CP Violation in Non-leptonic $B$ Decays as a Portal to New Physics

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DISCRETE 2020-2021
Bergen, Norway, 29 November - 3 December 2021
Setting the Stage
The Quark-Flavour Code

- Quark flavour physics and CP violation in the Standard Model (SM):

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

quark-mixing matrix, also known as the Cabibbo–Kobayashi–Maskawa (CKM) matrix

\[ \rightarrow \text{unitary matrix; complex phase } \rightarrow \text{CP violation} \]

\[ D \quad U \]
\[ \frac{}{V_{UD}} \]
\[ W^- \]

\[ \overline{D} \quad \overline{U} \]
\[ \frac{}{V_{UD}^*} \]
\[ W^+ \]

- New Physics (NP):

\[ \rightarrow \text{typically new sources for flavour and CP violation.} \]

\[ \Rightarrow \text{encoded in weak decays of } K, D \text{ and } B \text{ mesons ...} \]
Hope for New Physics (NP) ...

• We have indications that the SM cannot be complete:
  – Neutrino masses \( \neq 0 \): suggest see-saw mechanism, GUT scenarios ...
  – Baryon asymmetry of the Universe (SM cannot generate it ...)
  – The long-standing problem of dark matter (?)

• Fundamental theoretical questions/problems:
  – Hierarchy problem
  – Fine-tuning problem \( \rightarrow \) suggest NP in the TeV regime (!?)

• Popular specific models for physics beyond the SM:
  – Supersymmetry (SUSY)
  – Universal extra dimension (UED)
  – Warped extra dimension (WED)
  – Little Higgs models (LH, with T parity LHT)
  – \( Z' \) models
  – ... \( \rightarrow \) new sources of flavour & CP violation
Where Do We Stand?

- **Data taking @ LHC:** → discovery of “Higgs-like” particle, but ...
  - No SM deviations seen @ ATLAS/CMS.
  - CP violation & flavour sector globally consistent with the CKM picture.
  - But “anomalies” are emerging in $B$-meson decays $[+(g-2)_{\mu}]$:
    * Semi-leptonic $B$ decays: → talk by Wolfgang Altmannshofer
    * Non-leptonic $B$ decays: → this talk ...

- **Implications for the structure of NP:**
  \[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{NP}}(\varphi_{\text{NP}}, g_{\text{NP}}, m_{\text{NP}}, ...) \]
  - Large characteristic NP scale $\Lambda_{\text{NP}}$, i.e. not just $\sim$ TeV, which would be challenging for direct searches at ATLAS and CMS, or / and ...
  - Symmetries prevent large NP effects in FCNCs and the flavour sector.

- **Much more is yet to come:** LHC run II data + run III + ... Belle II ...


**Hierarchy of Scales**

\[ \Lambda_{NP} \sim 10^{(0\ldots?)} \text{ TeV} \gg \Lambda_{EW} \sim 10^{-1} \text{ TeV} \gg \gg \Lambda_{QCD} \sim 10^{-4} \text{ TeV} \]

(very) short distances

long distances

- Powerful theoretical concepts/techniques:

  - “Effective Field Theories”

  - Heavy degrees of freedom (NP particles, top, \(Z\), \(W\)) are “integrated out” from appearing explicitly: \(\rightarrow\) short-distance loop functions.

  - Calculation of *perturbative QCD corrections*.

  - *Renormalization group* allows the summation of large \(\log(\mu_{SD}/\mu_{LD})\).

- Applied to the SM and various NP scenarios, such as the following:

  - MSSM, UED, WED, LH, LHT, \(Z'\) models, ...

  - Standard Model Effective Theories (SMEFT): very popular ...
Theoretical Tool: Low-Energy Effective Hamiltonians

- Separation of short-distance from long-distance contributions (OPE):

\[
\langle f | H_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_j \lambda^j_{\text{CKM}} \sum_k C_k(\mu) \langle f | Q^j_k(\mu) | B \rangle
\]

\[G_F: \text{Fermi’s constant, } \lambda^j_{\text{CKM}}: \text{CKM factors, } \mu: \text{renormalization scale}\]

- Short-distance physics: [Buras et al.; Martinelli et al. ('90s); ...]

\[\rightarrow \text{Wilson coefficients } C_k(\mu) \rightarrow \text{perturbative quantities } \rightarrow \text{known!}\]

- Long-distance physics:

\[\rightarrow \text{matrix elements } \langle f | Q^j_k(\mu) | B \rangle \rightarrow \text{non-perturbative } \rightarrow \text{“unknown”!?}\]

[Lattice QCD: impressive recent progress, but still challenging for non-leptonic B decays]
Key Player for CP Violation: Non-Leptonic $B$ Decays

\[ |A_j|e^{i\delta_j} \propto \sum_k \left\{ \frac{C_k(\mu)}{\text{pert. QCD}} \right\} \times \langle f | Q^j_k(\mu) | B \rangle \]

- **QCD factorization (QCDF):**
  
  Huber et al. (2016); Bell et al. (2020); Bordone et al. (2020); Beneke et al. (2001)

- **Perturbative Hard-Scattermg (PQCD) Approach:**
  
  Li & Yu ('95); Cheng, Li & Yang ('99); Keum, Li & Sanda ('00); ... 

- **Soft Collinear Effective Theory (SCET):**
  
  Bauer, Pirjol & Stewart (2001); Bauer, Grinstein, Pirjol & Stewart (2003); ...

- **QCD sum rules:**
  
  Khodjamirian (2001); Khodjamirian, Mannel & Melic (2003); ... 

Lots of progress, still generally a theoretical challenge ...
Circumvent the \( \langle f | Q^j_k(\mu) | B \rangle \) in CP-B Studies:

- Amplitude relations allow us in fortunate cases to eliminate the hadronic matrix elements (\( \rightarrow \) typically strategies to determine the UT angle \( \gamma \)):
  - **Exact relations**: class of pure “tree” decays (e.g. \( B \rightarrow DK \)).
  - **Approximate relations**, which follow from the **flavour symmetries of strong interactions**, i.e. \( SU(2) \) isospin or \( SU(3)_F \):
    \[
    B \rightarrow \pi\pi, \ B \rightarrow \pi K, \ B_{(s)} \rightarrow KK.
    \]

- Decays of neutral \( B_d \) or \( B_s \) mesons:
  - Interference effects through \( B^0_q - \overline{B}^0_q \) mixing:
  - Lead to “mixing-induced” CP violation \( S(f) \), in addition to “direct” CP violation \( C(f) \) (caused by interference between decay amplitudes).
  - If one CKM amplitude dominates:
    \[
    \Rightarrow \text{hadronic matrix elements cancel in } S(f), \text{ while } C(f) = 0.
    \]
CP Violation $B$-Decay Benchmark Processes

- **Unitarity Triangle:**

  \[ B \rightarrow \pi\pi \text{ (isospin), } B \rightarrow \rho\pi, \ B \rightarrow \rho\rho \]

  \[ R_b \ (b \rightarrow u, c\ell\bar{\nu}_\ell) \]

  \[ R_t \ (B^0_q - \bar{B}^0_q \text{ mixing}) \]

  \[ \begin{align*}
  B_d &\rightarrow \pi^+\pi^- \\
  B_s &\rightarrow K^+K^- \\
  B^\pm_d &\rightarrow K^\pm D \\
  B^\pm_d &\rightarrow K^{*0}D \\
  B^\pm_c &\rightarrow D^\pm_s D \\
  \end{align*} \]

  only trees

  \[ B_d \rightarrow D(\ast)^{\pm}\pi^{\mp} : \gamma + 2\beta \]

  \[ B_s \rightarrow D_s^{\pm}K^{\mp} : \gamma + \phi_s \]

  only trees

  \[ \boxed{\text{New Physics} \Rightarrow \text{Discrepancies (!?)}} \]
Current Status of the Unitarity Triangle

The diagram illustrates the constraints on the parameters of the unitarity triangle, with regions excluded at a confidence level of 0.95. The parameters include $\sin 2\beta$, $\Delta m_d$, $\Delta m_s$, and $V_{ub}/V_{cb}$. The excluded area has a confidence level greater than 0.95, and the solutions with $\cos 2\beta < 0$ are highlighted.
Precision Measurements of the $B^0_q - \bar{B}^0_q$ Mixing Phases

[Many thanks to Kristof De Bruyn for updating the plots; see also his talk @ CKM 2021]
**Current Experimental Situation**

- $B_s^0 - \bar{B}_s^0$ mixing phase:

\[ \phi_s^{c\bar{c}s} \equiv \phi_s = \phi_s^{SM} + \phi_s^{NP} = -2\lambda^2 \eta + \phi_s^{NP} \]

- HFLAV *average* of the experimental data from various decay channels:

\[ \phi_s = -(2.86 \pm 1.09)^\circ \quad \text{vs.} \quad \phi_s^{SM} = -(2.120^{+0.046}_{-0.040})^\circ \]
Towards New Frontiers ... 

- **Crucial for resolving smallish effects of New Physics:**
  - Have a critical look at theoretical analyses and their approximations:
    - key issue: strong interactions: → “hadronic” effects
  - Match the experimental and theoretical precisions.

- **Benchmark decays for exploring CP violation:**
  \[ B^0_d \rightarrow J/\psi K_S, \quad B^0_s \rightarrow J/\psi \phi \]
  - Allow measurements of the \( B^0_{d,s} - \overline{B}^0_{d,s} \) mixing phases \( \phi_{d,s} \).
  - Uncertainties from doubly Cabibbo-suppressed *penguin* contributions.
  - These effects are usually neglected ...

\[ \Rightarrow \] *How big are they and how can they be controlled?*
CP Violation in $B^0_d \to J/\psi K_S$

- **SM corrections:** → doubly Cabibbo-suppressed penguins

$$A(B^0_d \to J/\psi K_S) = (1 - \lambda^2/2) A' \left[ 1 + \epsilon a' e^{i\theta'} e^{i\gamma} \right] \quad \epsilon \equiv \lambda^2/(1 - \lambda^2) \sim 0.05$$

- **Generalized expression for mixing-induced CP violation:**

$$\frac{S(B_d \to J/\psi K_S)}{\sqrt{1 - C(B_d \to J/\psi K_S)^2}} = \sin(\phi_d + \Delta \phi_d)$$

- $$\sin \Delta \phi_d \propto 2\epsilon a' \cos \theta' \sin \gamma + \epsilon^2 a'' \sin 2\gamma$$

- $$\cos \Delta \phi_d \propto 1 + 2\epsilon a' \cos \theta' \cos \gamma + \epsilon^2 a'' \cos 2\gamma$$

[S. Faller, R.F., M. Jung & T. Mannel (2008)]
The $B_s^0 \to J/\psi \phi$ Decay

- Final state is mixture of CP-odd and CP-even states:
  - disentangle through $J/\psi \to \mu^+ \mu^-$ and $\phi \to K^+ K^-$ angular distribution

- Impact of SM penguin contributions: $f \in \{0, \|, \perp\}$
  
  $$A(B_s^0 \to (J/\psi \phi)_f) = \left(1 - \frac{\lambda^2}{2}\right) A'_f \left[1 + \epsilon a'_f e^{i\theta'_f} e^{i\gamma}\right]$$

  - CP-violating observables $\implies \phi_{s,(\psi \phi)}^e = \phi_s + \Delta \phi_{s,(\psi \phi)}^f$

- Smallish $B_s^0 - \bar{B}_s^0$ mixing phase $\phi_s$ (as following from the data):
  
  $\implies \Delta \phi_{s,(\psi \phi)}^f \equiv \Delta \phi_s^f$ at the $1^\circ$ level would have a significant impact ...

[Faller, R.F. & Mannel (2008)]
Towards High-Precision Analyses

• Era of Belle II and the LHCb upgrades:
  – Experimental precision requires the control of the penguin corrections to reveal possible CP-violating NP contributions to $B^0_d - \bar{B}^0_d$ mixing.
  – The topic receives long-standing interest in the theory community:
    R.F., (99); Ciuchini, Pierini & Silvestrini (05, 11); Faller, R.F., Jung & Mannel (08); Gronau & Rosner (08); De Bruyn, R.F., Koppenburg; Jung (2012); De Bruyn, R.F. (15); Frings, Nierste & Wiebusch (15); Barel, De Bruyn, R.F., Malami (2020).

• The hadronic phase shifts $\Delta \phi_d, \Delta \phi^f_s$ cannot be reliably calculated:
  $\Rightarrow$ use “control channels” $B^0_s \rightarrow J/\psi K_S, B^0_d \rightarrow J/\psi \pi^0, B^0_s \rightarrow J/\psi \rho^0$:
  – Key feature: $\rightarrow$ “magnified” penguin parameters (no $\epsilon$ suppression)

  Example: $A(B^0_s \rightarrow J/\psi K_S) \propto [1 - ae^{i\theta} e^{i\gamma}]$

  – $U$-spin flavour symmetry: $ae^{i\theta} = a'e^{i\theta'}$
Combined Analysis

- **Interplay between the control channels:**

  \[
  B^0_d \rightarrow J/\psi K^0_S, \\
  B^0_d \rightarrow J/\psi \rho^0, \\
  B^0_d \rightarrow J/\psi \pi^0, \\
  \Delta \phi_d, \\
  \phi_d, \\
  B^0_s \rightarrow J/\psi K^0_S, \\
  B^0_s \rightarrow J/\psi \phi, \\
  \Delta \phi_s, \\
  \phi_s
  \]

**Assumptions:**

- Neglect exchange and penguin-annihilation topologies.
- Ignore polarisation-dependent effects in \( B_{(s)} \rightarrow J/\psi V \) modes:
  
  current lack of data → future measurements...

- Assume \( SU(3) \) flavour symmetry relations.

[Barel, R.F., De Bruyn, Malami (2021)]
$a = 0.14^{+0.17}_{-0.11}, \theta = (173^{+35}_{-45})^\circ, \phi_d = (44.4^{+1.6}_{-1.5})^\circ$

$a_V = 0.044^{+0.085}_{-0.038}, \theta_V = (306^{+48}_{-112})^\circ, \phi_s = -(4.2 \pm 1.4)^\circ$
Searching for New Physics

- **Experimental value:** \( \phi_{d,J/\psi K^0}^{\text{eff}} = (43.6 \pm 1.4)^\circ \)

- **Taking penguin effects into account:**
  \[
  \Delta \phi_s = (0.14^{+0.54}_{-0.70})^\circ, \quad \phi_s = \phi_s^{\text{SM}} + \phi_s^{\text{NP}} = -(4.2 \pm 1.4)^\circ
  \]

- **UT as determined above:** \( \phi_s^{\text{SM}} = -2\lambda^2 \eta = -(2.01 \pm 0.12)^\circ \)

  \[\Rightarrow\] \( \phi_s^{\text{NP}} = -(2.2 \pm 1.4)^\circ \)
Illustration of Future Prospects

• **Inputs and assumptions:**

  – Mixing phases:
    \[ \phi^\text{SM}_d = 2\beta = (45.7 \pm 2.8)^\circ, \quad \phi^\text{SM}_s = -(2.01 \pm 0.12)^\circ \]

  – Similar experimental improvements in \( B_0^s \rightarrow J/\psi \phi \) and \( B_0^d \rightarrow J/\psi \rho^0 \):

• **Observations:**

  – We could reveal CP-violating NP effects with \( 5\sigma \) significance.
  
  – The SM prediction of \( \phi_d \) is a limiting factor, making \( \phi_d \) less favourable.
\[ \sqrt{2}A(B_d^0 \to J/\psi\pi^0) = \frac{G_F}{\sqrt{2}} V_{cd}^* V_{cb} m_{J/\psi} f_{J/\psi} f_{B_d^0 \to \pi} (m_{J/\psi}^2) (p_{B_d^0} + p_{K^0}) \cdot \varepsilon_{J/\psi}^\mu \times (1 - ae^{i\theta} e^{i\gamma}) \times a_2(B_d^0 \to J/\psi\pi^0) \]

- Effective colour-suppression factor: \( a_2(B_d^0 \to J/\psi\pi^0) \) agrees (surprisingly) well with “naive factorization”: \( a_2 = 0.21 \pm 0.05 \)  
  [Buras & Silvestrini (1998)]

- Non-factorizable \( SU(3) \)-breaking effects:
  - Deviations from factorisation at (30-40)% level, with \( SU(3) \)-breaking effects at 20% level \( \Rightarrow \) non-fact. \( SU(3) \)-breaking effects at 5% level:
    - supported by plot on the right-hand side panel \( \rightarrow \) robust method!
The $B^0_s \rightarrow D^\mp_s K^\pm$ Puzzle

[R.F. and Eleftheria Malami (2021); see also Eleftheria’s talk © CKM 2021]
Topologies & Interference

• Colour-allowed tree topologies in the Standard Model:

\[ \begin{align*}
\Gamma(B_s^0(t) \rightarrow D_s^+ K^-) - \Gamma(\bar{B}_s^0(t) \rightarrow D_s^+ K^-) \\
\Gamma(B_s^0(t) \rightarrow D_s^+ K^-) + \Gamma(\bar{B}_s^0(t) \rightarrow D_s^+ K^-) \\
= C \cos(\Delta M_s t) + S \sin(\Delta M_s t) \\
\cos(y_s t/\tau_{B_s}) + A_{\Delta \Gamma} \sinh(y_s t/\tau_{B_s})
\end{align*} \]

- Decay width difference: \( y_s \equiv \Delta \Gamma_s/(2 \Gamma_s) = 0.062 \pm 0.004 \)
- Similar structure for CP-conjugate final state \( D_s^- K^+ \): \( \bar{C}, \bar{S} \) and \( \bar{A}_{\Delta \Gamma} \)
Closer Look at Observables

- Observables of the time-dependent rate asymmetries:

\[ C = \frac{1 - |\xi|^2}{1 + |\xi|^2}, \quad S = \frac{2 \text{Im} \xi}{1 + |\xi|^2}, \quad A_{\Delta\Gamma} = \frac{2 \text{Re} \xi}{1 + |\xi|^2} \]

\[ \Rightarrow \xi \text{ (and } \bar{\xi} \text{) can be unambiguously determined from the data.} \]

- Interference effects are described by the following observables:

\[ \xi = -e^{-i\phi_s} \left[ e^{i\phi_{CP}} \frac{A(\bar{B}_s^0 \to D_s^+ K^-)}{A(B_s^0 \to D_s^+ K^-)} \right] = -e^{-i(\phi_s + \gamma)} \left[ \frac{1}{x_s e^{i\delta_s}} \right] \text{ hadronic} \]

\[ \bar{\xi} = e^{-i\phi_s} \left[ e^{i\phi_{CP}} \frac{A(\bar{B}_s^0 \to D_s^- K^+)}{A(B_s^0 \to D_s^- K^+)} \right] = -e^{-i(\phi_s + \gamma)} \left[ x_s e^{i\delta_s} \right] \text{ hadronic} \]

\[ \Rightarrow x_s, \delta_s \text{ cancel in } \xi \times \bar{\xi} = e^{-i2(\phi_s + \gamma)} \Rightarrow \text{clean extraction of } \phi_s + \gamma \]

[Aleksan, Dunietz, Kayser (1992); R.F. (2003); De Bruyn et al. (2012)]
LHCb Analysis

- Measurements of the CP-violating observables and fit to the data:

Assume Standard Model relations: $|\bar{\xi}| = 1 / |\xi| \Rightarrow C + \bar{C} = 0$

⇒ Puzzling situation!? → R.F. and Eleftheria Malami (2021)
Transparent Determination of Parameters

• We use the following relations:

\[
\tan(\phi_s + \gamma) = -\left[ \frac{\overline{S} + S}{A_{\Delta\Gamma} + A_{\Delta\Gamma}} \right], \quad \tan \delta_s = \left[ \frac{\overline{S} - S}{A_{\Delta\Gamma} + A_{\Delta\Gamma}} \right]
\]

\[
\Rightarrow \phi_s + \gamma = \left[ ( -55^{+22}_{-18} )^\circ \right] \lor (125^{+22}_{-18})^\circ, \quad \delta_s = \left[ (182^{+34}_{-22})^\circ \right] \lor (2^{+34}_{-22})^\circ
\]

\[
|\xi| = \sqrt{\frac{1 - C}{1 + C}} \Rightarrow |\xi| = 2.53^{+1.43}_{-0.59}, \quad |\overline{\xi}| = 0.40 \pm 0.13
\]

• LHCb fit (without the solutions mod 180°):

\[
\phi_s + \gamma = (126^{+17}_{-22})^\circ, \quad \delta_s = (-2^{+13}_{-14})^\circ, \quad x_s = |\overline{\xi}| = 0.37^{+0.10}_{-0.09}
\]

excellent agreement:
How Could New Physics Enter?

• New Physics in $B^0_s - \bar{B}^0_s$ mixing:

  - Mixing parameters $\Delta M_s$ and $\Delta \Gamma_s$: time evolution → measured.
  - New sources of CP violation may affect the mixing phase $\phi_s$:
    
    \[ \rightarrow \text{included through } B^0_s \rightarrow J/\psi\phi \text{ (and penguin control modes):} \]
    
    \[ \text{Data (see above)} \Rightarrow \phi_s = \phi_{s}^{\text{SM}} + \phi_{s}^{\text{NP}} \Rightarrow \gamma = (131^{+17}_{-22})^\circ \]

• New Physics in decay amplitudes:

  - The puzzling value of $\gamma$ would require NP contributions with new sources of CP violation at the decay amplitude level:
    
    \[ \Rightarrow \text{explore the } \bar{B}^0_s \rightarrow D_s^+ K^- \text{ and } B^0_s \rightarrow D_s^+ K^- \text{ branching ratios} \]
Branching Ratios

• “Theoretical” branching ratios: \( \rightarrow B_s^0 - \bar{B}_s^0 \) mixing is “switched off”:

\[
B_{th} \equiv \frac{1}{2} \left[ B(\bar{B}_s^0 \rightarrow D_s^+ K^-)_{th} + B(B_s^0 \rightarrow D_s^+ K^-)_{th} \right]
\]

– Observable \( \xi \) allows us to disentangle the decay paths:

\[
B(\bar{B}_s^0 \rightarrow D_s^+ K^-)_{th} = 2 \left( \frac{|\xi|^2}{1 + |\xi|^2} \right) B_{th}
\]

\[
B(B_s^0 \rightarrow D_s^+ K^-)_{th} = 2 \left( \frac{1}{1 + |\xi|^2} \right) B_{th}
\]

• Standard Model relations:

\[
B(\bar{B}_s^0 \rightarrow D_s^+ K^-)_{th} \overset{SM}{=} B(B_s^0 \rightarrow D_s^- K^+)_{th}
\]

\[
B(B_s^0 \rightarrow D_s^+ K^-)_{th} \overset{SM}{=} B(\bar{B}_s^0 \rightarrow D_s^- K^+)_{th}
\]

\[
\Rightarrow B_{th} \overset{SM}{=} \bar{B}_{th}.
\]
• **“Experimental” branching ratios:** \( \rightarrow \) **time-integrated quantities:**

\[
B_{\text{exp}} = \frac{1}{2} \int_0^\infty \left[ \Gamma(\bar{B}_s^0(t) \to D_s^+ K^-) + \Gamma(B_s^0(t) \to D_s^+ K^-) \right] \, dt
\]

\[
B_{\text{th}} = \left[ \frac{1 - y_s^2}{1 + A_{\Delta\Gamma} y_s} \right] B_{\text{exp}}
\]

• Unfortunately, only a measurement of the following average is available:

\[
B^{\text{exp}}_\Sigma \equiv B_{\text{exp}} + \bar{B}_{\text{exp}} \equiv 2 \langle B_{\text{exp}} \rangle = (2.27 \pm 0.19) \times 10^{-4}
\]

• **Assuming the SM (as LHCb), we have the following relation:**

\[
B_{\text{th}} = \bar{B}_{\text{th}} = \left[ \frac{1 - y_s^2}{1 + y_s \langle A_{\Delta\Gamma} \rangle} \right] \langle B_{\text{exp}} \rangle \quad \text{with} \quad \langle A_{\Delta\Gamma} \rangle_+ \equiv \frac{\bar{A}_{\Delta\Gamma} + A_{\Delta\Gamma}}{2}
\]

• Finally, we obtain the following branching ratios from the data:

\[
B(\bar{B}_s^0 \to D_s^+ K^-)_{\text{th}} = (1.94 \pm 0.21) \times 10^{-4}
\]

\[
B(B_s^0 \to D_s^+ K^-)_{\text{th}} = (0.26 \pm 0.12) \times 10^{-4}
\]
Theoretical Interpretation

- Factorization provides the theoretical framework: \( b \to c \bar{u}s \) process:

\[
A(\bar{B}_s^0 \to D_s^+ K^-)|_{SM} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cb} f_K F_0^{B_s \to D_s} (m_K^2)(m_{B_s}^2 - m_{D_s}^2) a_1^{D_s K}
\]

\[[V_{us}^* V_{cb}: \text{CKM factor}; \ f_K: \text{kaon decay constant}; \ F_0^{B_s \to D_s} (m_K^2): \text{form factor}]\]

- Deviation from naive factorization:

\[
a_1^{D_s K} = a_1^{D_s K} \left(1 + \frac{E_{D_s K}}{T_{D_s K}}\right)
\]

  - Non-factorizable effects in the color-allowed tree amplitude \( T_{D_s K} \):

  “Show-case” example for factorization: \( |a_1^{D_s K}| = 1.07 \pm 0.02 \)

  - Moreover, non-factorizable exchange topology \( E_{D_s K} \):

\[
Experimental \ data: \ r_E^{D_s K} \equiv \left|1 + \frac{E_{D_s K}}{T_{D_s K}}\right| = 1.00 \pm 0.08
\]

- Amplitude with similar structure for the \( b \to u \bar{c}s \) mode \( \bar{B}_s^0 \to K^+ D_s^- \):

  - Here factorization appears on less solid ground: \( |a_1^{K D_s}| = 1.1 \pm 0.1 \)
Minimizing CKM and Form Factor Uncertainties

• Introduce ratios with appropriate semileptonic decay information:

\[ R_{D_s^+K^-} \equiv \frac{\mathcal{B}(\bar{B}_s^0 \to D_s^+K^-)_{th}}{d\mathcal{B}(\bar{B}_s^0 \to D_s^+\ell^-\bar{\nu}_\ell)/dq^2|_{q^2=m_K^2}} = 6\pi^2 f_K^2 |V_{us}|^2 |a_{1\text{eff}}^{D_s^0K}|^2 X_{D_sK} \]

\[ X_{D_sK} \equiv \Phi_{\text{ph}} \left[ F_0^{B_s\to D_s(m_K^2)} / F_1^{B_s\to D_s(m_K^2)} \right]^2 \]

– \(|V_{cb}| \) cancels: discrepancy between inclusive/exclusive determinations.
– \(\Phi_{\text{ph}}\) is a calculable phase-space factor, very close to 1.
– Impact of non-perturbative hadronic form factors is minimal due to

\[ F_0^{B_s\to D_s}(0) = F_1^{B_s\to D_s}(0) \]

– Implementation using experimental data: \(\Rightarrow\) \(|a_{1\text{eff}}^{D_s^0K}| = 0.82 \pm 0.11\)

• Similar analysis for \(\bar{B}_s^0 \to K^+D_s^-\), where the partner is \(\bar{B}_s^0 \to K^+\ell\bar{\nu}_\ell\):

– \(\bar{B}_s^0 \to K^+\ell^-\bar{\nu}_\ell\) observed but no differential rate measurement reported.
– Use \(\bar{B}_d^0 \to \pi^+\ell^-\bar{\nu}_\ell\) with \(SU(3)\) flavour symmetry:

\(\Rightarrow\) \(|a_{1\text{eff}}^{K^0D_s}| = 0.77 \pm 0.19\)
• Using analogous ratios with respect to semileptonic decays, we have performed a detailed analysis of $B_{(s)}$ decays with similar dynamics:

- $\bar{B}_d^0 \to D_d^+ K^-$ stands out, showing a discrepancy of $4.8\sigma$ significance.
- Surprisingly small branching ratios of some channels noted before: R.F., Tuning, Serra (2011); De Bruyn et al. (2012); Bordone et al. (2020).
- Detailed analyses within NP scenarios:
  Iguro & Kitahara (2020); Cai et al. (2021); Bordone, Greljo & Marzocca (2021)
- General NP effects in tree decays:
  Brod et al. (2015); Lenz & Tetlalmatzi-Xolocotzi (2019)

• Pattern of $|a_1|$ parameters complements CPV puzzle in $B_s^0 \to D_s^{\mp} K^{\pm}$:

⇒ Exciting new puzzle: manifestation of New Physics!?
New Physics: Amplitudes and Observables

- Generalization of the decay amplitudes:

\[
A(\bar{B}_s^0 \to D_s^+ K^-) = A(\bar{B}_s^0 \to D_s^+ K^-)_{\text{SM}} \left[ 1 + \bar{\rho} e^{i\delta} e^{i\varphi} \right]
\]

\[
\bar{\rho} e^{i\delta} e^{i\varphi} \equiv \frac{A(\bar{B}_s^0 \to D_s^+ K^-)_{\text{NP}}}{A(\bar{B}_s^0 \to D_s^+ K^-)_{\text{SM}}}
\]

[$\delta$: CP-conserving strong phase; $\varphi$: CP-violating NP phase]

- $\bar{B}_s^0 \to D_s^- K^+$ channel: \( \to \) analogous expression with $\rho$, $\varphi$ and $\delta$.

- Useful to introduce the following quantities:

\[
\bar{b} \equiv \frac{\langle R_{D_s K} \rangle}{6\pi^2 f_K'^2 |V_{us}|^2 |a_{1\text{ eff}}^{D_s K}|^2 X_{D_s K}} = 1 + 2 \bar{\rho} \cos \delta \cos \varphi + \bar{\rho}^2 = 0.58 \pm 0.16
\]

\[
b \equiv \frac{\langle R_{K D_s} \rangle}{6\pi^2 f_{D_s}^2 |V_{cs}|^2 |a_{1\text{ eff}}^{K D_s}|^2 X_{K D_s}} = 1 + 2 \rho \cos \delta \cos \varphi + \rho^2 = 0.50 \pm 0.26
\]

[$\langle R \rangle$: CP-averaged quantities]
• New Physics contributions may generate direct CP violation:

\[ A_{\text{CP}}^{\text{dir}} \equiv \frac{|A(B_s^0 \rightarrow D_s^+ K^-)|^2 - |A(\bar{B}_s^0 \rightarrow D_s^- K^+)|^2}{|A(B_s^0 \rightarrow D_s^+ K^-)|^2 + |A(\bar{B}_s^0 \rightarrow D_s^- K^+)|^2} = \frac{2 \rho \sin \delta \sin \varphi}{1 + 2 \rho \cos \delta \cos \varphi + \rho^2} \]

[Similar for the CP-conjugate asymmetry \( \bar{A}_{\text{CP}}^{\text{dir}} \), involving \( \bar{\rho} \) with \( \bar{\delta} \) and \( \bar{\varphi} \)]

• Generalisations for the observables \( \bar{\xi} \) and \( \xi \):

\[ \bar{\xi} = \bar{\xi}_{\text{SM}} \left[ \frac{1 + \rho e^{i\delta} e^{i\varphi}}{1 + \bar{\rho} e^{i\bar{\delta}} e^{-i\bar{\varphi}}} \right] = -|\bar{\xi}| e^{i\bar{\delta}_s} e^{-i(\phi_s + \gamma)} e^{i\Delta \bar{\varphi}} \]

[Similar expression for \( \xi \): interchange \( \bar{\rho}, \bar{\delta}, \bar{\varphi} \) and \( \rho, \delta, \varphi \)]

\[ \xi \times \bar{\xi} = e^{-i2(\phi_s + \gamma)} \left[ \frac{1 + \rho e^{i\delta} e^{i\varphi}}{1 + \rho e^{i\delta} e^{-i\varphi}} \right] \left[ \frac{1 + \bar{\rho} e^{i\bar{\delta}} e^{i\bar{\varphi}}}{1 + \bar{\rho} e^{i\bar{\delta}} e^{-i\bar{\varphi}}} \right] \]

\[ \frac{1 + \rho e^{i\delta} e^{i\varphi}}{1 + \rho e^{i\delta} e^{-i\varphi}} = e^{-i\Delta \Phi} \sqrt{1 - A_{\text{CP}}^{\text{dir}}} \left( \frac{1 - A_{\text{CP}}^{\text{dir}}}{1 + A_{\text{CP}}^{\text{dir}}} \right) \]

\[ |\xi \times \bar{\xi}|^2 = \left[ \frac{1 - A_{\text{CP}}^{\text{dir}}}{1 + A_{\text{CP}}^{\text{dir}}} \right] \left[ \frac{1 - \bar{A}_{\text{CP}}^{\text{dir}}}{1 + \bar{A}_{\text{CP}}^{\text{dir}}} \right] \]
Constraining NP Parameters

- **New Physics generalization of the “master equation”:**

\[
\xi \times \bar{\xi} = \sqrt{1 - 2 \left[I + \frac{C + \bar{C}}{(1 + C)(1 + \bar{C})}\right]} e^{-i[2(\phi_s + \gamma_{\text{eff}})]}
\]

\[
\gamma_{\text{eff}} \equiv \gamma + \frac{1}{2} (\Delta \Phi + \Delta \bar{\Phi}) = \gamma - \frac{1}{2} (\Delta \varphi + \Delta \bar{\varphi})
\]

\[
\tan \Delta \Phi = -\frac{2 \rho \cos \delta \sin \varphi + \rho^2 \sin 2\varphi}{1 + 2 \rho \cos \delta \cos \varphi + \rho^2 \cos 2\varphi}
\]

⇒ **UT angle γ enters as an “effective” angle with a NP phase shift!**

- **Takes possible direct CP violation into account:** \( C + \bar{C} \big|_{\text{SM}} = 0 \)

\[
\frac{C + \bar{C}}{(1 + C)(1 + \bar{C})} = \mathcal{A}_{\text{CP}}^{\text{dir}} + \bar{\mathcal{A}}_{\text{CP}}^{\text{dir}} + \mathcal{O}(\mathcal{A}_{\text{CP}}^{\text{dir}})^2)
\]
Assume $\delta = \bar{\delta} = 0^\circ$ to be consistent with LHCb, i.e. $C + \bar{C} = 0$

- **Branching ratios:**
  \[ \Rightarrow \bar{b} \text{ and } b \text{ give } \bar{\rho} \text{ and } \rho \text{ as functions of } \bar{\varphi} \text{ and } \varphi, \text{ respectively.} \]

- **CP violation:**
  \[ \Rightarrow \Delta\varphi = \Delta\bar{\varphi} = \gamma - \gamma_{\text{eff}} = -(61 \pm 20)^\circ, \]
  where we used $\gamma = (70 \pm 7)^\circ$ and $\gamma_{\text{eff}} = (131^{+17}_{-22})^\circ$, and
  \[
  \tan \Delta\varphi = \frac{\rho \sin \varphi + \bar{\rho} \sin \bar{\varphi} + \bar{\rho}\rho \sin(\bar{\varphi} + \varphi)}{1 + \rho \cos \varphi + \bar{\rho} \cos \bar{\varphi} + \bar{\rho}\rho \cos(\bar{\varphi} + \varphi)}.
  \]

- **Correlations between the NP parameters:**
  \[ b, \bar{b} \text{ and } \Delta\varphi \Rightarrow \varphi \text{ as function of } \bar{\varphi}, \text{ and } \rho \text{ as function of } \bar{\rho}. \]
• Central values of current measurements:

![Graph showing central values of current measurements.]

• Impact of uncertainties:

Interestingly, NP amplitudes in the (30-50)% range of the SM amplitudes could accommodate the data.

Intriguing situation: stay tuned...
The $B \rightarrow \pi K$ Puzzle

Closer Look at $B \rightarrow \pi K$ Decays

- **Example:**

\[ \propto A\lambda^4 R_b e^{i\gamma} \]
\[ \propto A\lambda^2 \]

\[ \lambda^2 R_b = \mathcal{O}(0.02) \Rightarrow \text{QCD penguins dominate} \]

- **As can straightforwardly be seen:**

This feature holds, in fact, for *all other* $B \rightarrow \pi K$ decays:

\[ B^+ \rightarrow \pi^+ K^0, \ B_d^0 \rightarrow \pi^0 K^0, \ B^+ \rightarrow \pi^0 K^+. \]
Electroweak Penguins

allow us to divide the \( B \to \pi K \) system into two classes:

- **EW penguins are colour-suppressed:** tiny contributions ...

- **EW penguins are colour-allowed:** sizeable, competing with trees!

Puzzle in CP Violation in $B_d^0 \to \pi^0 K_S$

- **CP-violating rate asymmetry:**

$$\frac{\Gamma(\bar{B}_d^0(t) \to \pi^0 K_S) - \Gamma(B_d^0(t) \to \pi^0 K_S)}{\Gamma(\bar{B}_d^0(t) \to \pi^0 K_S) + \Gamma(B_d^0(t) \to \pi^0 K_S)} = A_{CP}^{\pi^0 K_S} \cos(\Delta M_d t) + S_{CP}^{\pi^0 K_S} \sin(\Delta M_d t)$$

- **Isospin relation:**

$$\sqrt{2}A(B_d^0 \to \pi^0 K^0) + A(B_d^0 \to \pi^- K^+) \equiv 3A_{3/2}$$

$$3A_{3/2} = -(\hat{T} + \hat{C})e^{i\gamma} + (\hat{P}_{EW} + \hat{P}_{EW}^C) = -(\hat{T} + \hat{C})(e^{i\gamma} - qe^{i\phi}e^{i\omega})$$

- **$SU(3)$ flavour symmetry:**

  - Determination of $|\hat{T} + \hat{C}|$ from the $B^+ \to \pi^+ \pi^0$ branching ratio:

$$|\hat{T} + \hat{C}| = R_{T+C} |V_{us}/V_{ud}| \sqrt{2}|A(B^+ \to \pi^+ \pi^0)|$$

  - Electroweak penguin parameter:

$$qe^{i\phi}e^{i\omega} \equiv - \left[ \frac{\hat{P}_{EW} + \hat{P}_{EW}^C}{\hat{T} + \hat{C}} \right]_{SM} \equiv -\frac{3}{2\lambda^2 R_b} \left[ \frac{C_9 + C_{10}}{C_1 + C_2} \right] R_q = (0.68 \pm 0.05) R_q$$

($R_{T+C} = f_K/f_\pi$ and $R_q$ describe $SU(3)$-breaking effects)
Correlation between CP asymmetries of $B^0_d \rightarrow \pi^0 K_S$:

- Introduce the angle $\phi_{00} \equiv \arg(\bar{A}_{00}A_{00}^*)$ between the decay amplitude $A_{00} \equiv A(B^0_d \rightarrow \pi^0 K^0)$ and its CP-conjugate $\bar{A}_{00}$:

$$S_{\pi^0 K_S} = \sin(\phi_d - \phi_{00}) \sqrt{1 - (A_{\pi^0 K_S})^2}$$

- Isospin amplitude triangle relation and (minimal) $SU(3)$ input: $\implies$

$\phi_\pm \equiv \arg(\bar{A}_\pm A_{\pm}^*)$

⇒ Puzzling pattern!?
Modified Electroweak Penguin Sector

• Use further information from charged $B \to \pi K$ decays:
  
  – Satisfy also an isospin relation:
    
    $$\sqrt{2}A(B^+ \to \pi^0 K^+) + A(B^+ \to \pi^+ K^0) = 3A_{3/2}$$
  
  – Implies correlations in the $\phi-q$ plane of EW penguin parameters:

• Intriguing situation:
  
  – Yet another signal for new sources of CP violation?
  – Interesting playground for Belle II → look forward to data...

[R.F., Jaarsma, Malami & Vos (2016)]
Conclusions and Outlook
Towards New Frontiers with Non-Leptonic $B$ Decays

- $B^0_d \rightarrow J/\psi K_S$ and $B^0_s \rightarrow J/\psi \phi$ decays: \(\phi_d\) and \(\phi_s\)

  - Strategy to include doubly Cabibbo-suppressed penguins through “control channels” where these contributions are not suppressed.
  - Crucial in view of very impressive future precision for \(\phi_d\) and \(\phi_s\) ...
  - Interesting insights into hadronic physics: factorization...
  - \(\phi_s\) may still result in NP effects larger than 5\(\sigma\) significance.

- $B^0_s \rightarrow D^\mp K^\pm$ decays: \(Puzzle\)

  - Intriguing CP-violating observables...
  - Complemented through puzzling patterns in the $B^0_s \rightarrow D^\mp K^\pm$ branching ratios: consistent with other modes!
  - Model-independent framework to include and constrain NP effects.

- $B \rightarrow \pi K$ decays: \(Puzzle\)

  - Isospin relations with minimal $SU(3)$ input give intriguing correlations.
  - Mixing-induced CP violation in $B^0_d \rightarrow \pi^0 K_S$ plays a key role.
  - Modified electroweak penguin sector!? 
Intriguing Situation: Questions

- Could the puzzling patterns in the $B^0_s \rightarrow D^+_s K^\pm$ and $B \rightarrow \pi K$ decays actually be manifestations of the same kind of New Physics?

- Could there be links with the “$B$ decay anomalies” in semi-leptonic and leptonic rare $B$ decays receiving currently a lot of attention?

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**CERN COURIER**  
FLAVOUR PHYSICS NEWS

New data strengthens $R_K$ flavour anomaly  
23 March 2021

A report from the LHCb experiment

The principle that the charged leptons have identical electroweak interaction strengths is a distinctive feature of the Standard Model (SM). However, this lepton-flavour universality (LFU) is an accidental symmetry in the SM, which may not hold in theories beyond the SM (CERN Courier May/June 2019 p33). The LHCb collaboration has used a number of rare decays mediated by flavour-changing neutral currents, where the SM contribution is suppressed, to test for deviations from LFU. During the past few years, these and other measurements, together with results from B-factories, hint at possible departures from the SM.

![Comparison between $R_K$ measurements](image)

Comparison between $R_K$ measurements: In addition to the LHCb result, the measurements by the BaBar and Belle collaborations, which combine $B \rightarrow K \ell^+ \ell^-$ and $B \rightarrow K\ell^+\ell^-$ decays, are also shown. Credit: LHCb

- Specific models and links with collider physics and direct NP searches?

⇒ *Exciting times: stay tuned* ...
Greetings from the Nikhef Theory Group