

Numerical investigation of beam-driven wakefields in hollow plasma channels modelled with carbon ions

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Outline

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- Beam-driven wakefield
 - single CNT
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- X-ray driven wakefield
 - preliminary results
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Introduction

- Plasma-based accelerators → GeV/cm accelerating gradients (proportional to the plasma densities).
- Higher density materials (metallic crystals) might sustain \sim TV/cm wakefields.
- Carbon nanotubes (CNT) → good candidates as ultra-compact accelerating structures.
- CNT approximation → hollow plasma channels (electrons + “carbon ions”) in conventional PIC codes (in our case, [FBPIC¹](#)).
- Despite neglecting inherent properties of solid crystalline structures, hollow plasma channels have been adopted to model CNTs in several studies.

¹[Lehe, R., et al., Comput. Phys. Commun. 203, 66–82 \(2016\).](#)

Introduction

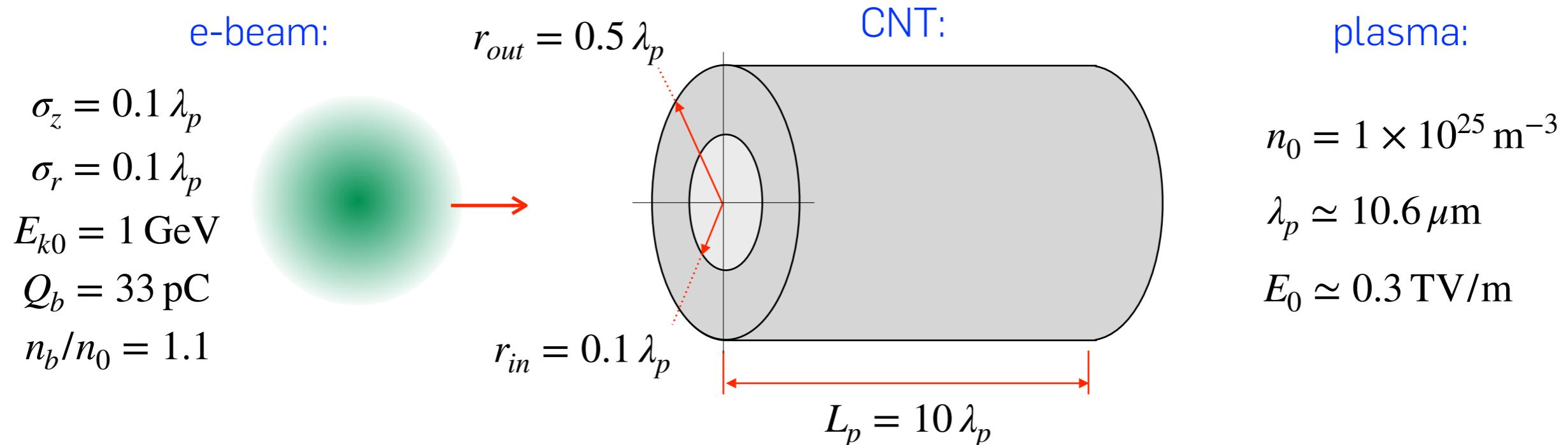
- In this work, properties of wakefields excited in single CNTs and CNT arrays are investigated using the hollow plasma channel approach.
- Both electron beams and laser X-rays are used as drivers.
- Parameter sets adopted in 2D cartesian simulations are revisited in a 2D axisymmetric geometry (additional azimuthal modes are used when required).
- Parameter scans (internal radius, wall thickness, beam density peak) are presented and discussed.
- A preliminary discussion about using an effective, constant plasma density to describe CNT arrays is introduced.

P.S.: for sake of simplicity, plasma hollow channels (even those with μm -scales) will be addressed as carbon nanotubes (CNTs).

Beam-driven wakefield - single CNT

Single CNT in 2D axisymmetric geometry

- Parameters based on Shin Young-Min², except with higher energy.
- Beam and CNT dimensions: parametrised as a function of λ_p ($\simeq 10.6 \mu\text{m}$).



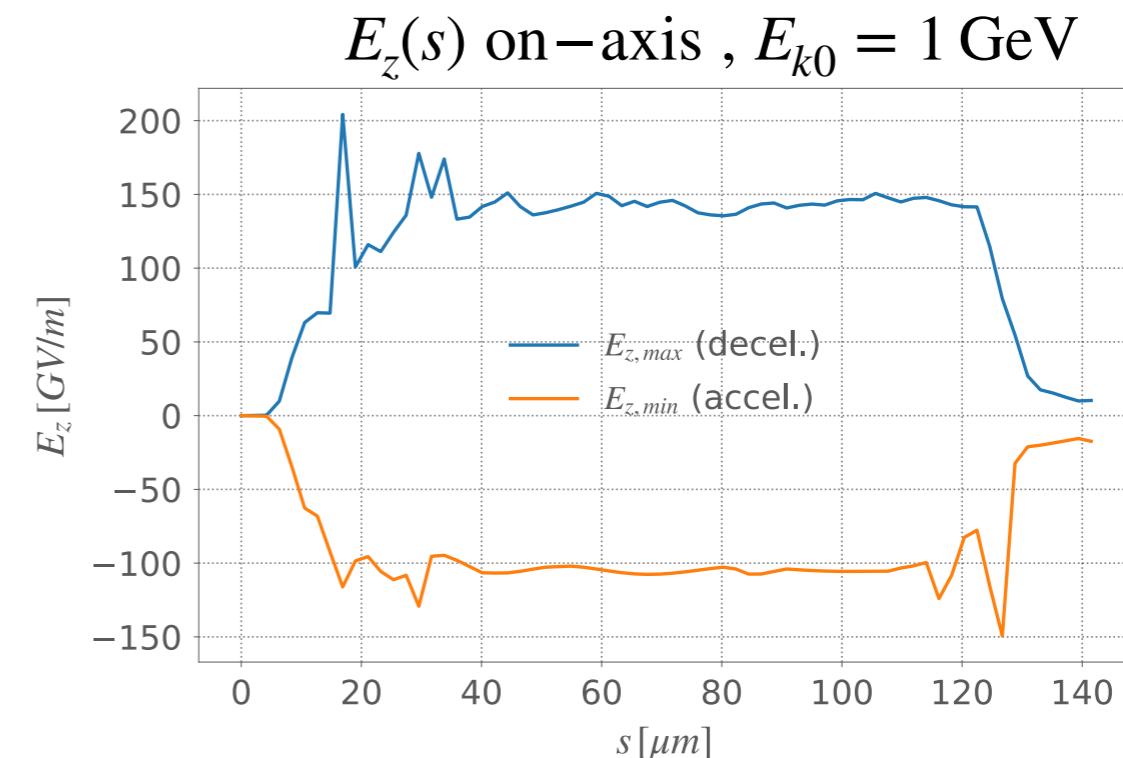
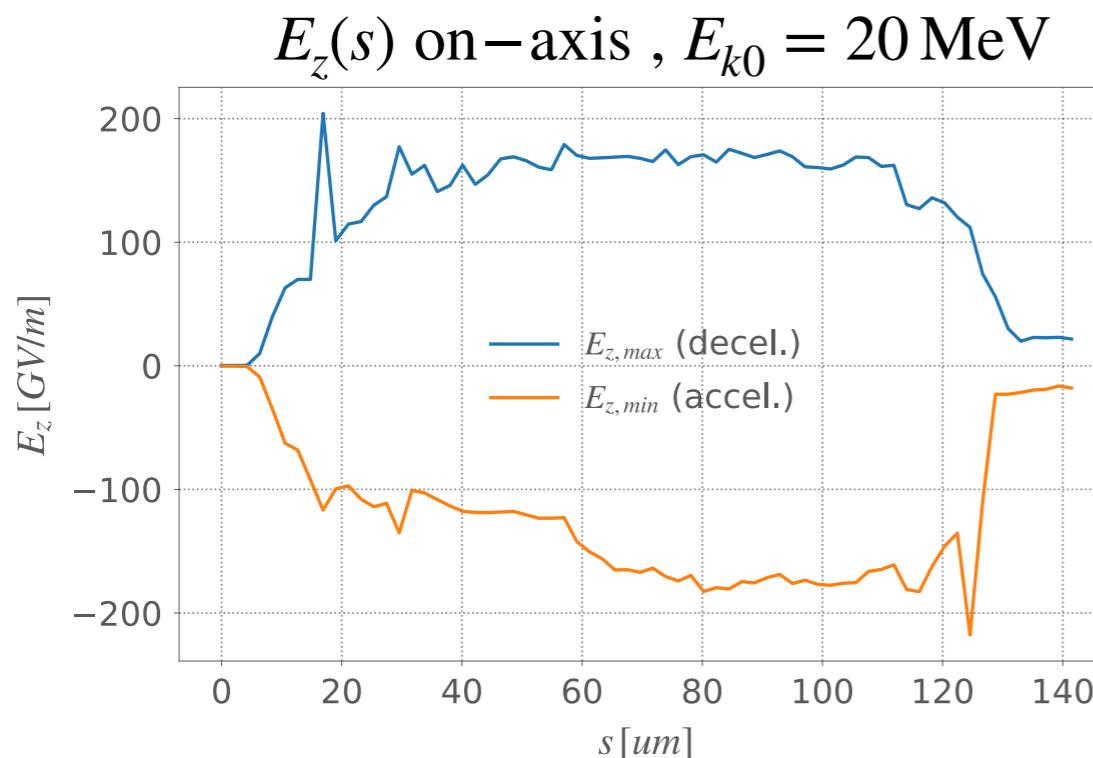
- Carbon ions:
 - $q = e$ (single-level ionisation);
 - $m_C \simeq 12 m_p$;

²Young-Min, S., Int. J. of Mod. Phys. A Vol. 34, No. 34, 1943005 (2019)

Beam-driven wakefield - single CNT

Single CNT in 2D axisymmetric geometry

- Parameters based on Shin Young-Min², except with **higher energy**.
- Beam and CNT dimensions: parametrised as a function of λ_p ($\simeq 10.6 \mu\text{m}$).

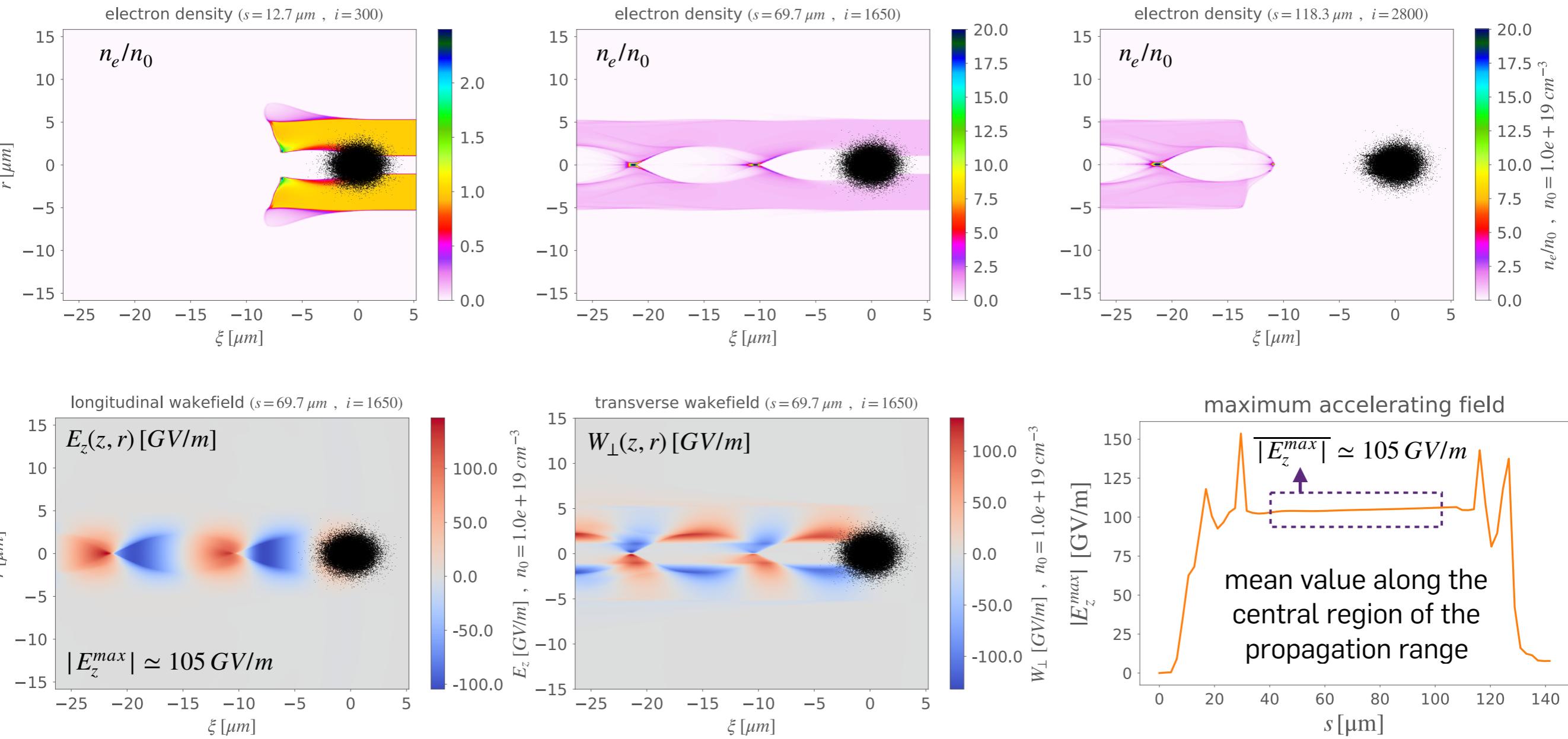


²Young-Min, S., Int. J. of Mod. Phys. A Vol. 34, No. 34, 1943005 (2019)

Beam-driven wakefield - single CNT

Single CNT in 2D axisymmetric geometry

- $r_{in} = 0.1 \lambda_p$, $r_{out} = 0.5 \lambda_p$, $n_0 = 10^{25} m^{-3}$, $\lambda_p = 10.6 \mu m$

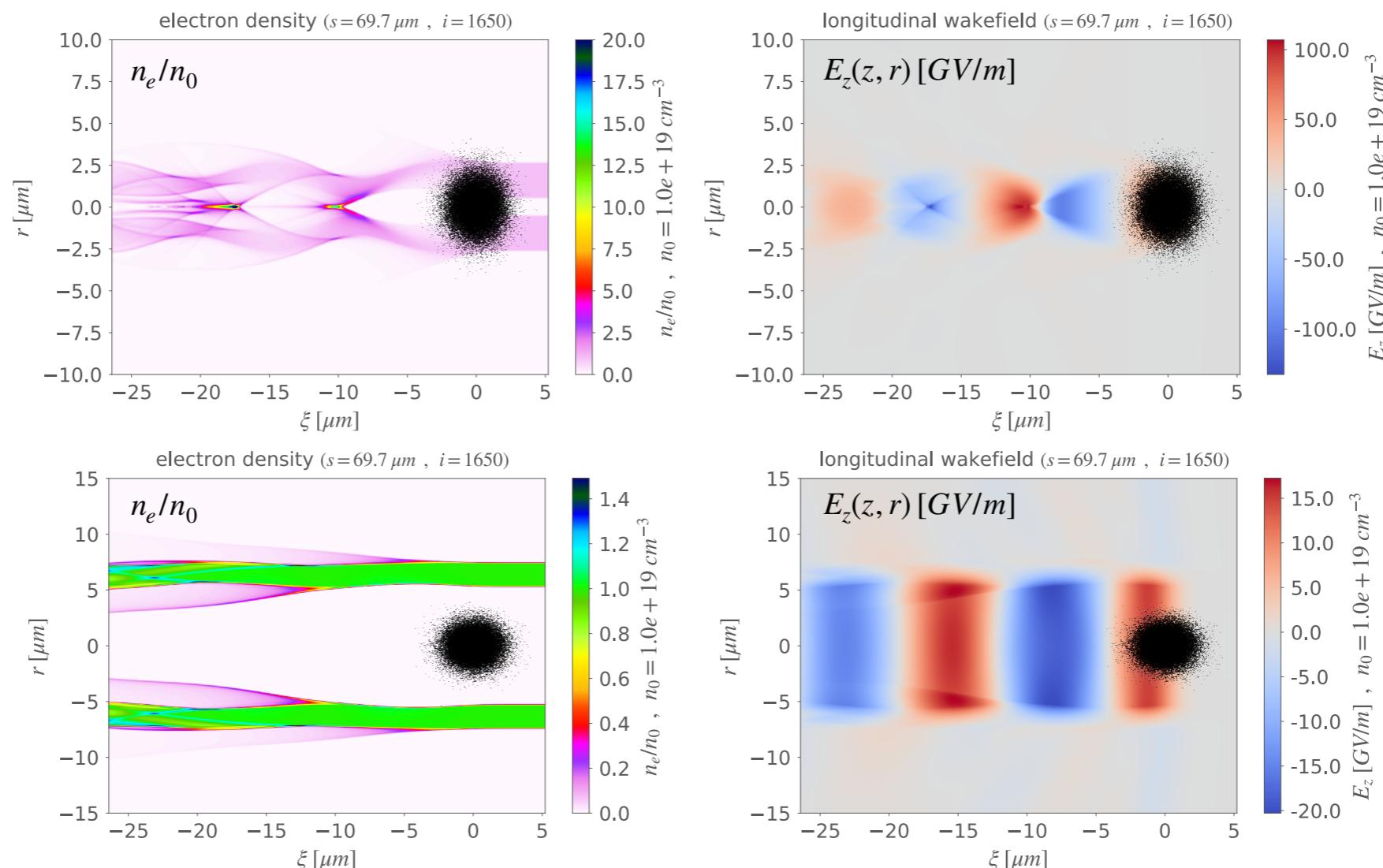


Beam-driven wakefield - single CNT

Internal radius (r_{in}) scan

- $r_{in} = 0.05 \lambda_p \sim 0.50 \lambda_p$ [$0.05 \mu\text{m} \sim 0.5 \mu\text{m}$, $n_0 = 10^{25} \text{ m}^{-3}$
 $0.17 \mu\text{m} \sim 1.7 \mu\text{m}$, $n_0 = 10^{26} \text{ m}^{-3}$]
- wall = $0.2 \lambda_p$ [$2.1 \mu\text{m}$, $n_0 = 10^{25} \text{ m}^{-3}$
 $0.7 \mu\text{m}$, $n_0 = 10^{26} \text{ m}^{-3}$]

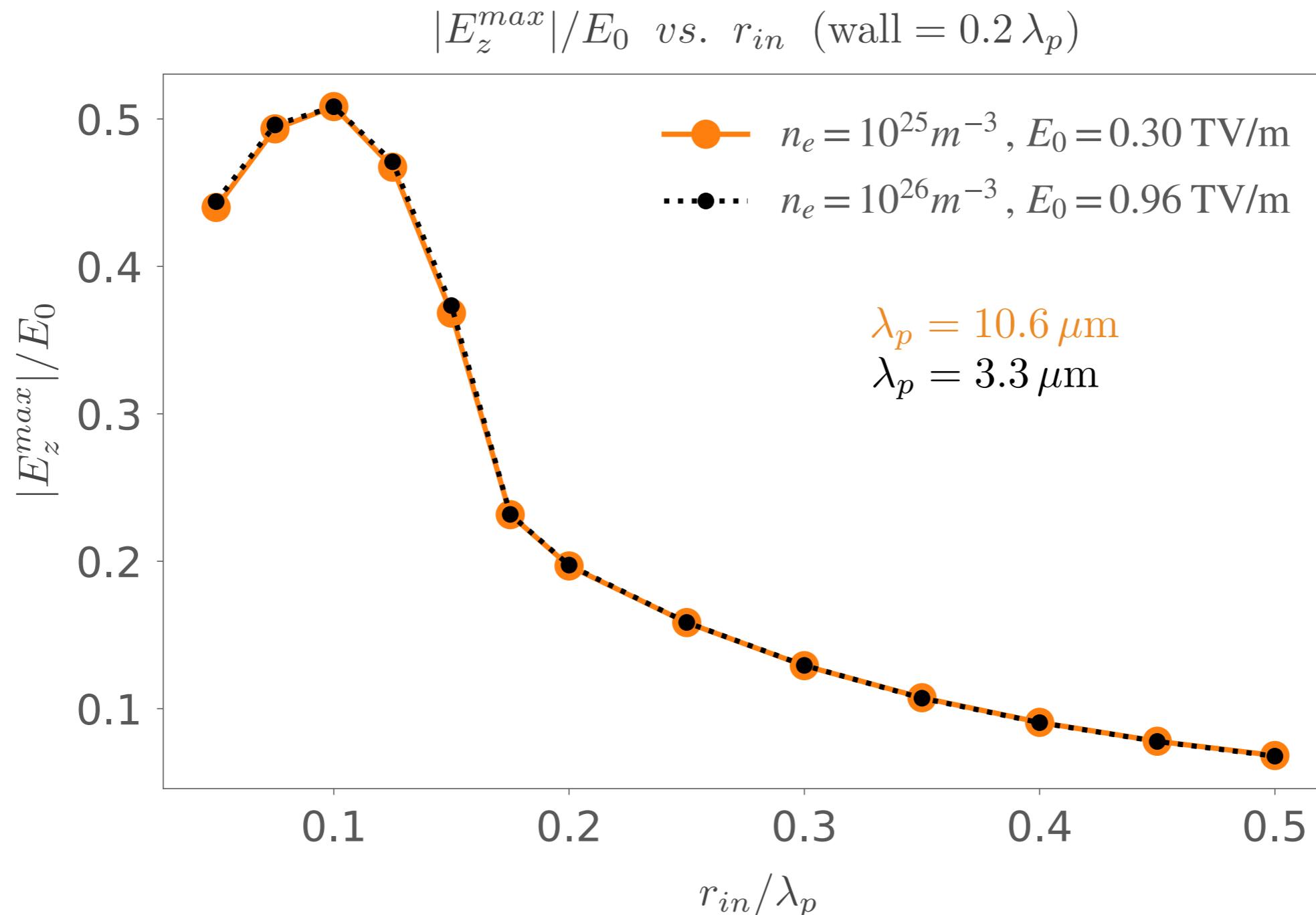
$r_{in} = 0.05 \lambda_p$
beam overlaps
CNT wall



$r_{in} = 0.5 \lambda_p$
CNT internal
radius is larger
than the beam
transverse size

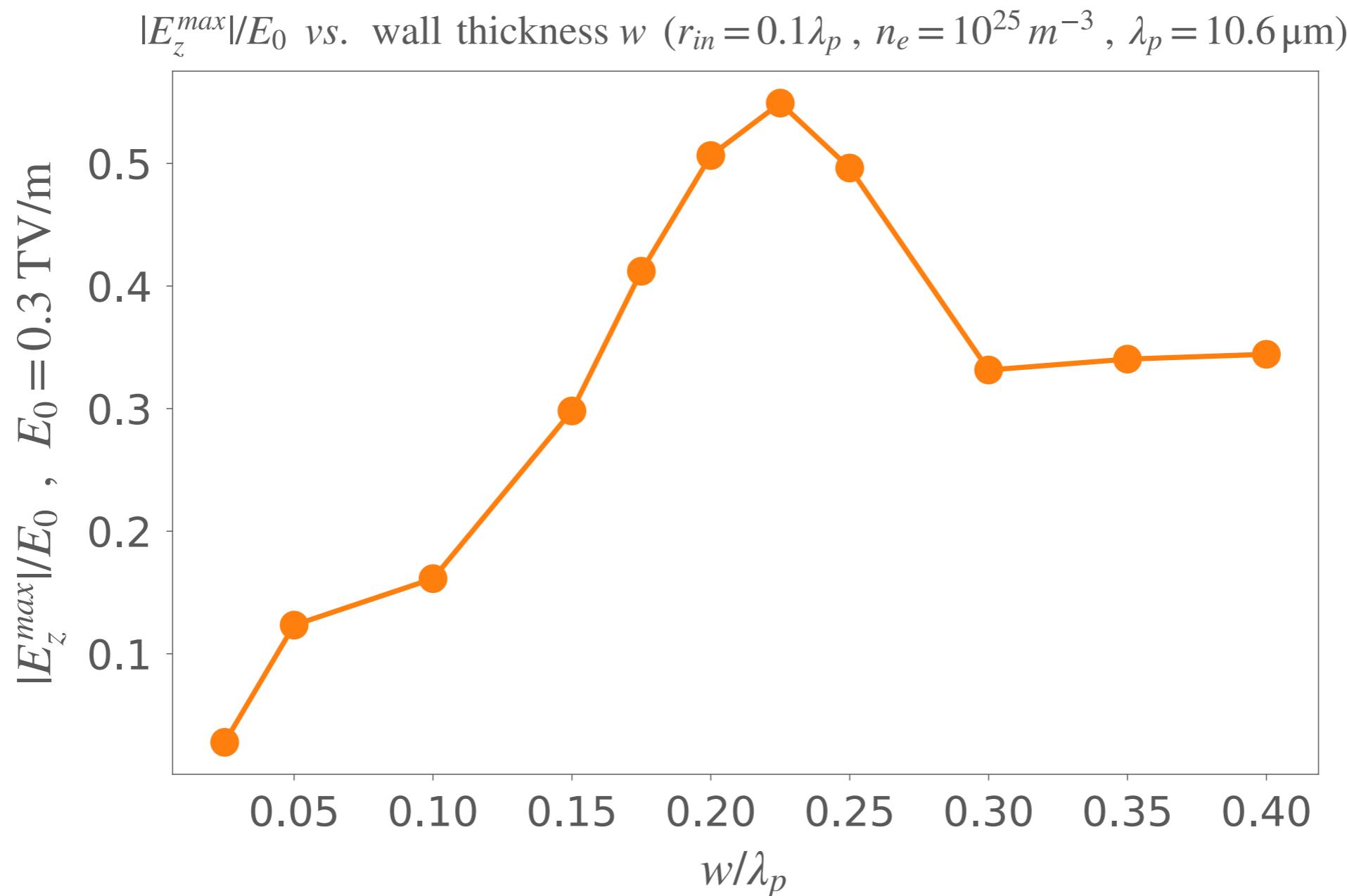
Beam-driven wakefield - single CNT

Internal radius (r_{in}) scan



Beam-driven wakefield - single CNT

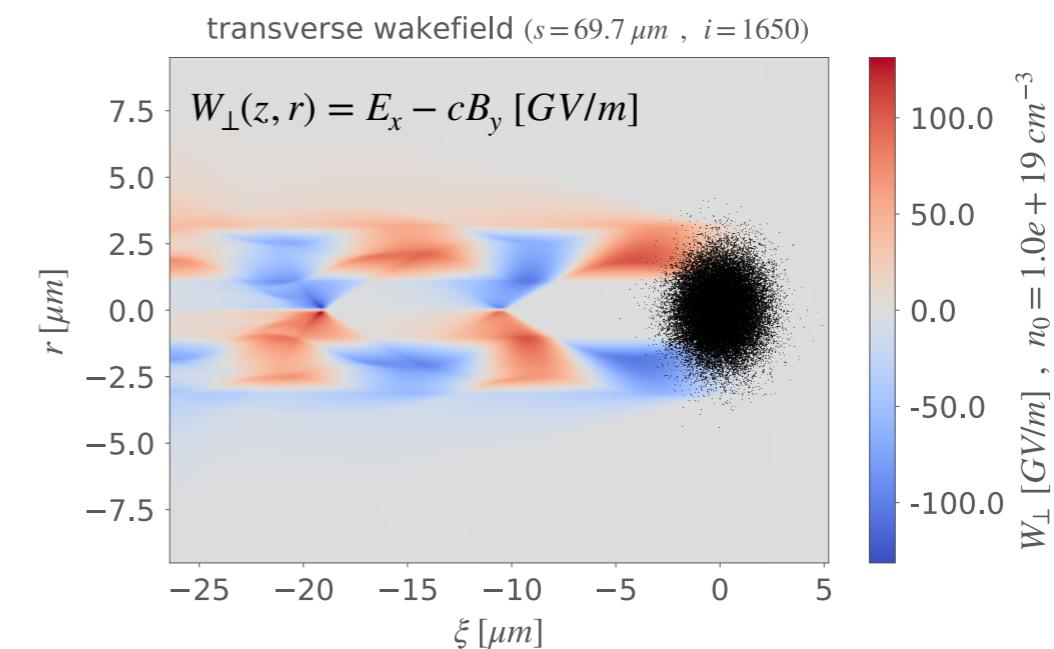
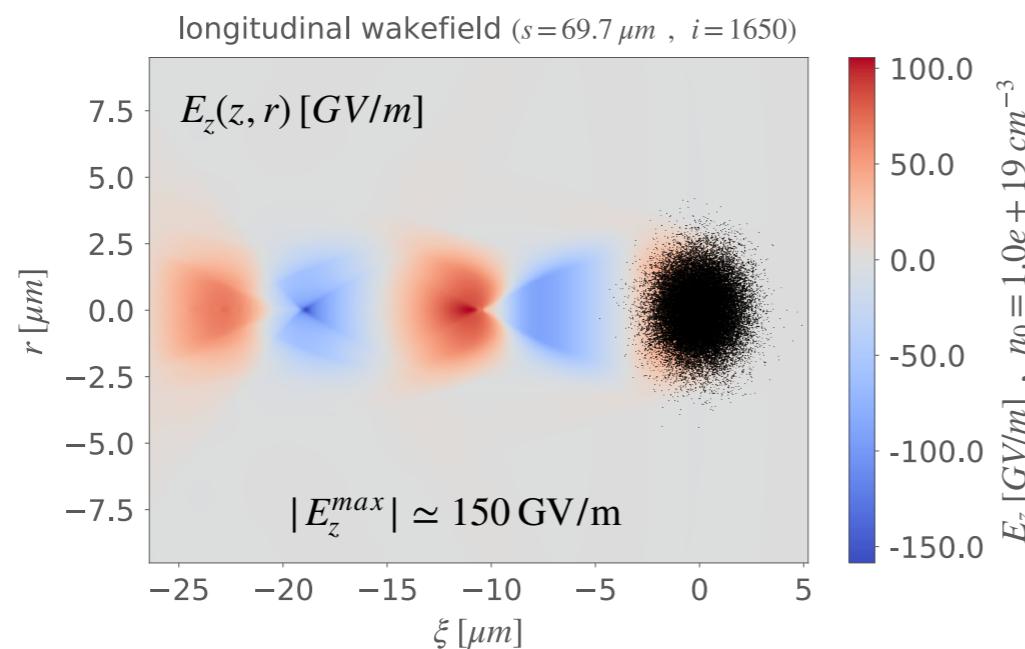
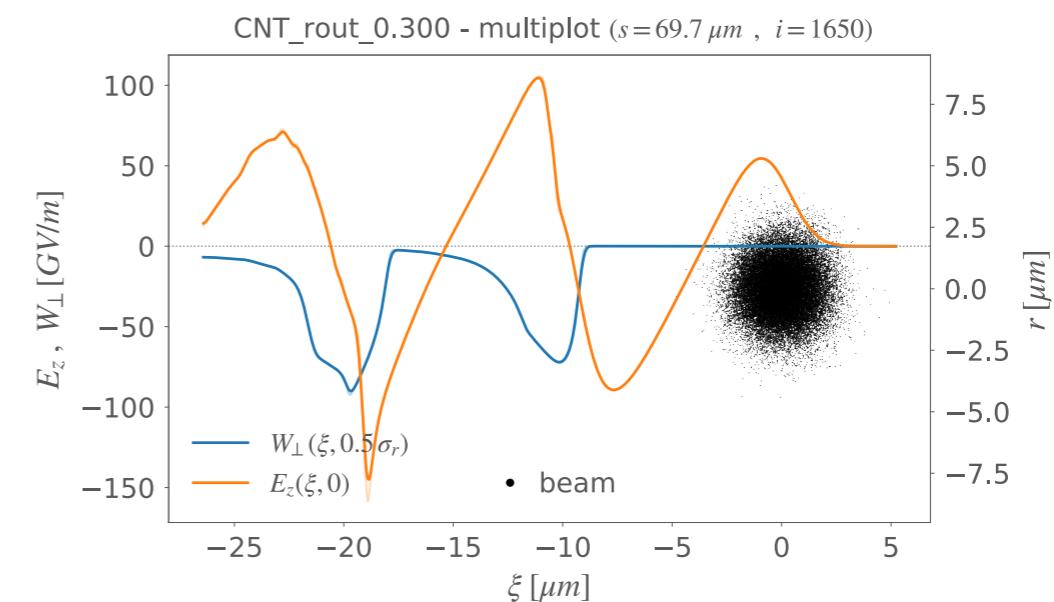
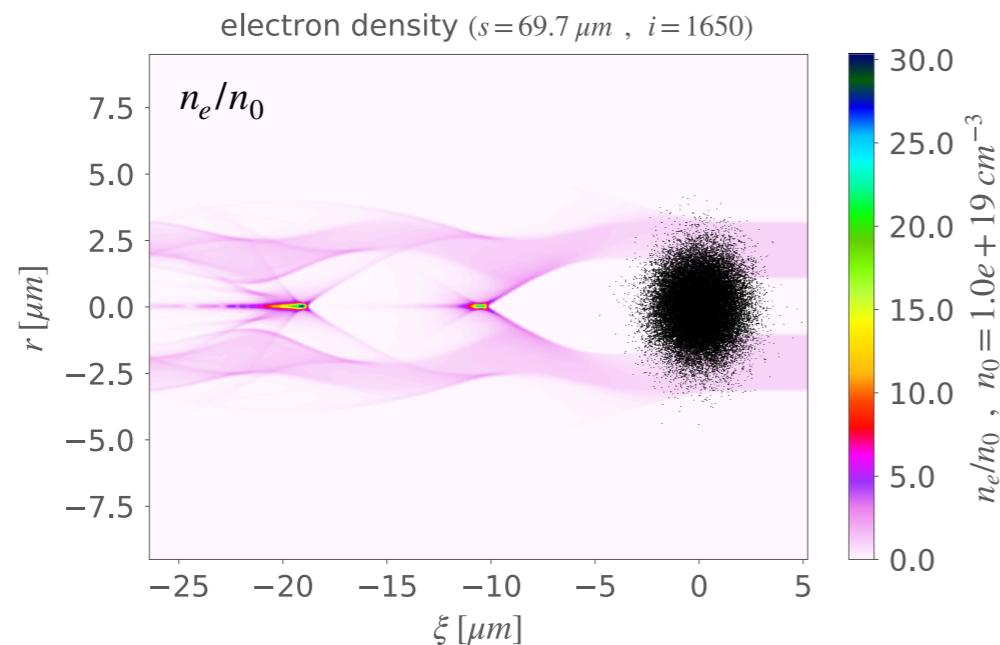
Wall thickness ($w = r_{out} - r_{in}$) scan , $r_{in} = 0.1 \lambda_p$



Beam-driven wakefield - single CNT

Optimal values of r_{in} and w

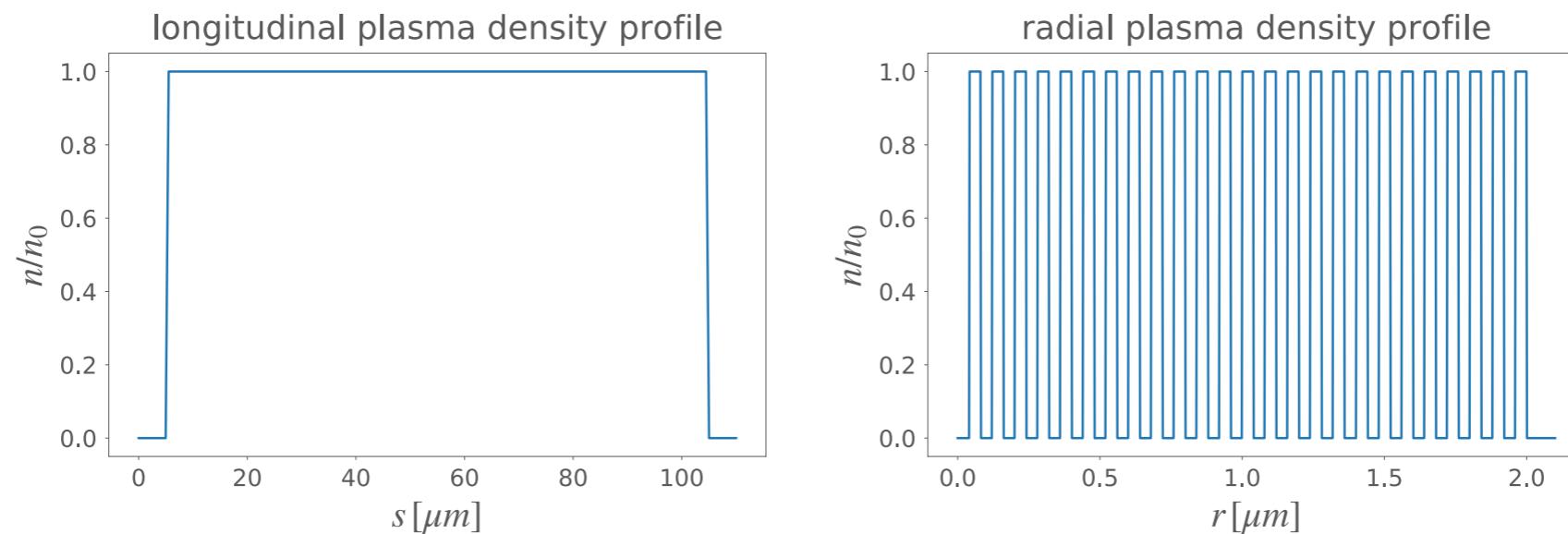
- $r_{in} = 0.1 \lambda_p$, $r_{out} = 0.3 \lambda_p$ ($w = 0.2 \lambda_p$) , $\lambda_p = 10.6 \mu\text{m}$



Beam-driven wakefield - CNT array

CNT array in 2D axisymmetric geometry

- Multiple concentric plasma hollow channels:



- Parameters from J. Resta-Lopez³,

CNT array:

$$L_p = 110 \mu\text{m}$$

$$n_0 = 10^{25} \text{ m}^{-3}$$

$$r_{in,0} = 20 \text{ nm}$$

$$\text{wall} = \text{gap} = 40 \text{ nm}$$

Beam:

$$n_b/n_0 = 1$$

$$\sigma_z = 0.5/k_p = 840 \text{ nm}$$

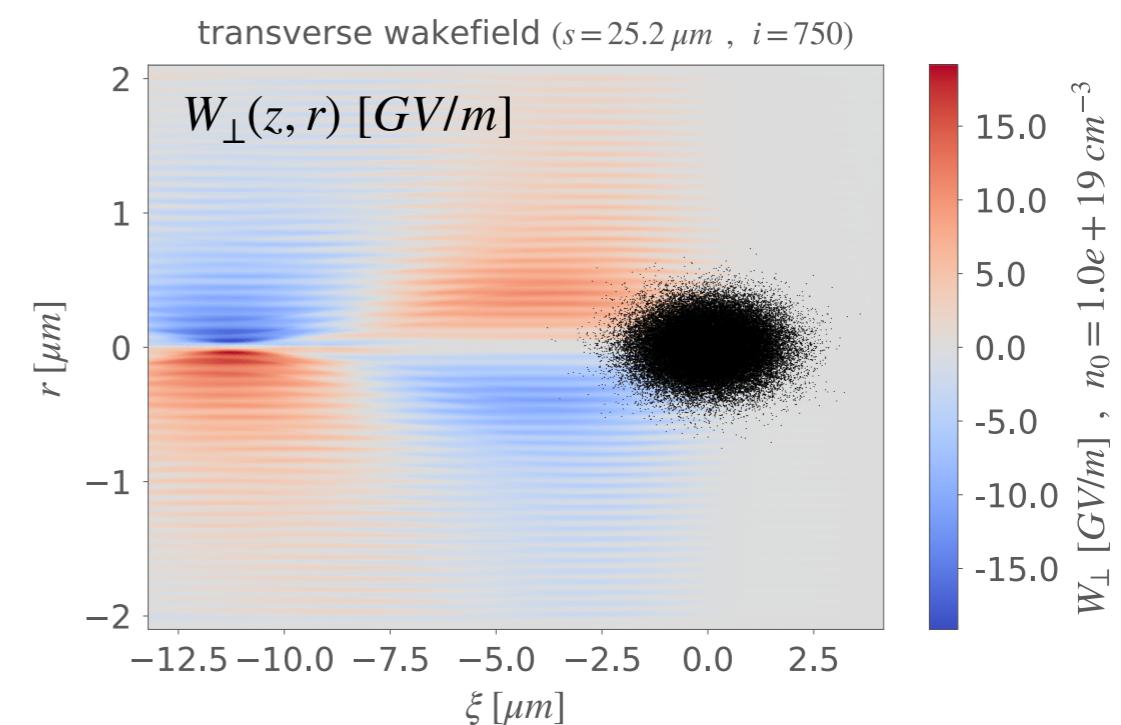
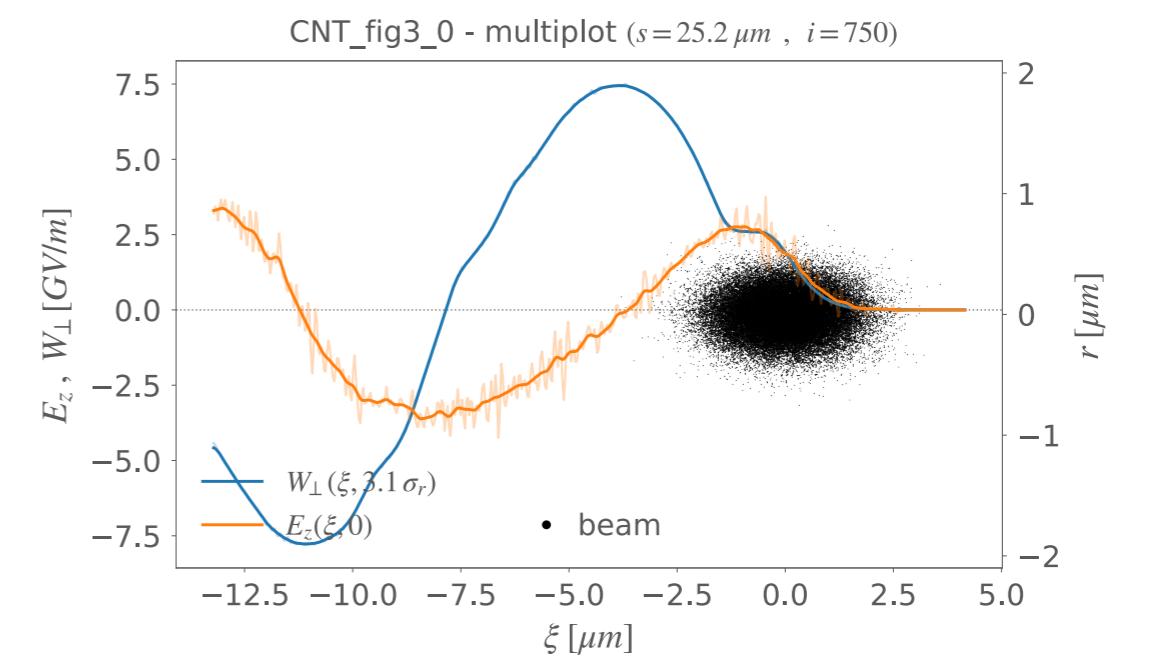
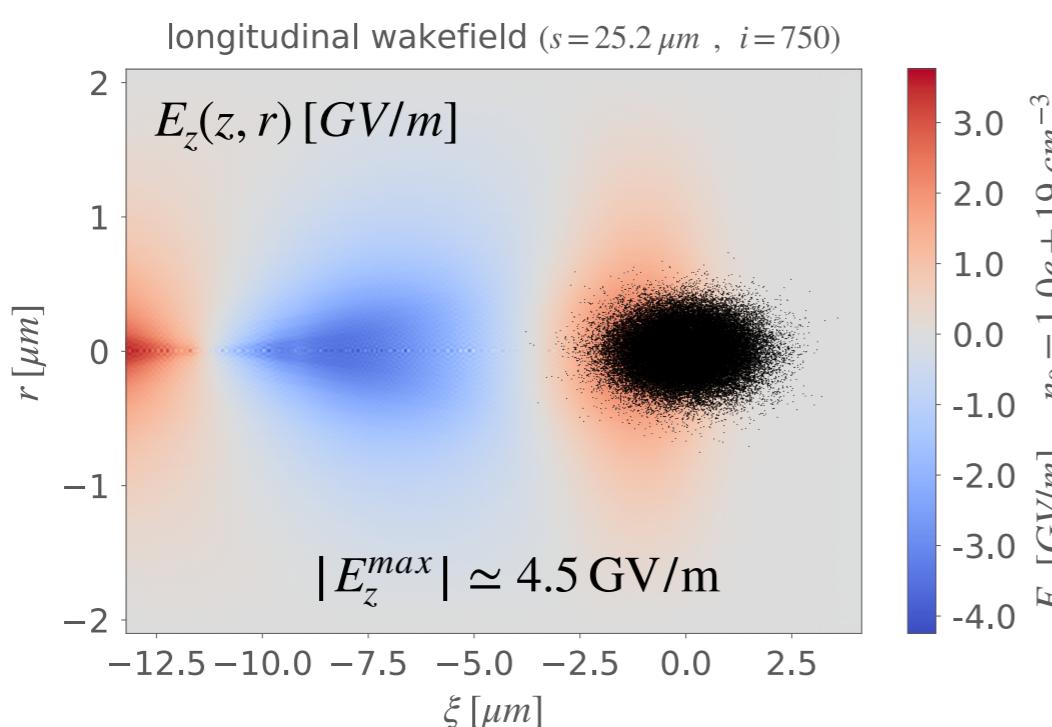
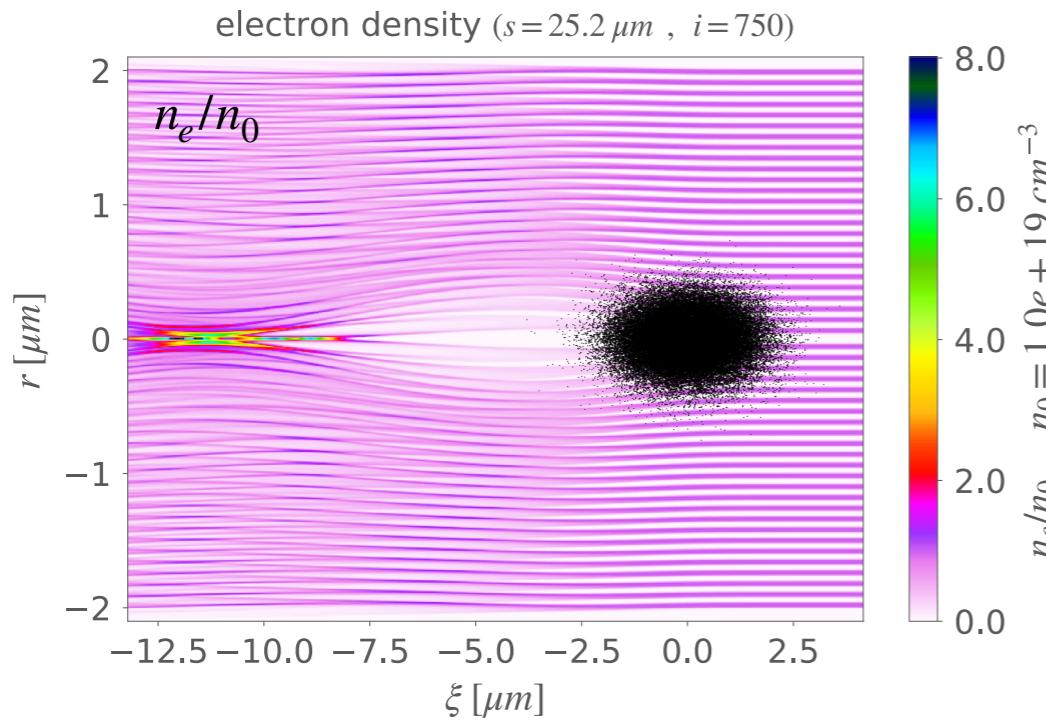
$$\sigma_r = 0.1/k_p = 168 \text{ nm}$$

$$E_{k0} = 200 \text{ MeV} (\delta = 1\%)$$

³[Resta-Lopez, J., et al., IPAC18 proceedings \(2018\)](#)

Beam-driven wakefield - CNT array

CNT array in 2D axisymmetric geometry



Beam-driven wakefield - CNT array

Beam density (n_b/n_0) scan

- Same parameters from J. Resta-Lopez³,

CNT array: $L_p = 110 \mu\text{m}$
 $n_0 = 10^{25} \text{ m}^{-3}$
 $r_{in,0} = 20 \text{ nm}$
wall = gap = 40 nm

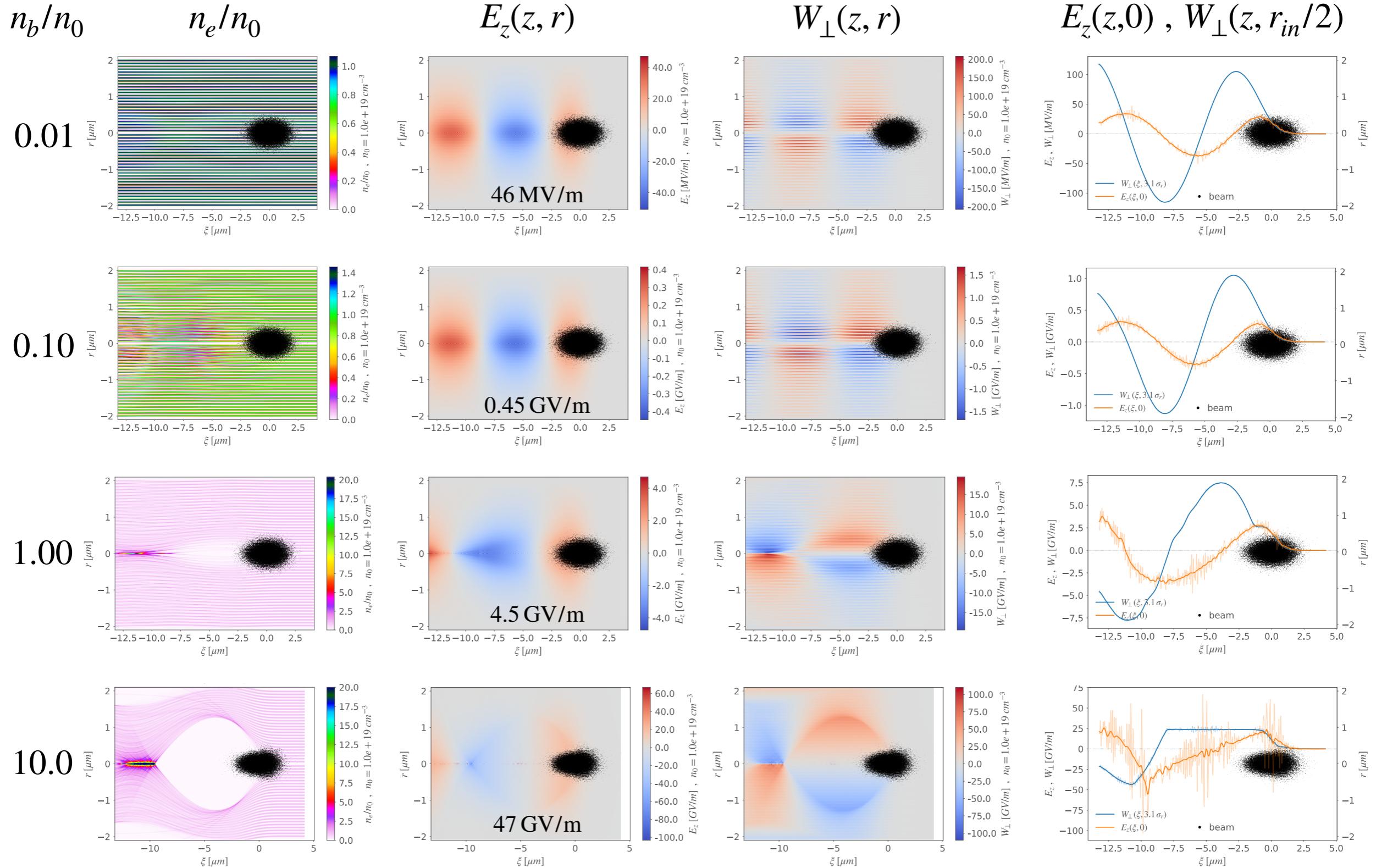
Beam: $\sigma_z = 0.5/k_p = 840 \text{ nm}$
 $\sigma_r = 0.1/k_p = 168 \text{ nm}$
 $E_{k0} = 200 \text{ MeV } (\delta = 1\%)$

$$n_b/n_0 = \{0.01, 0.1, 1, 10\}$$

³[Resta-Lopez, J., et al., IPAC18 proceedings \(2018\)](#)

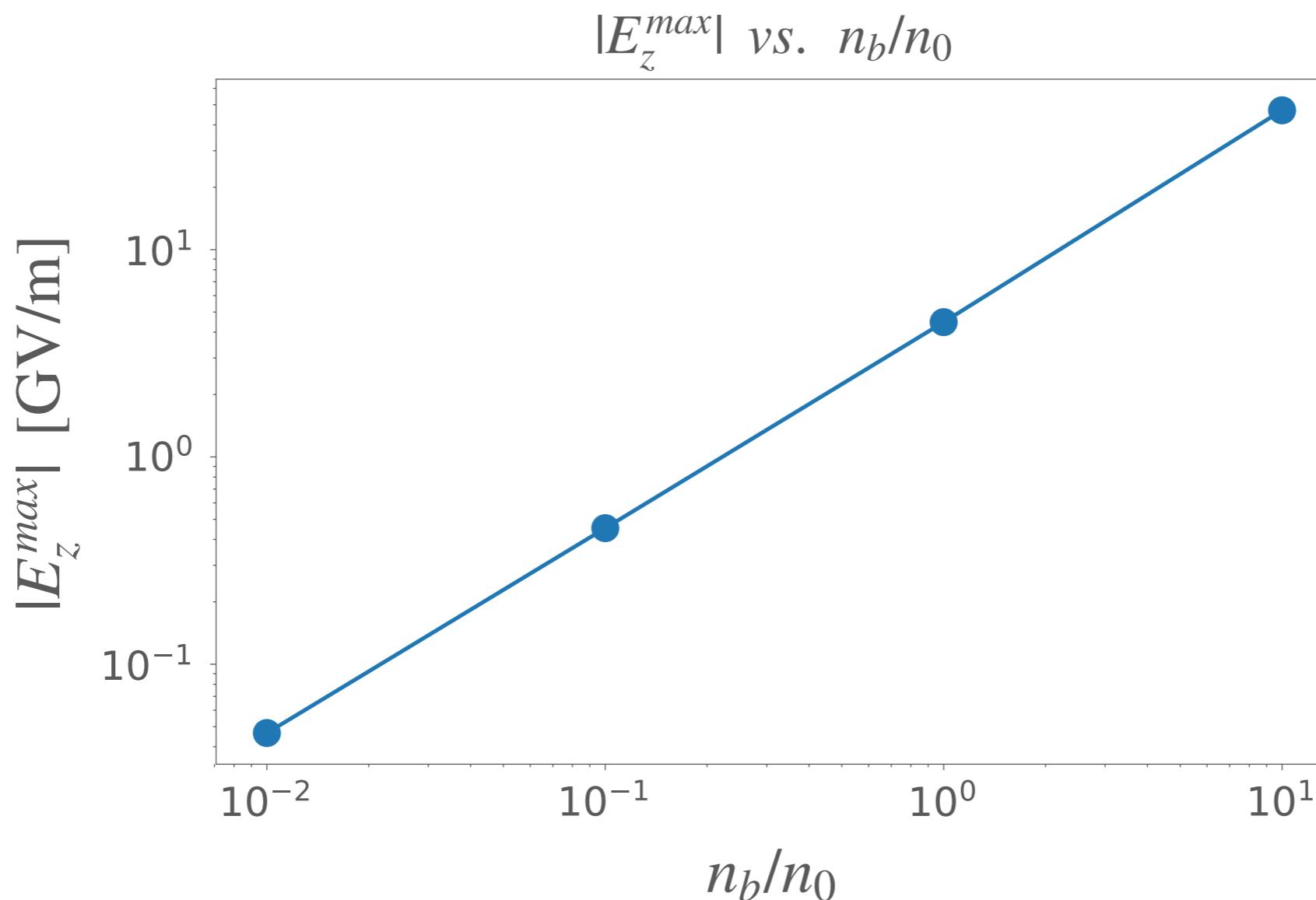
Beam-driven wakefield - CNT array

Beam density scan - $n_b/n_0 = [0.01, 0.1, 1, 10]$ - $s \approx 56 \mu\text{m}$



Beam-driven wakefield - CNT array

Beam density scan - $n_b/n_0 = [0.01, 0.1, 1, 10]$

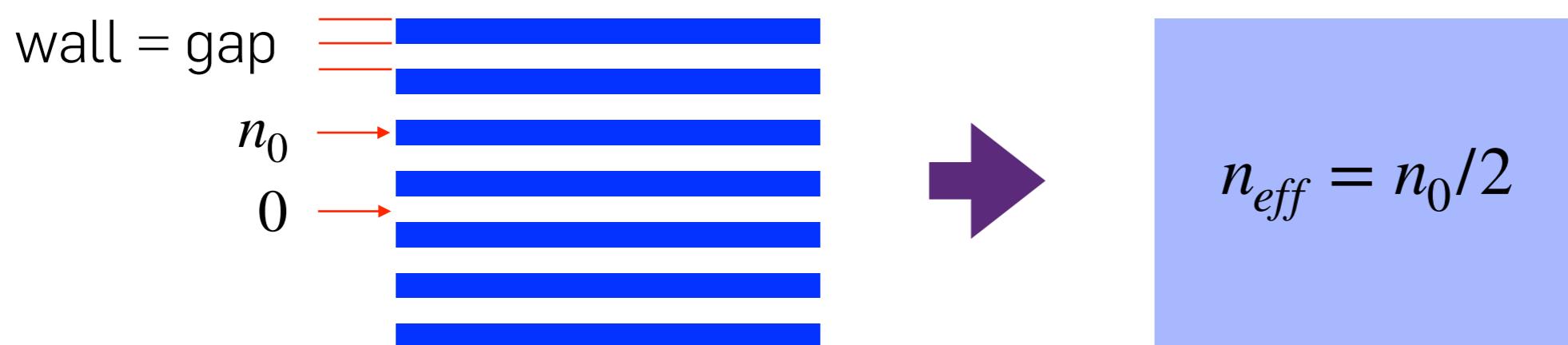


Beam-driven wakefield - CNT array

Beam density scan - $n_b/n_0 = [0.01, 0.1, 1, 10]$

Could the linear model, for a constant density plasma be used to estimate the wakefield amplitude in a CNT array?

- Plasma effective density (n_{eff}):



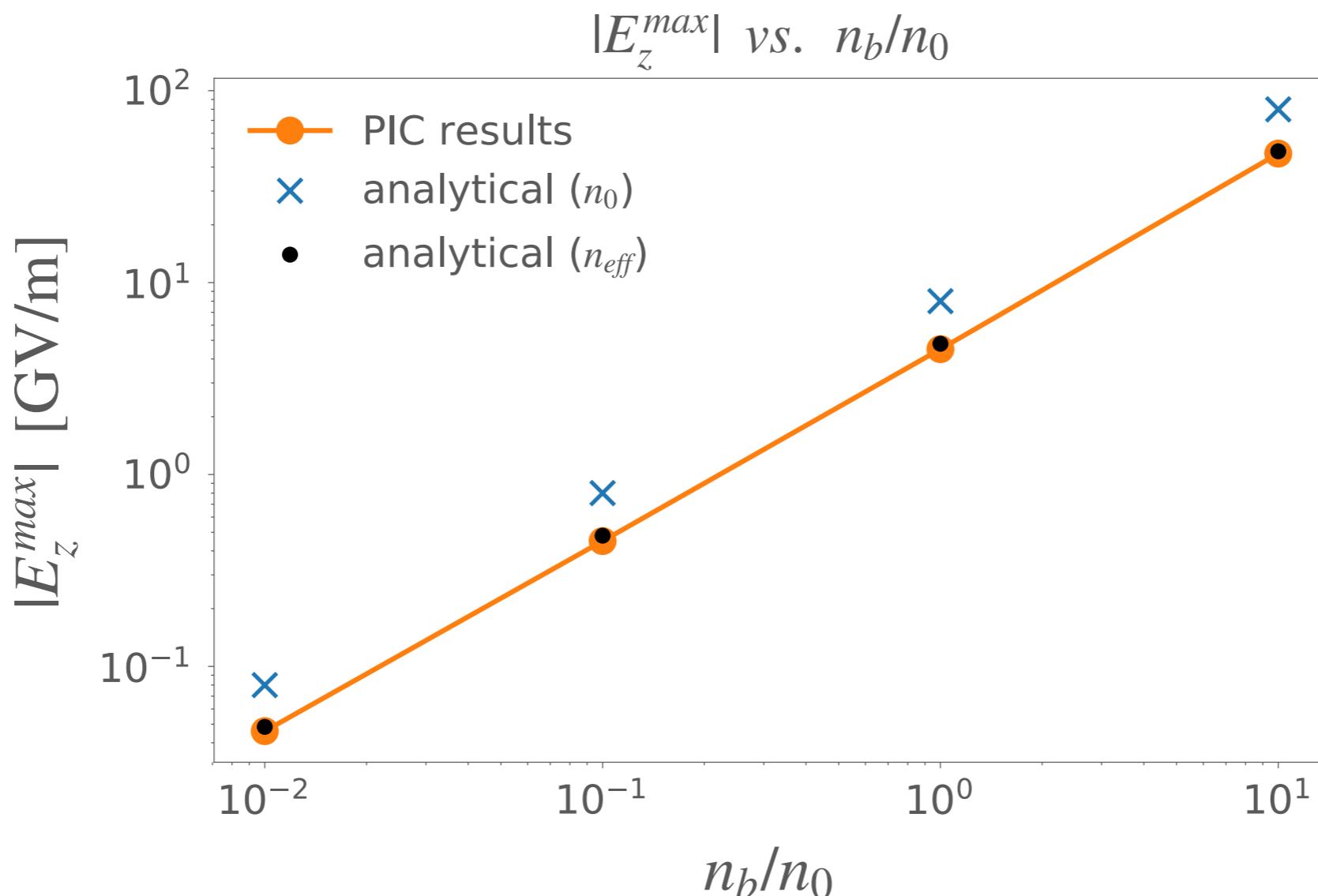
Beam-driven wakefield - CNT array

Beam density scan - $n_b/n_0 = [0.01, 0.1, 1, 10]$

n_b/n_0 (n_b/n_{eff})	analytical results: n_0 , n_b/n_0	analytical results: n_{eff} , n_{eff}/n_0	PIC simulation results
0.01 (0.02)	79.9 MV/m	48.4 MV/m	46 MV/m
0.1 (0.2)	0.80 GV/m	0.48 GV/m	0.45 GV/m
1 (2)	8.0 GV/m	4.8 GV/m	4.5 GV/m
10 (20)	79.9 GV/m	48.3 GV/m	47.0 GV/m

Beam-driven wakefield - CNT array

Beam density scan - $n_b/n_0 = [0.01, 0.1, 1, 10]$



$$n_{eff} = n_0/2 = 0.5 \times 10^{25} m^{-3}$$

$$n_b/n_{eff} = n_b/(n_0/2) = 2 n_b/n_0 = \{0.02, 0.2, 2, 20\}$$

X-ray driven wakefield - preliminary results

X-ray driven wakefield (Parameters based on X. Zhang, T. Tajima *et al.*⁴)

- X-ray laser:

$$\lambda_0 = 1 \text{ nm} \Rightarrow \epsilon_{photons} \simeq 1.2 \text{ KeV}$$

$$a_0 = 4 \Rightarrow I_0 = 2.2 \text{ W/cm}^2$$

$$\sigma_L = 3 \text{ nm}$$

$$r_0 = 5 \text{ nm (waist)}$$

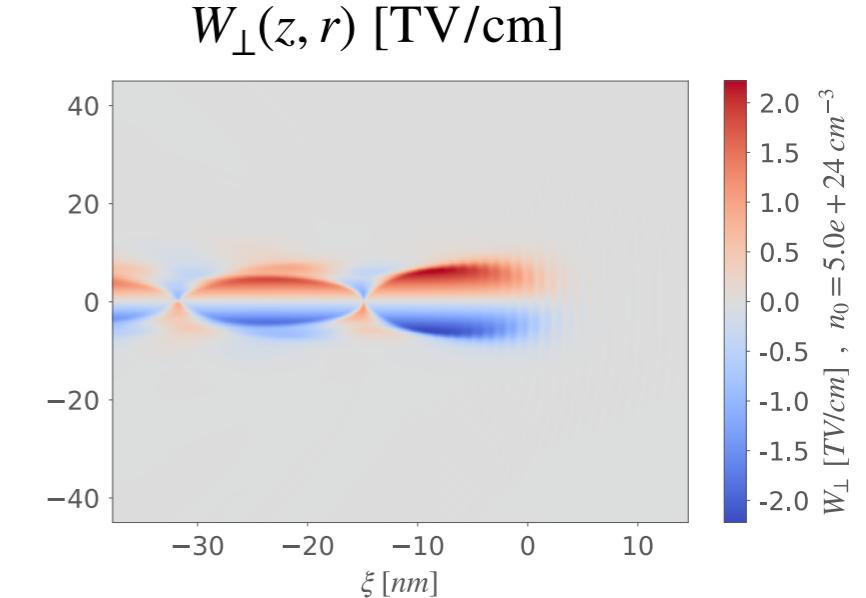
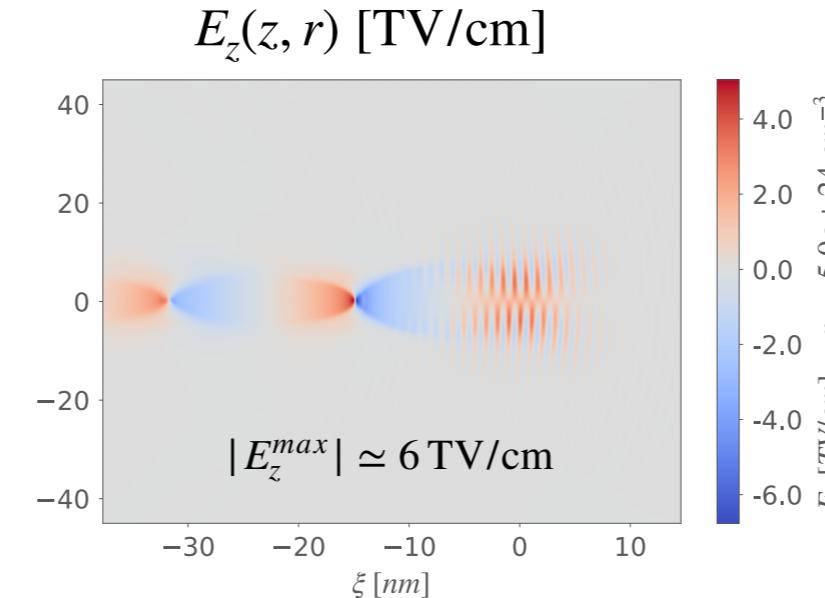
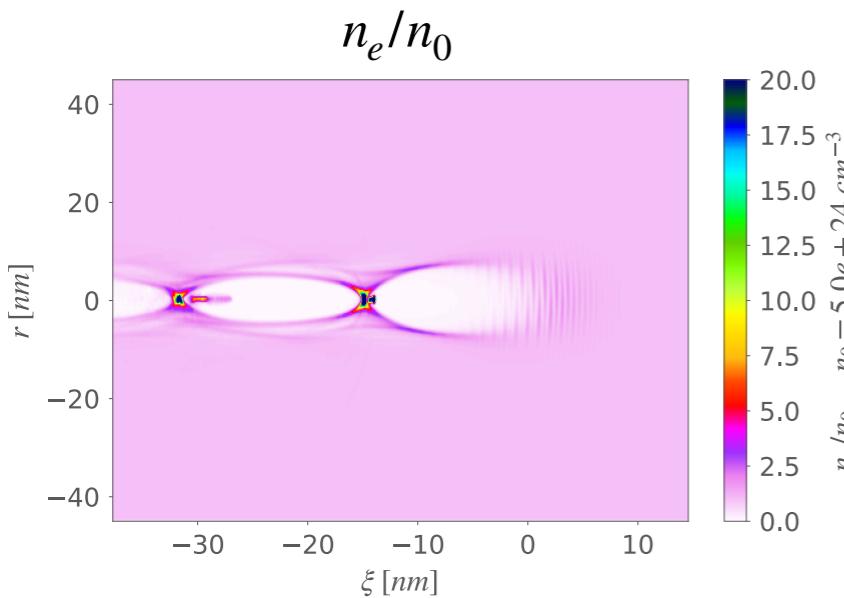
- Plasma:

$$n_0 = 5 \times 10^{30} \text{ m}^{-3}$$

$$L_p = 8000 \lambda_0 = 8 \mu\text{m}$$

$$\lambda_p \simeq 15 \text{ nm}$$

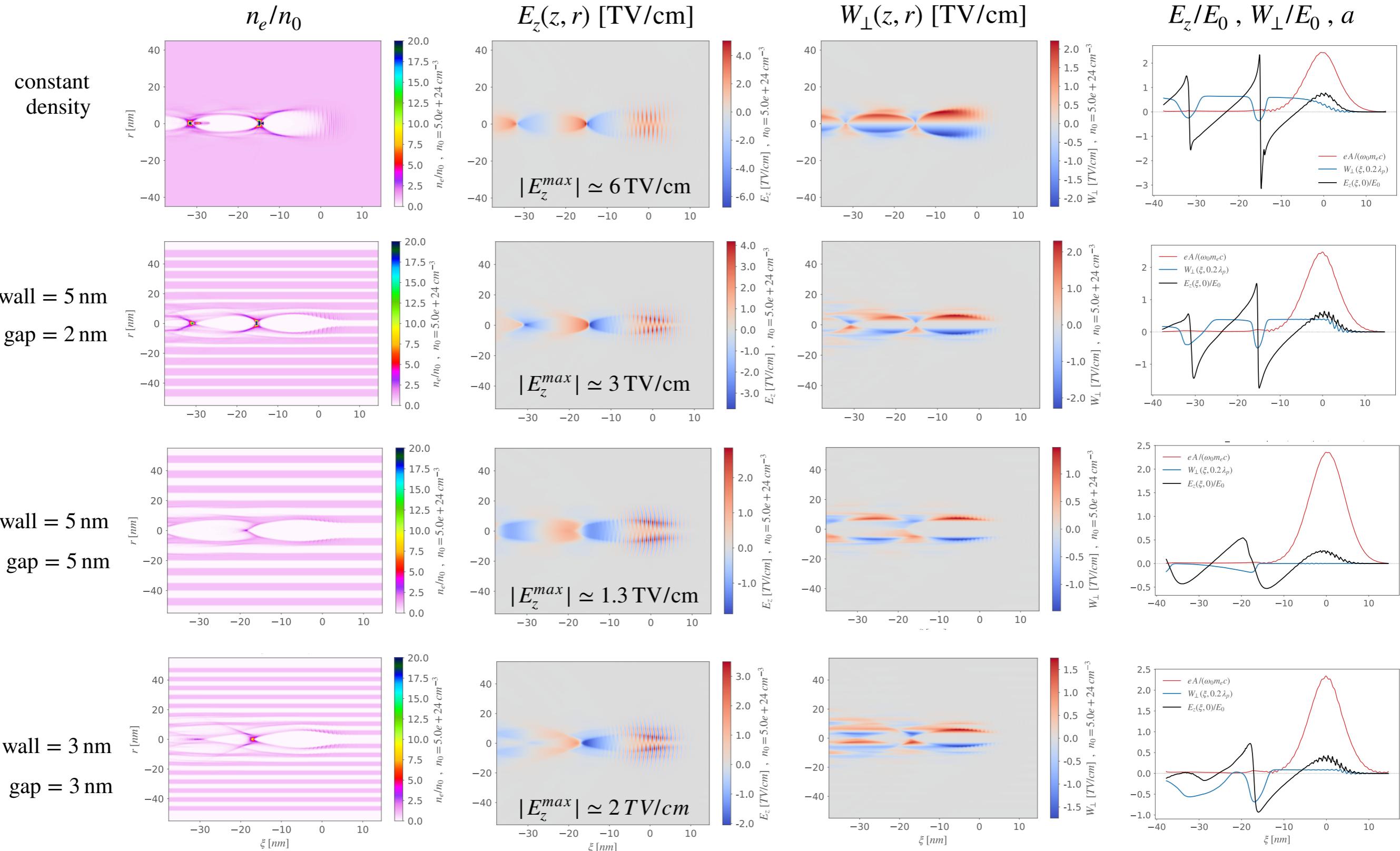
- Uniform plasma density



⁴Zhang, X.; Tajima, T., Phys. Rev. Accel. Beams 19, 101004 (2016)

X-ray driven wakefield - preliminary results

Constant plasma density \rightarrow CNT array ($s \simeq 170$ nm)



Summary

- Parameter scans might be helpful to determine the optimal CNT dimensions / aspect ratios to achieve higher amplitude wakefields.
- In comparison to 2D cartesian, the wakefield amplitudes obtained from 2D axisymmetric simulations are lower, matching analytical estimates from linear theory while in the linear regime.
- Since the wakefield is a collective, “macroscopic” effect, it might be possible use a uniform plasma, with an effective density n_{eff} , to describe the behaviour of a plasma with a micro-structure, such as a CNT array.
- In order to be valid, this approach requires some conditions to be attended (micro-structure must be periodic, with a small period if compared to the plasma and driver size).

Summary

- Regarding the ions, no appreciable motion was observed within the relevant timescales.
- For the investigated set of parameters, setting the ions as a fixed neutralising background would be accurate, and less computationally expensive.

Future plans

- Develop a way to constrain the transverse motion of plasma electrons within the CNT walls and its vicinities.
- Perform simulations adopting CNT dimensions and driver parameters compatible with facilities that could potentially perform experiments.
- In particular, we are interested in performing a parameter scan based on European X-FEL parameters.
- Further investigate the validity of the effective density approach to describe wakefields in CNT arrays.

Thank you

Thanks

Special thanks to:

- Javier Resta-López
(Univ. of Liverpool)
- Cristian Bontoiu
(Univ. of Liverpool)