On the measurement of the muon anomalous magnetic moment

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The ideas and formulas presented in the article will help to bring together the theoretical predictions for the anomalous magnetic moment of muon and the results of the "Muon g-2" experiment.
In Quantum Physics a state with spin perpendicular to a magnetic field can be expressed as a superposition of energy eigenstates with spins parallel and antiparallel to the field: the resultant spin precession is due to the energy difference between the two eigenstates.
If the state, like the muon, is unstable and can decay, it will have a natural energy spread. As a result the frequency of the spin precession can vary.
For a constant magnetic field the measured spin precession velocity will be spread according to the Lorentzian distribution with width $(\gamma\tau)^{-1}$, for Lorentz gamma factor $\gamma = E/m$, and particle lifetime *tau*.

Recently Fermilab's "Muon g-2" collaboration announced new results on the measurement of the muon anomalous magnetic moment.
Their measurement, which supports the previous result by the E821 experiment at Brookhaven National Laboratory differs significantly from the theoretical prediction of the muon anomalous magnetic moment.
The physics of spin precession in a magnetic field is well understood within **classical relativistic** physics.
However the muon in an **unstable** particle and as such it has a natural width, which can only be treated correctly within quantum physics.

When in the magnetic field a charged fermion has two independent energy eigenstates with spin parallel and anti parallel to the field. Those two states gain energy with opposite signs.
Due to that energy difference (ΔE) the direction of the spin, perpendicular to the magnetic field, is started to precess.

$$
\begin{aligned}
\psi^{\rightarrow} &= \frac{1}{\sqrt{2}}(\psi^{\uparrow} + \psi^{\downarrow}) \;, \quad \psi^{\leftarrow} = \frac{1}{\sqrt{2}}(\psi^{\uparrow} - \psi^{\downarrow}) & (1) \\
\psi(t) &= \frac{1}{\sqrt{2}} e^{-iE^{\uparrow}t}(e^{-i\frac{E^{\uparrow}-E^{\downarrow}}{2}t}\psi^{\uparrow} + e^{i\frac{E^{\uparrow}-E^{\downarrow}}{2}t}\psi^{\downarrow}) & (2) \\
&= \frac{1}{\sqrt{2}} e^{-iEt}(e^{-i\frac{\delta E}{2}t}\psi^{\uparrow} + e^{i\frac{\delta E}{2}t}\psi^{\downarrow}) & (3)
\end{aligned}
$$

$$
\begin{aligned}
P_{\psi_{\rightarrow}\Rightarrow\psi_{\rightarrow}} &= \cos^2\frac{\delta E\ t}{2} & (4) \\
P_{\psi_{\rightarrow}\Rightarrow\psi_{\leftarrow}} &= \sin^2\frac{\delta E\ t}{2} & (5)
\end{aligned}
$$

In the frame spin precession relative to the muon velocity direction

$$\delta E = \frac{g-2}{2}\frac{eB}{m} = a_{\mu}\frac{eB}{m} \quad (a_{\mu} = \frac{\alpha}{2\pi} + \dots) \qquad (6)$$

For unstable particle

$$\psi(t) = \frac{1}{\sqrt{2}} e^{-iEt - \frac{t}{2\gamma\tau}} (e^{-i\frac{\delta E}{2}t}\psi^{\uparrow} + e^{i\frac{\delta E}{2}t}\psi^{\downarrow}) \tag{7}$$

$$P_{\psi_{\rightarrow} \Rightarrow \psi_{\rightarrow}} = e^{-\frac{t}{\gamma\tau}} \; \cos^2 \frac{\delta E \;\, t}{2} \tag{8}$$

$$P_{\psi_{\rightarrow} \Rightarrow \psi_{\leftarrow}} = e^{-\frac{t}{\gamma\tau}} \; \sin^2 \frac{\delta E \;\, t}{2} \tag{9}$$

FNAL "Muon g-2" analyzes data according to these equations

$$\int_0^\infty P_{\psi_\rightarrow \Rightarrow \psi_\rightarrow} dt = \frac{\gamma\tau}{2} \frac{(\delta E)^2 + \frac{2}{(\gamma\tau)^2}}{(\delta E)^2 + \frac{1}{(\gamma\tau)^2}} \quad (10)$$

$$\int_0^\infty P_{\psi_\rightarrow \Rightarrow \psi_\leftarrow} dt = \frac{\gamma\tau}{2} \frac{(\delta E)^2}{(\delta E)^2 + \frac{1}{(\gamma\tau)^2}} \quad (11)$$

$$\int_0^\infty (P_{\psi_\rightarrow \Rightarrow \psi_\rightarrow} + P_{\psi_\rightarrow \Rightarrow \psi_\leftarrow}) dt = \gamma\tau \quad (12)$$

$$\int_0^\infty e^{-\frac{t}{\gamma\tau}} = \gamma\tau \quad (13)$$

$$A_{\rightarrow\rightarrow} = \frac{1}{2}[e^{-iE_0^{\uparrow}t - \frac{t}{2\gamma\tau}} + e^{-iE_0^{\downarrow}t - \frac{t}{2\gamma\tau}}] \quad (14)$$

$$A_{\rightarrow\leftarrow} = \frac{1}{2}[e^{-iE_0^{\uparrow}t - \frac{t}{2\gamma\tau}} - e^{-iE_0^{\downarrow}t - \frac{t}{2\gamma\tau}}] \quad (15)$$

$$A_{\rightarrow\rightarrow} = \frac{1}{4\pi i}\int \frac{e^{-iE^{\uparrow}t}dE^{\uparrow}}{E^{\uparrow} - (E + \frac{\delta E}{2}) + i\frac{1}{2\gamma\tau}} \quad (16)$$

$$+ \frac{1}{4\pi i}\int \frac{e^{-iE^{\downarrow}t}dE^{\downarrow}}{E^{\downarrow} - (E - \frac{\delta E}{2}) + i\frac{1}{2\gamma\tau}} \quad (17)$$

$$= A^{\uparrow} + A^{\downarrow} \quad (18)$$

$$A_{\rightarrow\leftarrow} = \frac{1}{4\pi i}\int \frac{e^{-iE^{\uparrow}t}dE^{\uparrow}}{E^{\uparrow} - (E + \frac{\delta E}{2}) + i\frac{1}{2\gamma\tau}} \quad (19)$$

$$- \frac{1}{4\pi i}\int \frac{e^{-iE^{\downarrow}t}dE^{\downarrow}}{E^{\downarrow} - (E - \frac{\delta E}{2}) + i\frac{1}{2\gamma\tau}} \quad (20)$$

$$= A^{\uparrow} - A^{\downarrow} \quad (21)$$

$$|A^{\uparrow}|^2 + |A^{\downarrow}|^2 = \frac{\gamma\tau}{2} \tag{22}$$

THANK YOU

$$A_{\rightarrow\rightarrow} = \frac{1}{(2\pi i)^2}\int\int \frac{e^{-i\frac{E^{\uparrow}+E^{\downarrow}}{2}t}\cos(\frac{E^{\uparrow}-E^{\downarrow}}{2}t) \quad dE^{\uparrow}dE^{\downarrow}}{(E^{\uparrow}-(E+\frac{\delta E}{2})+i\frac{1}{2\gamma\tau})(E^{\downarrow}-(E-\frac{\delta E}{2})+i\frac{1}{2\gamma\tau})} \tag{23}$$

$$A_{\rightarrow\leftarrow} = \frac{1}{(2\pi i)^2}\int\int \frac{e^{-i\frac{E^{\uparrow}+E^{\downarrow}}{2}t}\sin(\frac{E^{\uparrow}-E^{\downarrow}}{2}t) \quad dE^{\uparrow}dE^{\downarrow}}{(E^{\uparrow}-(E+\frac{\delta E}{2})+i\frac{1}{2\gamma\tau})(E^{\downarrow}-(E-\frac{\delta E}{2})+i\frac{1}{2\gamma\tau})} \tag{24}$$