Going beyond the Standard paradigm of Cosmology:

Torsion, gravitational anomalies and inflation without inflaton fields

Nick E. Mavromatos

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Athens, Greece
&
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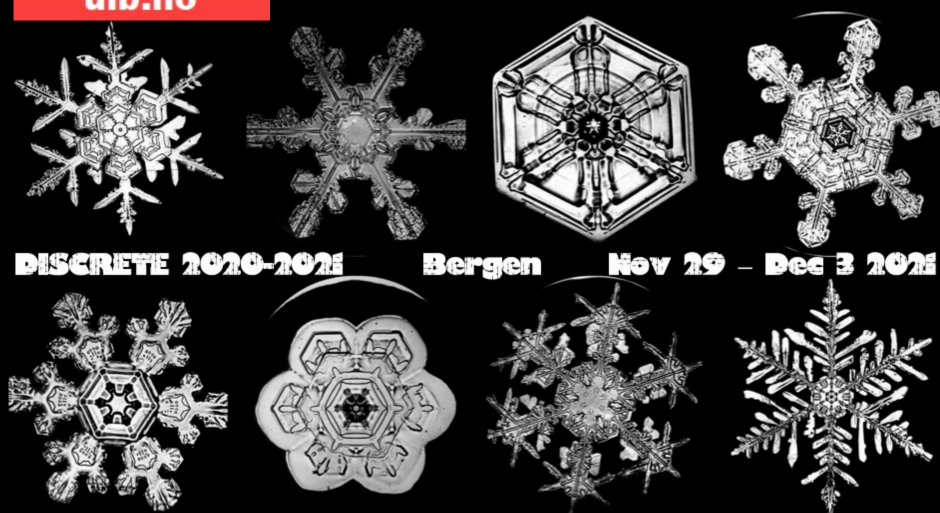
London, UK

CA18108 - Quantum gravity phenomenology in the multi-messenger approach



DISCRETE 2020-2021

Virtual Talk
November 29th 2021

1. Outline

Outline

- Motivation for Going Beyond ΛCDM

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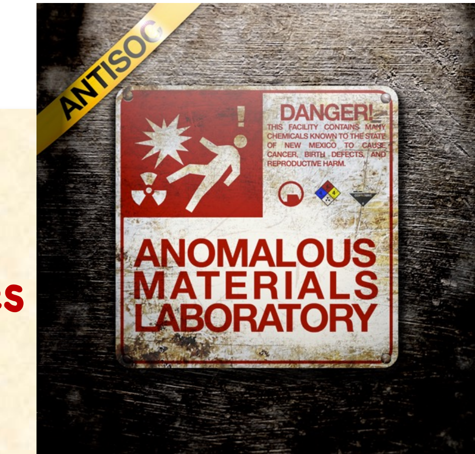
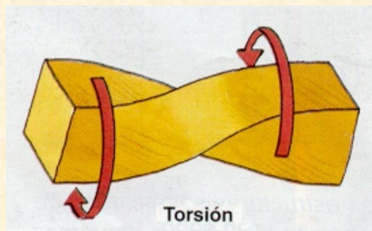


- **Motivation for Going Beyond ΛCDM**

- **String-inspired Cosmologies with Gravitational anomalies and torsion**



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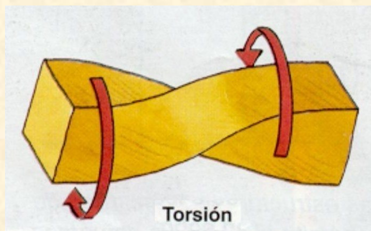

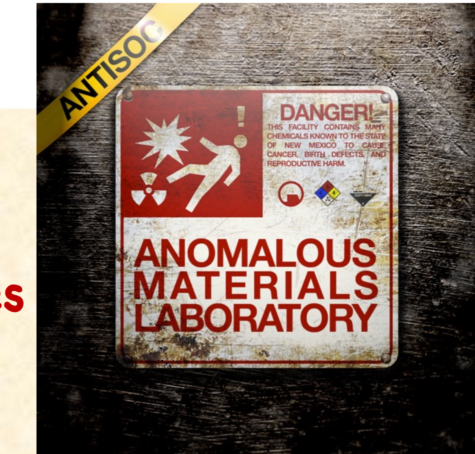

- Primordial Gravitational waves, Spontaneous-Lorentz-&-CPT-Violating anomaly condensation and ``running-vacuum-model (RVM)"-type dynamical inflation without inflaton fields

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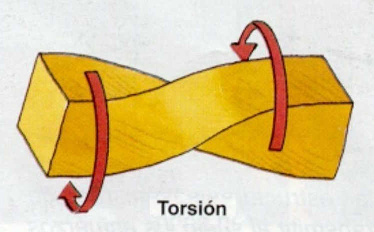

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- Post-inflationary epochs, Lorentz & CPT Violating Leptogenesis and Baryogenesis (matter-antimatter asymmetry in the Universe)

Sarben Sarkar's talk this afternoon

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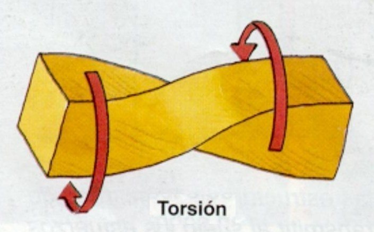

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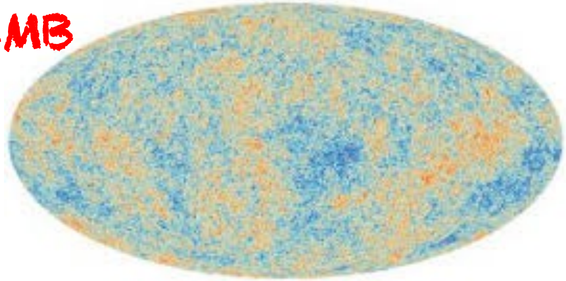
Sarben Sarkar's talk this afternoon

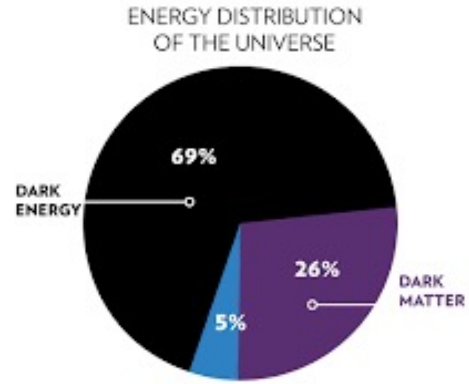
- RVM cosmology in modern era & potential resolution of data tensions

2. Motivation

Important (last ~ 20 yrs) Discoveries in Cosmology/Astronomy

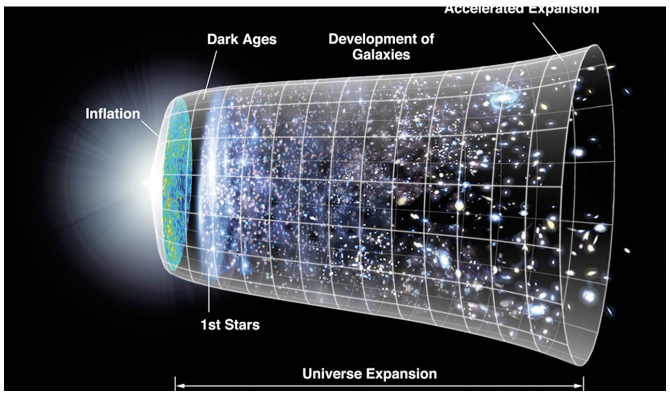
CMB

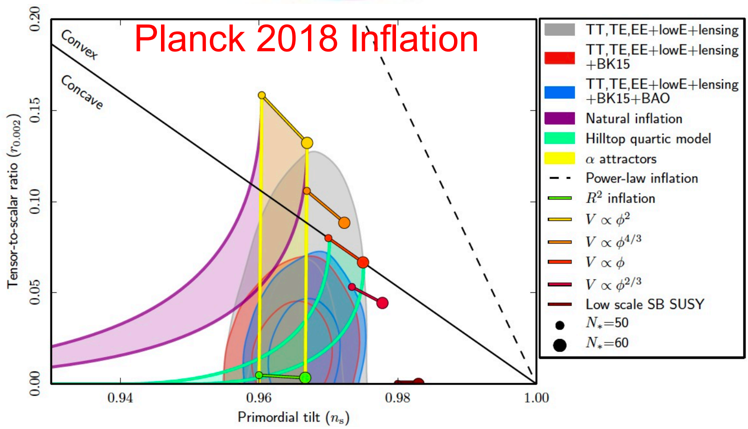




+ SnIa

Helped towards better understanding of evolution of Universe, showed current acceleration ← cosmol. constant (?) dominance





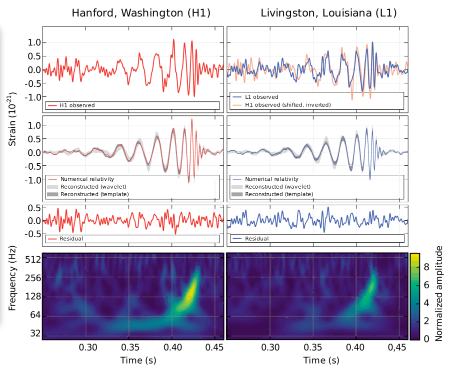
Cosmic time →

Inflation (de Sitter) → radiation-dominance → matter dominance → de Sitter (?) again

Gravitational Waves from Black Hole mergers

``Heard'' (2015) for the first time by LIGO Inteferometer

Open new era in Astronomy



Important (last ~ 20 yrs) Discoveries in Cosmology/Astronomy

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What still we do not know/did not observe:

Nature of Dark Energy

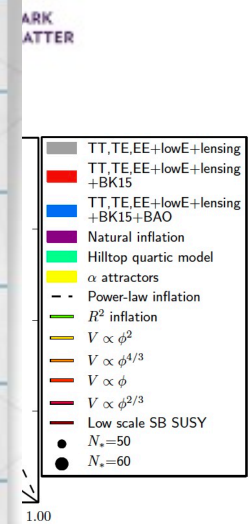
Nature of Dark matter

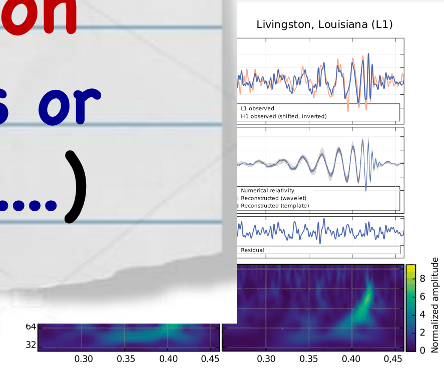
Primordial Gravitational Waves

(through detection of B-mode polarisation
in CMB from very early Universe)

Microscopic models of Inflation

(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)





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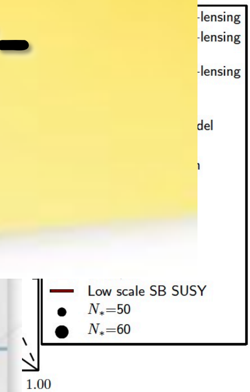
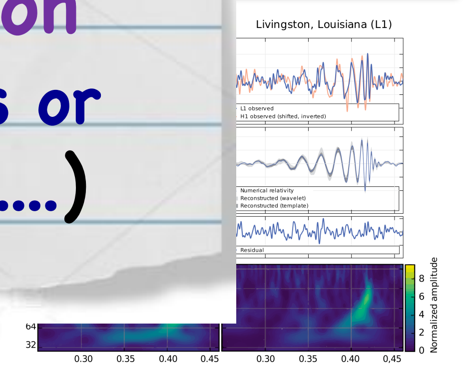
ΛCDM appears
to be in tension with
local measurements of
present-era H_0
& also σ_8 galaxy-
growth data ?

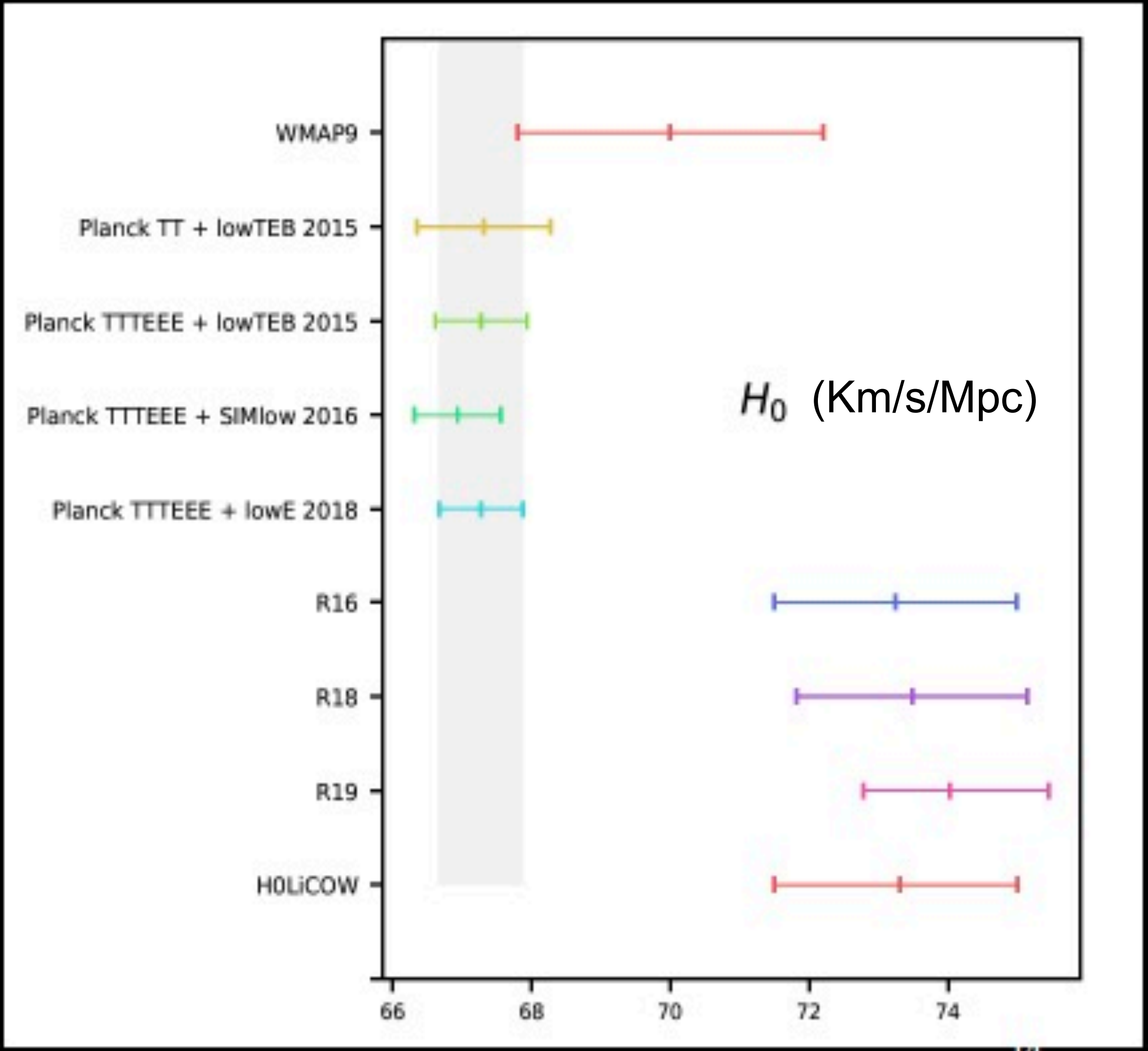




The H0 tension

We have two different blocks giving estimates of the Hubble constant in tension with each other:

- CMB (WMAP, Planck, ground based telescopes), BAO, BBN, Pantheon;
- Direct local distance ladder measurements (HST, SH0ES) and Strong lensing (H0LiCOW).

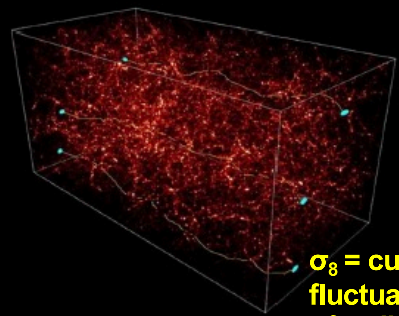


E. Di Valentino

E. Di Valentino

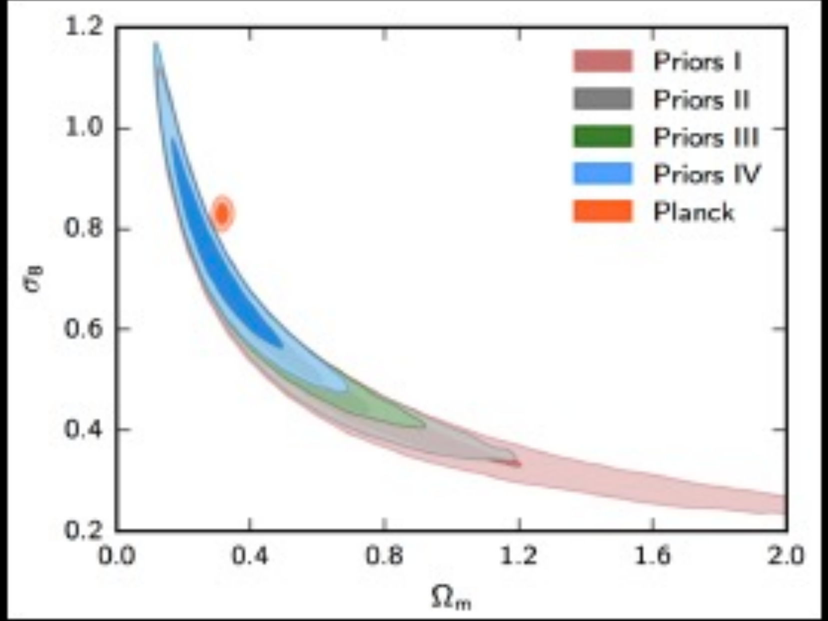
September 10th, 2019
VII Meeting on Fundamental Cosmology
Madrid

S8 tension



σ_8 = current matter density rms fluctuations within spheres of radius $8h^{-1}$ ($h = H_0/100$ = reduced Hubble constant)

$$S_8 \equiv \sigma_8 \sqrt{\Omega_m / 0.3}$$

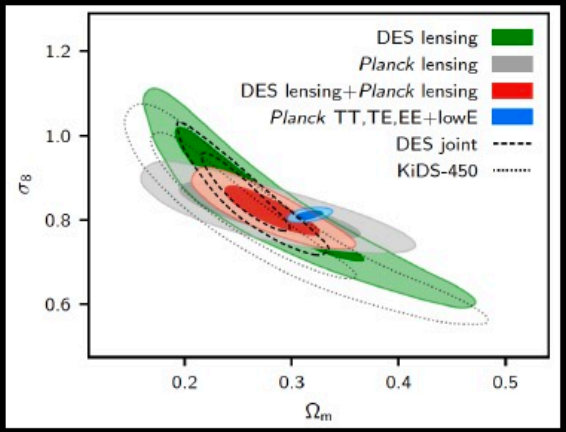


Joudaki et al, arXiv:1601.05786

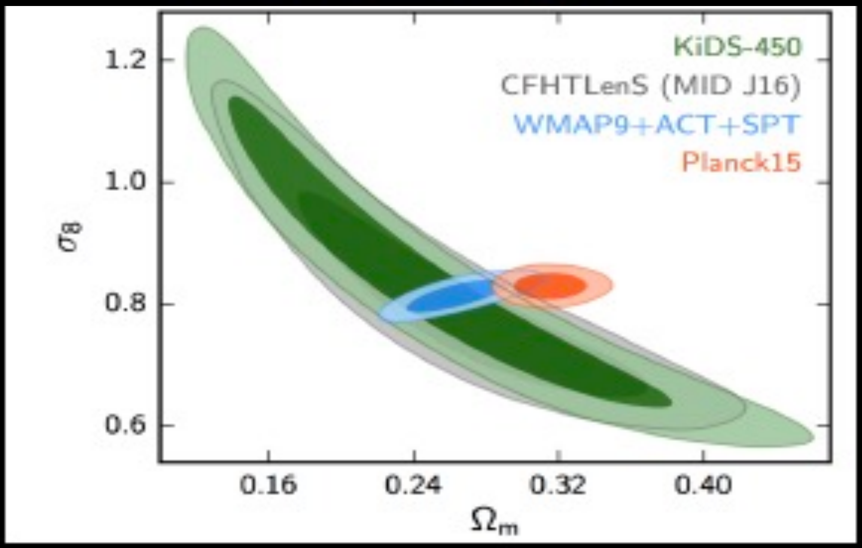
A tension on S8 is present between the Planck data in the ΛCDM scenario and the cosmic shear data.

S8 tension

Planck 2018, Aghanim et al., arXiv:1807.06209 [astro-ph.CO]



While there is no tension with DES galaxy lensing, a tension at about 2.5 sigma level is present for the DES results that include galaxy clustering.



Hildebrandt et al., arXiv:1606.05338.

The S8 tension is at about 2.6 sigma level between the Planck data in the ΛCDM scenario and CFHTLenS survey and KiDS-450.

Important (last ~ 20 yrs) Discoveries in Cosmology/Astronomy

CM

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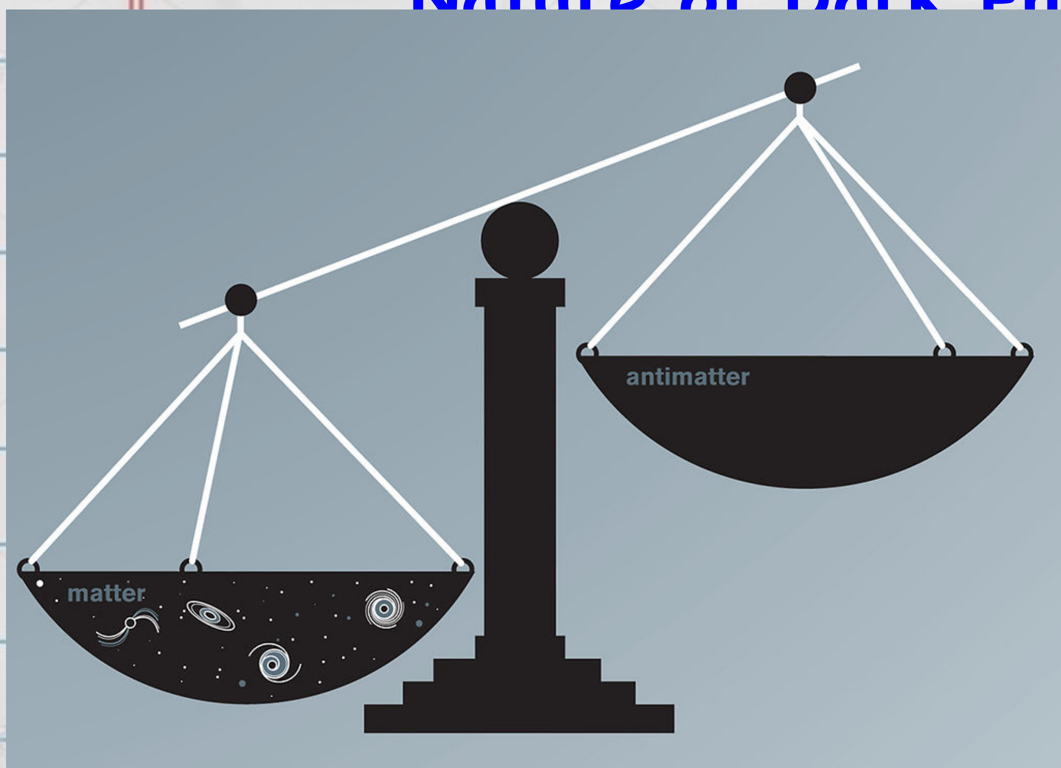
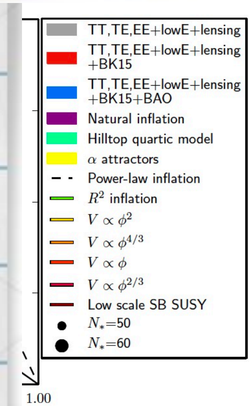
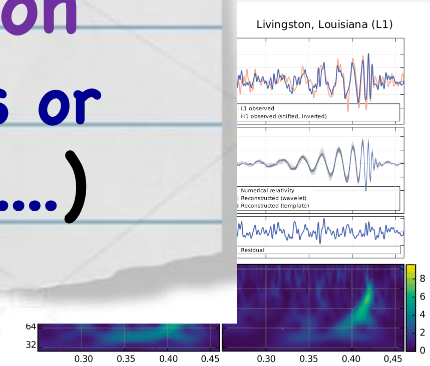
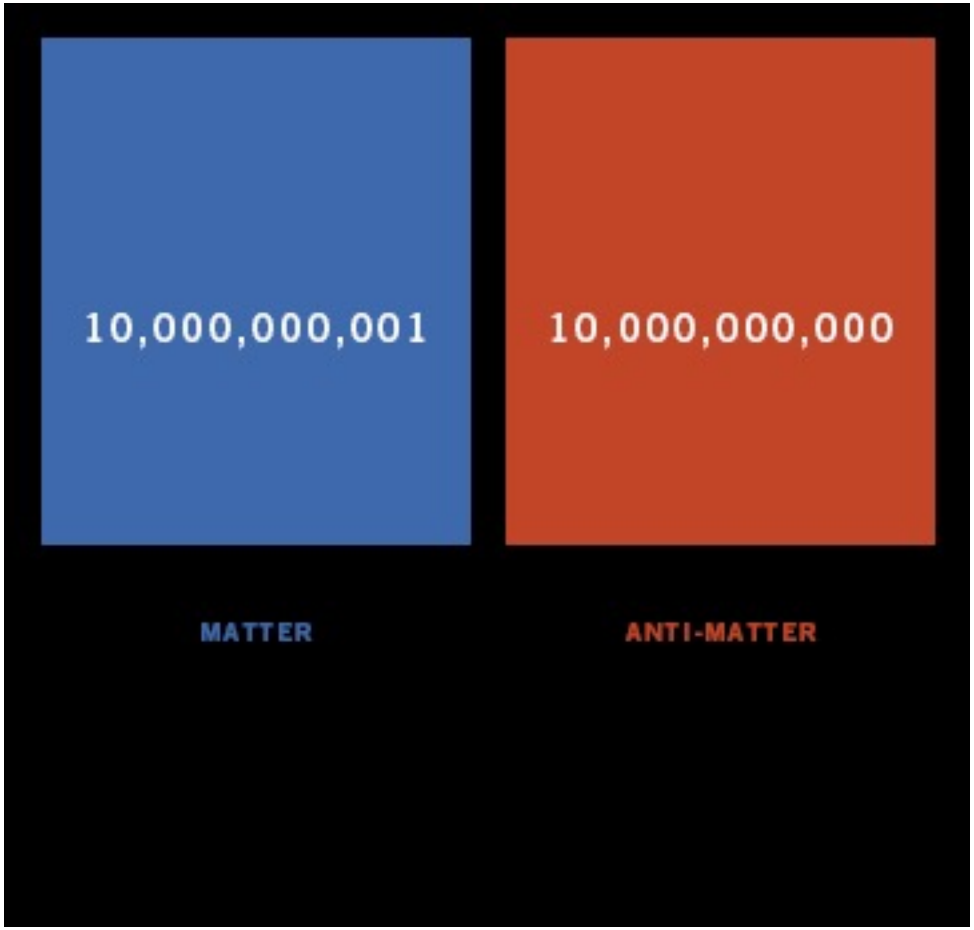
Microscopic understanding of **Matter/Antimatter asymmetry** in the Universe?









Microscopic understanding of Matter/Antimatter asymmetry in the Universe?

Baryon density in the Universe:

From CMB (Planck 2018 data)
$\Omega_b\, h^2 = 0.0224 \pm 0.0001$

From Big Bang Nucleosynthesis
$\Omega_b\, h^2 = 0.0214 \pm 0.002$

The (observed) Baryon Asymmetry

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

s = entropy density of Universe

Attempts at Explanation – Sakharov 's Conditions

Baryon number violation

C-violation

and CP violation

Departure from thermodynamic equilibrium (non-stationary system)

Standard Model (SM) satisfies these conditions but not at the ...right magnitude : the CP violation in the quark sector of the standard model is ...some ten orders of magnitude less than the one required for the observed matter-antimatter asymmetry

CP violation in lepton sector not yet observed

Need new physics beyond the SM → new sources of CP violation?

Assume CPT

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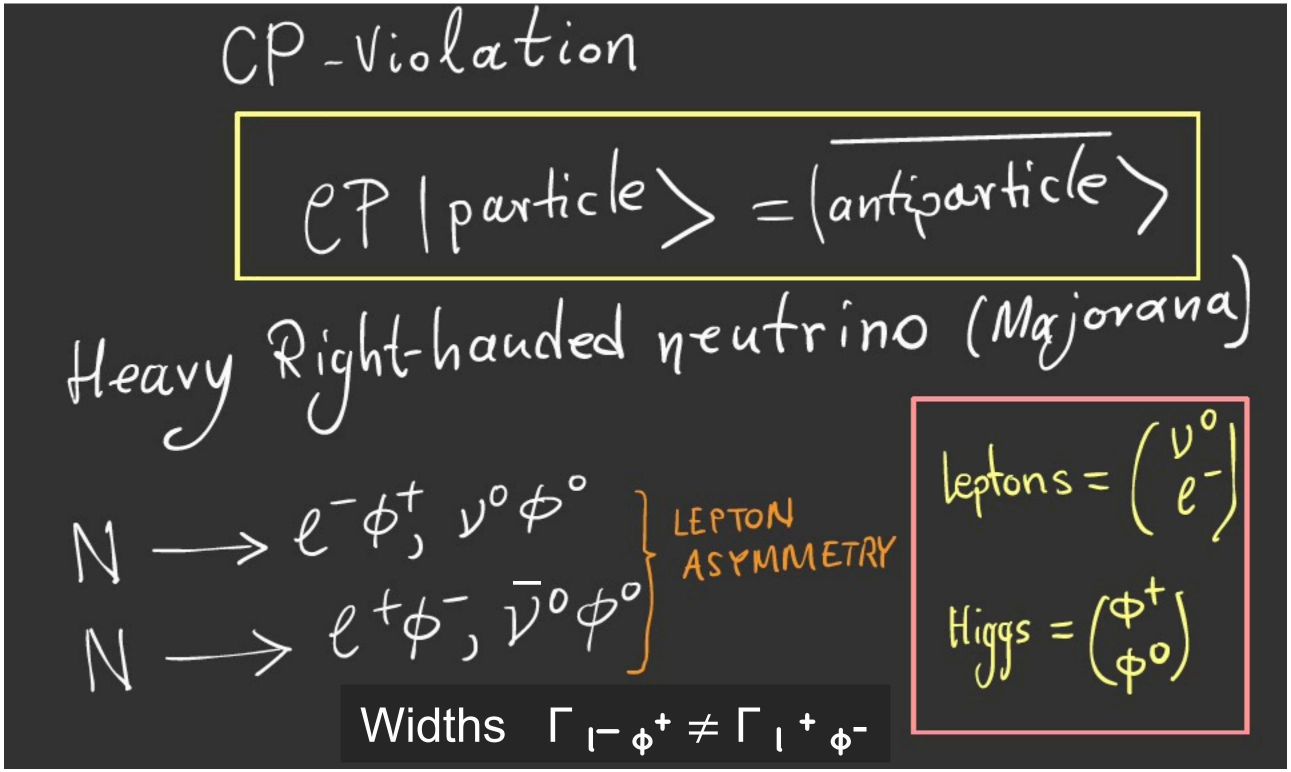
Role of Neutrinos ν?

- Several Ideas to go beyond the SM (e.g. GUT models, Supersymmetry, extra dimensional models etc.)
- Massive ν are **simplest** extension of SM
- Right-handed supermassive ν may provide extensions of SM with:

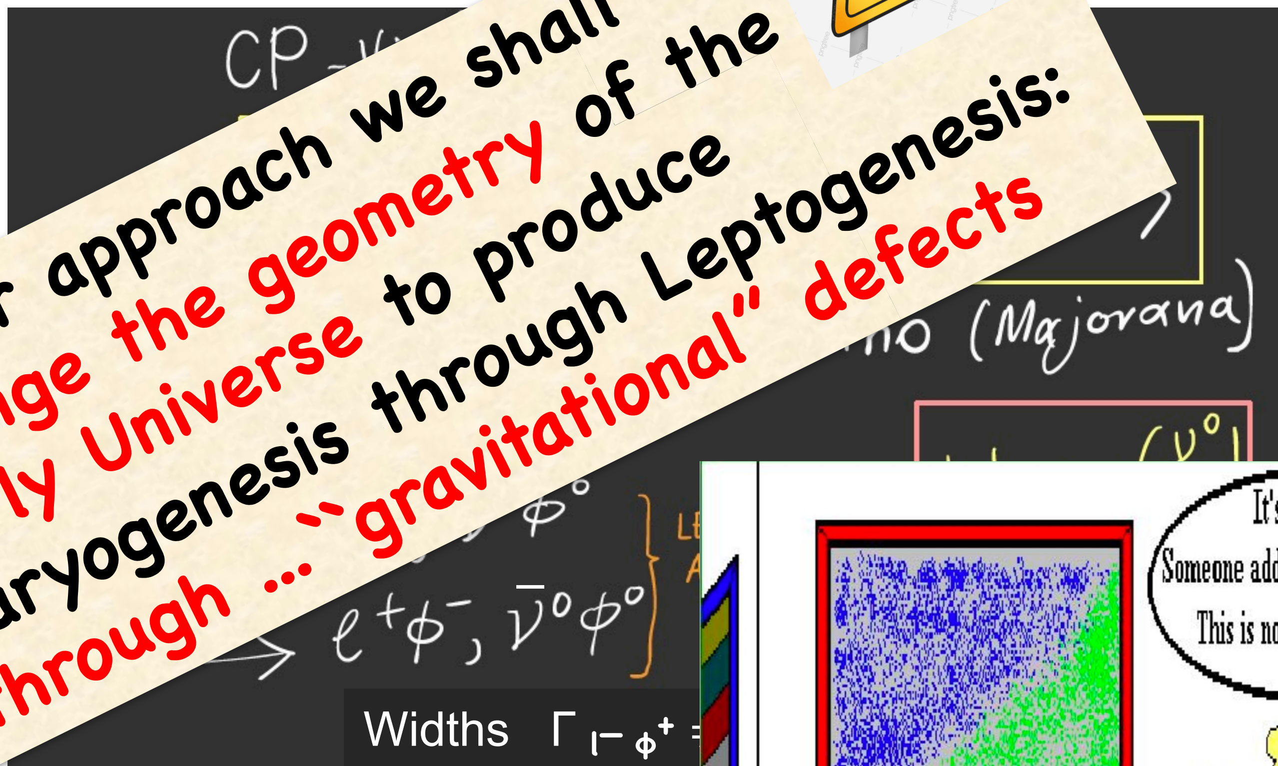
extra CP Violation and thus Origin of Universe's **matter-antimatter asymmetry** due to neutrino masses, **Dark Matter**

Baryogenesis through Leptogenesis
In models with heavy right-handed neutrinos

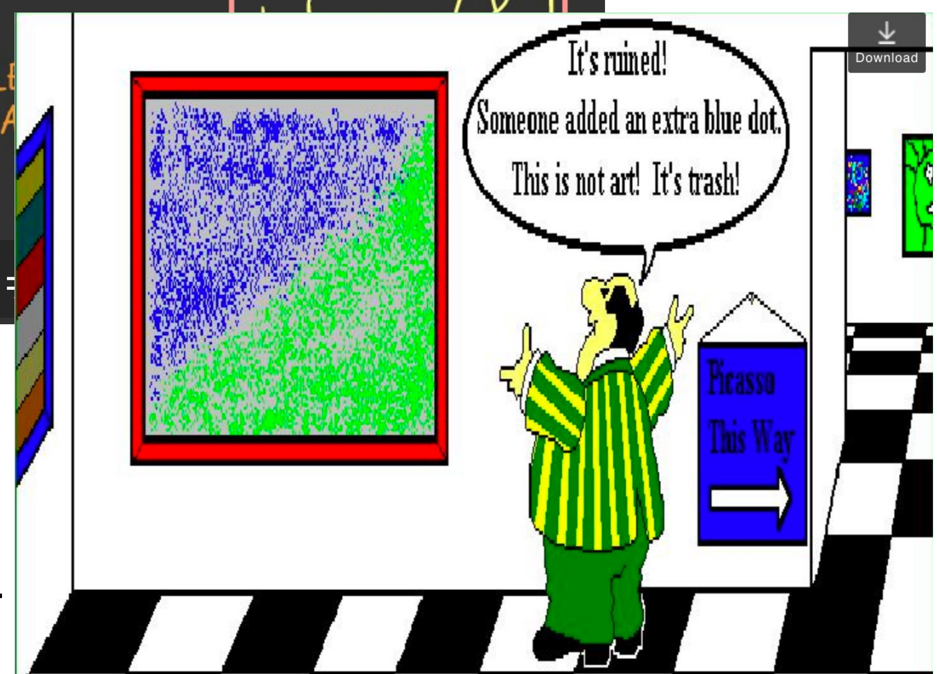


Lepton Asymmetry is then communicated to the Baryon sector by equilibrated sphaleron processes in the standard model which conserve the difference Baryon (B) – Lepton (L) number, but violate B + L → Baryogenesis = Baryon Asymmetry

Baryogenesis through L… …nesis
In models with heavy ri… …neutrinos

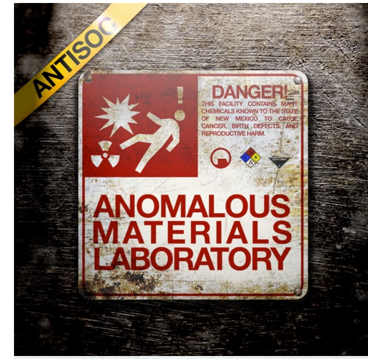


In our approach we shall
Change the geometry of the
Early Universe to produce
Baryogenesis through Leptogenesis:
through … "gravitational" defects



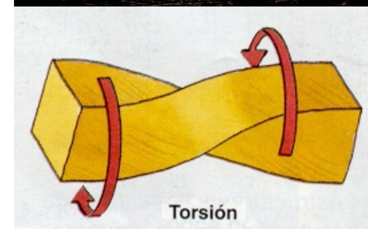
K Turzynski (ITP, Warsaw), talk :
https://www.slideserve.com/finola/why-is-there-something-rather-than-nothing-baryogenesis-and-leptogenesis

Specifically, we shall argue that in contorted string-inspired cosmologies (UV complete)



Gravitational Wave condensation in early Universe →

Gravitational Anomaly condensation →



Torsion-induced axion background *b*(x) with constant (or, at most, slowly varying with cosmic time) time derivative ←→ Spontaneous Lorentz Violation

STANDARD MODEL EXTENSION EFT ← Lorentz & CPT Violation

Kostelecky, Bluhm, Colladay, Lehnert, Potting, Russell *et al.*

$$\mathcal{L} = \tfrac{1}{2} i \bar{\psi} \Gamma^{\nu} \overleftrightarrow{\partial}_{\nu} \psi - \bar{\psi} M \psi ,$$

$$M \equiv m + a_{\mu}\gamma^{\mu} + b_{\mu}\gamma_5\gamma^{\mu} + \tfrac{1}{2} H^{\mu\nu}\sigma_{\mu\nu}$$

Spontaneous Violation of Lorentz Symmetry
(*LV coefficients are v.e.v. of tensor-valued field quantities*)
b_0 ≈ constant is string-inspired H-torsion background in our model : $b_{\mu} \equiv \partial_{\mu} b$, *b = axion dual of* H

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Gravitational Anomaly condensation →

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Relevant for Leptogenesis

cf. Sarben Sarkar talk this afternoon

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THIS TALK:

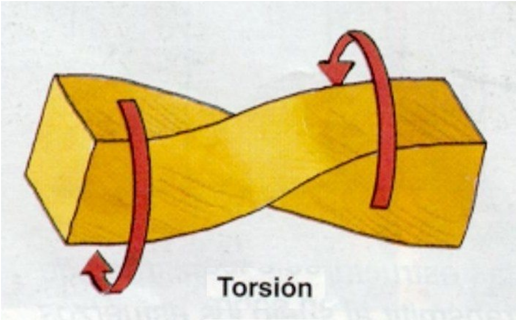
a microscopic (string-inspired) model for RVM Universe....

Basilakos, NEM, Solà
(i) JCAP 12 (2019) 025
(ii) IJMD28 (2019) 1944002
(iii) Phys.Rev.D 101 (2020) 045001
(iv) Phys.Lett.B 803 (2020) 135342
(v) Universe 2020,6(11), 218
NEM, Solà
(vi) EPJST 230 (2021), 2077
(vii) arXiv: 2105.02659 EPJPlus
NEM
(viii) arxiv: 2111.05675, *Universe* to appear
(ix) arxiv: 2108.02152, Phil.Trans. A (RS UK)

Links with :
spontaneous Lorentz violation
(via (gravitational axion)
backgrounds)
and
Matter-Antimatter Asymmetry
in theories with
Right-Handed Neutrinos

(i) NEM & Sarben Sarkar, EPJC 73 (2013), 2359
(ii) John Ellis, NEM & Sarkar, PLB 725 (2013), 407
(iii) De Cesare, NEM & Sarkar, EPJC 75 (2015), 514
(iv) Bossingham, NEM & Sarkar, EPJC 78 (2018), 113; 79 (2019), 50
(v) NEM & Sarben Sarkar, EPJC 80 (2020), 558

3. Torsion matters



Einstein-Cartan Theory with (Quantum) Torsion

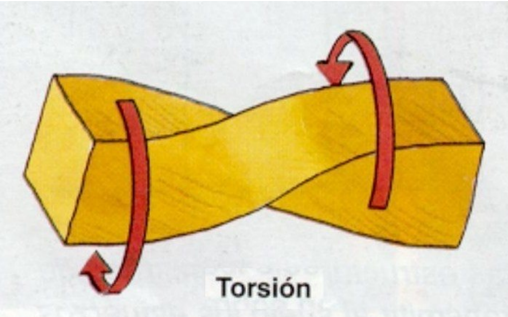
$$\mathrm{T}^a = \mathrm{d}\mathrm{e}^a + \overline{\omega}^a{}_b \wedge \mathrm{e}^b \equiv \overline{\mathrm{D}}\,\mathrm{e}^a \neq 0$$

Torsion 2-form

$$g_{\mu\nu} = \eta_{ab}\, e^a{}_\mu\, e^b{}_\nu \qquad g^{\mu\nu} = \eta^{ab}\, E^\mu{}_a\, E^\nu{}_b$$

metric vielbeins inverse vielbeins

$$E^a_\mu\, e^\mu{}_b = \delta^a{}_b \text{ and } E^a_\mu\, e^\nu{}_a = \delta_\mu{}^\nu$$



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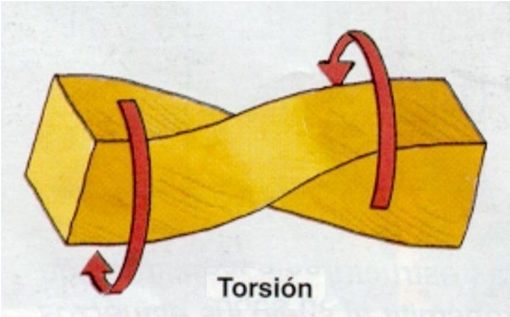
Palatini formalism: in presence of torsion, spin connection and vielbeins independent

Spin connection with torsion $\overline{\omega}^a_{\mu\ b} = \omega^a_{\mu\ b} + K^a_{\mu\ b}.$



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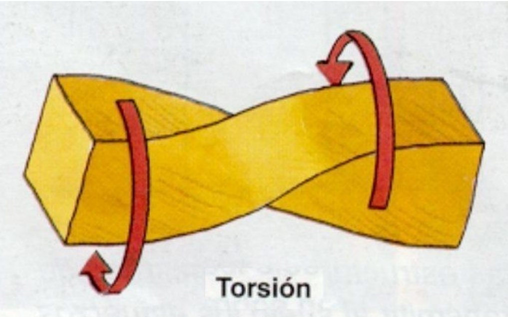
Riemannian Spin connection (Einstein theory) **Contorsion**

Einstein-Cartan Theory with (Quantum) Torsion



$$T^a = de^a + \overline{\omega}^a{}_b \wedge e^b \equiv \overline{D}\, e^a \neq 0$$ **Torsion 2-form**

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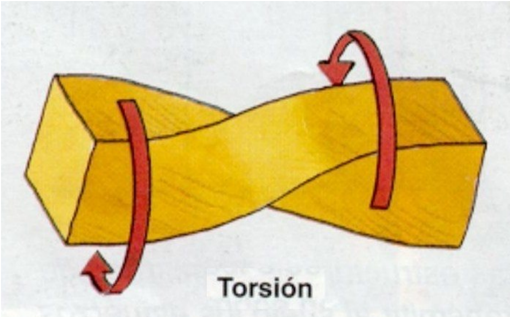
Riemannian Spin connection (Einstein theory) **Contorsion**

$$De^a \equiv de^a + \omega^a_b \wedge e^b = 0$$ **Metricity postulate for Einstein-Riemann spacetimes**

$$\rightarrow \quad \Gamma^\rho{}_{\sigma\beta} = \Gamma^\rho{}_{\beta\sigma} \equiv \frac{1}{2} g^{\alpha\rho}\left[g_{\alpha\beta,\sigma} + g_{\alpha\sigma,\beta} - g_{\beta\sigma,\alpha}\right]$$



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Contorsion

Generalised curvature 2-form

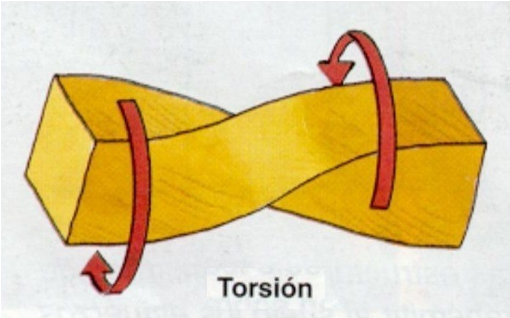
$$K^{ab}_\mu = e^a_\nu e^b_\rho K^{\nu\rho}_\mu\ , \quad K^{\nu\rho}_\mu = -K^{\rho\nu}_\mu$$

$$R^{ab}{}_{\mu\nu} = 2\partial_{[\mu,}\overline{\omega}^{ab}{}_{\nu]} + 2\overline{\omega}^a{}_{c[\mu}\overline{\omega}^{cb}{}_{\nu]}$$

$$K^\nu_{\rho\mu} = \frac{1}{2}\left(T^\nu_{\rho\mu} - T_\rho{}^\nu{}_\mu - T_\mu{}^\nu{}_\rho\right) = -K^\nu{}_{\mu\rho}$$

$$\overline{R}^\lambda{}_{\rho\mu\nu} = E^\lambda{}_a\, e^b{}_\rho\, \overline{R}^a{}_{b\mu\nu} = \partial_\mu \Gamma^\lambda{}_{\rho\nu} + \Gamma^\lambda{}_{\sigma\mu}\Gamma^\sigma{}_{\rho\nu} - (\mu \leftrightarrow \nu), \quad \lambda, \mu, \nu, \rho = 0, \dots 3\ ,$$

Torsion tensor $T^\lambda{}_{\mu\nu} = \overline{\Gamma}^\lambda{}_{\mu\nu} - \overline{\Gamma}^\lambda{}_{\nu\mu} \neq 0$



Einstein-Cartan Theory with (Quantum) Torsion

$$\mathrm{T}^a = \mathrm{d}\mathrm{e}^a + \overline{\omega}^a{}_b \wedge \mathrm{e}^b \equiv \overline{\mathrm{D}}\,\mathrm{e}^a \neq 0$$

Torsion 2-form

$$g_{\mu\nu} = \eta_{ab}\, e^a{}_\mu\, e^b{}_\nu \qquad g^{\mu\nu} = \eta^{ab}\, E^\mu{}_a\, E^\nu{}_b$$

metric **vielbeins** **inverse vielbeins**

$$E^a_\mu\, e^\mu{}_b = \delta^a{}_b \text{ and } E^a_\mu\, e^\nu{}_a = \delta_\mu{}^\nu$$

Palatini formalism: in presence of torsion, spin connection and vielbeins independent

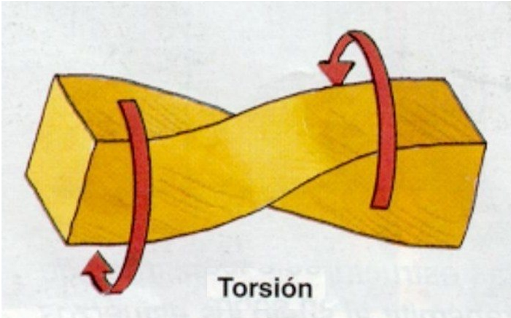
Spin connection with torsion $\overline{\omega}^a_{\mu\ b} = \omega^a_{\mu\ b} + K^a_{\mu\ b}.$

Generalised curvature 2-form

$$K^{\nu}_{\rho\mu} = \frac{1}{2}\left(T^{\nu}_{\rho\mu} - T_{\rho}{}^{\nu}{}_{\mu} - T_{\mu}{}^{\nu}{}_{\rho}\right) = -K^{\nu}{}_{\mu\rho}$$

$$\overline{\mathrm{R}}^a{}_b = \mathrm{d}\overline{\omega}^a{}_b + \overline{\omega}^a{}_c \wedge \overline{\omega}^c{}_b = \frac{1}{2}\overline{R}^a{}_{bcd}\mathrm{e}^c\,\mathrm{e}^d = \frac{1}{2}\overline{R}^a{}_{b\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu, \quad \overline{\omega}^a{}_b \equiv \overline{\omega}^a_{\mu\ b}\, dx^\mu.$$

$$\overline{\mathrm{R}}^a{}_b = \mathrm{R}^a{}_b + \mathrm{D}\,\mathrm{K}^a{}_b + \mathrm{K}^a{}_c \wedge \mathrm{K}^c{}_b$$



Einstein-Cartan Theory with (Quantum) Torsion

$$\mathrm{T}^a = \mathrm{d}\mathrm{e}^a + \overline{\omega}^a{}_b \wedge \mathrm{e}^b \equiv \overline{\mathrm{D}}\,\mathrm{e}^a \neq 0$$ **Torsion 2-form**

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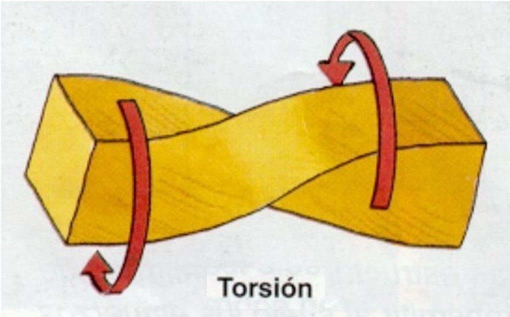
Generalised curvature 2-form

Gravitational action quadratic in (con)torsion, up to boundary terms

$$\mathcal{S}_{\mathrm{G}} = \frac{1}{2\kappa^2}\int d^4x\, \overline{\mathrm{R}}_{ab} \wedge \star(\mathrm{e}^a \wedge \mathrm{e}^b) = \frac{1}{2\kappa^2}\int d^4x \Big(\mathrm{R}_{ab} + \mathrm{K}_{ac} \wedge \mathrm{K}^c{}_b\Big) \wedge \star(\mathrm{e}^a \wedge \mathrm{e}^b)$$

Hodge star $\star(\mathrm{e}^{a_1}\ldots\mathrm{e}^{a_p}) = \frac{1}{(4-p)!}\epsilon^{a_1\ldots a_p}{}_{c_1\ldots c_{4-p}}\,\mathrm{e}^{c_1} \wedge \cdots \wedge \mathrm{e}^{c_{4-p}}$



Einstein-Cartan Theory with (Quantum) Torsion

$$\mathrm{T}^a = \mathrm{d}\mathrm{e}^a + \overline{\omega}^a{}_b \wedge \mathrm{e}^b \equiv \overline{\mathrm{D}}\,\mathrm{e}^a \neq 0$$ **Torsion 2-form**

$$g_{\mu\nu} = \eta_{ab}\, e^a{}_\mu\, e^b{}_\nu \qquad g^{\mu\nu} = \eta^{ab}\, E^\mu{}_a\, E^\nu{}_b$$

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Generalised curvature 2-form $K^\nu_{\rho\mu} = $ ... $K^\nu{}_{\mu\rho}$

Non-derivative terms of (con)torsion

Gravitational action quadratic in (con)torsion, up to boundary terms

$$\mathcal{S}_{\mathrm{G}} = \frac{1}{2\kappa^2}\int d^4x\, \overline{\mathrm{R}}_{ab} \wedge \star(\mathrm{e}^a \wedge \mathrm{e}^b) = \frac{1}{2\kappa^2}\int d^4x \Big(\mathrm{R}_{ab} + \mathrm{K}_{ac} \wedge \mathrm{K}^c{}_b\Big) \wedge \star(\mathrm{e}^a \wedge \mathrm{e}^b)$$

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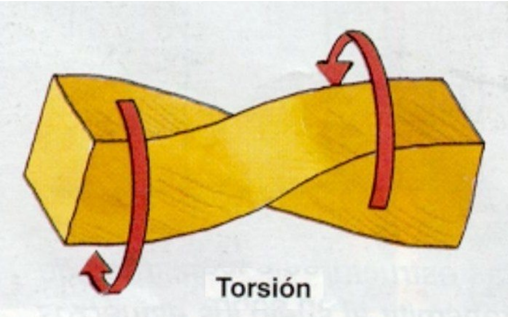


Einstein-Cartan Theory with Quantum Torsion

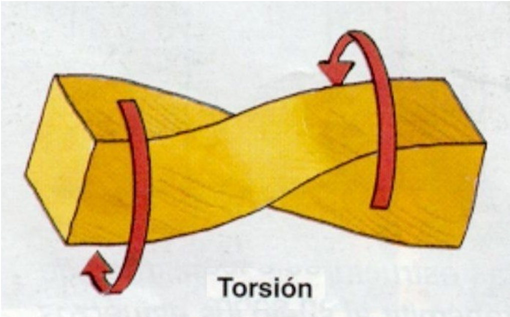
Fermions

$$\mathcal{S}_\psi^{\rm torsion} = \int d^4x \sqrt{-g}\left(\overline{\psi}\,\frac{i}{2}\,\gamma^\mu \overline{\mathcal{D}}_\mu\,\psi - \frac{i}{2}(\overline{\mathcal{D}}_\mu \overline{\psi})\,\gamma^\mu\,\psi - m\,\overline{\psi}\,\psi\right)$$

$$\overline{\mathbf{D}}\psi = \mathrm{d}\psi - \frac{i}{2}\,\overline{\omega}_{ab}\,\sigma^{ab}\,\psi\,, \qquad \sigma^{ab} = \frac{i}{2}\Big[\gamma^a\,,\,\gamma^b\Big]$$

$$\gamma^a\,\gamma^b\,\gamma^c = \eta^{ab}\,\gamma^c + \eta^{bc}\,\gamma^\alpha - \eta^{ac}\,\gamma^b - i\epsilon^{abcd}\,\gamma^5\,\gamma_d$$

$$\mathcal{S}_\psi^{\rm torsion} = \int d^4x \sqrt{-g}\left(\overline{\psi}\,\frac{i}{2}\,\gamma^\mu D_\mu\,\psi - \frac{i}{2}(D_\mu \overline{\psi})\,\gamma^\mu\,\psi - m\,\overline{\psi}\,\psi\; - \frac{1}{4}\,\epsilon^{abcd}\,\overline{\psi}\,\gamma^5\,\gamma_d\,\psi\,K_{abc}\right)$$



Einstein-Cartan Theory with Quantum Torsion

Fermions

$$\mathcal{S}_{\psi}^{\rm torsion} = \int d^4x\, \sqrt{-g}\, \Big(\overline{\psi}\, \frac{i}{2}\, \gamma^{\mu} \overline{\mathcal{D}}_{\mu}\, \psi - \frac{i}{2} (\overline{\mathcal{D}}_{\mu} \overline{\psi})\, \gamma^{\mu}\, \psi - m\, \overline{\psi}\, \psi \Big)$$

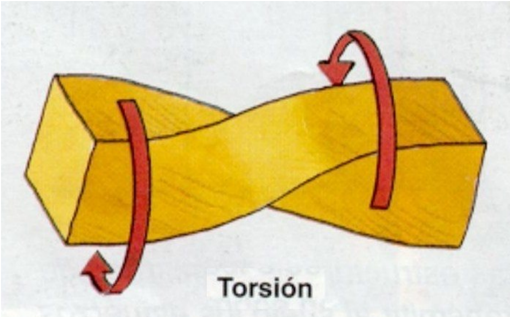
$$\overline{\mathbf{D}}\psi = \mathbf{d}\psi - \frac{i}{2}\, \overline{\omega}_{ab}\, \sigma^{ab}\, \psi\, , \qquad \sigma^{ab} = \frac{i}{2} \Big[\gamma^{a}\, ,\, \gamma^{b} \Big]$$

$$\gamma^a\, \gamma^b\, \gamma^c = \eta^{ab}\, \gamma^c + \eta^{bc}\, \gamma^{\alpha} - \eta^{ac}\, \gamma^b - i\epsilon^{abcd}\, \gamma^5\, \gamma_d$$

$$\mathcal{S}_{\psi}^{\rm torsion} = \int d^4x\, \sqrt{-g}\, \Big(\overline{\psi}\, \frac{i}{2}\, \gamma^{\mu} D_{\mu}\, \psi - \frac{i}{2} (D_{\mu} \overline{\psi})\, \gamma^{\mu}\, \psi - m\, \overline{\psi}\, \psi\, - \frac{1}{4}\, \epsilon^{abcd}\, \overline{\psi}\, \gamma^5\, \gamma_d\, \psi\, K_{abc} \Big)$$

Only totally antisymmetric part of torsion couples to fermionic matter

$$T_{[abc]} = -2K_{[abc]}$$



Einstein-Cartan Theory with Quantum Torsion

Fermions

$$S_\psi^{\rm torsion} = \int d^4x\,\sqrt{-g}\,\Big(\overline{\psi}\,\frac{i}{2}\,\gamma^\mu \overline{\mathcal{D}}_\mu\,\psi - \frac{i}{2}(\overline{\mathcal{D}}_\mu\overline{\psi})\,\gamma^\mu\,\psi - m\,\overline{\psi}\,\psi\Big)$$

$$\overline{\mathbf{D}}\psi = \mathrm{d}\psi - \frac{i}{2}\,\overline{\omega}_{ab}\,\sigma^{ab}\,\psi\,, \qquad \sigma^{ab} = \frac{i}{2}\big[\gamma^a\,,\,\gamma^b\big]$$

$$\gamma^a\,\gamma^b\,\gamma^c = \eta^{ab}\,\gamma^c + \eta^{bc}\,\gamma^\alpha - \eta^{ac}\,\gamma^b - i\epsilon^{abcd}\,\gamma^5\,\gamma_d.$$

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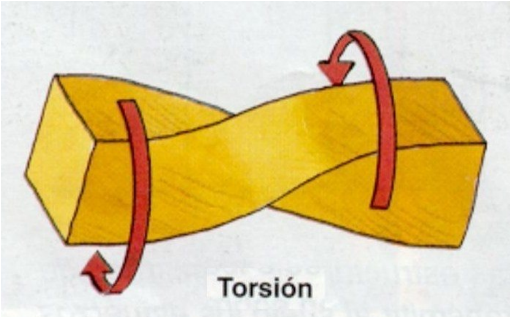
Torsion 3-form

$\mathbf{T} = \frac{1}{3!}T_{abc}\,\mathbf{e}^a\wedge\mathbf{e}^b\wedge\mathbf{e}^c,$ Its dual 1 form $\mathbf{S} = \star\mathbf{T},$ with components $S_d = \frac{1}{3!}\,\epsilon^{abc}{}_d\,T_{abc}$

Torsion couples with axial fermion current

$$S_\psi \ni -\frac{3}{4}\int d^4x\,\sqrt{-g}\,S_\mu\,\overline{\psi}\,\gamma^\mu\,\gamma^5\,\psi = -\frac{3}{4}\int \mathbf{S}\wedge\star\mathbf{J}^5 \qquad \mathbf{J}^5 = J^5_\mu\,\mathrm{d}x^\mu,$$



Einstein-Cartan Theory with **Quantum** Torsion

Ferm

$$\mathcal{S}_\psi^{\text{torsion}} = \int d^4 \ldots \big(\overline{\mathcal{D}}_\mu \overline{\psi}\big)\gamma^\mu \psi - m\overline{\psi}\psi\big)$$

$$\ldots \frac{i}{\ldots}\big[\gamma^a, \gamma^b\big]$$

$$\gamma^a\gamma^b\gamma^c \ldots$$

$$\mathcal{S}_\psi^{\text{torsion}} = \int \ldots {}^d\,\overline{\psi}\gamma^5\gamma_d\psi K_{abc}\big)$$

$$\ldots = -2K_{[abc]}$$

In Einstein-Cartan theory with fermions (quantum) torsion becomes equivalent to a pseudoscalar d.o.f.

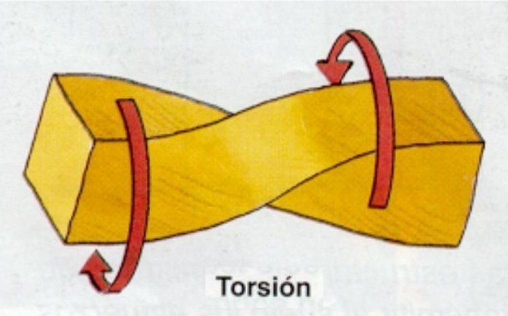
Duncan, Kaloper & Olive
(1992)

Torsion 3-form

$$\mathbf{T} = \frac{1}{3!}T_{abc}\,\mathbf{e}^a \wedge \mathbf{e}^b \wedge \mathbf{e}^c, \ldots \epsilon^{abc}{}_d T_{abc}$$

Torsion couples with axi

$$\mathcal{S}_\psi \ni -\frac{3}{4}\int d^4x\sqrt{-g}\,S \ldots \gamma^5\psi = -\frac{3}{4}\int \mathbf{S}\wedge\star\mathbf{J}^5 \qquad \mathbf{J}^5 = J^5_\mu\,\mathbf{d}x^\mu,$$



Einstein-Cartan Theory with Quantum Torsion

Fermions

$\mathcal{S}_\psi^{\text{torsion}} = \int d^4x \sqrt{-\dots}$ ... $\dots\overline{\psi})\,\gamma^\mu\,\psi - m\,\overline{\psi}\,\psi\big)$

... $\gamma^a, \gamma^b\big]$

$\gamma^a\,\gamma^b$ ...

$\mathcal{S}_\psi^{\text{torsion}} =$ ... $\gamma^5\,\gamma_d\,\psi\,K_{abc}\big)$

... $= -2K_{[abc]}$

How does (quantum) Torsion arise naturally, as an independent field (equivalent to a pseudoscalar), in String-Inspired Theories ?

Gross, Sloan, Metsaev, Tseytlin, Duncan, Kaloper & Olive, Svrcek & Witten

Torsion 3-form ...

$\mathbf{T} = \frac{1}{3!}T_{abc}\,\mathbf{e}^a \wedge \mathbf{e}^b \wedge \dots$ ... $\epsilon^{abc}{}_d\,T_{abc}$

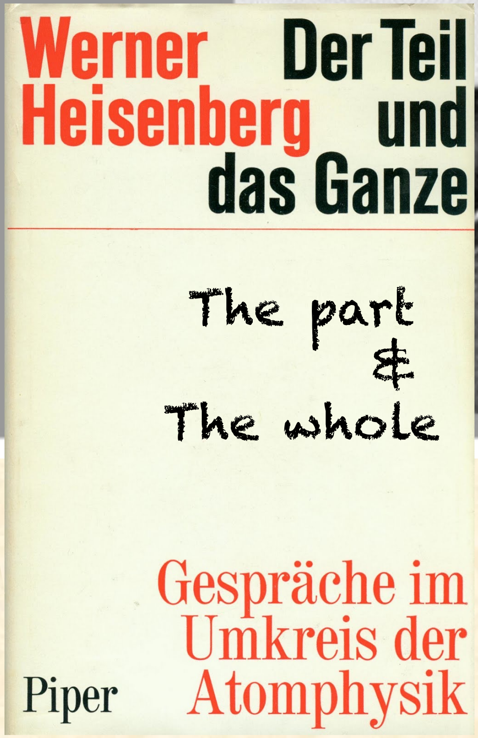
Torsion couples with a ... ent

$\mathcal{S}_\psi \ni -\frac{3}{4}\int d^4x\,\sqrt{-g}\,S_\mu\,\overline{\psi}\,\gamma^\mu\,\gamma^5\,\psi = -\frac{3}{4}\int \mathbf{S}\wedge\star\mathbf{J}^5 \qquad \mathbf{J}^5 = J^5_\mu\,\mathrm{d}x^\mu,$

"There is a fundamental error in separating the parts from the whole, the mistake of atomizing what should not be atomized.

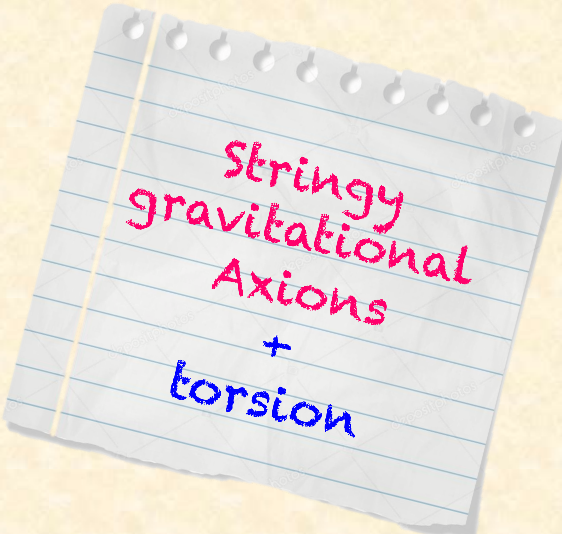
Unity and complementarity constitute reality"

Werner Karl Heisenberg
German Scientist & Nobel Prize
1901-1976



Quantum Torsion is an inseparable part of string cosmology

The Parts



Gross and Sloan, Metsaev and Tseytlin

Massless Gravitational multiplet of (closed) strings:

spin 0 scalar (dilaton Φ)

spin 2 traceless symmetric rank 2 tensor (graviton $g_{\mu\nu}$)

spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD $B_{\mu\nu} = -B_{\nu\mu}$

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$$\mathrm{U(1) - symmetry} : B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\theta(x)_{\nu]}$$

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4-DIM action

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R + 2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}e^{-4\Phi}H_{\lambda\mu\nu}H^{\lambda\mu\nu} + \ldots\Big)$$

$\kappa^2 = 8\pi\ G$

Green, Schwarz

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu}\,B_{\nu\rho]} \quad \Rightarrow \quad \boxed{\boldsymbol{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\Big(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\Big)}$$

$$\Omega_{3\mathrm{L}} = \omega^a_{\ c}\wedge \mathrm{d}\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c}\wedge\omega^c_{\ d}\wedge\omega^d_{\ a}, \qquad \Omega_{3\mathrm{Y}} = \mathbf{A}\wedge\mathbf{dA} + \mathbf{A}\wedge\mathbf{A}\wedge\mathbf{A},$$

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$$\overline{R}(\overline{\Gamma})$$

generalised curvature

Φ = constant throughout
Consistent in string models

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

Contorsion

Stringy gravitational Axions + torsion

Gross and Sloan, Metsaev and Tseytlin

Massless Gravitational multiplet of (closed) strings:

- **spin 0 scalar (dilaton Φ)**
- **spin 2 traceless symmetric rank 2 tensor (graviton $g_{\mu\nu}$)**
- **spin 1 antisymmetric rank 2 tensor**

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4-DIM action

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generalised curvature

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Contorsion

quantum torsion → gravitational axion b "dual" to H torsion

Svrcek & Witten

To this end…

Bianchi Identity for $H_{\mu\nu\rho}$

Svrcek & Witten

$\kappa^2 = 8\pi G$

Green, Schwarz

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$$\Omega_{3\mathrm{L}} = \omega^a_{\ c} \wedge \mathbf{d}\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c} \wedge \omega^c_{\ d} \wedge \omega^d_{\ a}, \qquad \Omega_{3\mathrm{Y}} = \mathbf{A} \wedge \mathbf{dA} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A},$$

$$\varepsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma}(x)_{;\,\mu} \propto R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}_{\ \ \ \rho\sigma} \qquad \widetilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

Dual tensors

To this end…

Bianchi Identity for $H_{\mu\nu\rho}$

Svrcek & Witten

$\kappa^2 = 8\pi G$

Green, Schwarz

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} \quad \Rightarrow \quad \boldsymbol{H} = \mathrm{d}\mathbf{B} + \frac{\alpha'}{8\kappa}\left(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\right)$$

$$\Omega_{3\mathrm{L}} = \omega^a_{\ c} \wedge \mathrm{d}\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c} \wedge \omega^c_{\ d} \wedge \omega^d_{\ a}, \quad \Omega_{3\mathrm{Y}} = \mathbf{A} \wedge \mathrm{d}\mathbf{A} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A},$$

$$\varepsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma}(x)_{;\,\mu} \propto \underbrace{R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\,\widetilde{F}^{\mu\nu}}$$

$$\equiv \mathcal{G}(\omega, \mathbf{A})$$

mixed anomalies

To this end...

Bianchi Identity for $H_{\mu\nu\rho}$

Svrcek & Witten
Duncan, Kaloper, Olive

$\kappa^2 = 8\pi\, G$

Green, Schwarz

String A... requires modification in definition of $H_{\mu\nu\rho}$

Implement in a Path-integral via Lagrange multiplier Pseudoscalar b(x) field

$$H_{\mu\nu\rho} = \partial_{[} \ldots \frac{\alpha'}{8\kappa}\left(\Omega_{3L} - \Omega_{3Y}\right)$$

$$\Omega_{3L} = \omega^a_{\ c} \wedge d\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c} \wedge \omega \ldots \quad \ldots = \mathbf{A} \wedge d\mathbf{A} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A},$$

$$\varepsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma}(x)_{;\,\mu} \propto R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\, \widetilde{F}^{\mu\nu}$$

$$\equiv \mathcal{G}(\omega, \mathbf{A})$$

mixed anomalies

Bianchi

$$\Pi_x\, \delta\Big(\varepsilon^{\mu\nu\rho\sigma}\, \boldsymbol{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big) = \int Db \exp\Big[i \int d^4x \sqrt{-g}\, \frac{1}{\sqrt{3}} b(x)\Big(\varepsilon^{\mu\nu\rho\sigma} H(x)_{\nu\rho\sigma;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$= \int Db\ \exp\Big[-i \int d^4x \sqrt{-g}\, \Big(\partial^\mu b(x)\, \frac{1}{\sqrt{3}} \epsilon_{\mu\nu\rho\sigma}\ \boldsymbol{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}} \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$\mathcal{Z} = \int DH\, Db\, \exp(-H \wedge *H + c_1 b(dH - \mathcal{G}) + \dots)$$

Effective action after H-torsion (exact) path-integration

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}} b(x)\Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

KR-axion

cf. classically in 4 dim:

$$-3\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

Bianchi

$$\Pi_x \, \delta\Big(\varepsilon^{\mu\nu\rho\sigma} \, \boldsymbol{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big) = \int Db \exp\left[i \int d^4x \sqrt{-g} \frac{1}{\sqrt{3}} b(x) \Big(\varepsilon^{\mu\nu\rho\sigma} H(x)_{\nu\rho\sigma;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big)\right]$$

$$= \int Db \, \exp\left[- i \int d^4x \sqrt{-g} \Big(\partial^\mu b(x) \frac{1}{\sqrt{3}} \epsilon_{\mu\nu\rho\sigma} \, \boldsymbol{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}} \mathcal{G}(\omega, \mathbf{A})\Big)\right]$$

$$\mathcal{Z} = \int DH \, Db \, \exp(-H \wedge *H + c_1 b(dH - \mathcal{G}) + \dots)$$

Effective action after H-torsion (exact) path-integration

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[- \frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b \, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}} b(x) \Big(R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu} \, \widetilde{F}^{\mu\nu}\Big) + \dots \Big]$$

KR-axion anomalous CP-Violating interaction

cf. classically in 4 dim:

$$-3\sqrt{2}\partial_\sigma b \; = \; \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

Bianchi anomalies

$$\Pi_x\, \delta\Big(\varepsilon^{\mu\nu\rho\sigma}\, \boldsymbol{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big) = \int Db \exp\Big[i \int d^4x \sqrt{-g}\, \frac{1}{\sqrt{3}} b(x) \Big(\varepsilon^{\mu\nu\rho\sigma} H(x)_{\nu\rho\sigma;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$= \int Db\ \exp\Big[-i \int d^4x \sqrt{-g}\, \Big(\partial^\mu b(x)\, \frac{1}{\sqrt{3}}\, \epsilon_{\mu\nu\rho\sigma}\, \boldsymbol{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}} \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$\mathcal{Z} = \int DH\, Db\, \exp(-H \wedge * H + c_1 b(dH - \mathcal{G}) + \dots)$$

Effective action after H-torsion (exact) path-integration

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2} \partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}} b(x) \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

KR-axion anomalous CP-Violating interaction

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$$-3\sqrt{2}\partial_\sigma b \;=\; \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

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$$= \int Db\ \exp\left[-i \int d^4x \sqrt{-g}\, \Big(\partial^\mu b(x)\, \frac{1}{\sqrt{3}}\, \epsilon_{\mu\nu\rho\sigma}\ \boldsymbol{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}} \mathcal{G}(\omega, \mathbf{A})\Big)\right]$$

$$\mathcal{Z} = \int DH\, Db\, \exp(-H \wedge *H + c_1 b(dH - \mathcal{G}) + \dots)$$

Effective action after H-torsion (exact) path-integration

$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2}\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}} b(x) \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

Total derivatives

shift symmetry: *b(x)* → *b(x)* + **const.**

cf. classically in 4 dim: $$-3\sqrt{2}\partial_\sigma b \ = \ \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

$$\Pi_x\, \delta\Big(\varepsilon^{\mu\nu\rho\sigma}\, H_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big) = \int Db\, \exp\Big[\ldots \frac{1}{\ldots} b(x)\Big(\varepsilon^{\mu\nu\rho\sigma} H(x)_{\nu\rho\sigma;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$$= \int Db\ \exp\Big[- i \ldots + \frac{b(x)}{\sqrt{3}}\mathcal{G}(\omega, \mathbf{A})\Big)\Big]$$

$\ldots H\, Db\, e \ldots$

Totally antisymmetric (quantum) H-torsion ⟷ axionic propagating d.o.f. (NB: similar situation with totally antisymmetric component of torsion in Einstein-Cartan theory with fermions)

Effective action after H-torsion (exact) path-integration

$$\ldots + \frac{1}{2}\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}} b(x) \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\, \widetilde{F}^{\mu\nu} \Big) + \ldots \Big]$$

Total derivatives

shi… …metry: $b(x) \rightarrow b(x)$ + const.

cf. classically in 4 dim:

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\, \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\partial_\mu b\right)J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

KR-axion anomalous CP-Violating interaction

$$\mathcal{F}^d = \varepsilon^{abcd}e_{b\lambda}\partial_a e^\lambda_c;$$ vielbeins

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$ **Axial Current**

After integrating out *H* field with Bianchi constraint

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

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or Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}e_{b\lambda}\partial_a e^\lambda_c,$$ vielbeins

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$ Axial Current

KR-axion anomalous CP-Violating interaction

After integrating out *H* field with Bianchi constraint

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$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}b(x)\left(R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

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$$\mathcal{F}^d = \varepsilon^{abcd}e_{b\lambda}\partial_a e^\lambda_c,$$ vielbeins

$$J^{5\mu} = \bar\psi_j\gamma^\mu\gamma^5\psi_j$$ Axial Current

KR-axion anomalous CP-Violating interaction

After integrating out H field with Bianchi constraint

NB: $$\mathcal{L} = \sqrt{-g}\left(i\,\bar\psi\,\gamma^a\overline{D}_a\psi - m\,\bar\psi\psi\right)$$

$$\overline{D}_a = \left(\partial_a - \frac{i}{4}\overline{\omega_{bca}}\sigma^{bc}\right),$$

$$\overline{\omega_{bca}} = e_{b\lambda}\left(\partial_a e^\lambda_c + \overline{\Gamma^\lambda_{\gamma\mu}}e^\gamma_c e^\mu_a\right).$$

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_I = \sqrt{-g}\bar\psi\left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5 B_a\right]\psi,$$

$$B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e^\lambda_c + \overline{\Gamma^\lambda_{\alpha\mu}}e^\alpha_c e^\mu_a\right)$$

$$\overline{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \frac{\kappa}{\sqrt{3}}H^\mu_{\nu\rho} \neq \overline{\Gamma}^\mu_{\rho\nu}$$

$$B^d \ni \epsilon^{abcd}H_{bca}$$

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\partial_\mu b\right)J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

Repulsive ints. characteristic of torsion

$$\mathcal{F}^d = \varepsilon^{abcd}e_{b\lambda}\partial_a e_c^\lambda,$$ vielbeins

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$ **Axial Current**

After integrating out *H* field with Bianchi constraint

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots \Big];$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots \Big] + \dots$$

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

$$\mathcal{F}^d = \varepsilon^{abcd}\, e_{b\lambda}\partial_a e^\lambda_c;$$

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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All fermion species

$$\mathcal{F}^d = \varepsilon^{abcd}\,e_{b\lambda}\partial_a e^\lambda_c\,,$$

Vanishes for Friedmann-Lemaitre-Roberston-Walker backgrounds

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\, \partial_\mu b\, \partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\, b(x)\, \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots \Big]$$

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$$J^{5\mu} = \bar{\psi}_j\, \gamma^\mu\, \gamma^5 \psi_j$$

All fermion species

The Model

Anomaly terms

$$S^{\rm eff}_B = \int d^4x \sqrt{-g}\Big[-\frac{1}{2\kappa^2} R + \frac{1}{2}\partial_\mu b\, \partial^\mu b + \frac{2\,\alpha'}{96\,\kappa\sqrt{3}} b(x) \left(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\right) + \dots$$

$$+ S^{Free}_{Dirac} + \int d^4x \sqrt{-g} \Big(\quad \frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x \sqrt{-g}\, J^5_\mu J^{5\mu} + \dots \Big] + \dots$$

$$J^{5\mu} = \bar{\psi}_j\, \gamma^\mu\, \gamma^5 \psi_j$$

All fermion species

4. Gravitational Anomalies, Lorentz Violation & RVM Inflation

depositphotos

The Parts

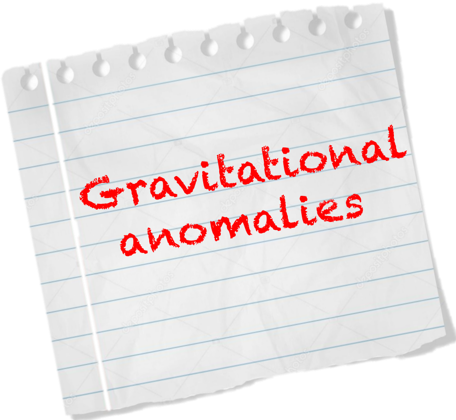
Stringy gravitational Axions

+

torsion

Gravitational anomalies

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x \sqrt{-g}\, b(x) \left(R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu} \widetilde{F}^{\mu\nu} \right)$$

Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory?)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x \sqrt{-g}\, b\, R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \Big] = 4 \int d^4x \sqrt{-g}\, \mathcal{C}^{\mu\nu}\, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g}\, \mathcal{C}_{\mu\nu}\, \delta g^{\mu\nu}$$

Cotton tensor

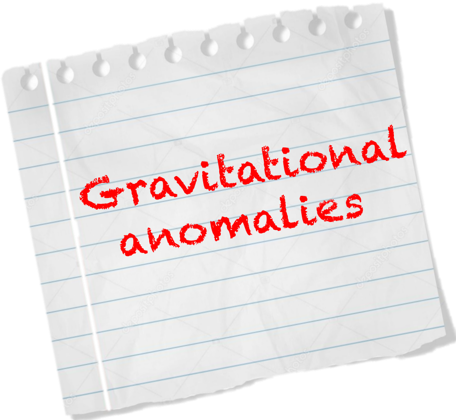
$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma \left(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha} \right) + v_{\sigma\tau} \left(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \right) \Big] = -\frac{1}{2}\Big[\left(v_\sigma \widetilde{R}^{\lambda\mu\sigma\nu} \right)_{;\lambda} + (\mu \leftrightarrow \nu) \Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \;\; v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless $\quad g_{\mu\nu}\, \mathcal{C}^{\mu\nu} = 0$

Jackiw, Pi (2003)

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x \sqrt{-g}\, b(x) \left(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - c_1 F_{\mu\nu}\, \widetilde{F}^{\mu\nu} \right)$$

Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory?)

Topological, does NOT contribute to stress tensor

$$\delta\left[\int d^4x \sqrt{-g}\, b\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \right] = 4 \int d^4x \sqrt{-g}\, \mathcal{C}^{\mu\nu}\, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g}\, \mathcal{C}_{\mu\nu}\, \delta g^{\mu\nu}$$

Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\left[v_\sigma \left(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha} \right) + v_{\sigma\tau} \left(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \right) \right] = -\frac{1}{2}\left[\left(v_\sigma\, \widetilde{R}^{\lambda\mu\sigma\nu} \right)_{;\lambda} + (\mu \leftrightarrow \nu) \right]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

not necessarily positive contributions to vacuum energy !

Traceless $\quad g_{\mu\nu}\, \mathcal{C}^{\mu\nu} = 0$

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = -\frac{1}{8} v^\nu R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\kappa^2 T^{\mu\nu}_{\rm matter\,;\mu} = -\,\mathcal{C}^{\mu\nu}{}_{;\mu} \neq 0$$

Diffeomoprphism invariance breaking by gravitational anomalies ?

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = -\frac{1}{8} v^\nu R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\kappa^2 T^{\mu\nu}_{{\rm matter}\,;\mu} + \mathcal{C}^{\mu\nu}{}_{;\mu} = 0$$

No problem with diffeo

Conserved Modified stress-energy tensor →
exchange of energy between matter & anomaly

The Parts

Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Spontaneous Lorentz Violation from anomaly condensates

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S^{\rm eff}_B = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Non-trivial if GW present

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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Primordial Gravitational Waves

The Model in Early Universe:
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$$= \int d^4x\, \sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\, \partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha^\prime}{96\,\kappa}\, \partial_\mu b(x)\, \mathcal{K}^\mu + \dots \Big]\,,$$

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

NEM, Solà
EPJ-ST
(2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

(i) Primordial Black Hole merging

NEM, Solà EPJ-ST (2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$
$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

Primordial Gravitational Waves

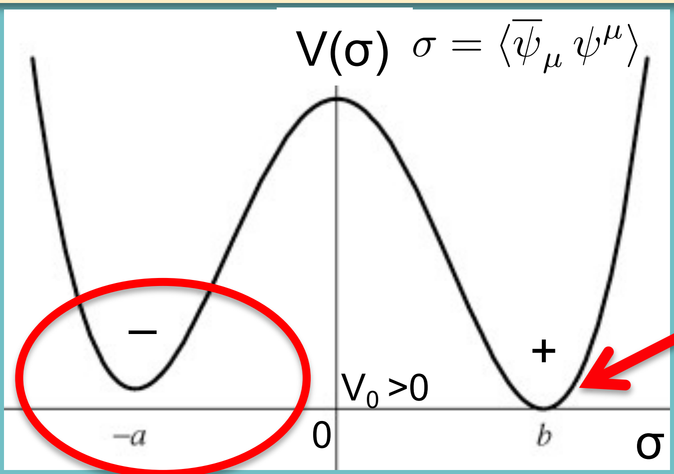
Potential Origins in pre-inflationary era?

(ii) Collapse/collisions of Domain walls formed in theories with (approximate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

NEM, Solà EPJ-ST (2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Statistical bias (percolation) in occupation probabilities of the +,- vacua

Lalak, Ovrut, Lola, G. Ross, Thomas

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

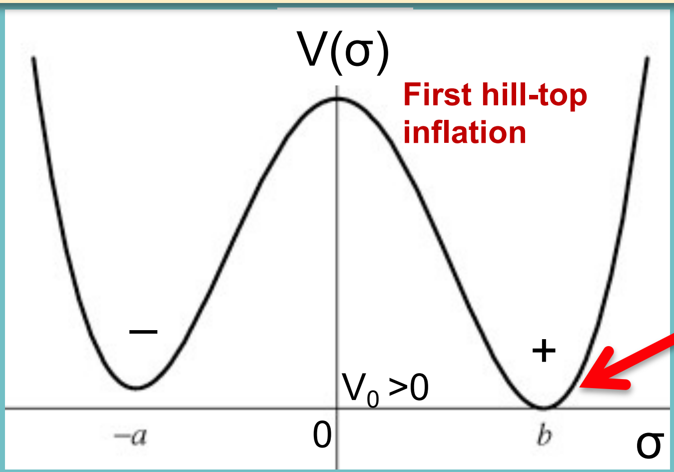
(ii) Collapse/collisions of Domain walls formed in theories with (approximate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → **Imaginary parts** → **instabilities**

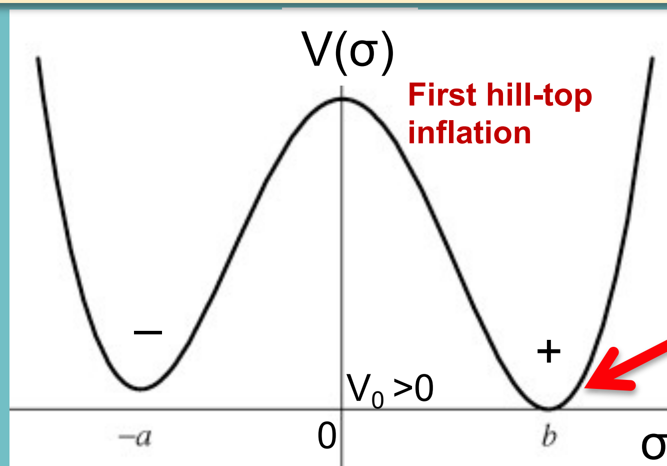
First Hill-top inflation = finite life –time → System **tunnels** to **RVM inflationary vacuum (GW condense)**

NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

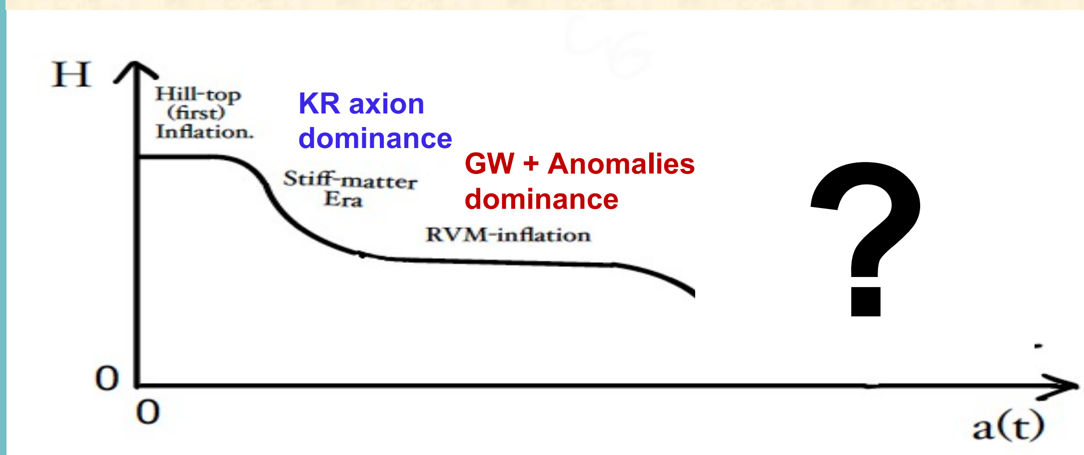
Basilakos, NEM, Solà (2019-20)



SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → *Imaginary parts* → **instabilities**

First Hill-top inflation = finite life –time → System **tunnels** to *RVM inflationary vacuum (GW condense)*

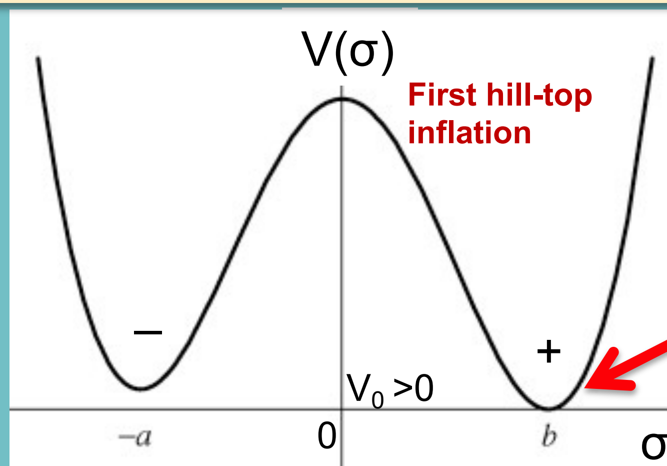


NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)



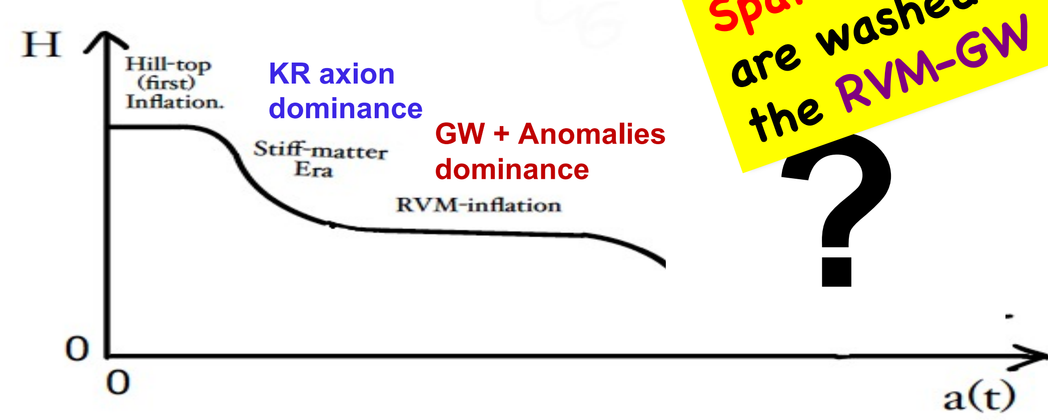
SUGRA broken gravitino Condensate stabilised → RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → **Imaginary parts** → **instabilities**

First Hill-top inflation = finite life –ti

System **tunnels** to **RVM inflationary va**

First inflation ensures any Spatial inhomogeneities are washed out before the RVM-GW inflation



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The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

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Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$
$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

Primordial Gravitational Waves →
Condensate < ...> of Gravitational Anomalies

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\int d^4x\,\sqrt{-g}\,\Big(\langle\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

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$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

$$+\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\int d^4x\sqrt{-g}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Condensate < ...> of Gravitational Anomalies

Cosmological-Constant-like

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\int d^4x\,\sqrt{-g}\,\Big(\langle\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

$\sqrt{-g}\,\mathcal{K}^{\mu}(\omega)_{;\mu}$

Effective action contains **CP violating axion-like coupling**

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad '\Big) + \dots\Big]$$

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$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

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$$ds^2 = dt^2 - a^2(t)\Big[(1-h_+(t,z))\,dx^2 + (1+h_+(t,z))\,dy^2 + 2h_\times(t,z)\,dx\,dy + dz^2\Big]$$

Average over inflationary space time in the presence of **primordial Gravitational waves**

b(x)=b(t)

Alexander, Peskin, Sheikh -Jabbari

Effective action contains **CP violating axion-like coupling**

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Average over inflationary space time in the presence of **primordial Gravitational waves**

integrate over graviton modes with momenta up to a UV cutoff μ

$b(x)=b(t)$

Alexander, Peskin, Sheikh -Jabbari

$$\langle R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \rangle = \frac{16}{a^4} \kappa^2 \int^{\mu} \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} k^4 \Theta + O(\Theta^3)$$

$$\Theta = \sqrt{\frac{2}{3}} \frac{\kappa^3}{12} H \dot{b} \ll 1$$

$$\kappa = M_{\rm Pl}^{-1}, \quad \dot{b} \equiv db/dt, \quad a(t) \sim e^{Ht}$$

$H \approx$ const. (inflation)

$$\partial_\mu\left(\sqrt{-g}\,\mathcal{K}^\mu(\omega)\right)$$

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b(x)=b(t)

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$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2\int^{\mu}\frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + O(\Theta^3)$$

Homogeneity & Isotropy

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H ≈ const. *(inflation)*

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Solutions (backgrounds) to the Eqs of Motion

$$\alpha' = M_s^{-2}$$

$$\partial_\alpha \left[\sqrt{-g} \left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}} \frac{\alpha'}{96\,\kappa} \mathcal{K}^\alpha(t) \right) \right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}} \frac{\alpha'}{96\,\kappa} \mathcal{K}^0$$

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time evolution of Anomaly

μ = UV k-momentum Cut-off

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$$\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\exp\left[-3Ht\left(1 - 0.73\times 10^{-4}\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)\right]$$

Solutions (backgrounds) to the Eqs of Motion

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≈ 0 during inflation

$$\frac{\mu}{M_s} \simeq 15\left(\frac{M_{\rm Pl}}{H}\right)^{1/2}$$

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Spontaneous LV solution ⚠

Planck Data $H/M_{\rm Pl} < 10^{-4}$ ⇒ **to ensure constant anomaly $\mu / M_s = O(10^3)$**

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}$$

$$\dot{\bar{b}} \propto \epsilon^{ijk} H_{ijk} = \text{constant}$$

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Using **slow-roll assumption** *b*

$$\varepsilon = \frac{1}{2} \frac{1}{(H M_{\rm Pl})^2} \dot{\bar{b}}^2 \sim 10^{-2}$$

Planck Data

@ end of Inflationary era

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathscr{K}^\alpha(t)\Big)\Big]=0 \quad\Rightarrow\quad \dot{\bar{b}}=\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathscr{K}^0\sim\text{constant}$$

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$$H=H_{\rm infl}\simeq\text{const.}$$

Constant anomaly during inflation, no transplanckian modes !

Restricts M_s range

$$2.6\times 10^{-5}\,M_{Pl}<M_s\leq 10^{-4}\,M_{Pl}$$

The Parts

Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Lorentz-Violating Leptogenesis & matter-antimatter Asymmetry

Dynamical Inflation without external inflatons

Spontaneous Lorentz Violation from anomaly condensates

The Parts


Dark Energy
("running vacuum model type")
Stringy gravitational Axions
+
torsion
Gravitational anomalies
Primordial gravitational waves
Lorentz-Violating Leptogenesis
&
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Dynamical Inflation
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Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

e-foldings $\mathcal{O}$(55-70)

Positive Cosmological Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7\, (\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

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Dark Energy
("running
vacuum model
type" (RVM))
Shapiro-Solà
Solà + ...

Can show analytically the De-Sitter type equation of state but with H = H(t) slightly varying with cosmic time:

$$p_{\rm total}(H(t)) = -\rho_{\rm total}(H(t))$$

$$\text{total} = b - \text{terms} + \text{Grav. Chern} - \text{Simons terms}$$
$$+ \text{Condensate}$$

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

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Dark Energy ("running vacuum model type" (RVM))

No external inflaton fields

RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate (cf. spares RVM evol)

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2}\frac{1}{(H M_{\rm Pl})^2}\dot{b}^2 \sim 10^{-2}$

CF.

RUNNING VACUUM MODEL

Shapiro + Solà
Sola + ...(2000)

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = 8\pi G\, (T^{\text{matter}}_{\mu\nu} + g_{\mu\nu}\, \frac{\Lambda(t)}{8\pi G})$$

$$G = \frac{\hbar\, c}{M_P^2}, \quad M_P = 1.22 \times 10^{19}\, \text{GeV} c^{-2}$$

$$1/\sqrt{8\,\pi\, G} = M_{Pl} = \text{reduced Planck mass}$$

$$T_{\mu\nu} = p\, g_{\mu\nu} + (\rho + p) U_\mu\, U_\nu \equiv T^{\text{matter}}_{\mu\nu} + \frac{g_{\mu\nu}\, \Lambda(t)}{8\pi\, G}$$

total stress-energy tensor, including vacuum terms

Total energy: $\rho^{\text{total}} = \rho^{\Lambda}_{\text{RVM}} + \rho^{\text{dust}} + \rho^{\text{radiation}}$

Running vacuum: Λ → Λ(t) cosmic-time dependent

$$\nabla^\mu\, T_{\mu\nu} = 0 \quad \text{energy − momentum density conservation}$$

CF.

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Total energy: $\rho^{\rm total} = \rho^{\Lambda}_{\rm RVM} + \rho^{\rm dust} + \rho^{\rm radiation}$

$$p^{\Lambda}_{\rm RVM} = -\rho^{\Lambda}_{\rm RVM} \qquad \rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha \frac{H^4}{H_I^2}\right)$$

$$\nabla^{\mu}\, T_{\mu\nu} = 0 \qquad \text{energy} - \text{momentum density conservation}$$

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$$\nabla^\mu T_{\mu\nu} = 0 \quad \text{energy – momentum density conservation}$$

CF.

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent

$\omega = \rho_m / p_m$ m = matter, radiation

$$\rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha \frac{H^4}{H_I^2}\right)$$

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega)H\rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$$\dot{H} + \frac{3}{2}(1+\omega)H^2\left(1 - \nu - \frac{c_0}{H^2} - \alpha\frac{H^2}{H_I^2}\right) = 0$$

Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$$D > 0$$

Early de Sitter (unstable) $D a^{4(1-\nu)} \ll 1$ ⇒ $H^2 = (1-\nu)H_I^2/\alpha$

Radiation $D a^{4(1-\nu)} \gg 1$ ⇒ $H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$

$\omega = 1/3$

Late dark-Energy dominated era $H^2(a) = H_0^2\left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$ $\tilde{\Omega}_{\Lambda 0}$ dominant

CF.

Cosmological Evolution of RVM

NEM, Solà

$\omega = \rho_m / p_m$ m = **stiff axion in stringy RVM**

$$\rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha \frac{H^4}{H_I^2}\right)$$

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega) H \rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$$\dot{H} + \frac{3}{2}(1+\omega) H^2 \left(1 - \nu - \frac{c_0}{H^2} - \alpha \frac{H^2}{H_I^2}\right) = 0$$

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CF.

Cosmological Evolution of RVM

$\omega = \rho_m / p_m$ *m* = matter, radiation, stiff axion

$$\rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha \frac{H^4}{H_I^2}\right)$$

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Gravitational Anomaly Condensates → Dynamical Inflation

NEM, Solà

Cannot obtain such terms in **ordinary Quantum Field Theories**

You need the **condensate of the gravitational anomalies** which have **CP-violating couplings** with the **gravitational axions**

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

Dark Energy ("running vacuum model type" (RVM))

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2} \frac{1}{(H M_{\rm Pl})^2} \dot{b}^2 \sim 10^{-2}$

Gravitational Anomaly Condensates → Dynamical Inflation

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Another important role of CP-violation in Early Universe

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Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive
Cosmological
Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

Negative coefficient ν < 0
due to CS anomaly
in early Universe, unlike
late-era RVM

RVM-like terms
drive inflation
contain scalar d.o.f.
from the anomaly
condensate

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2} \frac{1}{(H M_{\rm Pl})^2} \dot{b}^2 \sim 10^{-2}$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}$$

Undiluted KR axion background at the end of Inflation

@ end of Inflationary era

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

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Undiluted KR axion background at the end of Inflation

Spontaneous LV

@ end of Inflationary era

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Important for Leptogenesis @ radiation era

5.

Post-RVM-Inflationary Era

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

Cancellation of Gravitational Anomalies in Radiation Era by:

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Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\alpha'}{\kappa}b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu - \sqrt{\frac{3}{8}}J^{5\mu}\Big)\Big] + \dots,$$

$$J^{5\mu} = \sum_j \bar{\psi}_j\gamma^\mu\gamma^5\psi_j;$$

Chiral current, including RHN

(Mixed) Anomaly equation

$$\nabla_\mu J^{5\mu} = \frac{e^2}{8\pi^2}F^{\mu\nu}\widetilde{F}_{\mu\nu} - \frac{1}{192\pi^2}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} \equiv G(\mathbf{A},\omega)$$

includes possible chiral U(1) or QCD-type anomalies

Gauge terms do **not** contribute to stress tensor
→ do **not** affect diffeomorphism invariance

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\alpha'}{\kappa}b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu - \sqrt{\frac{3}{8}}J^{5\mu}\Big)\Big] + \dots,$$

$$J^{5\mu} = \sum_j \bar{\psi}_j\gamma^\mu\gamma^5\psi_j,$$

Chiral current, including RHN

$$\partial_\mu\Big[\sqrt{-g}\Big(\sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}J^{5\mu} - \frac{\alpha'}{\kappa}\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu\Big)\Big] = \sqrt{\frac{3}{8}}\frac{\alpha'}{\kappa}\Big(\frac{\alpha_{\rm EM}}{2\pi}\sqrt{-g}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} + \frac{\alpha_s}{8\pi}\sqrt{-g}\,G^a_{\mu\nu}\,\widetilde{G}^{a\mu\nu}\Big)$$

chiral U(1) Gluon QCD

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chiral U(1)

Gluon QCD

instanton generated potential + mass for KR axion b-field during matter dominance

potential role of KR axion as aDM candidate

Cosmic Time

forward direction

CF: **Summary of (stringy-RVM) Cosmological Evolution**

Basilakos, NEM, Solà

$2.6 \times 10^{-5}\, M_{Pl} < M_s \leq 10^{-4}\, M_{Pl}$

$$V_b^{QCD} \simeq \Lambda_{QCD}^4 \Big(1 - \cos(\frac{b}{f_b})\Big), \quad f_b \equiv \sqrt{\frac{8}{3}}\frac{\kappa}{\alpha'} = \sqrt{\frac{8}{3}}\Big(\frac{M_s}{M_{Pl}}\Big)^2 M_{Pl}$$

$$\Lambda_{QCD} \sim 218 \text{ MeV}$$

@ QCD Era

T~ 200 MeV

$$S_b^{eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}\partial_\mu b\, \partial^\mu b - \frac{\alpha'}{\kappa}\sqrt{\frac{3}{8}}\frac{\alpha_s}{8\pi} b(x)\, G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}\Big]$$

$$m_b = \mathcal{O}(10^{-4})\, \text{eV}$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

Possible potential generation for b → axion Dark matter

Cosmic Time

forward direction

CF: **Summary of (stringy-RVM) Cosmological Evolution**

Basilakos, NEM, Solà

KR (gravitational or model-independent) axions connected to "torsion" in string theory
→ Geometrical origin of Dark Matter

Matter Era

Possible potential generation for b → axion Dark matter

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation

$$S^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\alpha'}{\kappa}b(x)\nabla_\mu\Big(\sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu - \sqrt{\frac{3}{8}}J^{5\mu}\Big)\Big] + \dots,$$

$$J^{5\mu} = \sum_j \bar{\psi}_j \gamma^\mu \gamma^5 \psi_j;$$

Chiral current, including RHN

$$\partial_\mu\Big(\sqrt{-g}\Big[\sqrt{\frac{3}{8}}J^{5\mu} - \sqrt{\frac{2}{3}}\frac{1}{96}\mathcal{K}^\mu\Big]\Big) = \text{“chiral U(1) anomalies”}.$$

Possibly also QCD type

Eqs of Motion for b-field → $\partial_\mu\Big(\sqrt{-g}\,\partial^\mu b(x)\Big) = \text{“chiral U(1) anomalies”},$

Scale factor ***a(t) ~ T⁻¹***

Possibly also QCD

$$\dot{\bar{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

and/or QCD type

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and/or QCD type

Consistent with stringent phenomenological limits of torsion, LV & CPTV today in the context of standard model extension

CF.

In presence of fermions KR (approx. constant) background couples to axial-fermion current

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{\bar{b}}\,\delta_{\mu 0}$$

$$\propto \underbrace{\partial_\mu b}_{B_\mu} J^{5\mu}$$

STANDARD MODEL EXTENSION EFT

Lorentz & CPT Violation

Kostelecky, Bluhm, Colladay, Lehnert, Potting, Russell *et al.*

$$\mathcal{L} = \tfrac{1}{2} i \bar{\psi} \Gamma^\nu \overleftrightarrow{\partial}_\nu \psi - \bar{\psi} M \psi,$$

B_0

$$M \equiv m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \tfrac{1}{2} H^{\mu\nu} \sigma_{\mu\nu}$$

Spontaneous Violation of Lorentz Symmetry

(LV coefficients are v.e.v. of tensor-valued field quantities)

$B_0 \approx$ constant is H-torsion background in our model

$\mathbf{B^0}$: (scale factor a(t) $\propto 1/T$), T=cosmic temperature

$$B^0 \sim M_{\rm Pl}^{-1} \dot{\overline{b}}$$

matching with exit from inflation & requirement for leptogenesis (see below)

If chiral U(1) anomalies present

$$B^0 \sim T^2$$

$$B_0\Big|_{\rm today} \sim 2.435 \times 10^{-34}\,{\rm eV}$$

vs

$$B_{0\,\rm today} = \mathcal{O}\left(10^{-44}\right)\,{\rm meV}$$

chiral anomalies Absent" $B^0 \sim T^3$

Quite safe from stringent Experimental Bounds

$$|B^0| < 10^{-2}\,{\rm eV}$$
$$B_i \equiv b_i < 10^{-31}\,{\rm GeV}$$

Taking into account Earth's Veliocity relative to CMB frame (400 - 800 km/s) :

$$B_i' = \left(\frac{v_i^{\rm CMB}}{c}\right) B_0$$

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$$\dot{b} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

Viewed as sufficiently slow varying to induce Leptogenesis

Bossingham, NEM, Sarkar (2018)

The Parts

Dark Energy
("running vacuum model type")

Stringy gravitational Axions
+
torsion

Gravitational anomalies

Primordial gravitational waves

Lorentz-Violating Leptogenesis
&
matter-antimatter Asymmetry

Dynamical Inflation
without external inflatons

Spontaneous Lorentz Violation
from anomaly condensates

The Parts

Dark Energy
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&
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Dynamical Inflation
without external inflatons

Spontaneous Lorentz Violation
from anomaly condensates

For more details, see:
Sarben Sarkar's
talk this afternoon

Lorentz- & CPT-Violating

Leptogenesis →

→ Baryogenesis

in models with Massive Right-handed Neutrinos

NEM, Sarkar,
+ de Cesare, Bossingham

Early Universe
$T \gg T_{EW}$

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

Heavy RHN interact with axial constant background

with only temporal component $B_0 \neq 0$ $\quad B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

NEM, Sarkar,
+ de Cesare, Bossingham

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sufficiently slowly varying during leptogenesis (brief) epoch →
Approximately constant B_0-background

NEM, Sarkar,
+ de Cesare, Bossingham

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CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

Early Universe
$T \gg T_{EW}$

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

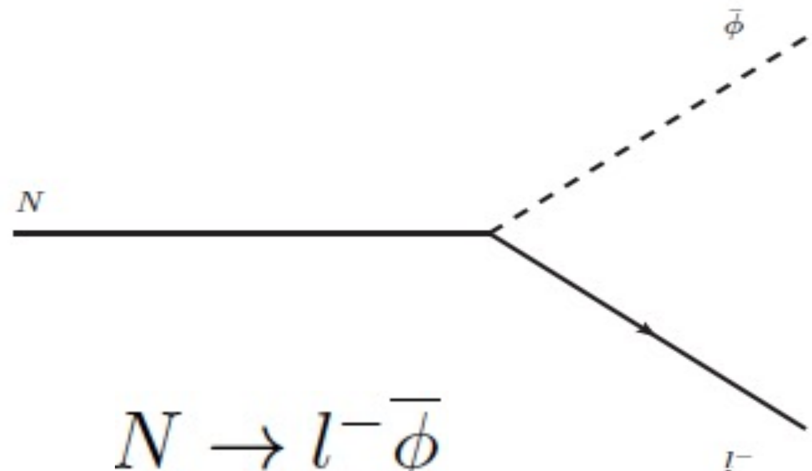
Heavy RHN interact with axial constant background

with only temporal component $B_0 \neq 0$

Produce Lepton asymmetry

Lepton number & CP Violations
@ tree-level due to
Lorentz/CPTV Background

$$N \to l^+\phi \qquad\qquad N \to l^-\overline{\phi}$$



$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega + B_0}{\Omega - B_0} \neq \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2}\frac{m^2}{\Omega}\frac{\Omega - B_0}{\Omega + B_0}$$

$B_0 \neq 0$

CPV & LV

$$\Omega = \sqrt{B_0^2 + m^2}$$

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Constant B_0 Background

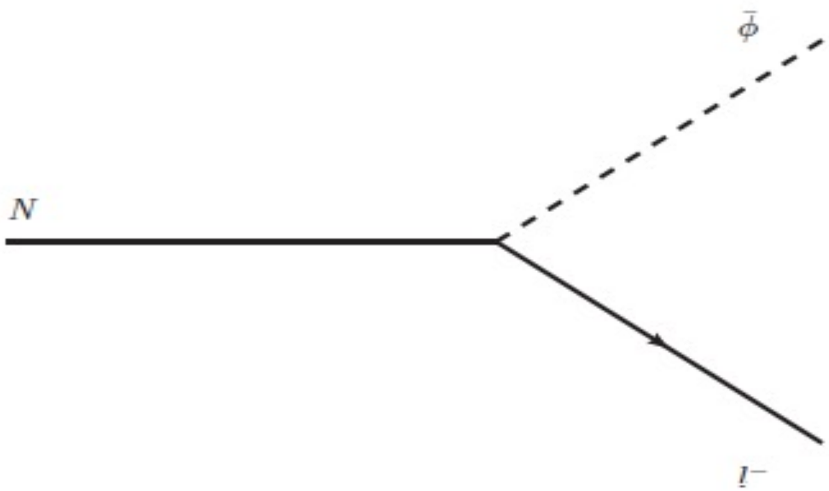
Early Universe
$T \gg T_{EW}$

CPT Violation

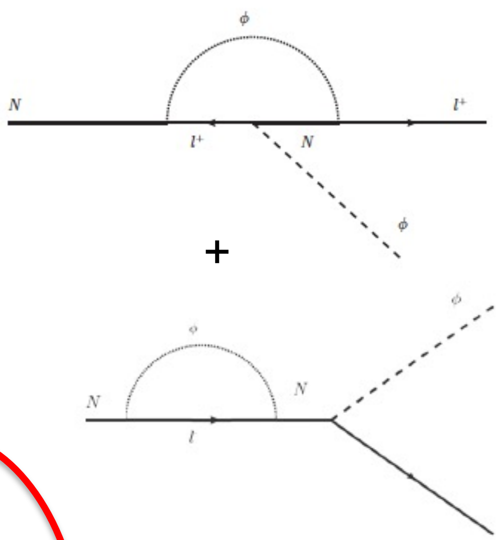
Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

Produce Lepton asymmetry



Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

$$\mathcal{L} = i\overline{N}\not\partial N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not B\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Constant B_0 Background

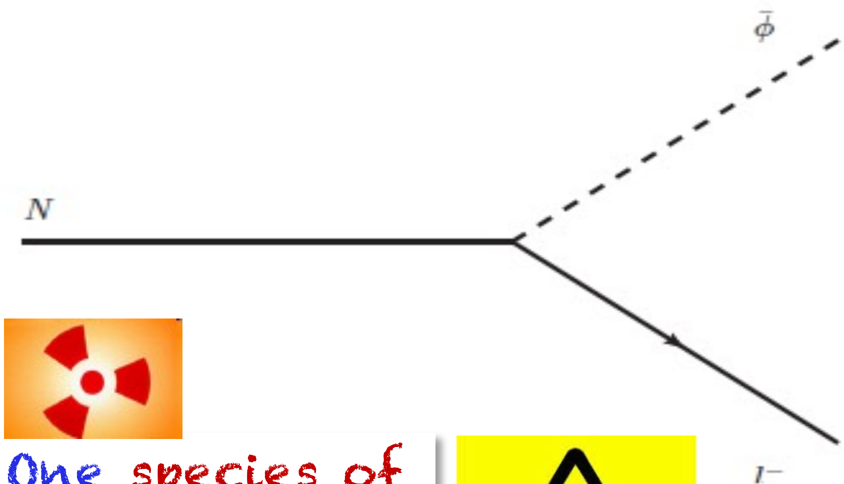
CPT Violation

Early Universe
$T \gg T_{EW}$

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

Produce Lepton asymmetry

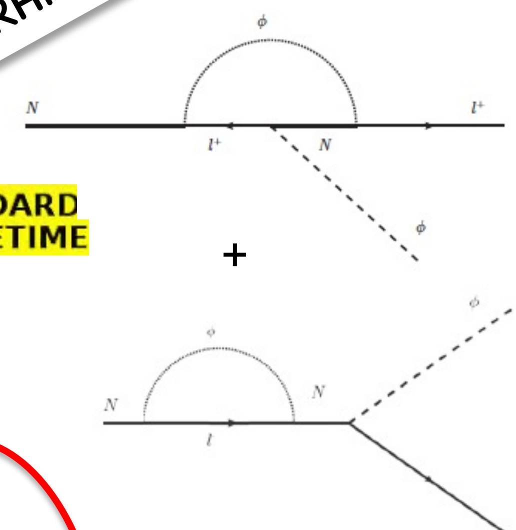


One species of RHN suffices

Contrast ... loop ... al ... ogenesis ... ce of H-torsion)

More than one species of RHN required to produce CPV

STANDARD SPACETIME



Fukugita, Yanagida,

6.

Conclusions & Outlook

Starting from an anomalous gravitational theory, which arises in the low-energy limit of string theory (The WHOLE), we have shown:

(i) how the (totally antisymmetric) Kalb-Ramond torsion in (3+1)-dimensions is equivalent to an axion-like field

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

(ii) how spontaneous LIV and CPTV can arise from condensation of gravitational waves, which in turn leads to condensation of the anomalous gravitational CP-violating Chern-Simons terms in the action.

(iii) how these considerations lead to a consistent cosmology of ``running vacuum model (RVM)", leading to:

(a) inflation without inflatons,

(b) LIV & CPTV Leptogenesis in this non-Riemannian geometric setting at post inflationary epochs.

(c) RVM Cosmology at modern eras – observable deviations from ΛCDM
Resolution of cosmological data tensions?

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

No initial singularity
(stringy reasons- higher curvature terms)

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

We exist because of Anomalies !

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type Running Dark Energy

forward direction

Cosmological (stringy RVM) Evolution: the Whole & its Parts

Cosmic Time

forward direction

No initial singularity
(stringy reasons- higher curvature terms)

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves

Gravitational anomaly (GA)

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Paraphrasing C. Sagan: we are anomalously made of star stuff !

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KR axial backround

Primordial
Gr
W

Gravitational

$$\rho_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu\left(\frac{H}{M_{\rm Pl}}\right)^2\right.$$

Lep
RH

Ma

matter

Modern de-Sitter Era

Paraphrasing
C. Sagan:

Can be shown that that **stringy RVM** characterizes also **post inflationary era** but with **v >0** (e.g. due to **cosmic e/m field contributions**)

Basilakos, NEM, Solà

RVM-type
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Basilakos, NEM, Solà

Distinguishing feature from ΛCDM
Alleviate current-epoch data tensions

$$\rho_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu\left(\frac{H}{M_{\rm Pl}}\right)^2 \right.$$

$$0 < \nu = \mathcal{O}(10^{-3})$$

$$\frac{3}{\kappa^2}\,c_0 \simeq 10^{-122}\, M_{\rm Pl}^4$$

Gómez-Valent Solà

Le RH

Ma

xio matter

Modern de-Sitter Era

RVM-type Running Dark Energy

Gomez-Valent, Solà

NB :

Could **Alleviate** Tensions in Data, e.g. H_0, σ_8 tensions

Consistent with cosmo data

$$0 < \nu = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \beta \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa^2} c_0 \simeq 10^{-122} M_{\rm Pl}^4$$

$$\rho_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu\left(\frac{H_0}{M_{\rm Pl}}\right)^2 + \beta\frac{H^4}{M_{\rm Pl}^4}\right), \quad \beta > 0.$$

Running RVM Dark Energy

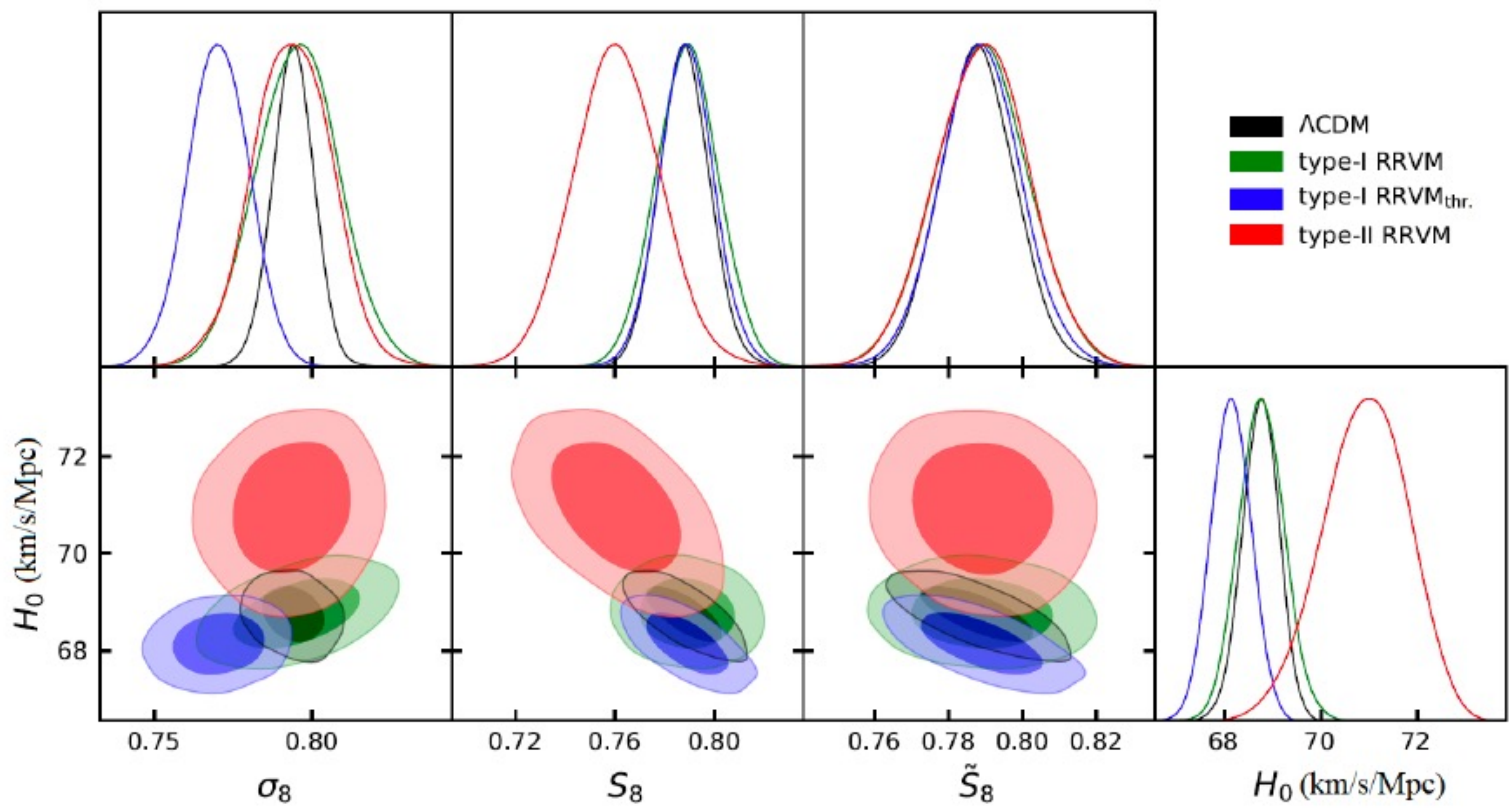
Not dominant today

This Photo by Unknown Author is licensed under CC BY-SA

Basilakos, Tsiapi
(Planck data 2015)

Solà, Gomez-Valent, Cruz, Moreno-Pulido, Planck 2018

Alleviation of the S_8, σ_8 tension by RVM model



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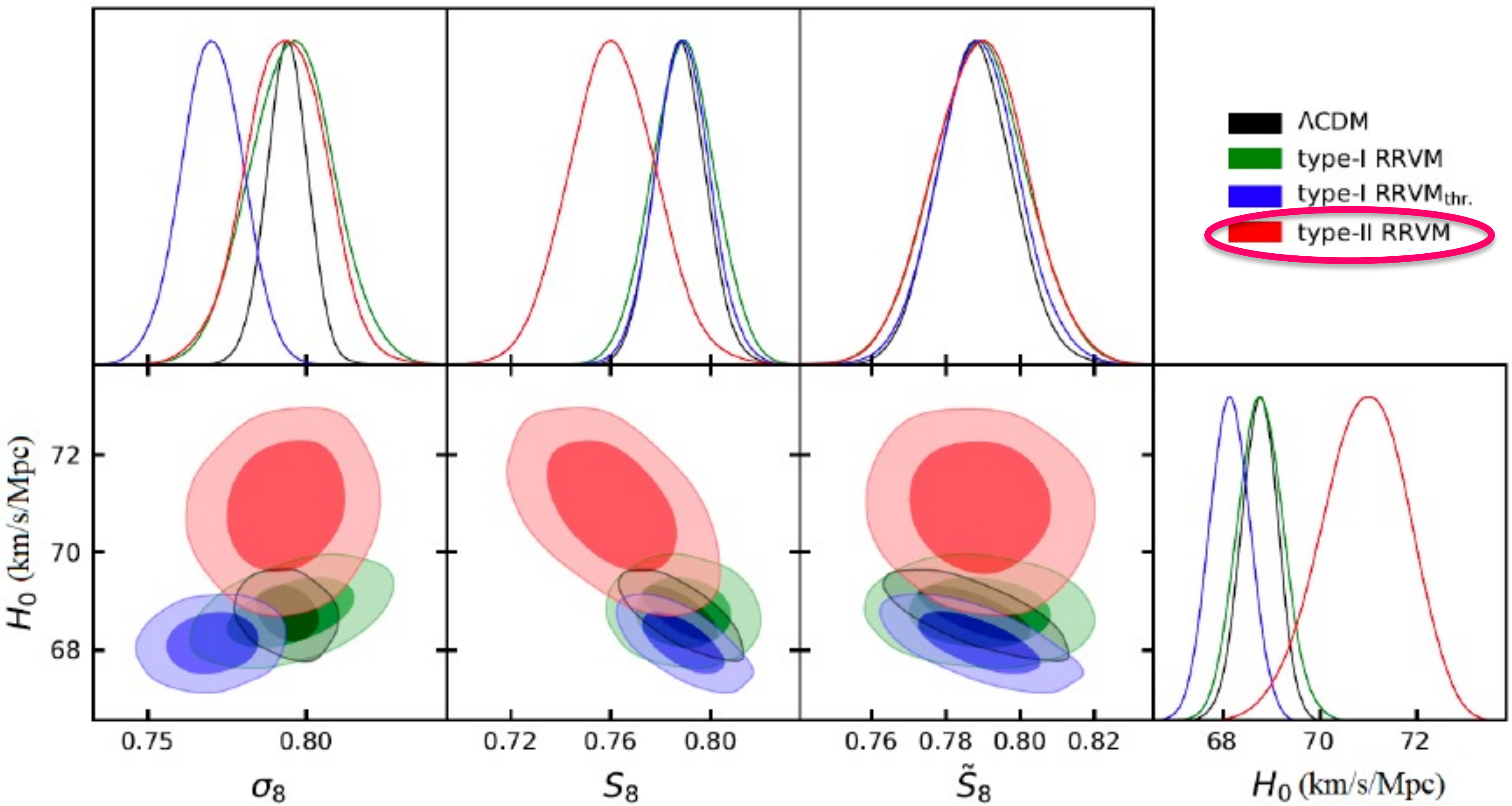

NB: **Type II RVM:** **mild (e.g. logarithmic) dependence of Gravitational ``constant''**
$\kappa^2 = \kappa^2(H)$

$$\rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha \frac{H^4}{H_I^2}\right)$$

$$\nu = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \alpha \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa^2}\, c_0 \simeq 10^{-122}\, M_{\rm Pl}^4$$

STRINGY RUNNING VACUUM MODEL & RVM type II

NEM + Solà (2021), NEM

Implemented in stringy RVM via graviton quantum fluctuations which result in ln(H) corrections in coefficients ν & α in the effective potential of (one-loop) QG but not κ(H) → difference from type II RVM

Alexandre, Houston NEM (2014)

NB: Type II RVM: mild (e.g. logarithmic) dependence of Gravitational ``constant" $\kappa^2 = \kappa^2(H)$

$$\rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha \frac{H^4}{H_I^2}\right)$$

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Leptogenesis induced by RHN (tree-level) decays

Spontaneous

OUTLOOK: (i) Incorporate other model-dependent stringy axions → Axiverse
Interesting Cosmology
(*eg* Marsh 2015)
could be ultralight → AION etc

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type
Running Dark Energy

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forward direction

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KR axial backround

RVM Inflationary (de Sitter) Phase

Primordial
Gravitational
Waves

Gravitational
anomaly (GA)

ist because
alies !

OUTLOOK: (ii) Look for imprints of the LV & CPT KR axial background in CMB in early eras.

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type
Running Dark Energy

Cosmological (stringy RVM) Evolution: the Whole & its Parts

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RVM Inflationary (de Sitter) Ph...

Undiluted constant KR axial backround

Primordial Gravitational Waves

Gravitational anomaly (GA)

We ... cause ...

OUTLOOK: (iii) Can we also get evidence of ν < 0 coefficient of H^2 during RVM inflation?

$$\rho_{\rm RVM}^{\rm string} \simeq 3\, M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \mathcal{O}(10^7) \Big(\frac{H}{M_{\rm Pl}}\Big)^4 \Big]$$

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

RVM-type
Running Dark Energy

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RV[illegible]ationa[illegible]y (de Sitter) Ph[illegible]

Primordial Gravitational Waves

Gravitational anomaly (GA)

Undiluted constant KR axial backround

OUTLOOK: (iv) *Possible Effects of stringy axions with* **periodic (instanton-induced) potentials** in abundant production of primordial BH during RVM inflation & effects on GW spectrum?
NEM, arxiv: 2111.05675, *Universe* to appear

Leptogenesis induced by RHN (tree-level) decays

Spontaneous Lorentz and CPT Violation

Matter Era

axion Dark matter

Modern de-Sitter Era

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RVM Inflationary (de Sitter) Phase

Undilut... nt
KR ... nd

Primordial Gravitational Waves

Gravitational anomaly (GA)

Thank you !

...K: Incorporate other

Leptog...
RHN...

Spontaneous L...

coul...

Matter E...

axion Dark matter

Modern de-Sitter Era

RVM-type
Running Dark Energy

Spares

The Parts

Dark Energy ("running vacuum model type")

Stringy gravitational Axions + torsion

Gravitational anomalies

Primordial gravitational waves

Dark Matt

orentz-Violating Leptogenesis & matter-antimatter Asymmetry

Dynamical Inflation without external inflatons

Spontaneous Lorentz Violation from anomaly condensates

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violation

Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

Produce Lepton asymmetry

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Solving system of Boltzmann eqs

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m} \simeq 0.007\,\frac{B_0}{m}$$

$$Y_k \sim 10^{-5}$$

$$m \geq 100\text{TeV} \to$$

$$B^0 \sim 1\text{MeV}$$

$$T_D \simeq m \sim 100\ \text{TeV}$$

consistent with :
light neutrino masses in SM +
stability of Higgs vacuum

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

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Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

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Baryogenesis

?

CPTV Thermal l

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CPT Violation

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$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Equilibrated electroweak B+L violating sphaleron interactions

B-L conserved

$$L = \frac{2}{M} l_L l_L \phi\phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Environmental Conditions Dependent

Observed Baryon Asymmetry In the Universe (BAU)

Fukugita, Yanagida,

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

The Parts

Dark Energy
("running vacuum model type")

Stringy gravitational Axions
+
torsion

Gravitational anomalies

Primordial gravitational waves

Dark Matter

Lorentz-Violating Leptogenesis
&
matter-antimatter Asymmetry

Dynamical Inflation
without external inflatons

Spontaneous Lorentz Violation
from anomaly condensates

GENERIC REMARKS ON GENERAL RELATIVITY

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = 8\pi \mathrm{G}\, (T_{\mu\nu}^{\mathrm{matter}} + g_{\mu\nu}\, \frac{\Lambda}{8\pi \mathrm{G}})$$

$\mathrm{G} = \frac{\hbar\, c}{M_{\mathrm{P}}^2}, \quad M_{\mathrm{P}} = 1.22 \times 10^{19}\, \mathrm{GeV} c^{-2}$ **(3+1)-dim Planck mass**

$1/\sqrt{8\,\pi\,\mathrm{G}} = M_{\mathrm{Pl}} =$ reduced Planck mass

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Vacuum energy density

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$$1/\sqrt{8\,\pi\,\mathrm{G}} = M_{\mathrm{Pl}} = \text{reduced Planck mass}$$

$$T_{\mu\nu} = p\, g_{\mu\nu} + (\rho + p) U_\mu\, U_\nu \equiv T_{\mu\nu}^{\mathrm{matter}} + \frac{g_{\mu\nu}\Lambda}{8\pi\, \mathrm{G}}$$

total stress-energy tensor, including vacuum terms

ρ = energy density, p = pressure density

U_μ = observer's velocity w.r.t. cosmic frame

$\nabla^\mu\, T_{\mu\nu} = 0$ energy – momentum density conservation

GENERIC REMARKS ON GENERAL RELATIVITY

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = 8\pi \mathrm{G}\, (T^{\rm matter}_{\mu\nu} + g_{\mu\nu}\, \frac{\Lambda(t)}{8\pi \mathrm{G}})$$

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total stress-energy tensor, including vacuum terms

Running vacuum: Λ → Λ(t) cosmic-time dependent

$\nabla^{\mu}\, T_{\mu\nu} = 0$ energy – momentum density conservation

RUNNING VACUUM MODEL (RVM) for the UNIVERSE

Shapiro + Sola
Sola + ...(2000)

Vacuum energy density assumed de Sitter like but with time-dependent Cosmological parameter Λ(t) :

$$\rho^{\Lambda}_{\rm RVM}(t) = \Lambda(t)/\kappa^2 \qquad \kappa = \sqrt{8\pi {\rm G}} = M_{\rm Pl}^{-1}$$

$$p(t)_{\rm RVM} = -\rho^{\Lambda(t)}_{\rm RVM}(t)$$

Renormalization-Group-like equation for the evolution of **vacuum energy density**
Hubble parameter H(t) ←→ RG scale *μ*

$$\frac{d\,\rho^{\rm RVM}_{\Lambda}(t)}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_{i=F,B}\Big[a_i M_i^2 H^2 + b_i H^4 + \mathcal{O}\big(\frac{H^6}{M_i^2}\big)\Big]$$

general covariance →
even powers of *H*

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Also in non-critical strings
Ellis NEM Nanopoulos ('98)

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Sola, Gomez Valent ...

Relevant for Cosmological observation/phenomenology up to and including H^4

$$\rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha\frac{H^4}{H_I^2}\right)$$

$$\nu = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \alpha \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa^2}\,c_0 \simeq 10^{-122}\,M_{\rm Pl}^4$$

Total energy: $\rho^{\rm total} = \rho^{\Lambda}_{\rm RVM} + \rho^{\rm dust} + \rho^{\rm radiation}$

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent + ... (2013 - 2018)

$\omega = \rho_m / p_m$ *m* = matter, radiation

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega)H\rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$$\dot{H} + \frac{3}{2}(1+\omega)H^2\left(1 - \nu - \frac{c_0}{H^2} - \alpha\frac{H^2}{H_I^2}\right) = 0$$

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Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$$D > 0$$

Early de Sitter (unstable) $Da^{4(1-\nu)} \ll 1 \Rightarrow H^2 = (1-\nu) H_I^2 / \alpha$

Radiation $Da^{4(1-\nu)} \gg 1 \Rightarrow H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$

$\omega = 1/3$

Late dark-Energy dominated era $H^2(a) = H_0^2 \left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$ $\tilde{\Omega}_{\Lambda 0}$ dominant

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Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\ldots}}}$$

Current phenomenology

$$\nu = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \alpha \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa^2} c_0 \simeq 10^{-122} M_{\rm Pl}^4$$

Early de Sitter (unstable) $Da\ldots$

Deviation from ΛCDM – running Vacuum format

Radiation $Da\ldots$

Late dark-Energy dominated era $H^2(a) = H_0^2 \left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$ $\tilde{\Omega}_{\Lambda 0}$ dominant

Cosmological Evolution of RVM

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RUNNING VACUUM MODEL (RVM) for the UNIVERSE

Shapiro + Sola
Sola + ...

Dominant in early Universe → drives inflation
→ no need for external inflatons

Basilakos, Lima ,Sola + ...

Relevant for Cosmological observation/phenomenology up to and including H^4

$$\rho^{\Lambda}_{\rm RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha \frac{H^4}{H_I^2}\right)$$

$$\nu = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \alpha \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa^2}\, c_0 \simeq 10^{-122}\, M_{\rm Pl}^4$$

Total energy: $\rho^{\rm total} = \rho^{\Lambda}_{\rm RVM} + \rho^{\rm dust} + \rho^{\rm radiation}$

RUNNING VACUUM MODEL (RVM) for the UNIVERSE

Shapiro + Sola
Sola + ...

This talk

Dominant in early Universe → drives inflation
→ no need for external inflatons

Basilakos,
Lima ,Sola + ...

includes scalar d.o.f.
``anomaly condensate''

Basilakos
NEM, Sola
(2019-20)

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RUNNING VACUUM MODEL (RVM) for the UNIVERSE

Shapiro + Sola
Sola + ...

NB: In generic running vacuum the coefficients ν and α are constant and positive throughout the evolution

Dominant in early Universe → drives inflation
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IN THIS TALK

NB: In string-inspired running vacuum the coefficients ν and α are not constant throughout the evolution:

Inflationary phase: $\nu < 0$, $\alpha > 0$

post inflationary phases: $\nu > 0$, $\alpha > 0$

Dominant in early Universe → drives inflation → no need for external inflatons

Basilakos, Lima ,Sola + ...

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Basilakos NEM, Sola (2019-20)

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