On the equivalence between Starobinsky and Higgs inflation in gravity and supergravity

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WHY INFLATION in THE EARLY UNIVERSE?

- Cosmological inflation (a phase of quasi-de-Sitter accelerated expansion with an exit) was proposed to explain homogeneity and spatial flatness of our Universe at large scales, its large size and entropy; inflation can explain the almost scale-invariant spectrum of CMB radiation; cosmological perturbations from quantum fluctuations during inflation can seed the CMB anisotropy and the LSS.
- Inflation is a paradigm, not a theory! Theoretical mechanisms of inflation use a driver (called inflaton field) with proper scalar potential.
- The physical nature and origin of inflaton and scalar potential, as well as its interactions with other fields are the big mysteries.
- There is a more fundamental (vs. phenomenological) way of thinking about inflation, and it is given by *supergravity* and *string theory*.
  Inflation is the very HEP phenomenon (10^{13} GeV) !

WHY SUPERGRAVITY?

- Supergravity is a field theory with local SUSY that automatically implies GR.
- Supergravity is the only way to consistently describe a spin-3/2 field in GR;
- Supergravity remains the primary candidate for new physics beyond the SM; it connects gravity to particle physics, unifies bosons and fermions, and severely restricts their couplings; but the scale of SUSY breaking is unknown (well above TeV scale).
- SUSY leads to a cancellation of quadratic divergences in quantum loops;
- Some supergravity theories arise as the low-energy effective actions in (compactified) superstring theory (quantum gravity) in String Landscape; it leads to their UV-completion and possible protection against quantum corrections! Supergravity can be considered as a bridge between classical and quantum gravity.
- Supergravity as a more fundamental theoretical framework to the phenomenological model building (though not ultimate one) around the GUT scale! Supergravity with spontaneously broken SUSY has particle candidate (LSP) for DM.

PLAN of TALK

- Review of Starobinsky inflation
- Review of Higgs Inflation
- Review of the equivalence of Starobinsky and Higgs inflationary models
- Starobinsky and Higgs inflation in supergravity
- On the equivalence of Higgs and Starobinsky inflation in supergravity
- Comments and Conclusion

Starobinsky model

The Starobinsky model of inflation is defined by the action (Starobinsky,1980)

$$S_{\mathrm{Star.}} = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} \left(R + \frac{1}{6m^2} R^2 \right) , \tag{1}$$

where we have introduced the reduced Planck mass $M_{\mathrm{Pl}} = 1/\sqrt{8\pi G_{\mathrm{N}}} \approx 2.4 \times 10^{18}$ GeV, and the scalaron (inflaton) mass m as the only parameter. We use the spacetime signature $(-,+,+,+,)$.

The $(R{+}R^2)$ gravity model (1) can be considered as the simplest extension of the standard Einstein-Hilbert action in the context of modified $F(R)$ gravity theories with an action

$$S_F = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g}\, F(R) , \tag{2}$$

in terms of the function $F(R)$ of the scalar curvature R.

Equivalence between $f(R)$ gravity and scalar-tensor gravity I

The $F(R)$ gravity action (2) is classically equivalent to

$$S[g_{\mu\nu},\chi] = \frac{M^2_{\rm Pl}}{2}\int d^4x\sqrt{-g}\,\left[F'(\chi)(R-\chi)+F(\chi)\right] \qquad (3)$$

with the real scalar field χ, provided that $F'' \neq 0$ that we always assume. The primes denote the derivatives with respect to the argument.

The equivalence is easy to verify because the χ-field equation implies $\chi = R$. In turn, the factor F' in front of the R in (3) can be (generically) eliminated by a Weyl transformation of metric $g_{\mu\nu}$, which transforms the action (3) into the action of the scalar field χ minimally coupled to Einstein gravity and having the scalar potential

$$V = \left(\frac{M^2_{\rm Pl}}{2}\right)\frac{\chi F'(\chi)-F(\chi)}{F'(\chi)^2} \quad . \qquad (4)$$

Equivalence between $f(R)$ gravity and scalar-tensor gravity II

The kinetic term of χ becomes canonically normalized after the field redefinition $\chi(\varphi)$ as

$$F'(\chi) = \exp\left(\sqrt{\tfrac{2}{3}}\varphi/M_{\rm Pl}\right) \ , \quad \varphi = \frac{\sqrt{3}M_{\rm Pl}}{\sqrt{2}} \ln F'(\chi) \ , \tag{5}$$

in terms of the canonical inflaton field φ, with the total acton

$$S_{\rm quintessence}[g_{\mu\nu},\varphi] = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right] \ . \tag{6}$$

The classical and quantum stability conditions of $F(R)$ gravity theory are given by

$$F'(R) > 0 \quad \text{and} \quad F''(R) > 0 \ , \tag{7}$$

and they are obviously satisfied for Starobinsky model (1) for $R > 0$.

The inverse transformation

The inverse transformation reads

$$R = \left[\frac{\sqrt{6}}{M_{\rm Pl}} \frac{dV}{d\varphi} + \frac{4V}{M_{\rm Pl}^2} \right] \exp\left(\sqrt{\tfrac{2}{3}} \varphi / M_{\rm Pl} \right) , \tag{8}$$

$$F = \left[\frac{\sqrt{6}}{M_{\rm Pl}} \frac{dV}{d\varphi} + \frac{2V}{M_{\rm Pl}^2} \right] \exp\left(2\sqrt{\tfrac{2}{3}} \varphi / M_{\rm Pl} \right) . \tag{9}$$

In the case of Starobinsky model (1), one finds the famous potential

$$V(\varphi) = \frac{3}{4} M_{\rm Pl}^2 m^2 \left[1 - \exp\left(-\sqrt{\tfrac{2}{3}} \varphi / M_{\rm Pl} \right) \right]^2 . \tag{10}$$

This scalar potential is bounded from below (non-negative and stable), and it has the absolute minimum at $\varphi = 0$ corresponding to a Minkowski vacuum. The scalar potential (10) also has a plateau of positive height (related to the inflationary energy density), that gives rise to slow roll of inflaton during the inflationary era.

The inflationary features

A duration of inflation is measured in the slow roll approximation by the e-foldings number

$$N_e \approx \frac{1}{M_{\rm Pl}^2}\int_{\varphi_{\rm end}}^{\varphi_*} \frac{V}{V'} d\varphi \ , \tag{11}$$

where φ_* is the inflaton value at the reference scale (horizon crossing), and $\varphi_{\rm end}$ is the inflaton value at the end of inflation when one of the slow roll parameters

$$\varepsilon_V(\varphi) = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_V(\varphi) = M_{\rm Pl}^2\left(\frac{V''}{V}\right) \ , \tag{12}$$

is no longer small (close to 1).

The amplitude of scalar perturbations at horizon crossing is given by

$$A_s = \frac{V_*^3}{12\pi^2 M_{\rm Pl}^6 ({V_*}')^2} = \frac{3m^2}{8\pi^2 M_{\rm Pl}^2} \sinh^4\left(\frac{\varphi_*}{\sqrt{6}M_{\rm Pl}}\right) \ . \tag{13}$$

Starobinsky inflation and CMB (Planck)

The Starobinsky model (1) is in very good agreement with the Planck data. The Planck (2018) satellite mission measurements of the Cosmic Microwave Background (CMB) radiation give the scalar perturbations tilt as $n_s \approx 1 + 2\eta_V - 6\varepsilon_V \approx 0.9649 \pm 0.0042$ (68%CL) and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_V < 0.064$ (95%CL). The Starobinsky inflation yields $r \approx 12/N_e^2 \approx 0.004$ and $n_s \approx 1 - 2/N_e$, where N_e is the e-foldings number between 50 and 60, with the best fit at $N_e \approx 55$.

The Starobinsky model (1) is geometrical (based on gravity only), while its (mass) parameter m is fixed by the observed CMB amplitude (COBE, WMAP) given by $\log(10^{10} A_s) = 2.975 \pm 0.056$ (68%CL) (or $A_s \approx 1.96 \cdot 10^{-9}$) as

$$m \approx 3 \cdot 10^{13}\ \text{GeV} \quad \text{or} \quad \frac{m}{M_{\text{Pl}}} \approx 1.3 \cdot 10^{-5}\ . \tag{14}$$

A numerical analysis of (11) with the potential (10) yields (with $N_e \approx 55$)

$$\sqrt{\frac{2}{3}}\varphi_*/M_{\text{Pl}} \approx \ln\left(\frac{4}{3}N_e\right) \approx 5.5\ , \quad \sqrt{\frac{2}{3}}\varphi_{\text{end}}/M_{\text{Pl}} \approx \ln\left[\frac{2}{11}(4+3\sqrt{3})\right] \approx 0.5$$

More comments about Starobinsky inflation

- Universality for slow roll: see Eqs. (8) and (9);
- No free parameters (high predictive power);
- Einstein criterium ("*simple but not too simple*"):

Starobinsky potential (10) won against a power potential (Planck mission, 2018);

- Attractor solution with an exit: $H(t) \approx \left(\frac{M}{6}\right)^2 (t_{\text{end}} - t) + \ldots$ that is driven by the $+R^2$ term (scale invariance, no ghost; uniqueness in quadratically modified gravity); scalaron as the Nambu-Goldstone boson of spontaneously broken scale invariance.
- The UV-cutoff of $(R + R^2)$ gravity is $M_{\text{Pl}} \gg H_{\text{inf.}}$, after expanding the Starobinsky potential (10) in powers of ϕ;
- Starobinsky potential as the mass term: $\frac{3}{2}g(1 - e^{-\sqrt{2/3}\phi}) = \varphi$ yields the non-canonical kinetic term with a singularity at $\varphi_{\text{cr.}} = 3g/(2m)$ and the critical exponent $\alpha = \sqrt{2/3}$ (the universality again);
- Any viable inflationary model should be close to the Starobinsky model!

(among single-field models of slow-roll inflation)

Higgs inflation

Basic ideas (Bezrukov, Shaposhnikov, 2007):

(i) identify inflaton with Higgs particle,

(ii) no new physics beyond the SM up to Planck scale,

(iii) non-minimal coupling of Higgs to gravity.

The Lagrangian (in Jordan frame) reads $(M_{\mathrm{Pl}} = 1)$

$$\mathcal{L}_J = \sqrt{-g}\left[\frac{1}{2}(1+\xi\phi^2)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_H(\phi)\right] \quad (16)$$

where

$$V_H(\phi) = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2 \quad (17)$$

Details of Higgs inflation

- going from Jordan frame to Einstein frame after

$$g_J^{\mu\nu} = g_E^{\mu\nu}(1+\xi\phi^2) \tag{18}$$

- getting a canonical scalar kinetic term for $\varphi = \varphi(\phi)$ after

$$\frac{d\varphi}{d\phi} = \frac{\sqrt{1+\xi(1+6\xi)\phi^2}}{1+\xi\phi^2} \tag{19}$$

  This yields the standard (quintessence) Lagrangian

$$\mathcal{L}_E = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\varphi)\right] \tag{20}$$

  with the potential

$$V(\varphi) = \frac{V_H(\phi(\varphi))}{\left[1+\xi\phi^2(\varphi)\right]^2} \tag{21}$$

The large field approximation

- In the large field approximation, $\varphi \gg \xi^{-1}$, a solution to (19) is

$$\varphi \approx \sqrt{\frac{3}{2}} \ln\left(1 + \xi\phi^2\right) \tag{22}$$

so that we get

$$V(\varphi) = \frac{\lambda}{4\xi^2}\left(1 - e^{-\sqrt{2/3}\varphi}\right)^2 \tag{23}$$

that coincides with the Starobinsky inflationary potential.

- The (CMB) phenomenology requires $\xi/\sqrt{\lambda} \approx 5 \cdot 10^4$ with the inflaton mass $m = \sqrt{\frac{\lambda}{3}}\xi^{-1} \approx 10^{-5}$.

Comments about Higgs inflation I

- Actually, the SM Higgs field H is a doublet, though one can choose the unitary gauge in which $H = \phi/\sqrt{2}$ in the Higgs Lagrangian

$$\mathcal{L}_H = \sqrt{-g}\left[\frac{1}{2}R + \xi H^\dagger H R - g^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2\right] \quad (24)$$

- il the large field approximation and during slow roll (inflation) we can ignore the scalar kinetic term and simplify the potential as

$$\mathcal{L}_H \approx \sqrt{-g}\left[\frac{1}{2}(1+\xi\phi^2)R - \frac{\lambda}{4}\phi^4\right] \quad (25)$$

Then varying with respect to ϕ yields $\xi\phi R = \lambda\phi^3$ or

$$\phi^2 = \frac{\xi}{\lambda}R \quad (26)$$

Substituting it into $\mathcal{L}_H$ gives the Starobinsky model again:

$$\mathcal{L}_H \approx \sqrt{-g}\left(\frac{1}{2}R + \frac{\xi^2}{4\lambda}R^2\right)$$

Comments about Higgs inflation II

- This established correspondence in the theory of gravity is known in the literature as the asymptotic duality between the Higgs and Starobinsky models of inflation.

- There is no correspondence in the small field approximation. Reheating is also different. For instance, the reheating temperature $T_H \approx 10^{13}$ GeV, whereas $T_S \approx 10^9$ GeV.

- The question arises: does the correspondence also hold in supergravity theory?

  The answer is more difficult because supergravity realizations of Starobinsky and Higgs inflationary models are non-trivial.

4. Inflaton in a massive vector N=1 multiplet

The Inflaton (scalaron) can also belong to a massive vector multiplet V that has a single physical scalar. The scalar potential of a vector multiplet is given by the D-term instead of the F-term, while any desired values of the CMB observables (n_s and r) are derivable from the inflaton potential proportional to the derivative squared of arbitrary real function $J(\mathrm{g}V)$ (starting from Van Proeyen 1989). The Lagrangian is

$$\mathcal{L} = \int d^2\theta 2\mathcal{E} \left\{ \tfrac{3}{8}(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8\mathcal{R}) e^{-\frac{2}{3}J} + \tfrac{1}{4} W^\alpha W_\alpha \right\} + \text{h.c.} \,, \tag{5}$$

and its bosonic part in Einstein frame reads

$$e^{-1}\mathcal{L} = \tfrac{1}{2}R - \tfrac{1}{4}F_{mn}F^{mn} - \tfrac{1}{2}J''\partial_m C\partial^m C - \tfrac{\mathrm{g}^2}{2}J''B_m B^m - \tfrac{g^2}{2}J'^2 \,, \tag{6}$$

where $C = V|$ is the real scalar inflaton field and $J = J(C)$.

The D-type scalar potential of the Starobinsky inflationary model is obtained with ($M_{\mathrm{Pl}} = 1$)

$$J(C) = \tfrac{3}{2}(C - \ln C) \quad \text{and} \quad C = \exp\left(\sqrt{2/3}\phi\right) \,.$$

Super-Higgs mechanism

Consider the master function $J(V)$ as a function $\tilde{J}(He^{2V}\overline{H})$ where we have introduced the Higgs chiral superfield H. The $\tilde{J}$ is invariant under the gauge transformations

$$H \to e^{-iZ}H \ , \quad \overline{H} \to e^{i\overline{Z}}\overline{H} \ , \quad V \to V + \tfrac{i}{2}(Z - \bar{Z}) \ , \qquad (6)$$

whose gauge parameter Z itself is a chiral superfield. The original theory of the massive vector multiplet governed by the master function J is recovered in the supersymmetric gauge $H = 1$.

We can now choose the different (Wess-Zumino) supersymmetric gauge in which $V = V_1$, where V_1 describes the irreducible massless vector multiplet minimally coupled to the dynamical Higgs chiral multiplet H (Aldabergenov, SVK, 2017). The *standard* Higgs mechanism appears when choosing the *canonical* function $J = \frac{1}{2}He^{2V}\bar{H}$ that corresponds to a *linear* function $\tilde{J}$.

R^2 inflation from our model I

The relevant part of our supergravity model (5) *before* Weyl rescaling to Einstein frame reads ($M_{\rm Pl}=1$)

$$e^{-1}\mathcal{L} = \exp\left(-\tfrac{2}{3}J\right)\left(\frac{1}{2}R\right) - \tfrac{1}{2}g^2 \exp\left(-\tfrac{4}{3}J\right)(J')^2 \quad , \tag{9}$$

where $J=J(C)$, $C=V|_{\theta=0}$, $J(C)=\frac{3}{2}(C-\ln C)$, and $C=e^{\sqrt{\frac{2}{3}}\phi}$, and we have *ignored* the kinetic term of C. This implies

$$e^{-\frac{2}{3}J} = Ce^{-C} \equiv \Omega > 0$$

and

$$e^{-1}\mathcal{L} = \Omega\left(\tfrac{1}{2}R\right) - \tfrac{1}{2}\left(\tfrac{3}{2}g\right)^2 \Omega^2 \left(1-C^{-1}\right)^2 \quad ,$$

R^2 inflation from our model II

where $C = C(\Omega)$ is given by *Lambert* function, and Ω is the *auxiliary* field. Varying $\mathcal{L}$ with respect to Ω yields

$$\tfrac{1}{2}R = \left(\tfrac{3}{2}g\right)^2 \Omega \left(1 - \frac{2}{C(\Omega)}\right) \approx \left(\tfrac{3}{2}g\right)^2 \Omega \left(1 + \frac{2}{\ln \Omega}\right) \quad , \tag{11}$$

where in the *large* field approximation, $C^{-1} \ll 1$ and $|1/\ln \Omega| \ll 1$, so that in the leading order we get

$$\tfrac{1}{2}R \approx \left(\tfrac{3}{2}g\right)^2 \Omega \ . \tag{12}$$

Substituting it back into the Lagrangian yields

$$e^{-1}\mathcal{L} \approx \tfrac{1}{8}\left(\tfrac{3}{2}g\right)^{-2} R^2$$

as the leading term.

R^2 inflation from our model III

After including the *next-to-leading term* we find

$$e^{-1}\mathcal{L} \approx \frac{1}{8}\left(\frac{3}{2}g\right)^{-2} R^2 \left[1 + \frac{2}{\ln\left(\frac{2}{9}R/g^2\right)}\right] \qquad (14)$$

so that the modified R^2 inflation is reproduced in the gauge $H = 1$.

When using the *Wess-Zumino* gauge $V = V_1$ with the charged Higgs (Stueckelberg) superfield H and the function $\tilde{J}(\bar{H}e^{2gV_1}H)$, *the same* R^2 inflation is reproduced along the same lines with another function $\exp[-\frac{2}{3}\tilde{J}(\bar{H}H)] = \Omega$ after ignoring both the H-kinetic term and the gauge field dependence in V_1 in the large field approximation.

Comments I

- The $\int R^2$ term is most relevant for inflation in both gravity and supergravity. It is distinguished by its two features: (a) *scale invariance*, and (b) *no ghosts*.

- The inflaton terms in our supergravity model ($M_{\mathrm{Pl}} = 1$)

$$-\frac{1}{2}J''(\partial C)^2 - \frac{1}{2}g^2(J')^2 \tag{15}$$

can be transformed by a field redefinition $gJ'(C) = m\varphi$ into a sum of the *non-canonical* kinetic term and the *mass term* as

$$-\frac{m^2}{2g^2}(J'')^{-1}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 \quad .$$

Comments II

- In the Starobinsky case, we have $J''(C) = \frac{2}{3}C^{-2} = \frac{2}{3}\exp\left(-2\sqrt{2/3}\phi\right)$ and $\frac{3}{2}g(1-C^{-1}) = m\varphi$. Therefore, the existence of a *plateau* in the canonical potential $V(\phi)$ gets translated into the existence of a *singularity* at a *finite* value of $\varphi_{\text{cr.}} = 3\; g/(2m)$ in the kinetic term of φ in (16). The "*critical exponent*" in the Starobinsky case is $\alpha = \sqrt{2/3}$.

- Starobinsky model can be extended to the so-called α-*attractors* = the inflationary models with the "critical exponent" α in the potential proportional to $(1-e^{-\alpha\phi})$ in the large field approximation (Kallosh, Linde, Roest 2013).

Comments III

- Starobinsky-like models in *higher* (D) spacetime dimensions are based on $(R + R^n)$ gravity with $n = D/2$. The presence of an n-form field F is required with a *flux compactification* on a sphere and the *warp* factor. It yields (Nakada and SVK 2017)

$$\alpha = \sqrt{\frac{D-2}{D-1}} \quad \text{and} \quad r = \frac{8(D-1)}{(D-2)N_e^2} \quad . \tag{17}$$

- Minkowski vacuum is uplifted to a *dS vacuum* in our $D = 4$ supergravity models via the alternative *FI term* (Aldabergenov and SVK 2018). It leads to a spontaneous SUSY breaking *after* inflation with $< D >= \xi$ and the *cosmological constant* $\Lambda = \frac{1}{2}\xi^2 = \Lambda_0$.

Comments IV

- The Starobinsky and Higgs inflationary models as Quantum Field Theories are *non-renormalizable* and need a UV completion. The UV *cut-off* of $(R+R^2)$ gravity is $\Lambda_{\mathrm{S}} = M_{\mathrm{Pl}}$, whereas $\Lambda_{\mathrm{H}} = M_{\mathrm{Pl}}/\xi$. Hence, the Higgs inflation is *much more sensitive* to quantum corrections. *Extra* massive scalar may *increase* Λ_{H}, as long as that do not spoil inflation.

- There exist a *D-brane-antibrane* configuration that reproduces the Starobinsky potential in supergravity by the D-term (Binetruy, Dvali, Kallosh, Van Proeyen 2004). This gives a *UV-completion* of the proposed supergravity model in string theory (quantum gravity).

Conclusion

- The Starobinsky inflation and the Higgs inflation belong to the same *universality* class of the inflationary models.

- In supergravity, the Starobinsky picture and the Higgs picture of inflation appear in *the two different gauges* of the *same* supergravity model, modulo the subleading corrections.

- Key CMB measurements *needed*: the values of the *tensor-to-scalar ratio* r and *non-gaussianity*.
  BICEP/Keck Array, Simons Observatory, LiteBIRD, etc.

- Thank you very much for your attention!