Challenges in supersymmetric cosmology

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Problem of scales

- describe high energy (SUSY?) extension of the Standard Model
  unification of all fundamental interactions
- incorporate Dark Energy
  simplest case: infinitesimal (tuneable) +ve cosmological constant [4]
- describe possible accelerated expanding phase of our universe
  models of inflation (approximate de Sitter) [5]

⇒ 3 very different scales besides $M_{\text{Planck}}$: [6]

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<th>DarkEnergy</th>
<th>ElectroWeak</th>
<th>Inflation</th>
<th>QuantumGravity</th>
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<tbody>
<tr>
<td>meV</td>
<td>TeV</td>
<td>$M_I$</td>
<td>$M_{\text{Planck}}$</td>
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Supersymmetry

A well motivated proposal

addressing several open problems of the Standard Model

- natural elementary scalars
- realise unification of the three Standard Model forces
- natural dark matter candidate (lightest supersymmetric particle)
- addressing the hierarchy problem
- prediction of light Higgs ($\lesssim 130$ GeV)
- soft UV behavior and important ingredient of string theory

But no experimental indication of any BSM physics at LHC

It is likely to be there at some (more) fundamental level
Relativistic dark energy 70-75% of the observable universe

negative pressure: $p = -\rho \Rightarrow$ cosmological constant

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab} \Rightarrow \rho_\Lambda = \frac{c^4 \Lambda}{8\pi G} = -p_\Lambda$$

Two length scales:

- $[\Lambda] = L^{-2} \leftarrow$ size of the observable Universe
  
  $\Lambda_{\text{obs}} \simeq 0.74 \times 3H_0^2/c^2 \simeq 1.4 \times (10^{26} \text{ m})^{-2}$

  Hubble parameter $\simeq 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$

- $[\frac{\Lambda}{G} \times \frac{c^3}{\hbar}] = L^{-4} \leftarrow$ dark energy length $\simeq 85 \mu\text{m}$ [2]
Inflation:

Theoretical paradigm consistent with cosmological observations

But phenomenological models with not real underlying theory [2]

introduce a new scalar field that drives Universe expansion at early times

Inflaton potential

slow-roll region with $V'$, $V''$ small compared to the de Sitter curvature
Problem of scales: connections

Direct connection of inflation and supersymmetry breaking:
identify the inflaton with the partner of the goldstino

Goldstone fermion of spontaneous supersymmetry breaking

while accommodating observed vacuum energy
Inflation in supergravity: main problems

Inflaton: part of a chiral superfield $X$

- slow-roll conditions: the eta problem $\Rightarrow$ fine-tuning of the potential

$$\eta = \frac{V''}{V}, \quad V_F = e^K(\lvert DW \rvert^2 - 3\lvert W \rvert^2), \quad DW = W' + K'W$$

$K$: Kähler potential, $W$: superpotential  
Planck units: $\kappa = 1$

- canonically normalised field: $K = X\bar{X} \Rightarrow \eta = 1 + \ldots$

- trans-Planckian initial conditions $\Rightarrow$ break validity of EFT

no-scale type models that avoid the $\eta$-problem

$$K = -3\ln(T + \bar{T}); \quad W = W_0 \Rightarrow V_F = 0$$

- stabilisation of the (pseudo) scalar companion of the inflaton

chiral multiplets $\Rightarrow$ complex scalars

- moduli stabilisation, de Sitter vacuum, \ldots
**Inflation from supersymmetry breaking**

I.A.-Chatrabhuti-Isono-Knoops '16, '17, '19

Inflaton: goldstino superpartner in the presence of a gauged R-symmetry

- **linear superpotential** \( W = f X \Rightarrow \text{no } \eta\)-problem

\[
V_F = e^K (|DW|^2 - 3|W|^2) \\
= e^K (|1 + K_X X|^2 - 3|X|^2) |f|^2 \\
= e^{|X|^2} (1 - |X|^2 + \mathcal{O}(|X|^4)) |f|^2 = \mathcal{O}(|X|^4) \Rightarrow \eta = 0 + \ldots
\]

linear \( W \) guaranteed by an R-symmetry

- **gauge R-symmetry**: (pseudo) scalar absorbed by the \( U(1)_R \)

- inflation around a maximum of scalar potential (hill-top) \( \Rightarrow \text{small field} \)

- no large field initial conditions

- vacuum energy at the minimum: tuning between \( V_F \) and \( V_D \)
Two classes of models

- Case 1: R-symmetry is restored during inflation (at the maximum)

- Case 2: R-symmetry is (spontaneously) broken everywhere and restored at infinity
  
  example: $S = \ln X$
Case 1: R-symmetry restored during inflation

maximum at the origin with small $\eta$ by a correction to the Kähler potential

$$\mathcal{K}(X, \bar{X}) = \kappa^{-2} X \bar{X} + \kappa^{-4} A (X \bar{X})^2 \quad A > 0 \quad [12][15]$$

$$\mathcal{W}(X) = \kappa^{-3} f X$$

$$f(X) = 1 \quad (+\beta \ln X \text{ to cancel anomalies but } \beta \text{ very small})$$

$$\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D$$

$$\mathcal{V}_F = \kappa^{-4} f^2 e^{X \bar{X}(1+AX\bar{X})} \left[ -3X\bar{X} + \frac{(1 + X\bar{X}(1 + 2AX\bar{X}))^2}{1 + 4AX\bar{X}} \right]$$

$$\mathcal{V}_D = \kappa^{-4} \frac{q^2}{2} \left[ 1 + X\bar{X}(1 + 2AX\bar{X}) \right]^2$$

Assume inflation happens around the maximum $|X| \equiv \rho \simeq 0 ~ \Rightarrow$
Predictions

slow-roll parameters \((q \simeq 0)\)

\[
\eta = \frac{1}{\kappa^2} \left( \frac{V''}{V} \right) = -4A + \mathcal{O}(\rho^2)
\]

\[
\epsilon = \frac{1}{2\kappa^2} \left( \frac{V'}{V} \right)^2 = 16A^2 \rho^2 + \mathcal{O}(\rho^4) \simeq \eta^2 \rho^2
\]

\(\eta\) naturally small since \(A\) is a correction

inflation starts with an initial condition for \(\phi = \phi_*\) near the maximum

and ends when \(|\eta| = 1\)

\[\Rightarrow \text{number of e-folds } N = \int_{\text{end}}^{\text{start}} \frac{V}{V'} = \kappa \int \frac{1}{\sqrt{2\epsilon}} \simeq \frac{1}{|\eta_*|} \ln \left( \frac{\rho_{\text{end}}}{\rho_*} \right)\]

Planck '15 data: \(\eta \simeq -0.02 \Rightarrow N \gtrsim 50\) naturally
amplitude of density perturbations \[ A_s = \frac{\kappa^4 V_*}{24\pi^2\epsilon_*} = \frac{\kappa^2 H_*^2}{8\pi^2\epsilon_*} \]

spectral index \[ n_s = 1 + 2\eta_* - 6\epsilon_* = 1 + 2\eta_* \]

tensor – to – scalar ratio \[ r = 16\epsilon_* \]

Planck ’15 data: \( \eta \simeq -0.02, A_s \simeq 2.2 \times 10^{-9}, N \gtrsim 50 \)

\[ \Rightarrow r \lesssim 10^{-4}, H_* \lesssim 10^{12} \text{ GeV} \quad \text{assuming } \rho_{\text{end}} \lesssim 1/2 \]

Question: can a ‘nearby’ minimum exist with a tiny +ve vacuum energy?

Answer: Yes in a ‘weaker’ sense: perturbative expansion \[^{[10]}\]

valid for the Kähler potential but not for the slow-roll parameters

need D-term contribution and next (cubic) correction in \( \mathcal{K} \)
Microscopic Model

Fayet-Iliopoulos model based on a $U(1)$ R-symmetry in supergravity

two chiral multiplets $\Phi_\pm$ of charges $q_\pm$ and mass $m$ and FI parameter $\xi$

$$W = m \Phi_+ \Phi_-$$

R-symmetry $\Rightarrow q_+ + q_- \neq 0$

Higgs phase: $\langle \Phi_- \rangle = v \neq 0$

Limit of small SUSY breaking compared to the $U(1)$ mass: $m^2 << q_- v^2$

integrate out gauge superfield $\rightarrow$ EFT for the goldstino superfield $\Phi_+$

$$W = mv\Phi_+ \; ; \; \; K = \bar{\Phi}_+ \Phi_+ + A (\bar{\Phi}_+ \Phi_+)^2 + B (\bar{\Phi}_+ \Phi_+)^3 + \cdots$$

parameter space allows realistic inflation

and a nearby minimum with tuneable energy
Two distinct cases:

- Standard Model superfields $\phi$ neutral under $U(1)_R \Rightarrow (\kappa = 1)$

$$W(X, \phi) = [f + w(\phi)]X \quad w(\phi) : \text{MSSM superpotential}$$

- SM particles neutral and superpartners charged $\Rightarrow U(1)_R \supset R$-parity:

$$W(X, \phi) = fX + w(\phi) \quad \text{I.A.-Knoops '16}$$

Both cases lead to similar results

Kinetic terms:  

$$K(X, \bar{X}, \phi, \bar{\phi}) = \sum [1 + \Delta \phi(X\bar{X})] \phi\bar{\phi} + J(X\bar{X})$$

$$J = X\bar{X} + \alpha(X\bar{X})^2 + \beta(X\bar{X})^3$$
Coupling with MSSM: constraints

- Viable inflation and (nearly vanishing) vacuum energy \( \Rightarrow q \gtrsim 0.8f \) [10]

- Positive scalar masses: 
  \[
m_0^2 = m_{3/2}^2 - \frac{1}{2} \langle 1 + \Delta \phi \rangle \langle \mathcal{D}_R \rangle^2 \geq 0
\]
  \( \Rightarrow \langle \Delta \phi \rangle \lesssim 0.15 \)

\( Z \equiv X \)
Gaugino masses from $U(1)_R$ anomaly cancellation:

$$e^{-1} \mathcal{L} \supset \frac{1}{8} \sum_{A=R,1,2,3} \text{Im}(f_A) F^A \tilde{F}^A ; \quad f_A = 1 + \beta_A \ln X$$

with $\beta_R = -\frac{g^2}{3\pi^2}$, $\beta_1 = -\frac{11g_1^2}{8\pi^2}$, $\beta_2 = -\frac{5g_2^2}{8\pi^2}$, $\beta_3 = -\frac{3g_3^2}{8\pi^2}$

Typical spectrum:

$$\alpha = 0.139 , \quad \beta = 0.6 , \quad g/f = 0.7371 , \quad f = 2.05 \times 10^{-7} \quad \Rightarrow$$

<table>
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<tr>
<th>$m_z, m_R$</th>
<th>$m_\zeta$</th>
<th>$m_{3/2}$</th>
<th>$m_0$</th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
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<td>$1.25 \times 10^{12}$</td>
<td>$6.15 \times 10^{11}$</td>
<td>$7.51 \times 10^{11}$</td>
<td>$2.68 \times 10^{11}$</td>
<td>$1.03 \times 10^{10}$</td>
<td>$6.54 \times 10^9$</td>
<td>$5.84 \times 10^9$</td>
</tr>
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masses (in GeV) of inflaton, inflatino, gravitino, and MSSM sparticles

$\{m_z, m_{3/2}, m_0\} > \{m_1, m_2, m_3\}$

$(H_{\inf} = 3 \times 10^{11} \text{ GeV})$
Inflaton decay and reheating

Dominant decay to scalars and inflatino

\[ \Gamma_{z \rightarrow \phi\phi}^{\text{tot}} = 5.8 \times 10^{-3} \text{ GeV} \]

\[ \Gamma_{z \rightarrow \lambda\lambda}^{\text{tot}} \approx \Gamma_{z \rightarrow \zeta\zeta} = 4.7 \times 10^{-4} \text{ GeV} \]

\[ \Rightarrow T_{\text{reh}} \sim \sqrt{M_P \Gamma_{\text{tot}}} = 1.26 \times 10^8 \text{ GeV} \]

Possible dark matter candidate: superheavy LSP

\[ m_{\text{LSP}} \sim 10^{10} \text{ GeV} \text{ with } T_{\text{reh}}/m_{\text{DM}} \sim 10^{-3} \]

Chung-Kolb-Riotto '99
Conclusions

General class of models with inflation from SUSY breaking:

**identify inflaton with goldstino superpartner**

- (gauged) R-symmetry restored
  
  small field, avoids the $\eta$-problem, no (pseudo) scalar companion
  
  a nearby minimum can have tuneable positive vacuum energy

- inflaton sector can be coupled to MSSM
  
  with gauge $U(1)_R$ containing the R-parity

- D-term inflation is also possible using a new FI term
  
  it can lead to large $r$ of primordial gravitational waves

Open question: string theory realisation