
75
100
Residual Distribution
a0 pz
pz
None
a0

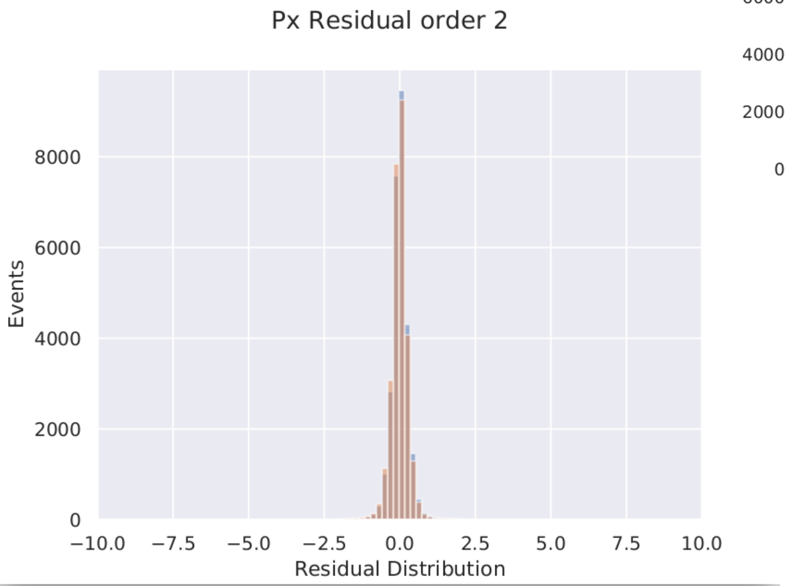

Py Residual order 2
0.0
0.1
0.2
0.3
−100
−75
−50
−25
0
25
50
75
Residual Distribution
a0 pz
pz
None
a0

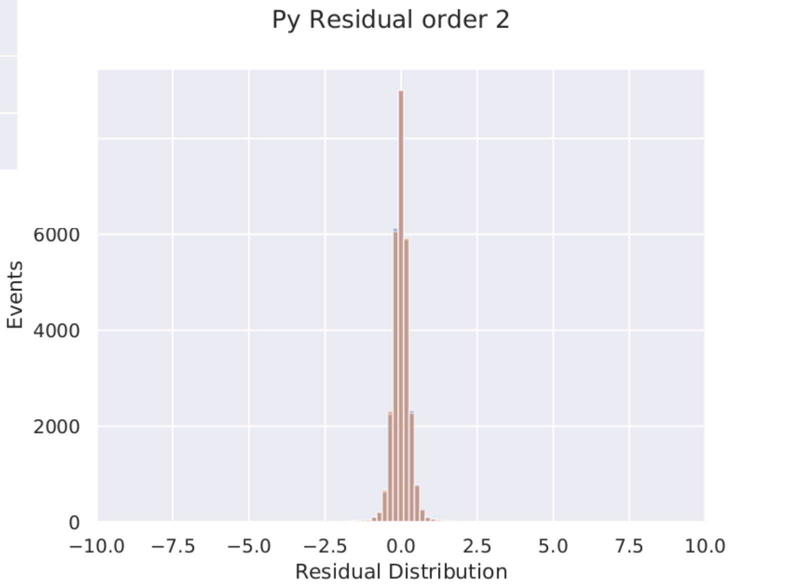
2nd Order


X Residual order 2
Events
0
1000
2000
3000
4000
5000
6000
7000
8000
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Residual Distribution


Y Residual order 2
5000
6000
7000
8000
10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Residual Distribution

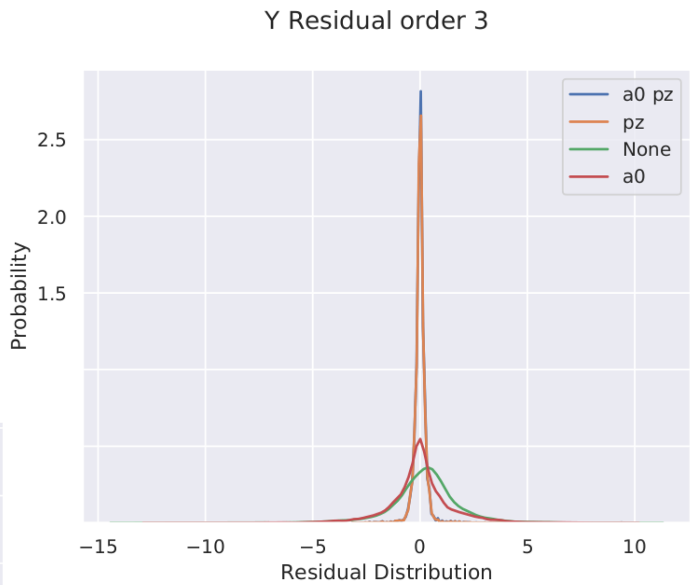

Pz Residual order 2
0
2000
4000
6000
8000
10000
12000
14000
−0.4
−0.2
0.0
0.2
Residual Distribution


Px Residual order 2
Events
0
2000
4000
6000
8000
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Residual Distribution

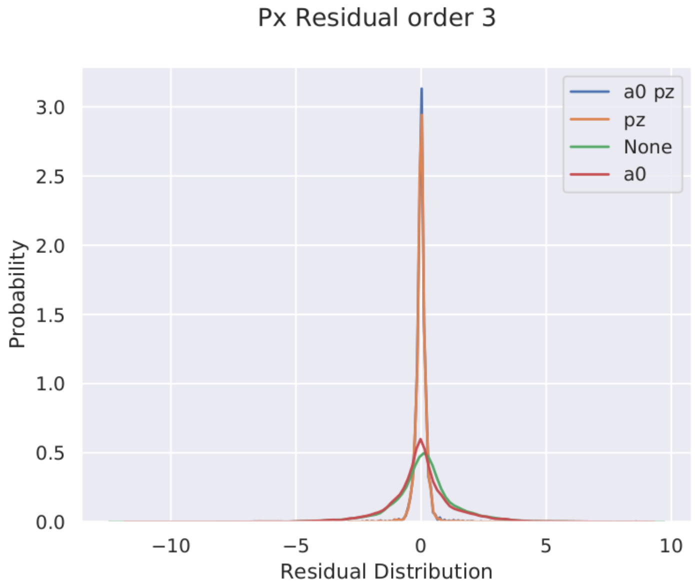

Py Residual order 2
Events
0
2000
4000
6000
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
Residual Distribution

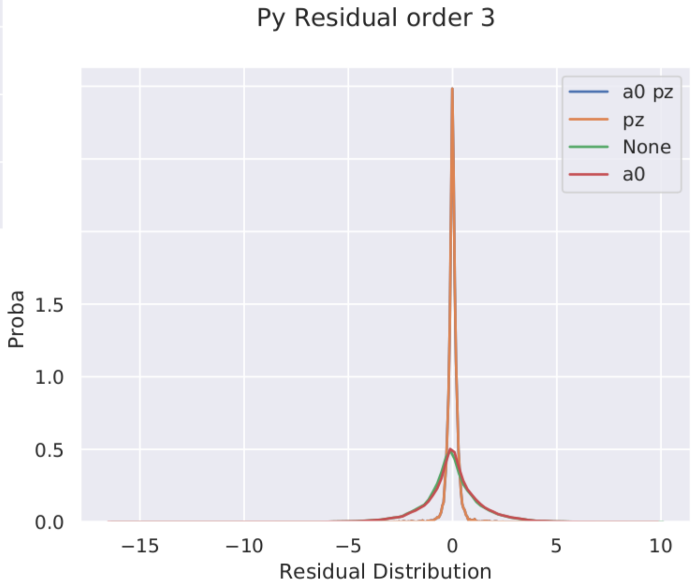
3rd Order


X Residual order 3
Probability
Residual Distribution
a0 pz
pz
None
a0

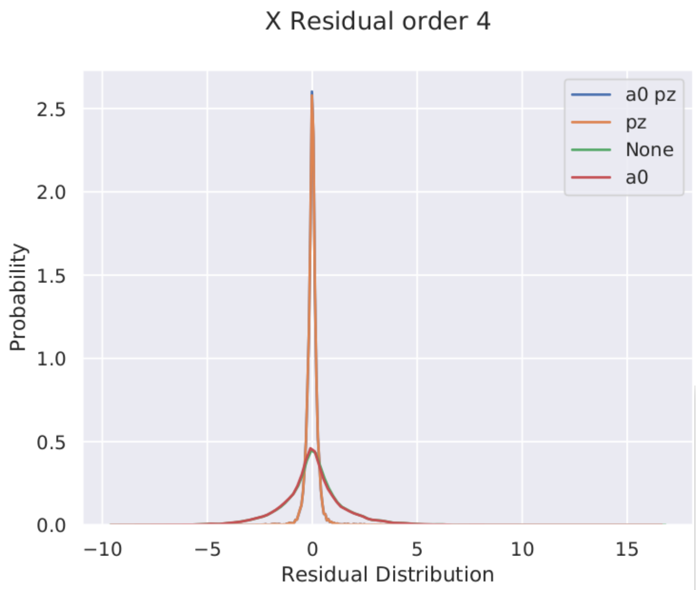

Y Residual order 3
Probability
Residual Distribution
a0 pz
pz
None
a0

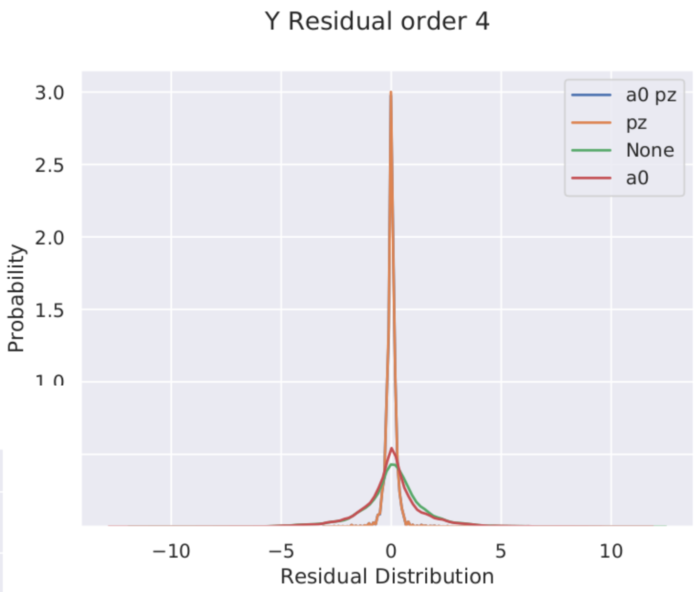

Pz Residual order 3
Probability
Residual Distribution
a0 pz
pz


Px Residual order 3
Probability
Residual Distribution
a0 pz
pz
None
a0

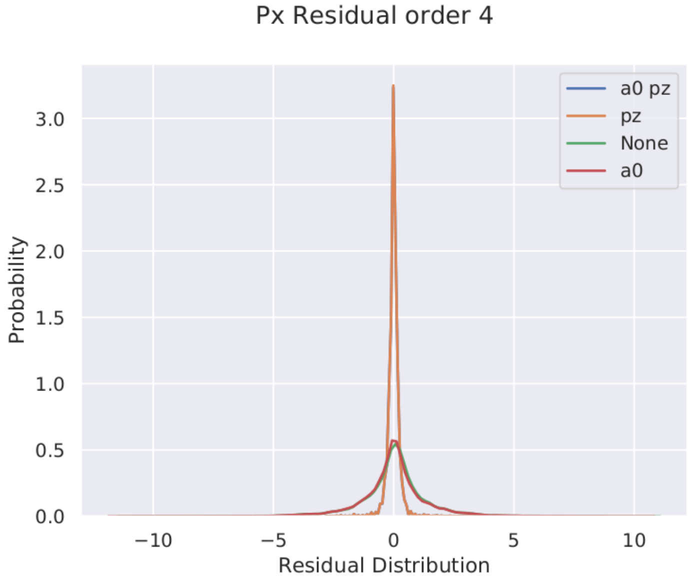

Py Residual order 3
Proba
Residual Distribution
a0 pz
pz
None
a0

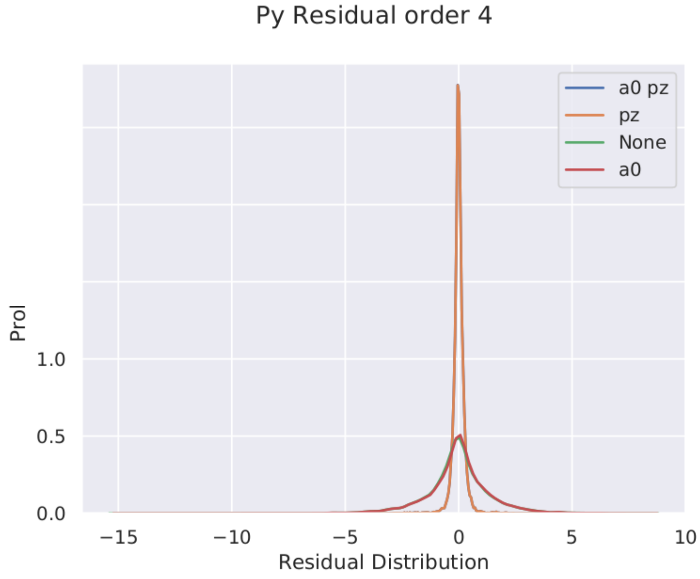
4^{th} Order


X Residual order 4
a0 pz
pz
None
a0
Probability
Residual Distribution

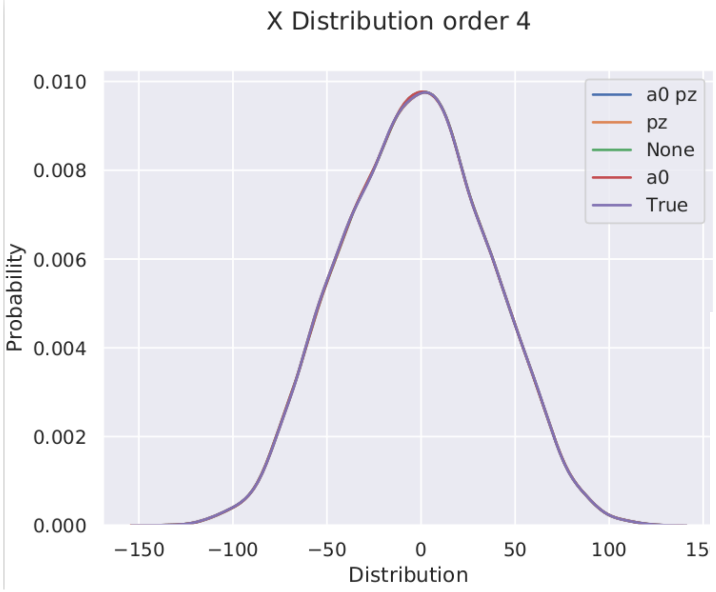

Y Residual order 4
a0 pz
pz
None
a0
Probability
Residual Distribution

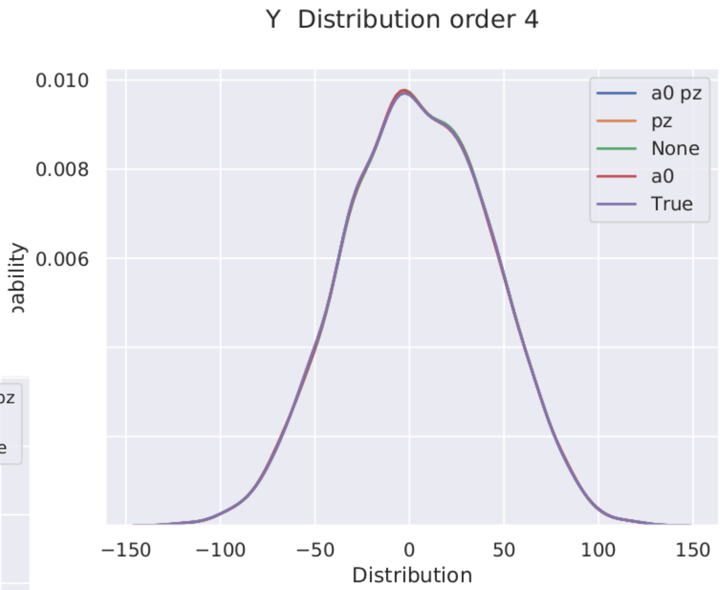

Pz Residual order 4
a0 pz
pz
Probability
Residual Distribution

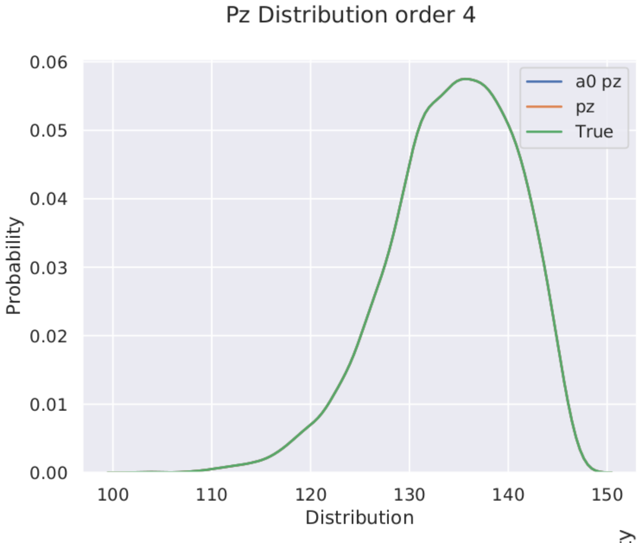

Px Residual order 4
a0 pz
pz
None
a0
Probability
Residual Distribution

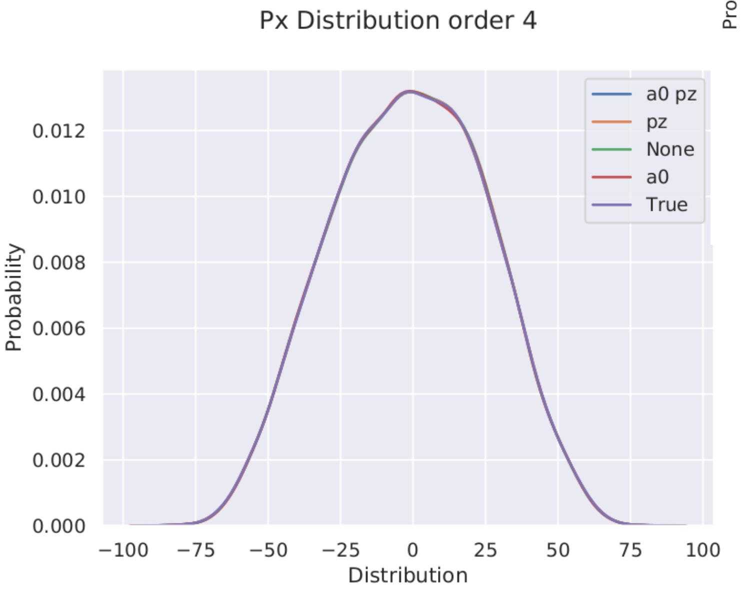

Py Residual order 4
a0 pz
pz
None
a0
Residual Distribution

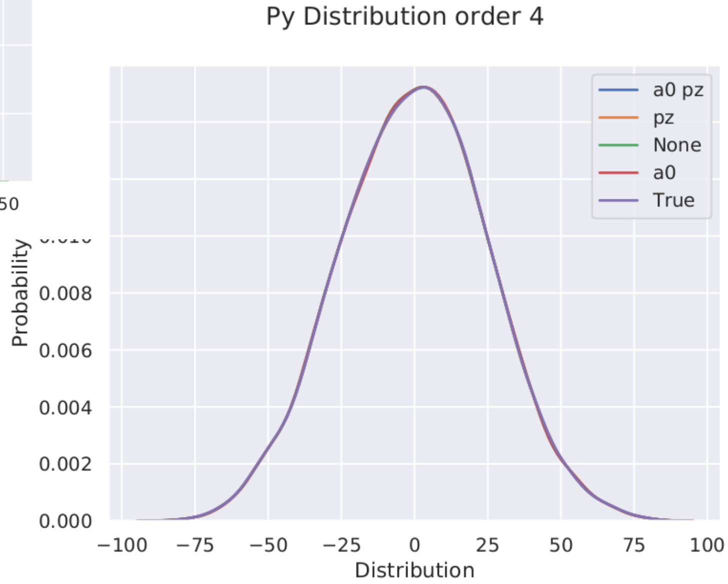
4^{th} Order


X Distribution order 4
Probability
Distribution
a0 pz
pz
None
a0
True


Y Distribution order 4
Distribution
a0 pz
pz
None
a0
True


Pz Distribution order 4
Probability
Distribution
a0 pz
pz
True

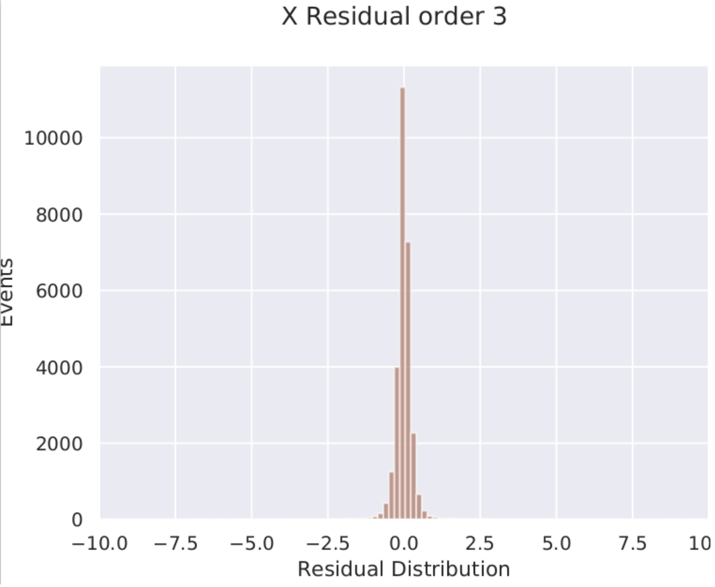

Px Distribution order 4
Probability
Distribution
a0 pz
pz
None
a0
True

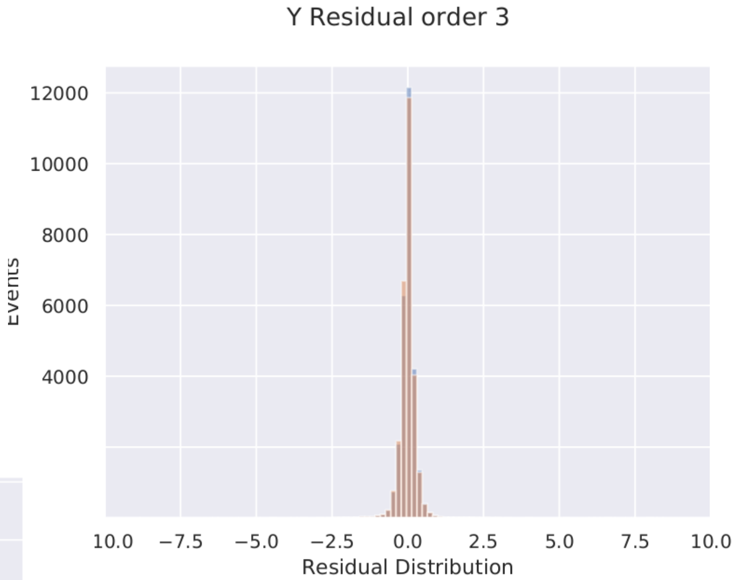

Py Distribution order 4
Probability
Distribution
a0 pz
pz
None
a0
True

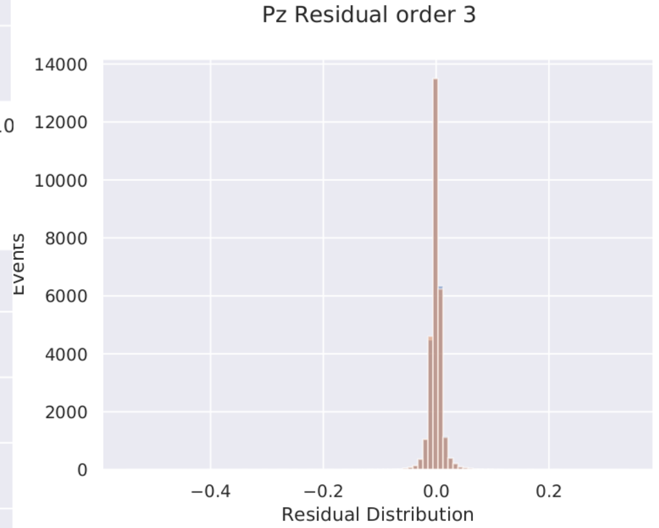
Transfer Matrix

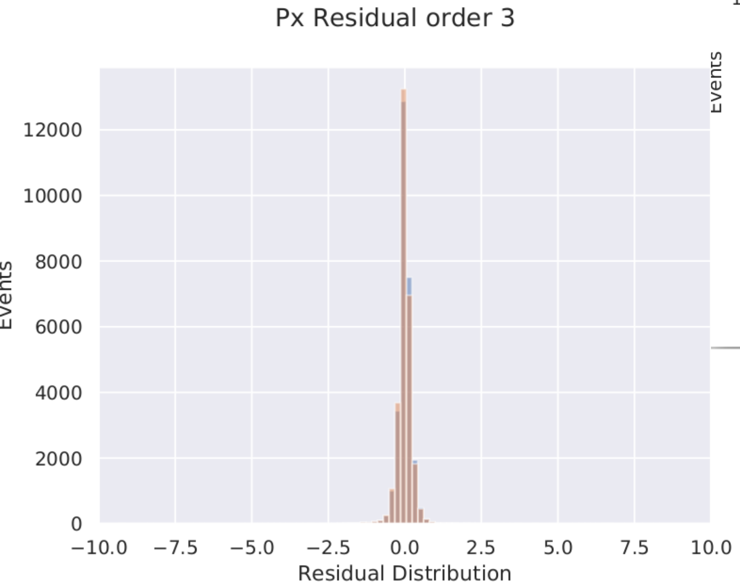
- After 3rd order no appreciable difference is noted in the residuals. Have reached resolution of tracker stations.
- The predicted distribution of the transfer matrix between TKU S2 and TKU S1 matches well with true experimental values.
- Will now apply transfer matrix to an independent run

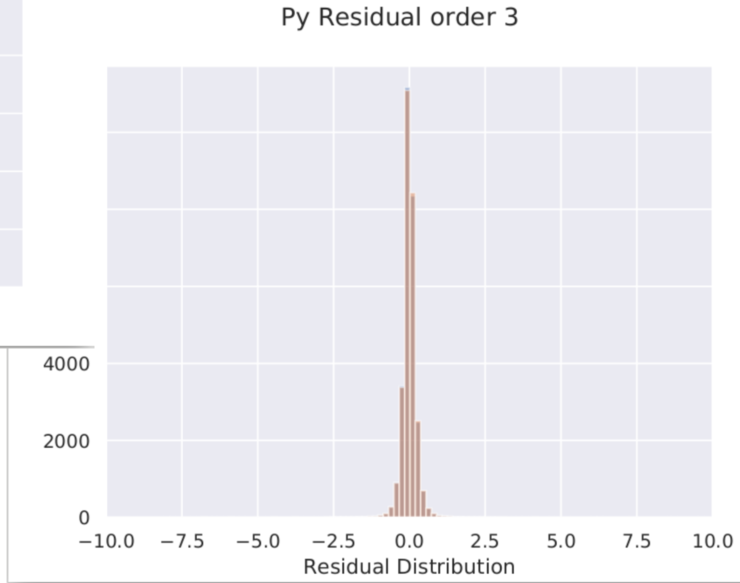
3rd Order


X Residual order 3
Events
Residual Distribution

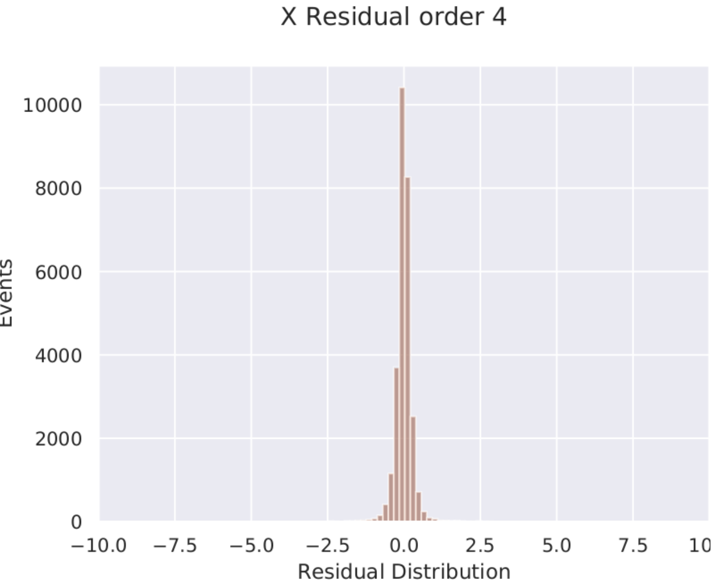

Y Residual order 3
Events
Residual Distribution

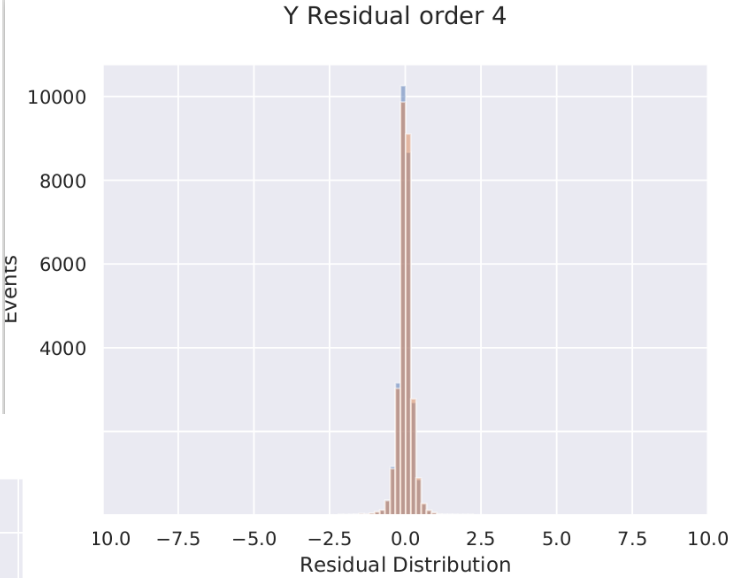

Pz Residual order 3
Events
Residual Distribution

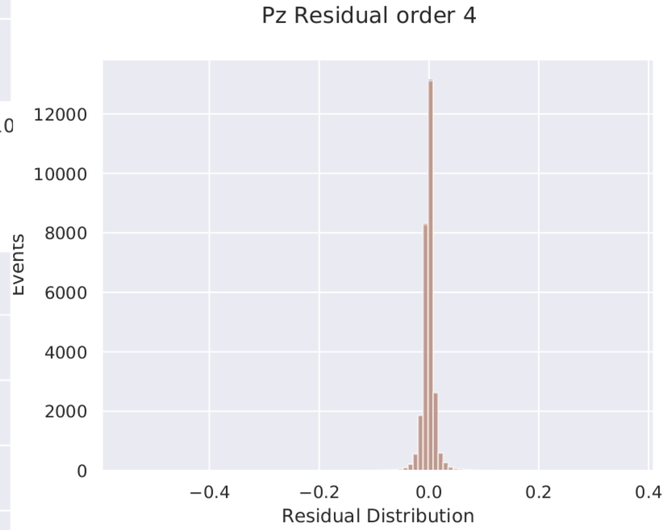

Px Residual order 3
Events
Residual Distribution

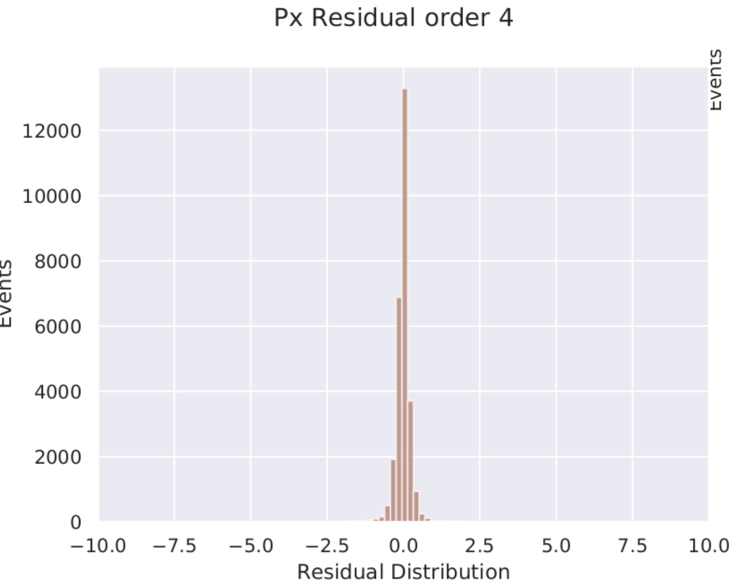

Py Residual order 3
Residual Distribution

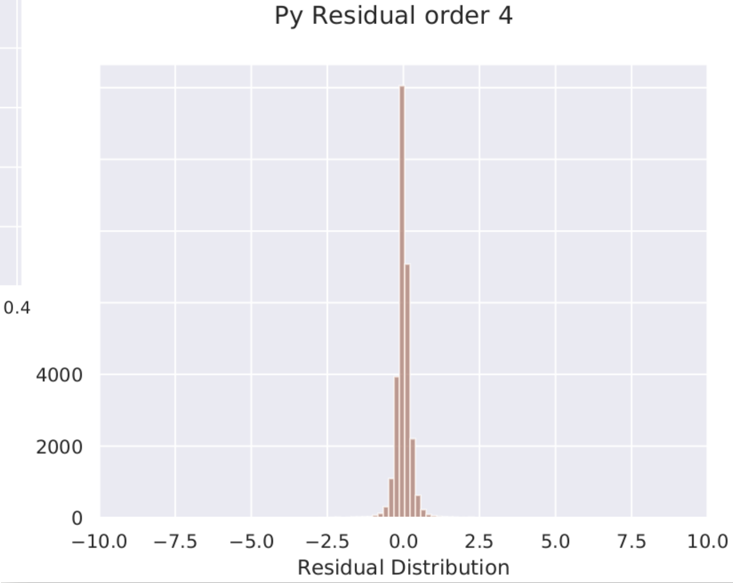
4^{th} Order


X Residual order 4
Events
Residual Distribution
0
2000
4000
6000
8000
10000
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10


Y Residual order 4
Events
Residual Distribution
4000
6000
8000
10000
10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0


Pz Residual order 4
Events
Residual Distribution
0
2000
4000
6000
8000
10000
12000
−0.4
−0.2
0.0
0.2
0.4


Px Residual order 4
Events
Residual Distribution
0
2000
4000
6000
8000
10000
12000
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

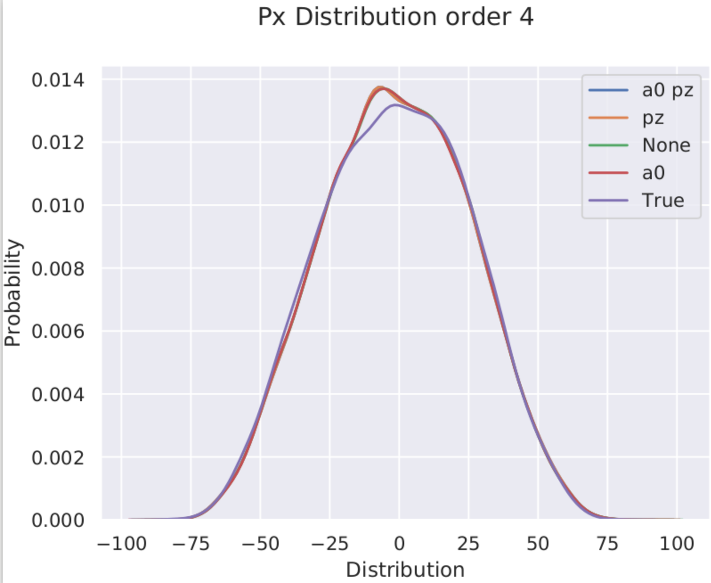

Py Residual order 4
Residual Distribution
0
2000
4000
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

4^{th} Order


X Distribution order 4
a0 pz
pz
None
a0
True
Probability
Distribution


Y Distribution order 4
a0 pz
pz
None
a0
True
Probability
Distribution


Pz Distribution order 4
a0 pz
pz
True
Probability
Distribution


Px Distribution order 4
a0 pz
pz
None
a0
True
Probability
Distribution


Py Distribution order 4
a0 pz
pz
None
a0
True
Distribution

Conclusion

- Transfer Matrix from 3rd Order appears to predict well where particles are transported from TKU S2 to TKU S1 within the resolution of the trackers
- This is if one excludes particles which have largely deviated i.e. they have scattered/scraped
- Improvement if include Pz => should use full 6D approach and include time
- Will extend approach to other stations where Energy straggling is larger, especially between TKU and TKD.

THE END