$B - \bar{B}$ mixing: decay matrix at high precision

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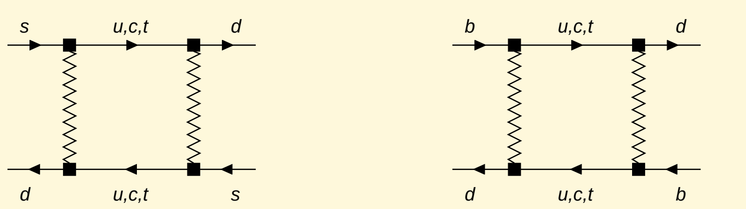
Meson-antimeson mixing

The flavoured neutral mesons $M = K, D, B_d, B_s$ mix with their antiparticles:

$\rightarrow$ mass eigenstates $\neq$ flavour eigenstates

$\rightarrow$ $M - \overline{M}$ oscillations

$\Rightarrow$ gold mine for studies of CP violation



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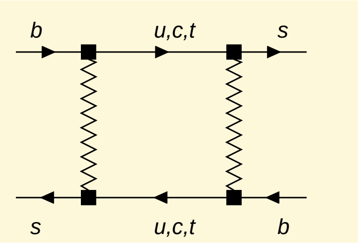
Diagonalise $M^s - i\,\frac{\Gamma^s}{2}$ to find the two mass eigenstates:

$$\begin{aligned} |B_L\rangle &= p|B_s\rangle + q|\overline{B}_s\rangle. \\ |B_H\rangle &= p|B_s\rangle - q|\overline{B}_s\rangle \end{aligned}$$

with masses $M^s_{L,H}$ and widths $\Gamma^s_{L,H}$.



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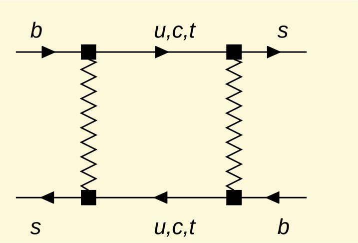
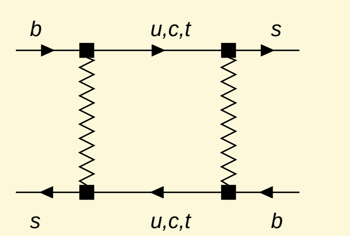


Mass and width differences:

$$\Delta m_s = M_H - M_L \simeq 2|M^s_{12}|,$$ equals oscillation frequency

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \simeq 2|\Gamma^s_{12}|\cos\phi^{(s)}$$ where $\phi^{(s)} \equiv \arg\left(-\frac{M^s_{12}}{\Gamma^s_{12}}\right) \sim 0$

The mass difference Δm_s is calculated from the dispersive part of the box diagram, which is dominated by virtual t quarks.



The width difference $\Delta\Gamma_s$ stems from the absorptive part of the box diagram, involving the light u,c quarks on the internal lines.

Theoretical predictions for Δm_s and $\Delta\Gamma_s$ are sums of terms looking like

$$|V_{tb} V_{ts}|^2 \times \text{perturbative coefficient} \times \text{hadronic matrix element}$$

$|V_{ts}| \simeq |V_{cb}|$ introduces a parametric uncertainty and is affected by the $|V_{cb}|$ tragedy that inclusive and exclusive decays give different values for $|V_{cb}|$ ($\Rightarrow$ 13% uncertainty in Δm_s).

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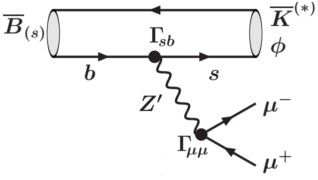
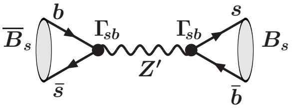
Matrix elements like $\langle B_s|\bar{s}\gamma^\mu(1-\gamma_5)b\ \bar{s}\gamma^\mu(1-\gamma_5)b|\overline{B}_s\rangle$ are calculated with lattice QCD or QCD sum rules and are sources of hadronic uncertainty.

Δm_s probes new physics from heavy virtual particles (with masses beyond 100 TeV).

Example:

A hypothetical Z' with FCNC coupling could accomodate the $b \to s\mu^+\mu^-$ anomalies, but this scenario is severely constrained by $B_s - \bar{B}_s$ mixing data:





$\Delta\Gamma_s$ is less competitive than Δm_s in probing heavy new physics, but probes new physics with light virtual or real particles.

Other virtue: We can normalise Δm_s to $\Delta\Gamma_s$!

Mistakes in hadronic calculations can mimick new physics in Δm_s. The $|V_{cb}|$ controversy feeds into $|V_{ts}|$ and limits the precision of the Δm_s prediction.

But: Information from $\Delta\Gamma_s$ can distinguish new physics from hadronic and parametric uncertainties.

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But: Information from $\Delta\Gamma_s$ can distinguish new physics from hadronic and parametric uncertainties.

$|V_{ts}|$ and most of the hadronic uncertainty drops out from $\frac{\Delta\Gamma_s}{\Delta m_s}$:

If $\frac{\Delta\Gamma_s^{\rm exp}}{\Delta m_s^{\rm exp}}$ agrees with the SM prediction, there is no new physics in Δm_s. Any potential discrepancy in Δm_s would then be due to $|V_{ts}| \simeq |V_{cb}|$ or incorrect assessments of hadronic uncertainty.

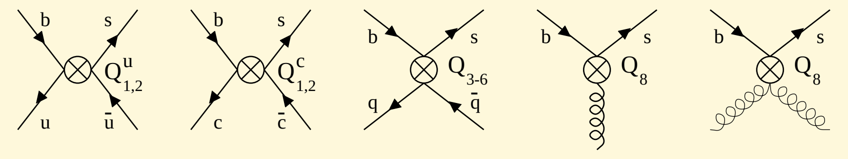
Effective hamiltonian

$$H^{|\Delta B|=1} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1}^{2} C_i \left(V_{ub} V_{us}^* Q_i^u + V_{cb} V_{cs}^* Q_i^c \right) + V_{tb} V_{ts}^* \sum_{i \geq 3} C_i Q_i \right]$$

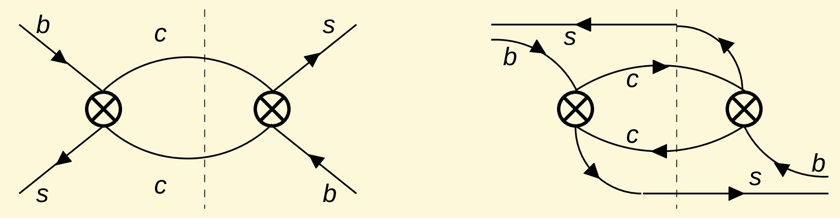
Q_i: effective $|\Delta B| = 1$ operators, e.g.
$Q_2^c = \overline{c}\gamma_\mu(1-\gamma_5)b\, \overline{d}\gamma^\mu(1-\gamma_5)c$

C_i: Wilson coefficients = effective couplings, contain short distance structure including QCD corrections, depend on $\frac{m_t}{M_W}$.

$Q_{3-6,8}$ are penguin operators

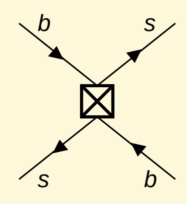


Leading contribution to $\Delta\Gamma_s$:



$\Delta\Gamma_s$ stems from Cabibbo-favoured tree-level $b \to c\bar{c}s$ decays.

Heavy Quark Expansion (HQE):
Exploit $m_b \gg \Lambda_{QCD}$ to express $\Delta\Gamma_s$ in terms of short-distance coefficients and matrix elements of local $|\Delta B| = 2$ operators.



$\Rightarrow$ expansion of $\Delta\Gamma_s$ in $\alpha_s(m_b)$ and Λ_{QCD}/m_b.

Operators at leading order in Λ_{QCD}/m_b (leading power):

$$Q = \bar{s}_i \gamma_\mu (1 - \gamma_5) b_i \; \bar{s}_j \gamma^\mu (1 - \gamma_5) b_j, \quad \widetilde{Q}_S = \bar{s}_i (1 + \gamma_5) b_j \; \bar{s}_j (1 + \gamma_5) b_i.$$

i, j are colour indices.

Matrix elements:

$$\langle B_s | Q | \overline{B}_s \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B$$

$$\langle B_s | \widetilde{Q}_S | \overline{B}_s \rangle \; = \; \frac{1}{3} M_{B_s}^2 f_{B_s}^2 \widetilde{B}_S'.$$

Here $f_{B_s} = (0.2307 \pm 0.0013)\,\text{GeV}$ is the B_s decay constant and $M_{B_s} = 5.37\,\text{GeV}$ is the B_s mass. In the $\overline{\text{MS}}$ scheme:

$$B = 0.813 \pm 0.034 \qquad \text{and} \qquad \widetilde{B}_S' = 1.31 \pm 0.09.$$

The HQE gives

$$\Delta\Gamma_s = \frac{G_F^2 m_b^2}{12\pi M_{B_s}} |V_{cs}^* V_{cb}|^2 \left| G' \langle B_s|Q|\bar{B}_s\rangle + \widetilde{G}_S \langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle \right|$$

with the perturbative coefficients G',$\widetilde{G}_S$.

The coefficients G',$\widetilde{G}_S$ emerging from the calculation correspond to the choice $m_b = m_b^{\mathrm{pole}}$ in the prefactor. Subsequently one may switch to the $\overline{\mathrm{MS}}$ definition $\bar{m}_b$ through e.g.

$$\widetilde{G}_S^{\overline{\mathrm{MS}}} \equiv \frac{m_b^{\mathrm{pole}\,2}}{\bar{m}_b^2} \widetilde{G}_S$$

and expanding in α_s to the order in which G',$\widetilde{G}_S$ are calculated.

Experiment (HFLAV 2021): $\Delta\Gamma^{\text{exp}} = (0.082 \pm 0.005)\,\text{ps}^{-1}$
average from LHCb, CMS, and CDF data.

Theory prediction with QCD corrections at next-to-leading order (NLO):

$$\Delta\Gamma_s = \left(0.077 \pm 0.015_{\text{pert}} \pm 0.002_{B,\widetilde{B}_S} \pm 0.017_{\Lambda_{QCD}/m_b}\right) \text{ GeV} \qquad \text{(pole)}$$

$$\Delta\Gamma_s = \left(0.088 \pm 0.011_{\text{pert}} \pm 0.002_{B,\widetilde{B}_S} \pm 0.014_{\Lambda_{QCD}/m_b}\right) \text{ GeV} \qquad (\overline{\text{MS}})$$

H.M. Asatrian, H.H. Asatryan, A. Hovhannisyan, S. Tumasyan, UN, A. Yeghiazaryan, Phys.Rev. D102 (2020) no.3, 033007

based on the NLO calculations in

M. Beneke, G. Buchalla, C. Greub, A. Lenz, UN, PLB459 (1999) 631
A. Lenz, UN, JHEP 0706 (2007) 072

The perturbative error exceeds the experimental error.

$\Rightarrow$ need NNLO!

NNLO

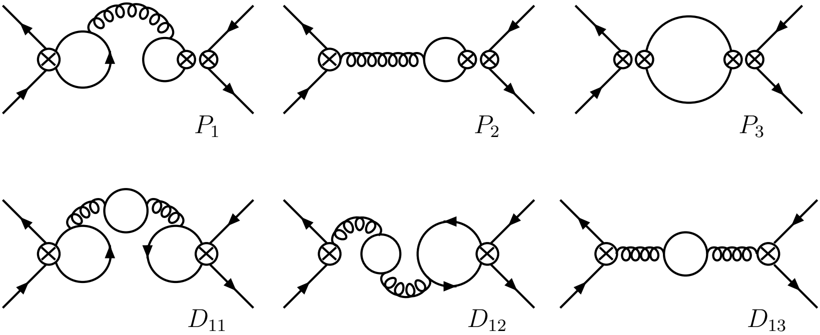
The NNLO calculation involves propagator-type three-loop diagrams with the two masses m_c and m_b.

First step: diagrams with closed fermion loop (large-N_f limit) and neglecting $\mathcal{O}(m_c^2/m_b^2)$ terms.

H.M. Asatrian, A. Hovhannisyan, A. Yeghiazaryan, UN, JHEP 1710 (2017) 191

Second step: NNLO penguin effects diagrams with closed fermion loop (large-N_f limit) and exact m_c dependence.

H.M. Asatrian, H.H. Asatryan, A. Hovhannisyan, S. Tumasyan, UN, A. Yeghiazaryan, Phys.Rev. D102 (2020) no.3, 033007.



The penguin coefficients $C_{3-6} = \mathcal{O}(0.05)$ are counted as α_s.

NNLO status

The large-N_f pieces of the NNLO corrections do not improve the predictions of $\Delta\Gamma_s$.

The full NNLO calculation of $\Delta\Gamma_s$ is needed!
This is an ongoing long-term project in the Collaborative Research Center *Particle Physics Phenomenology after the Higgs Disvovery (P^3H)*.

Team: Marvin Gerlach, Vlad Shtabovenko, Matthias Steinhauser, UN

First results:

- verification of the NLO results in a different operator basis
- NNLO penguin results beyond the large-N_f limit
  M.Gerlach et al., JHEP07(2021)043 [arXiv:2106.05979]
- 3-loop integrals for $m_c = 0$

Expect NNLO result for $\Delta\Gamma_s$ (with $m_c = 0$ in α_s^2 piece) within the next two months.

Summary and outlook

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- By confronting the measured $\Delta\Gamma_s^{\rm exp}, \Delta m_s^{\rm exp}$ with the calculated $\Delta\Gamma_s/\Delta m_s$ we can bypass the parametric and most of the hadronic uncertainty.
- All calculations equally apply to $\Delta\Gamma_d$.

Backup

CP asymmetries in flavour-specific (typically semi-leptonic) decays:

$$a_{\text{fs}}^q = \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to \bar{f})}{\Gamma(\bar{B}_q(t) \to f) + \Gamma(B_q(t) \to \bar{f})} \qquad \text{with } q = d, s$$

$$a_{\rm fs}^{s,\rm exp} = (60 \pm 280) \cdot 10^{-5}$$

from semi-leptonic decays.

NLO update:

$$a_{\rm fs}^{s} = (2.07 \pm 0.08_{\rm pert} \pm 0.02_{B,\widetilde{B}_S} \pm 0.05_{\Lambda_{QCD}/m_b} \pm 0.04_{\rm CKM}) \times 10^{-5} \quad (\text{pole}),$$

$$a_{\rm fs}^{s} = (2.04 \pm 0.09_{\rm pert} \pm 0.02_{B,\widetilde{B}_S} \pm 0.04_{\Lambda_{QCD}/m_b} \pm 0.04_{\rm CKM}) \times 10^{-5} \quad (\overline{\rm MS}),$$

H.M. Asatrian, H.H. Asatryan, A. Hovhannisyan, S. Tumasyan, UN, A. Yeghiazaryan, Phys.Rev. D102 (2020) no.3, 033007

based on the NLO calculations in

M. Beneke, G. Buchalla, A. Lenz, UN, PLB576 (2003) 173

A. Lenz, UN, JHEP 0706 (2007) 072

$$a_{\rm fs}^{d,\rm exp} = (-21 \pm 17) \cdot 10^{-4}$$

from semi-leptonic decays.

NLO update:

$$\begin{aligned} a_{\rm fs}^d &= -(4.71 \pm 0.18_{\rm pert} \pm 0.04_{B,\widetilde{B}_S} \pm 0.11_{\Lambda_{QCD}/m_b} \\ &\qquad \pm 0.10_{\rm CKM}) \times 10^{-4} \quad ({\rm pole}), \\ a_{\rm fs}^d &= -(4.64 \pm 0.21_{\rm pert} \pm 0.04_{B,\widetilde{B}_S} \pm 0.09_{\Lambda_{QCD}/m_b} \\ &\qquad \pm 0.10_{\rm CKM}) \times 10^{-4} \quad (\overline{\rm MS}). \end{aligned}$$

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