Discreteness and Determinism in Quantum Mechanics.

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Gerard 't Hooft

Bergen, Norway,

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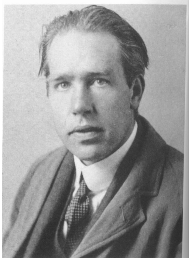
In the 1920s, a group of physicist, in their discussions at the physics institute in Copenhagen, reached agreements as to what the theory of quantum mechanics says, and how to work with it. There was one issue where agreement was more difficult to obtain:



What is really going on, in a quantum system as we describe it?

But finally they did agree: It is amazing how well the theory predicts all probabilities *without* the need to answer this last question. Therefore:

Do not ask the question; there is no way to answer it by doing experiments. *"Shut up and calculate!"*

Their package of prescriptions and equations is called the Copenhagen interpretation. It is entirely correct,

but ...

But I disagree with the last verdict !

By asking what it might be that is 'truly happening',
one will learn more about our physical world.

There are things that we cannot find out unless we try to explain what quantum mechanics is.

Take: The Standard Model of the Elementary Particles.

The interactions are determined by the coefficients for non-linear parts of the field equations: the coupling constants.

The field equations do not reveal what the strengths of these coupling constants should be expected to be, and why they have the values observed.

And then, there is the cosmological coupling constant, Λ.

Motivation of my work:

Understand the problem of "quantum gravity", which amounts to:
How to reconcile General Relativity (GR) with Quantum Mechanics,
and: what determines the SM constants of nature.

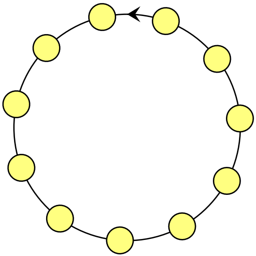
We understand GR quite well. *My claim:* Today's obstacle is our lack of understanding what quantum mechanics really is.

The aim of this talk is to do away with a popular view of quantum mechanics (QM) as "something mysterious that nobody can understand".

In fact, one can understand QM very well as a *vector representation* of any kind of ordinary, deterministic, realistic evolution law for physical data. There is no such thing as 'quantum logic'.

The 'truth' requires only standard logic.
This will be explained.

Basic Models



1. The periodic chain.

Ontological (= real) states:

$$|0\rangle,\ |1\rangle,\ \dots\ |N-1\rangle$$

Evolution law:

$$|k\rangle_{t+\delta t} = U(\delta t)\,|k\rangle_t$$

$$U(\delta t)|k\rangle = |k+1 \mod N\rangle$$

$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

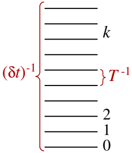
(Schrödinger Equation)

$$|n\rangle^E \stackrel{\text{def}}{=} \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} e^{2\pi ikn/N}|k\rangle^{\text{ont}}\ , \qquad k = 0,\cdots,\ N-1\ ;$$

$$n = 0,\cdots,\ N-1\ .$$

$$|k\rangle^{\text{ont}} = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} e^{-2\pi ikn/N}|n\rangle^E\ .$$

$$H = \frac{2\pi}{N\,\delta t}\,n = \omega n$$



QM was thought to require revision of logic. NOT SO. QM may well be nothing but the vector representation of an ordinary moving system.

In model previous slide: Given an initial state,

$$\psi(x,\, 0) = \delta(x - x_0)\ ,$$

The vector function of a classical system will keep this form. After k steps in time,

$$\psi(x,\, t) = \delta(x - x(t))\ .$$

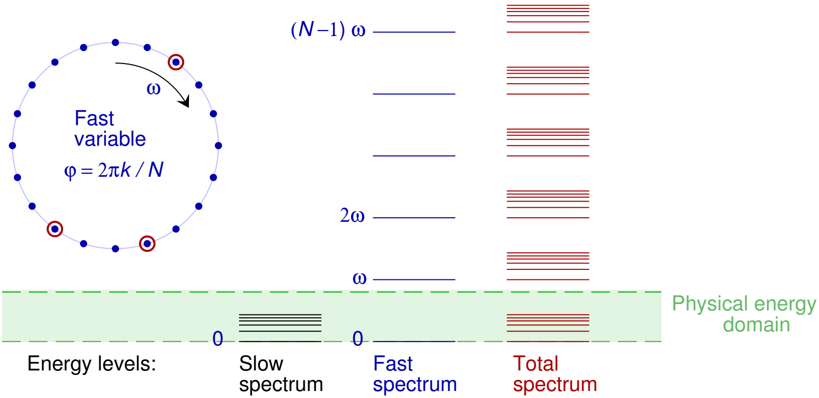
It is easy to find the H that does this.

Question: can a classically evolving system obey a Schrödinger equation that *does* smear the wave function?

Answer: *if hyper-fast classical variables are added to the system!*.

We now show how to understand the emergence of such a Schrödinger equation, without loosing locality.

Simplified approach: just one fast periodic variable $\varphi(t)$ with angular velocity $\omega \gg 10\,TeV$.



The slow spectrum can come from *interactions with the fast variable(s)!*
The zero-energy state of the fast variable is a *superposition* of real states.

Let's try to generate an equation as complicated as $H = \frac{1}{2}p^2 + V(x)$,

First, we have to put x on a lattice, and replace $\frac{1}{2}p^2 \to 1 - \cos p$.

Then the operator $\cos p$ is obtained by considering the permutations $P = \{|x\rangle \leftrightarrow |x+1\rangle\}$, separately for the even and the odd values of x.

Now this commutator would act at all time values $t = k\,\delta t$. That is much too fast.

Therefore, we postulate that this permutator only acts if *two* of the fast variables are in some given positions, η and η' see circles :

$$H_1 = \frac{\pi}{2\delta t}\sum_a \delta_{\varphi_1=\eta_a}\,\delta_{\varphi_2=\eta'_a}\,P\ , \quad \text{using} \quad e^{-\frac{1}{2}i\pi P} = -iP\ , \quad \text{if} \quad P = \pm 1$$

In the lowest energy state, $\langle \sum_a \delta_{\varphi_1=\eta_a}\,\delta_{\varphi_2=\eta'_a} \rangle = \dfrac{N_a}{N_1 N_2}$.

Note: we here use perturbation expansion. N_a/N_1N_2 is a small number.

Thus, the Hamiltonian generates 'quantum' superpositions!

Doing this separately for the even and the odd x values, we now are in the position to achieve the Schroedinger equation for $H = \frac{1}{2}p^2$, for a deterministically evolving system.

It generates superpositions in the wave functions of the effective slow variable x, using perturbation expansion (very accurate here).

To get a term $V(x)$, we can add a binary fast variable $\sigma(t)$ (a variable that flip-flops rapidly). This gives it two energy states, again widely separated. As with the other fast variable(s), only the lowest energy state is occupied. $V(x)$ may be postulated to contain a term that affects the speed of the flipflops of σ, it similarly has to be limited to special values of the other fast variables, otherwise, $V(x)$ gets unphysical, large values.

Proposal: continue along these lines to get the Schrödinger equation for the SM.

Quantum Mechanics uses complex numbers. Why are wave functions always complex?

Answer: complex numbers are pairs of real numbers: $\psi = \Re + i\Im$.

This means that there exists one single binary variable $\sigma(t)$ that takes two values. It flipflops as time proceeds, and so, it functions as our clock. Energy is dual to this binary clock. Quantum mechanics emerges as soon as we switch from the classical binary 'observable' $\sigma(t)$ to its dual observable, energy.
From this point of view, energy is not classically observable. On the other hand, energy = frequency, and of course observable.

Note: the faster our variable flipflops, the lower the energy.

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Our constructions are so easy that one may well wonder what the causes are of the numerous *quantum paradoxes:*

Quantum paradoxes:

- EPR Paradox
- Bell's theorem, Clauser - Horne - Shimony - Holt (CHSH) inequality
- Measurement problems, Collapse problem
- Entanglement
- Schrödinger's cat
- Greenberger - Horne - Zeilinger (GHZ) paradox (etc.)

Mostly come about out of lack of understanding how classical evolution laws may operate:

Most of these models use spin degrees of freedom, assuming nature has rotation symmetry. Are these to be worried about, if we formulate our theories on a lattice?

There is continuous rotation invariance, but only in the vector representation! Whether this holds up at different time scales is not known.

Our procedure: 1. Take a quantum theory, in a given basis.
Suppose the wave function of your initial state is a delta peak:
$\langle k|k_1\rangle = \delta_{k,k_1}$
Then, we compute the probabilities. No paradox yet.

Paradox comes if some of your observers change their minds as of what to observe.

But then, they choose to be in a quantum superposition ...

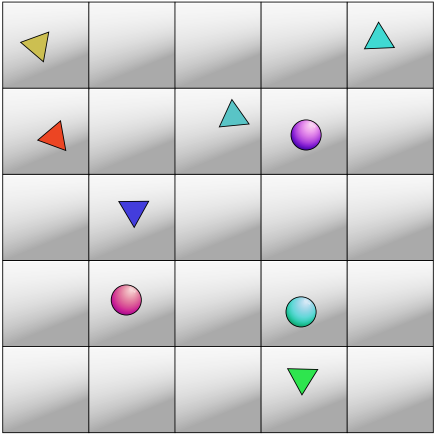
In the *Cellular Automaton Interpretation,* this is forbidden. Observers have the 'free will' to choose a classical state for the settings of their detectors, but they can't choose to be in a superposition.

We can construct a setting that will put the observers in a superposition,

But this requires a different basis of states, so that we use entirely different 'beables'. And then we also take away the observer's 'free will'.

Is there still a paradox? A strange feature of this theory is its *internal symmetries.* Rotations require the vector representation. Is this a 'feature' or a 'bug'?

The Cellular Automaton: Only *classical* evolution equations.



Quantum field lattice: same with *quantum* evolution equations.

The ‘fast variables’ are completely localised: at least one such variable in each CA cell.
In that case, there seems to be no problem with locality.
All *commutators* between all observables vanish outside the light cone.
This guarantees that no information can be sent faster than some limiting speed.

Unsolved problem: how to recover Lorentz invariance on a space-time lattice. It seems to be very hard to get such large symmetry groups.

This topic will be hard to reconcile with GR.

Conjecture: Most symmetries of the SM have become continuous symmetries because they act on vectors, not on the classical variables.

At the Planck scale, there may be more symmetry.

This one concludes by using the familiar mathematical technique used in QM: The vector representation.

Consider a scalar field $\phi(x)$, quantised as usual in a QFT.
Its contribution to the Hamiltonian H is:

$$H = \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}m^2\,\phi^2 + \tfrac{\lambda}{4!}\,\phi^4 \ .$$

Measured at the Planck scale, $m \approx 10^{-19}$. Various authors (Veltman, Jegerlehner, ...) point out that, for the scalar Higgs field, it so happens that λ runs towards ≈ 0 at the Planck scale.
This may imply a new, accidental, global symmetry at the Planck scale:

$$\phi(x) \rightarrow \phi(x) + C^{\mathrm{nst}} \ ,$$

leaving Hamiltonian H invariant.

This Goldstone symmetry holds whenever a light, scalar particle has a mass and a coupling strength that both run to zero at the Planck scale.

Qu: Can we employ these symmetries to identify the automaton evolution rule?

Conclusion

In its present form, QM just allows for a certain amount of ignorance in our understanding. But in spite of the existence of this vector representation, our world can still be **fully** deterministic. This may be helpful for understanding the details of the Standard Model.

The usual 'quantum paradoxes' arise when one assumes that an observer can 'freely choose' between different observables for measurements. This seems to be allowed according to the Copenhagen rules. We now propose a small modification of these rules: There exists a class of 'beables': states that are all orthonormal, such that beables can only evolve into other beables. The universe started as a single beable.

Alice and Bob can only choose freely for another beable. If they want to choose any *superimposed* state, they actually need to modify the initial state entirely. All beables have different beables in their past. This is usually ignored when Bell and followers assume 'statistical independence'. As for "free will": in spite of what many people think, our brains are controlled by laws of nature just like the rest of our world. Sorry about that, but this is an unavoidable assumption.

And it is an important issue: without explaining what 'free will' amounts to in its mathematics, one will fail to understand the underlying physics on which QM is based.

Equations: A cat can may seem to be in superposition of "dead" and "alive". People try to attach strange versions of logic to that – According to the CA interpretation, a cat is always in a beable state, where 'dead' and 'alive' are easy to distinguish.

Yet the equations seem to say that superpositions are possible. This needs to be explained.
Equations present QM as vector representation of reality. This representation allows for an interpretation in terms of probabilities. But it doesn't have to be that. Example: permutation group. Note: this group is discrete., while its vector space is continuous.

We cannot prove that our world is deterministic, but QM doesn't prove that it isn't.
I showed some simple examples as to what QM could be.

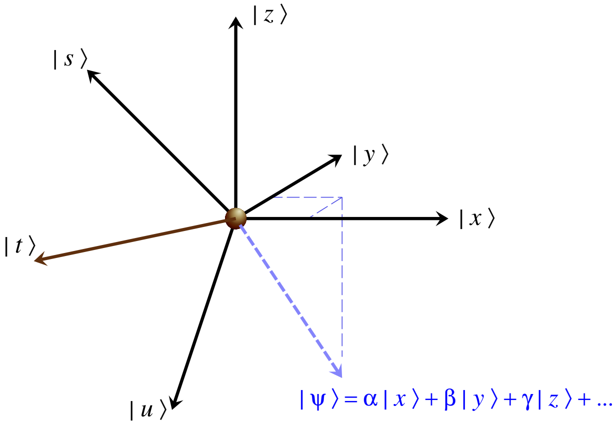
Cause of the quantum effects: many of the degrees of freedom of the nano-world fluctuate too fast to be followed by observers,

- We must work with MODELS. Then use *quantum notation*

If you believe in determinism, you have to believe it all the way. This is very hard for some people. They want their 'free will'.

Important problem: in spite of theory being discrete, symmetries can be continuous.

The vector representation for 6 beable states.



Schematic set-up of a 6 dimensional vector space showing the 6 basis elements $|x\rangle$, $|y\rangle$, $|z\rangle$, $|s\rangle$, $|t\rangle$ and $|u\rangle$, and a superimposed state $|\psi\rangle$. Nature can only be in one of the basis elements.

THANK YOU