Searching for New Physics in Rare K and B Decays without $|V_{cb}|$ and $|V_{ub}|$ uncertainties

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Based on arxiv:2109.11032
with Andrzej J. Buras

DISCRETE 2020-2021

$|V_{cb}|$ and $|V_{ub}|$ tensions

$|\mathbf{V}_{\mathbf{cb}}|_{\mathbf{inclusive}}$ ➡ $B \to X_c \ell \nu$

$|\mathbf{V}_{\mathbf{cb}}|_{\mathbf{exclusive}}$ ➡ $B \to D$ **and** $B \to D^*$

$|V_{cb}|$ and $|V_{ub}|$ tensions

$$|V_{cb}|_{\text{inclusive}} = (42.16 \pm 0.50) \cdot 10^{-3}$$

2107.00604
[Bordone, Capdevila, Gambino]

$$|V_{cb}|_{\text{exclusive}} = (39.09 \pm 0.68) \cdot 10^{-3}$$

1902.0819
[FLAG]

See also 1912.09335 [Bordone, Gubernari, van Dyk, Jung]

$$|V_{ub}|_{\text{inclusive}} = (4.10 \pm 0.28) \cdot 10^{-3}$$

Belle 2021

$$|V_{ub}|_{\text{exclusive}} = (3.73 \pm 0.14) \cdot 10^{-3}$$

FLAG

$$|V_{ub}|_{\text{exclusive}} = (3.77 \pm 0.15) \cdot 10^{-3}$$

Light-cone sum rules
2102.07233 [Leljak, Melic, van Dyk]

Problematic $|V_{cb}|$ and $|V_{ub}|$ uncertainties

- $|\mathbf{V_{cb}}|$ and $|\mathbf{V_{ub}}|$ ➡ $\sim 7-10\,\%$ uncertainties
- Strong dependence of K and B decays on $|\mathbf{V_{cb}}|$ and $|\mathbf{V_{ub}}|$

⬇

Large uncertainties on K and B decays

- $|\mathbf{V_{cb}}|^2 \to 16\,\%$, e.g. in $\mathbf{Br}(\mathbf{B_{s,d}} \to \mu\mu)$ and $\mathbf{\Delta M_{s,d}}$
- $|\mathbf{V_{cb}}|^3 \to 24\,\%$, e.g. in $\mathbf{Br}(\mathbf{K^+} \to \pi^+\nu\nu)$ and $|\epsilon_\mathbf{K}|$
- $|\mathbf{V_{cb}}|^4 \to 32\,\%$, e.g. in $\mathbf{Br}(\mathbf{K_L} \to \pi^0\nu\nu)$ and $\mathbf{Br}(\mathbf{K_S} \to \mu\mu)$

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BUT

These are observables with:

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These are observables with:

1. **Precise theoretical predictions (NNLO QCD corrections)**

   $\to$ **Accuracy at** $1 - 2\,\%$

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These are observables with:

1. **Precise theoretical predictions (NNLO QCD corrections)**

   $\to$ **Accuracy at** $1-2\,\%$
2. **Precisely measured** $(\mathbf{Br(B_s \to \mu\mu), \Delta M_{s,d}})$
3. **Small difference between the experimental measurements and the SM predictions** $\to\ \sim 30\,\%$ **for** $\mathbf{Br(B_s \to \mu\mu)}$

➡ **High precision needed for NP study**

Idea: cancellation of the $|V_{cb}|$ dependence

$$\mathbf{R_{s(d)\mu} \equiv \frac{Br(B_{s(d)} \to \mu^+\mu^-)}{\Delta M_{s(d)}} = C\frac{\tau_{B_{s(d)}}}{\hat{B}_{s(d)}} f(x_t)}$$

C: $4.291 \cdot 10^{-10}$

Idea of:
0303060 [Buras]
2104.09521 [Bobeth, Buras]

Computing Ratios (in the SM) to obtain much clearer observables

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$\mathbf{C} \to 4.291 \cdot 10^{-10}$

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Computing Ratios (in the SM) to obtain much clearer observables

- **Cancellation of the full CKM dependence**
- **Cancellation also of the hadronic form factors** $F^2_{B_{s(d)}}$
- **Precise determination of** $\mathbf{\hat{B}_{s(d)}}$ **(LQCD) and of** $\mathbf{f(x_t)} = \dfrac{\mathbf{Y_0(x_t)^2}}{\mathbf{S_0(x_t)}}$ **(NLO QCD)**

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$\downarrow$

$4.291 \cdot 10^{-10}$

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Computing Ratios (in the SM) to obtain much clearer observables

$$[\mathbf{R_{s\mu}}]_{\mathbf{SM}} = \left(\mathbf{2.04^{+0.08}_{-0.06}}\right) \cdot \mathbf{10^{-10} ps}$$

vs

$$[\mathbf{R_{s\mu}}]_{\mathbf{exp}} = \left(\mathbf{1.61^{+0.19}_{-0.17}}\right) \cdot \mathbf{10^{-10} ps}$$

$\mathbf{2.1\sigma}$ **tension with SM**

WITHOUT CKM uncertainties

Idea: cancellation of the $|V_{cb}|$ dependence

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C ↓ $4.291 \cdot 10^{-10}$

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Computing Ratios (in the SM) to obtain much clearer observables

Generalization of the strategy

$\rightarrow$ other ratios of (functions of) observables

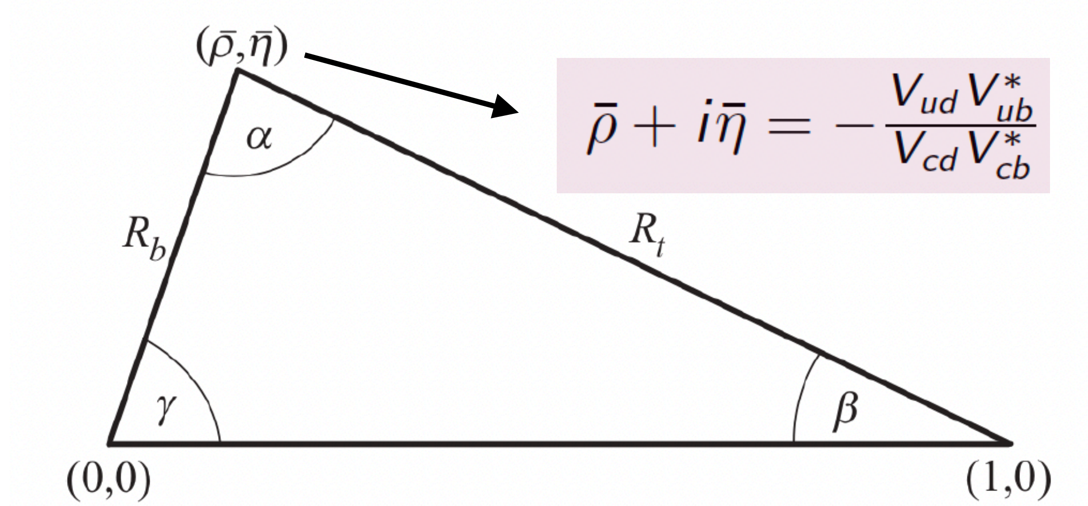
to eliminate the $|\mathbf{V_{cb}}|$ dependence

They will in general depend on other CKM parameters ➡

CKM parametrization

Cleanest set of CKM parameters → **Not explicit $|\mathbf{V_{ub}}|$ dependence**

$$\lambda = |\mathbf{V_{us}}|,\ \ |\mathbf{V_{cb}}|,\ \ \beta,\ \ \gamma$$



$$R_t \equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{(1-\bar\rho)^2+\bar\eta^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|, \longrightarrow R_t = \frac{\sin\gamma}{\sin(\beta+\gamma)} \approx \sin\gamma,$$

$$R_b \equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\bar\rho^2+\bar\eta^2} = (1-\frac{\lambda^2}{2})\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|. \longrightarrow R_b = \frac{\sin\beta}{\sin(\beta+\gamma)} \approx \sin\beta\,.$$

CKM parametrization

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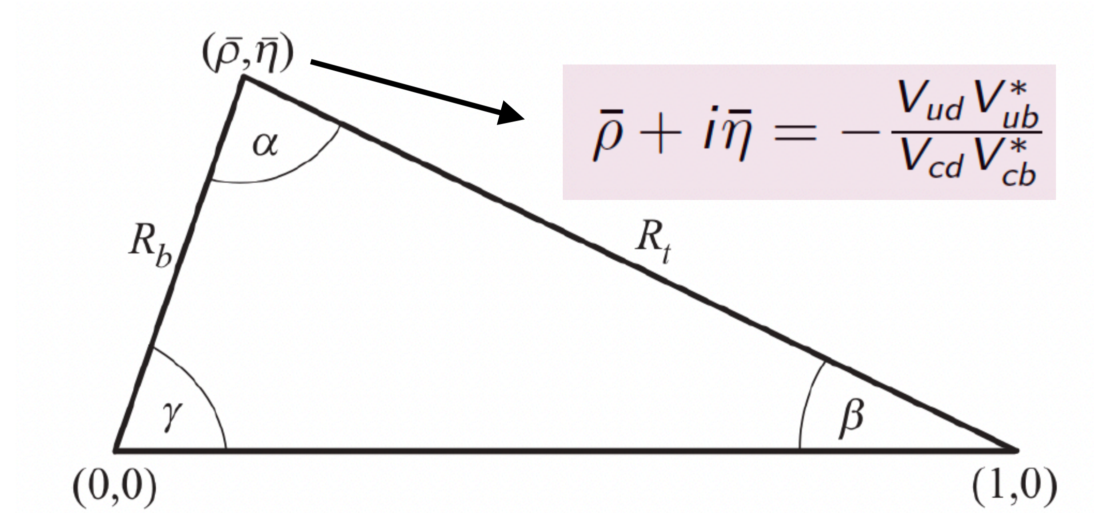

- $\lambda = |\mathbf{V_{us}}|$: **in general a weak dependence on it**
- β : **determined from** $\mathbf{S_{\psi K_S}} \rightarrow (22.2 \pm 0.7)^\circ$
- γ : **determined by LHCb through tree-level strategies** $\rightarrow (65.4^{+3.8}_{-4.2})^\circ$

CKM parametrization

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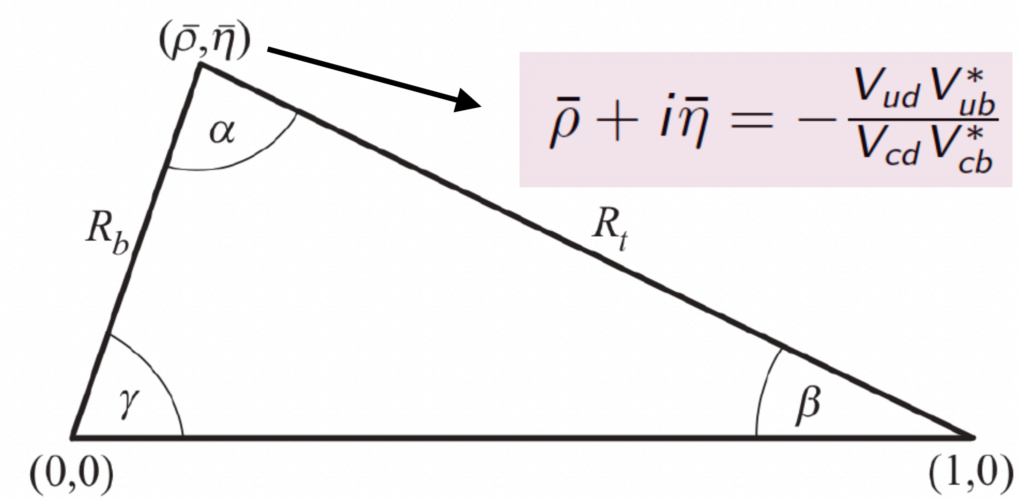
$$\lambda = |\mathbf{V}_{\mathbf{us}}|, \;\; |\mathbf{V}_{\mathbf{cb}}|, \;\; \beta, \;\; \gamma$$



$$|V_{td}| = \lambda |V_{cb}| \sin\gamma, \qquad |V_{ub}| = \lambda\sqrt{\sigma}|V_{cb}|\sin\beta, \qquad \sigma = \left(\frac{1}{1-\frac{\lambda^2}{2}}\right)^2,$$

$$|V_{ts}| = G(\beta,\gamma)|V_{cb}|, \qquad G(\beta,\gamma) = 1 + \frac{\lambda^2}{2}(1 - 2\sin\gamma\cos\beta)\,,$$

Dependence on $|\mathbf{V}_{\mathbf{ub}}|$, $|\mathbf{V}_{\mathbf{ts}}|$ and $|\mathbf{V}_{\mathbf{td}}|$ traded for the ones on λ, β, γ

Basic Strategy for $|\mathbf{V_{cb}}|$ independent ratios

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3. If the dependence on the CKM parameters is not a power-law function $\rightarrow$ Derivation of approximated power-law semi-numerical expressions with $\lesssim 2\,\%$ accuracy [Definition of *critical exponents*]

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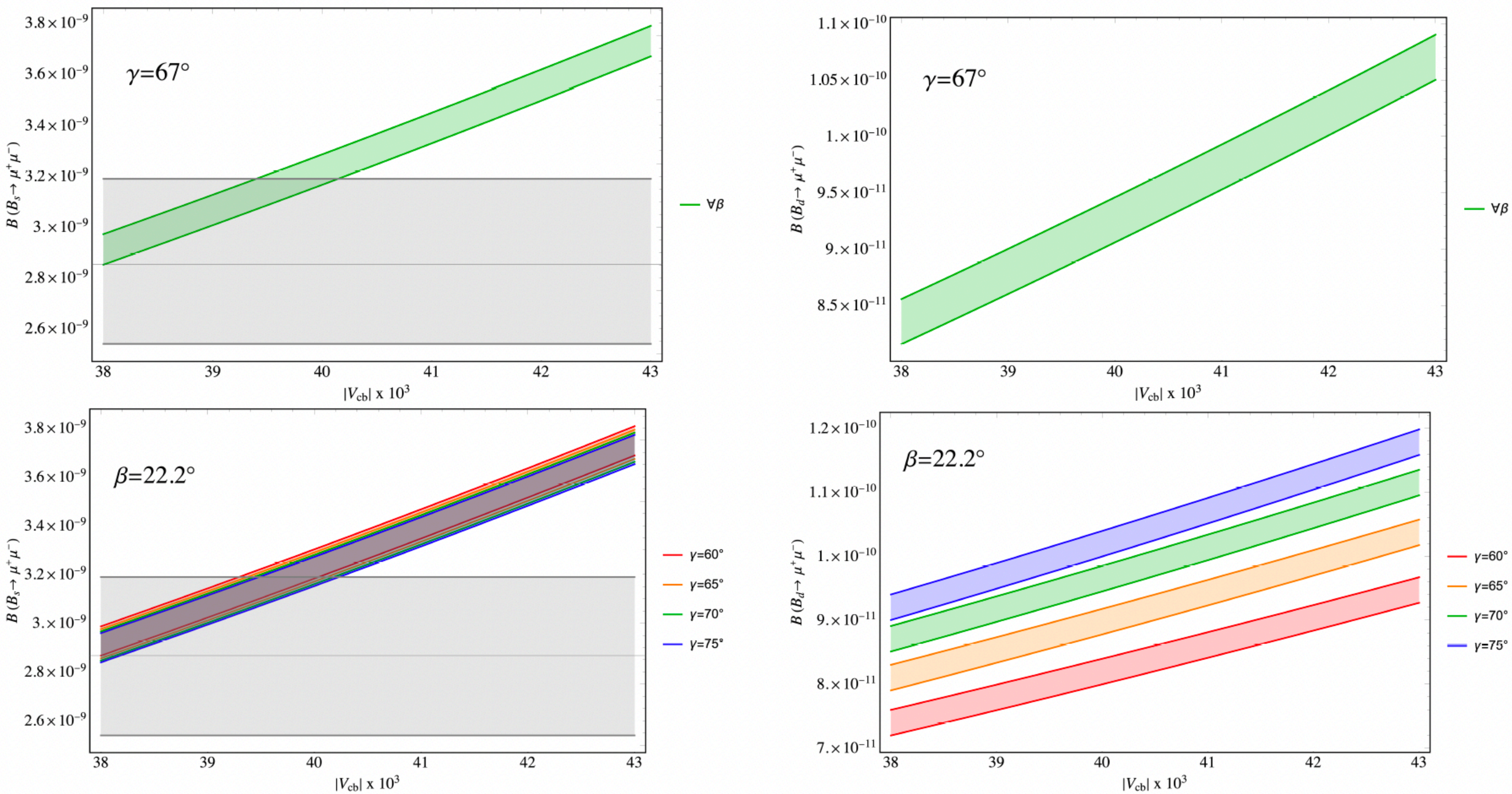
4. $|V_{cb}|$ independent ratios $\rightarrow$ Possible residual dependence on β and γ [Possible strategies to determine β and γ]

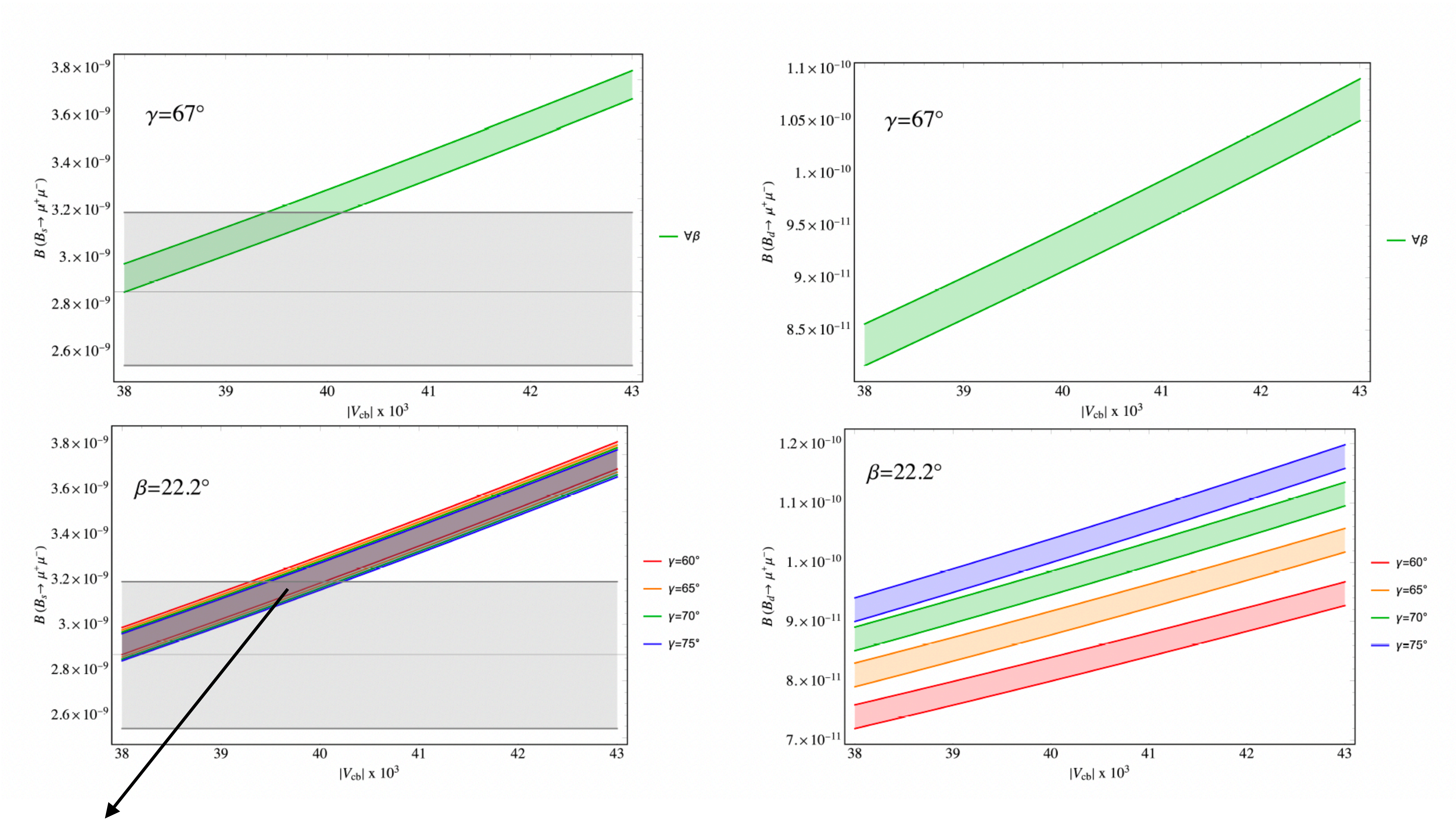
Br($B_{s,d} \to \mu\mu$)



Largest source of uncertainty: $|V_{cb}|$

Negligible β dependence

$Br(B_{s,d} \to \mu\mu)$



Tension SM-exp depends on $|V_{cb}|$, while in $R_{s\mu}$ it is $|V_{cb}|$ independent:

$|V_{cb}|_{inclusive} \to 2\sigma$ **anomaly,** $|V_{cb}|_{FLAG}$ **in agreement with SM**

$\mathbf{Br(B_{s,d} \to \mu\mu)}$

1. $$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\rm SM} = (3.39 \pm 0.06) \times 10^{-9} \left(\frac{F_{B_s}}{227.7\mathrm{MeV}}\right)^2 \left|\frac{V_{tb}^* V_{ts}}{0.0402}\right|^2 \quad \sim |\mathbf{V_{cb}}|^2$$

$$\mathcal{B}(B_d \to \mu^+\mu^-)_{\rm SM} = (0.973 \pm 0.02) \times 10^{-10} \left(\frac{F_{B_d}}{190.5\mathrm{MeV}}\right)^2 \left|\frac{V_{tb}^* V_{td}}{0.00848}\right|^2 \quad \sim |\mathbf{V_{cb}}|^2 (\sin\gamma)^2$$

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2. Exact $|\mathbf{V_{cb}}|$ independent correlations of $\mathrm{Br}(B_s \to \mu\mu)$ with $\mathrm{Br}(K^+ \to \pi^+\nu\nu)$

$$|V_{cb}|^2 = \frac{\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)}{2.09 \times 10^{-6} \bar{R}_s} \left(\frac{227.7\mathrm{MeV}}{F_{B_s}} \frac{1}{G(\beta,\gamma)}\right)^2$$

$|\mathbf{V_{cb}}|$ from $\mathrm{Br}(B_s \to \mu\mu)$

$$\mathcal{B}(K^+ \to \pi^+\nu\bar\nu) = (1+\Delta_{\rm EM})\frac{\kappa_+}{\lambda^8}\,|V_{cb}|^4 X(x_t)^2 \left[\frac{R_t^2 \sin^2\beta}{(1-\lambda^2/2)^2} + \left(1-\frac{\lambda^2}{2}\right)^2 \left(R_t\cos\beta + \frac{\lambda^4 P_c(X)}{|V_{cb}|^2 X(x_t)}\right)^2\right]$$

$$\kappa_+ = (5.173 \pm 0.025) \times 10^{-11} \left[\frac{\lambda}{0.225}\right]^8$$

$$P_c(X) = (0.405 \pm 0.024) \left[\frac{0.225}{\lambda}\right]^4$$

$\mathrm{Br}(\mathrm{B}_{s,d} \to \mu\mu)$

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$|\mathbf{V_{cb}}|$ from $\mathrm{Br}(\mathrm{B_s} \to \mu\mu)$

$$B_1 = \frac{X(x_t)^2}{\lambda^8} B_3^2 \left(\frac{227.7\mathrm{MeV}}{F_{B_s}} \frac{1}{G(\beta,\gamma)}\right)^4$$
$$\times \left[\sigma \sin^2\gamma \sin^2\beta + \frac{1}{\sigma}\left(\sin\gamma\cos\beta + \frac{\lambda^4 P_c(X)}{B_3 X(x_t)} \left(\frac{F_{B_s}}{227.7\mathrm{MeV}} G(\beta,\gamma)\right)^2\right)^2\right]$$

$$B_1 = \frac{\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})}{\kappa_+(1+\Delta_{\rm EM})},$$

$$B_3 = \frac{\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)}{2.09 \times 10^{-6} \bar{R}_s},$$

$\mathrm{Br}(\mathrm{B_{s,d}} \to \mu\mu)$

3. **Approximated power-law expression for** $\mathrm{Br}(\mathrm{K}^+ \to \pi^+ \nu\nu)$

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = (7.92 \pm 0.28) \times 10^{-11} \left[\frac{|V_{cb}|}{41.0 \times 10^{-3}}\right]^{2.8} \left[\frac{\sin\gamma}{\sin 67^\circ}\right]^{1.39};$$

Critical exponents

$$\mathrm{Br}(\mathrm{K}^+ \to \pi^+ \nu\nu) \propto |V_{cb}|^{2.8} \quad \& \quad \mathrm{Br}(\mathrm{B_{s,d}} \to \mu\mu) \propto |V_{cb}|^{2}$$

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$$R_1(\beta, \gamma) = \frac{\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})}{[\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)]^{1.4}}, \qquad R_2(\beta, \gamma) = \frac{\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})}{[\mathcal{B}(B_d \to \mu^+\mu^-)]^{1.4}}.$$

Approximately $|V_{cb}|$ independent !!!

$$\mathbf{R_1} \propto [\sin\gamma]^{1.39} [F_{B_s}]^{-2.8}$$
$$\mathbf{R_2} \propto [\sin\gamma]^{-1.41} [F_{B_d}]^{-2.8}$$

BUT not γ independent !

$\mathrm{Br}(B_{s,d} \to \mu\mu)$

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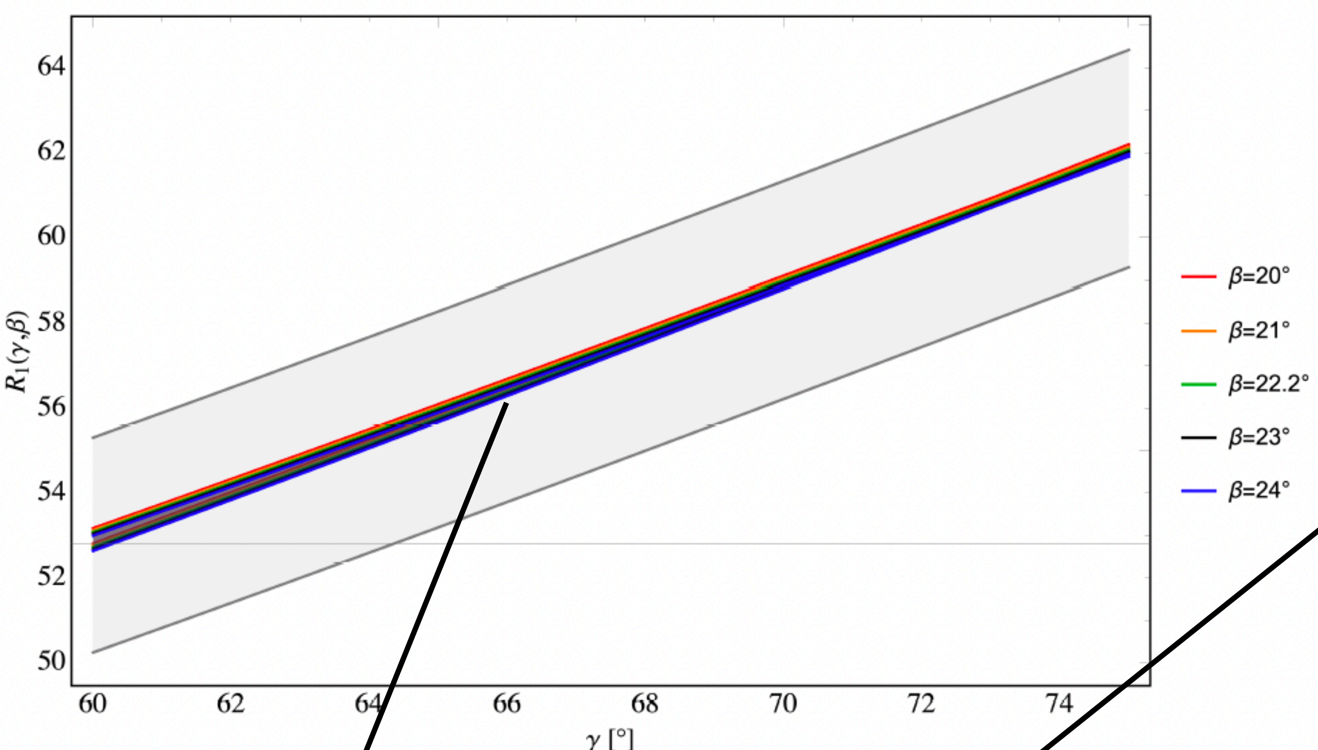

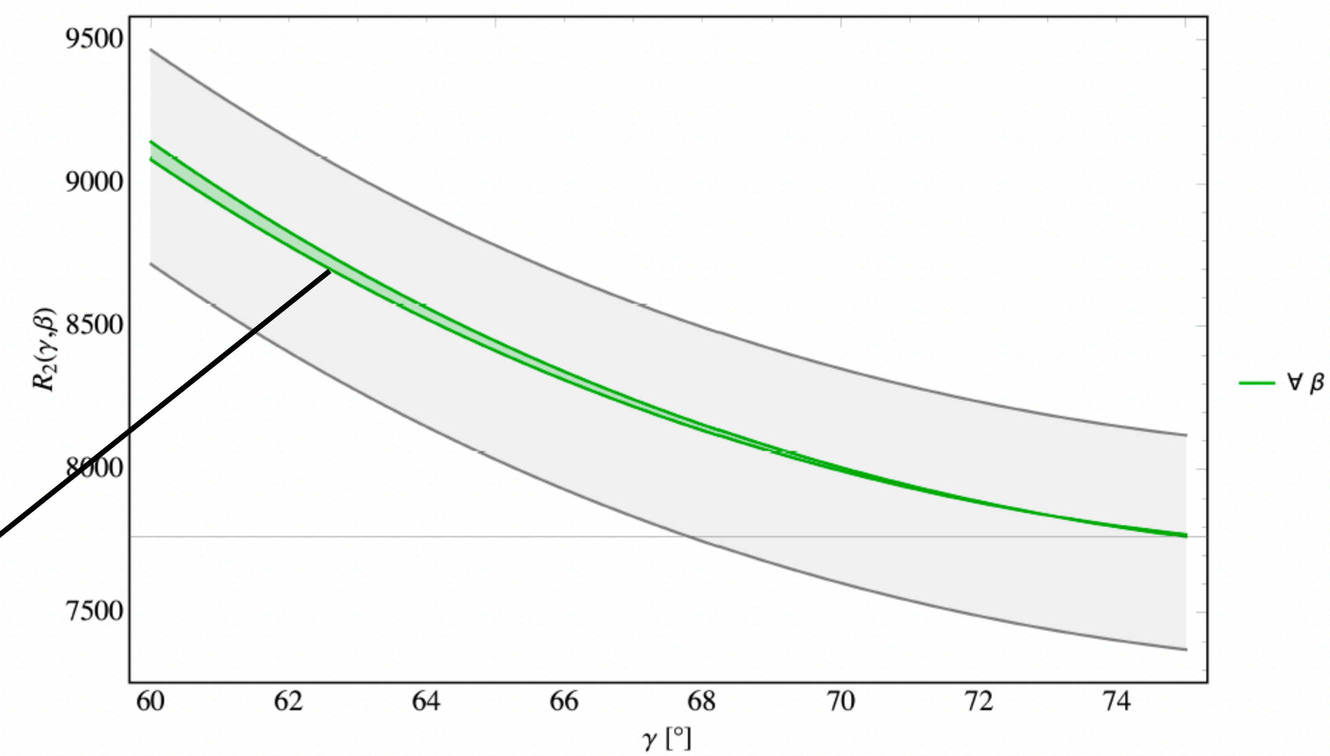

Colored regions: $38 < |V_{cb}|10^3 < 43$

$\to$ **Approximately** $|\mathbf{V_{cb}}|$ **independent !!!**

γ **: largest uncertainty**

$\to$ **Once** γ **will be precisely measured,**

R_1 **and** R_2 **will be very good tests of the SM**

$\mathbf{Br(B_{s,d} \to \mu\mu)}$

2. **Exact power-law expression for $\mathbf{Br(K_L \to \pi^0 \nu\nu)}$**

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) = (2.61 \pm 0.04) \times 10^{-11} \left[\frac{\sin\beta}{\sin(22.2^\circ)}\right]^2 \left[\frac{\sin\gamma}{\sin(67^\circ)}\right]^2 \left[\frac{|V_{cb}|}{41.0 \times 10^{-3}}\right]^4$$

$$\mathbf{Br(K_L \to \pi^0 \nu\nu)} \propto |V_{cb}|^4 \quad \& \quad \mathbf{Br(B_{s,d} \to \mu\mu)} \propto |V_{cb}|^2$$

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$$R_3(\beta,\gamma) = \frac{\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})}{[\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)]^2}, \qquad R_4(\beta,\gamma) = \frac{\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})}{[\mathcal{B}(B_d \to \mu^+\mu^-)]^2}.$$

Exactly $|\mathbf{V_{cb}}|$ independent !!!

$$\mathbf{R_3} \propto [\sin\beta]^2 [\sin\gamma]^2 [F_{B_s}]^{-4}$$
$$\mathbf{R_4} \propto [\sin\beta]^2 [\sin\gamma]^{-2} [F_{B_d}]^{-4}$$

BUT not β and γ independent !

$\mathbf{Br(B_{s,d} \to \mu\mu)}$

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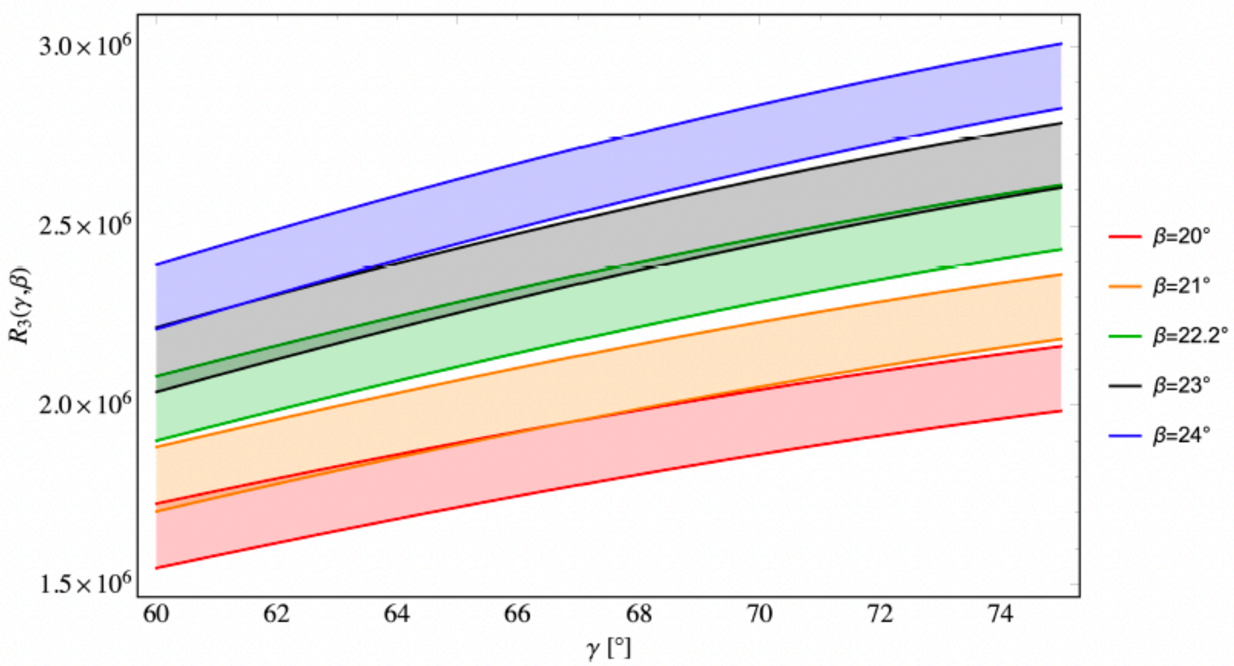
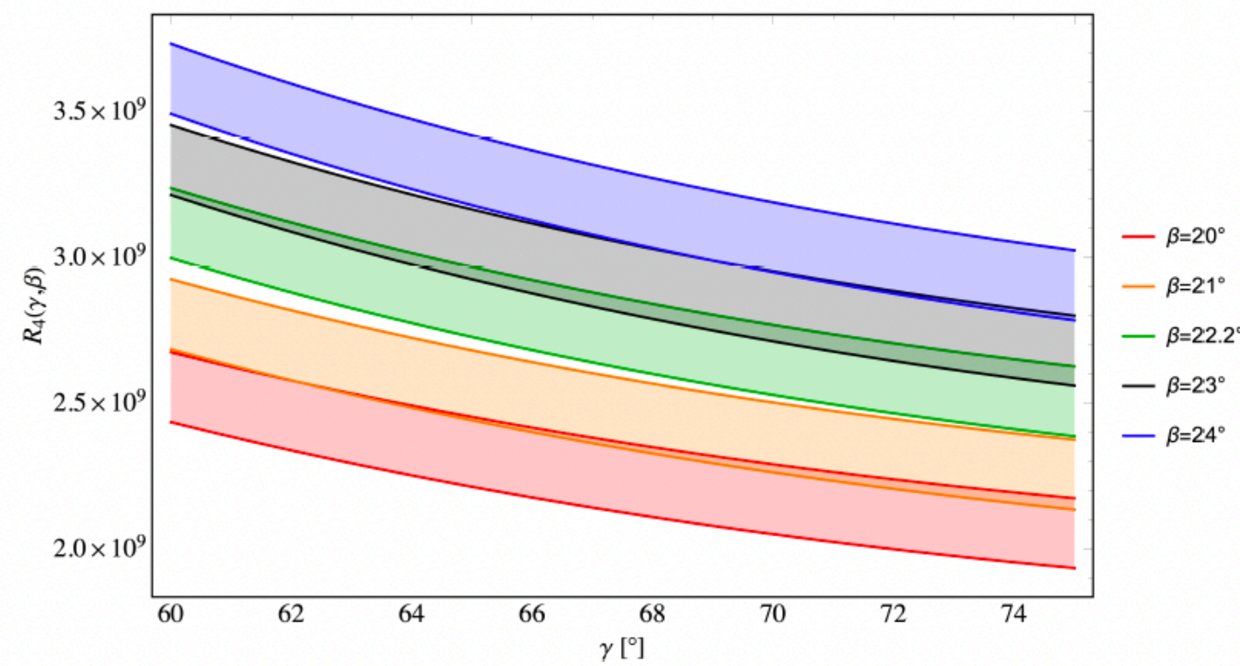




Exactly $|V_{cb}|$ independent !!!

Dominant uncertainties due to γ and β

Critical exponents for other observables

Observable ∝

$$\propto |V_{cb}|^{r_1}[\sin\gamma]^{r_2}[\sin\beta]^{r_3}$$

	Observable	r_1	r_2	r_3
	$K^+ \to \pi^+ \nu\bar{\nu}$,	2.8	1.39	0.0
	$K_L \to \pi^0 \nu\bar{\nu}$,	4.0	2.0	2.0
	$K_S \to \mu^+\mu^-$,	4.0	2.0	2.0
	$\lvert\varepsilon_K\rvert$,	3.4	1.67	0.87
★	$B_s \to \mu^+\mu^-$	4.0	0.0	0.0
	$B_d \to \mu^+\mu^-$	2.0	2.0	0.0
★	$B^+ \to K^+ \nu\bar{\nu}$	2.0	0.0	0.0
★	$B^0 \to K^{0*} \nu\bar{\nu}$	2.0	0.0	0.0
	ΔM_d	2.0	2.0	0.0
★	ΔM_s	2.0	0.0	0.0

16 $|V_{cb}|$ independent ratios

$$R_5 \equiv \frac{Br(K^+ \to \pi^+ \nu\nu)}{[Br(B^+ \to K^+ \nu\nu)]^{1.4}} \propto [\sin\gamma]^{1.39} \qquad R_6 \equiv \frac{Br(K^+ \to \pi^+ \nu\nu)}{[Br(B^0 \to K^{0*} \nu\nu)]^{1.4}} \propto [\sin\gamma]^{1.39}$$

$$R_7 \equiv \frac{Br(B^+ \to K^+ \nu\nu)}{Br(B_s \to \mu\mu)} \propto [F_{B_s}]^{-2} \qquad R_8 \equiv \frac{Br(B^0 \to K^{0*} \nu\nu)}{Br(B_s \to \mu\mu)} \propto [F_{B_s}]^{-2}$$

CKM independent (R_7)
1.8σ **tension SM-exp**

$$R_9 \equiv \frac{|\epsilon_K|}{(\Delta M_d)^{1.7}} \propto [\sin\gamma]^{-1.73}[\sin\beta]^{0.87} \qquad R_{10} \equiv \frac{|\epsilon_K|}{(\Delta M_s)^{1.7}} \propto [\sin\gamma]^{1.67}[\sin\beta]^{0.87}$$

$$R_{11} \equiv \frac{Br(K^+ \to \pi^+ \nu\nu)}{|\epsilon_K|^{0.82}} \propto [\sin\gamma]^{0.015}[\sin\beta]^{-0.71} \qquad R_{12} \equiv \frac{Br(K_L \to \pi^0 \nu\nu)}{|\epsilon_K|^{1.18}} \propto [\sin\gamma]^{0.03}[\sin\beta]^{0.98}$$

$$R_{s\mu} \equiv \frac{Br(B_s \to \mu\mu)}{\Delta M_s} \propto \text{const} \qquad R_{d\mu} \equiv \frac{Br(B_d \to \mu\mu)}{\Delta M_d} \propto \text{const}$$

$$R_0 \equiv \frac{Br(K^+ \to \pi^+ \nu\nu)}{Br(K_L \to \pi^0 \nu\nu)^{0.7}} \propto [\sin\beta]^{-1.4} \qquad R_{SL} \equiv \frac{Br(K_S \to \mu\mu)_{SD}}{Br(K_L \to \pi^0 \nu\nu)} \propto \text{const}$$

Almost depending only on β: accurately determined from $S_{\psi K_S}$ (R_{11}, R_{12}, R_0)

CKM independent ($R_{s\mu}$, $R_{d\mu}$, R_{SL})

Basic Strategy for $|V_{cb}|$ independent branching ratios

Determination of CKM parameters through precisely measured observables:

$$|\epsilon_K|, \Delta M_s, \Delta M_d, S_{\psi K_S}$$

↓

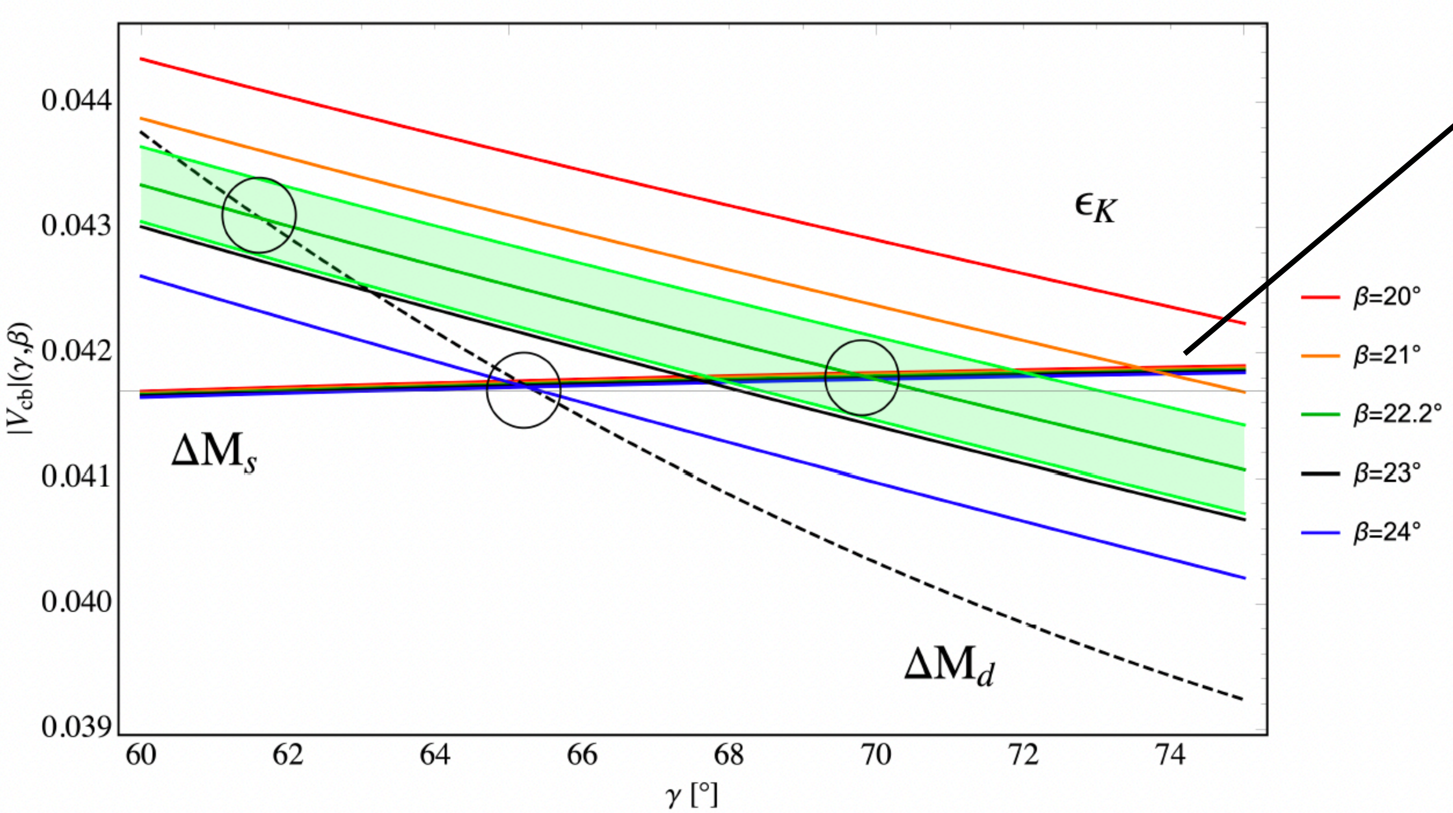
CKM dependence in K and B branching ratios traded for dependence on $|\epsilon_K|, \Delta M_s, \Delta M_d, S_{\psi K_S}$

Basic Strategy for $|V_{cb}|$ independent branching ratios

Determination of CKM parameters through precisely measured observables:

$$|\epsilon_K|,\ \Delta M_s,\ \Delta M_d,\ S_{\psi K_S}$$

CKM dependence in K and B branching ratios traded for dependence on $|\epsilon_K|,\ \Delta M_s,\ \Delta M_d,\ S_{\psi K_S}$



$|V_{cb}|$ from ΔM_s:

almost independent from γ and β

$60^\circ < \gamma < 75^\circ$ **and** $20^\circ < \beta < 24^\circ \rightarrow$

$$|V_{cb}| = 41.78(62) \cdot 10^{-3}$$

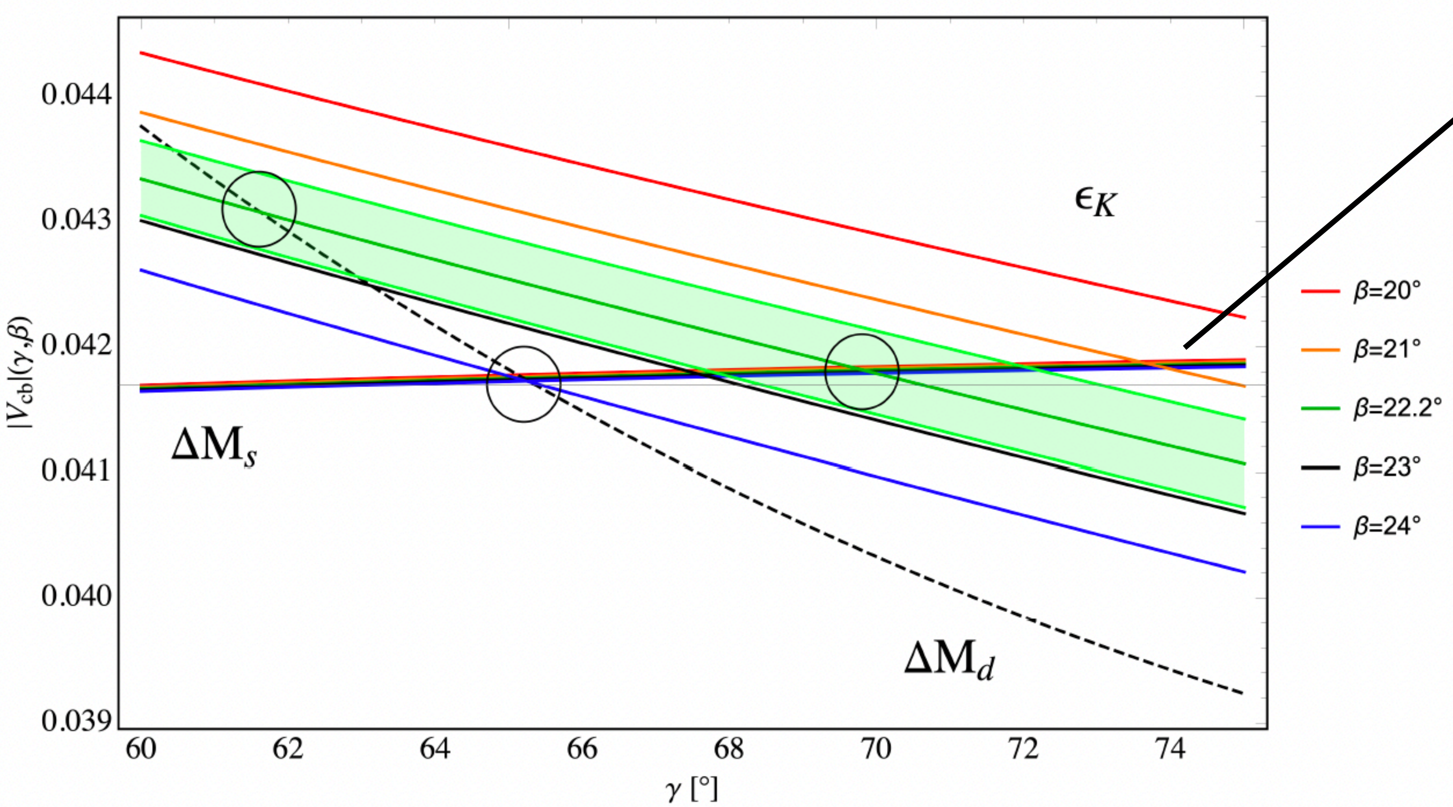
Most precise $|V_{cb}|$ determination from a single observable

Basic Strategy for $|V_{cb}|$ independent branching ratios

Determination of CKM parameters through precisely measured observables:

$|\epsilon_K|, \Delta M_s, \Delta M_d, S_{\psi K_S}$

CKM dependence in K and B branching ratios traded for dependence on $|\epsilon_K|, \Delta M_s, \Delta M_d, S_{\psi K_S}$



$|V_{cb}|$ from ΔM_s:
almost independent from γ and β

$60^\circ < \gamma < 75^\circ$ **and** $20^\circ < \beta < 24^\circ \rightarrow$

$|V_{cb}| = 41.78(62) \cdot 10^{-3}$

$\beta = 22.2(7)^\circ$ **from** $S_{\psi K_S}$

$\gamma = 69.8(26)^\circ$ **from** $|\epsilon_K|, \Delta M_s \rightarrow$

$|V_{cb}| = 41.81(61) \cdot 10^{-3}$

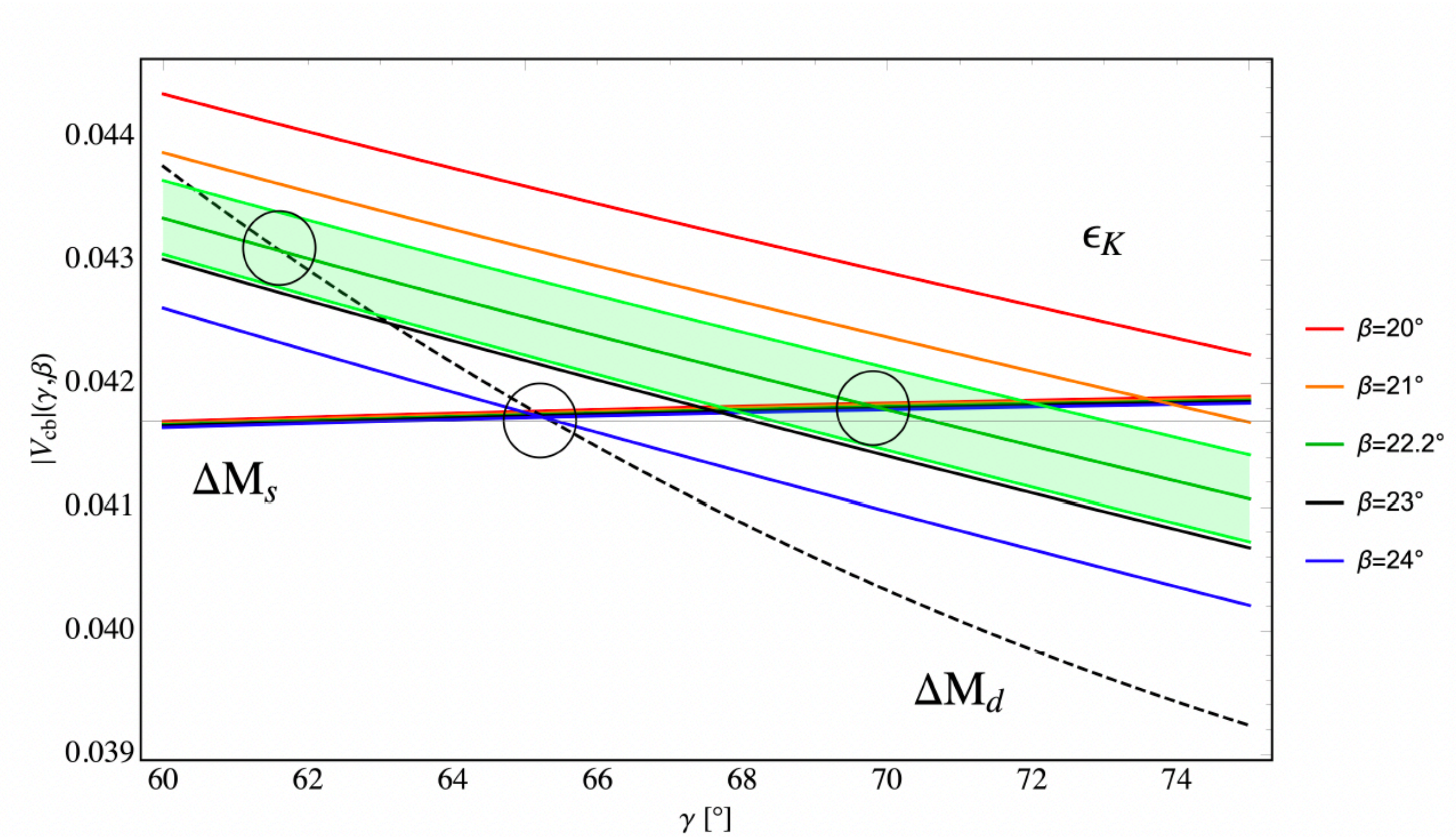
$|V_{ub}| = 3.65(12) \cdot 10^{-3}$

Basic Strategy for $|V_{cb}|$ independent branching ratios

Determination of CKM parameters through precisely measured observables:

$|\epsilon_K|$, ΔM_s, ΔM_d, $S_{\psi K_S}$

CKM dependence in K and B branching ratios traded for dependence on $|\epsilon_K|$, ΔM_s, ΔM_d, $S_{\psi K_S}$



$(|V_{cb}|, \gamma, \beta)$ **, from:**

- $|\epsilon_K|$, ΔM_s, ΔM_d
- $|\epsilon_K|$, ΔM_s, $S_{\psi K_S}$
- $|\epsilon_K|$, ΔM_d, $S_{\psi K_S}$

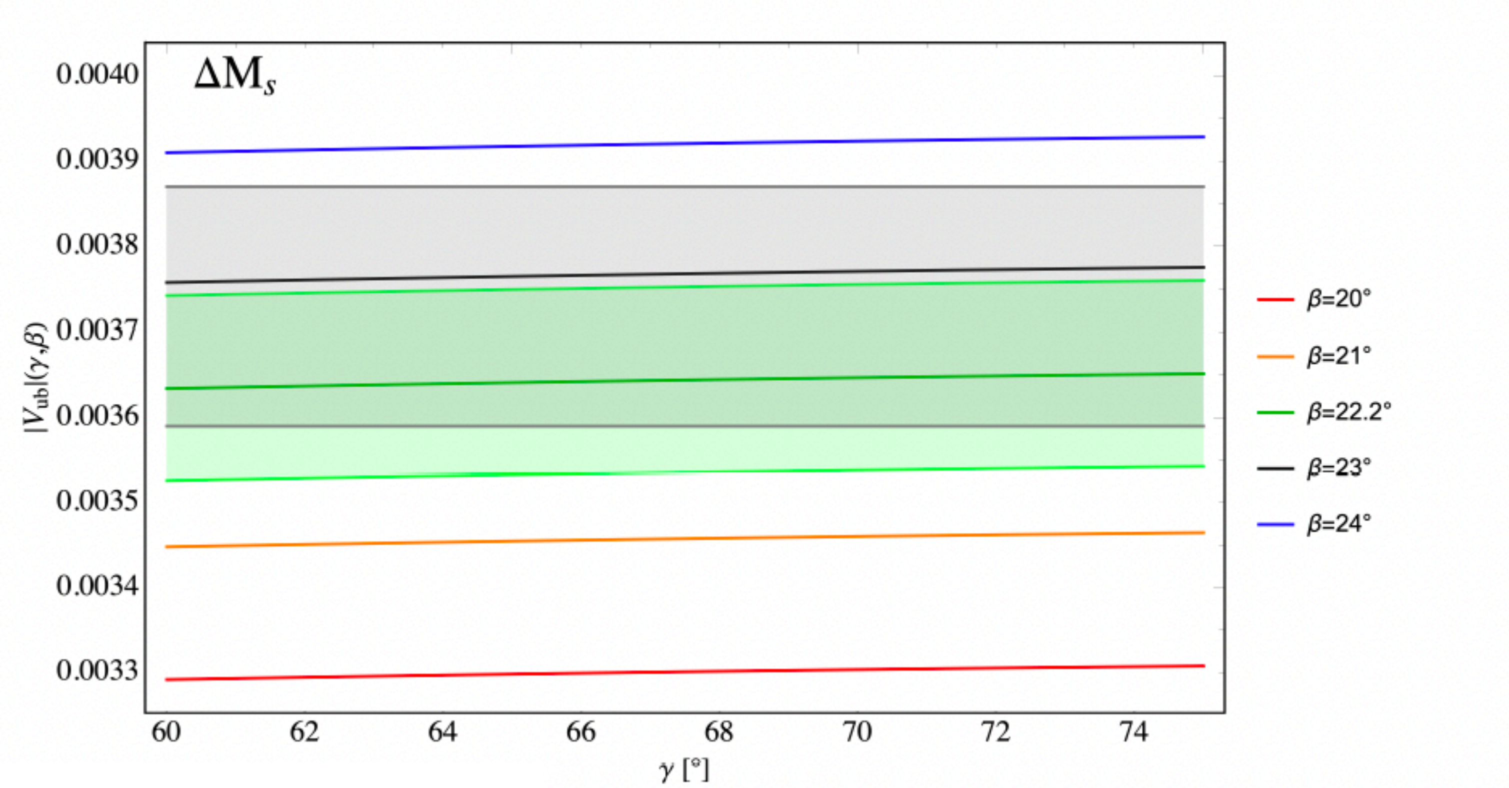
Tensions in the SM

Basic Strategy for $|V_{cb}|$ independent branching ratios

Determination of CKM parameters through precisely measured observables:

$|\epsilon_K|, \Delta M_s, \Delta M_d, S_{\psi K_S}$

CKM dependence in K and B branching ratios traded for dependence on $|\epsilon_K|, \Delta M_s, \Delta M_d, S_{\psi K_S}$



$\beta = 22.2(7)^\circ$ **from** $S_{\psi K_S}$

$\gamma = 69.8(26)^\circ$ **from** $|\epsilon_K|, \Delta M_s \rightarrow$

$|V_{ub}| = 3.65(12) \cdot 10^{-3}$

Consistent with FLAG:

$|V_{ub}| = 3.73(11) \cdot 10^{-3}$

Basic Strategy for $|V_{cb}|$ independent branching ratios

Determination of $|V_{cb}|$ and β through precisely measured observables:
$|\epsilon_K|$ ($|V_{cb}|$ for K decays), $\Delta M_{s,d}$ ($|V_{cb}|$ for B decays), $S_{\psi K_S}$ (β) [In the SM]

↓

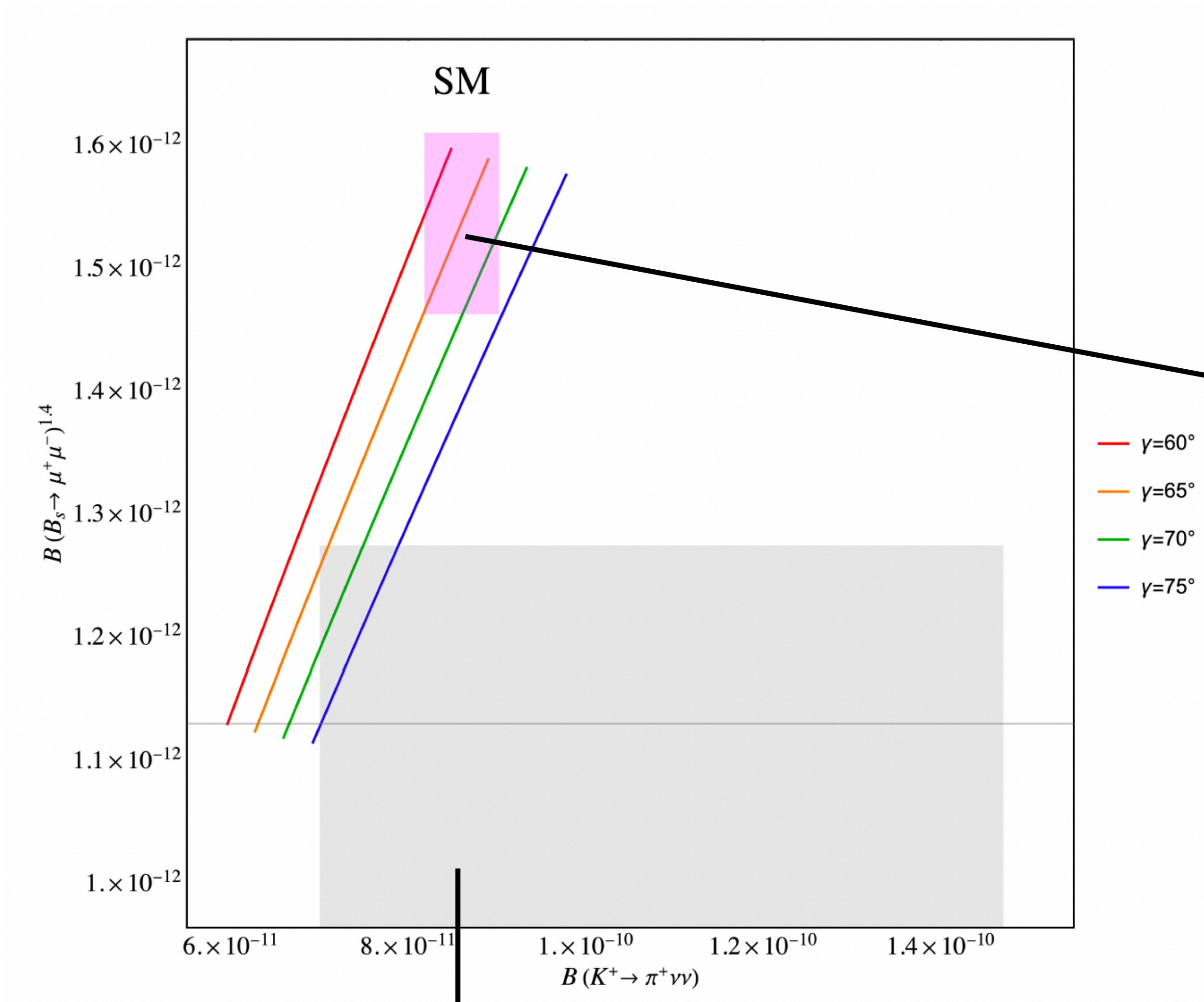
$|V_{cb}|$ dependence in K and B branching ratios traded for dependence on $|\epsilon_K|$, ΔM_s, ΔM_d

Decay	Branching Ratio	Decay	Branching Ratio
$K^+ \to \pi^+ \nu\bar{\nu}$	$(8.60 \pm 0.42) \times 10^{-11}$	$B_s \to \mu^+\mu^-$	$(3.62^{+0.15}_{-0.10}) \times 10^{-9}$
$K_L \to \pi^0 \nu\bar{\nu}$	$(2.94 \pm 0.15) \times 10^{-11}$	$B_d \to \mu^+\mu^-$	$(0.99^{+0.05}_{-0.03}) \times 10^{-10}$
$K_S \to \mu^+\mu^-$	$(18.5 \pm 1.0) \times 10^{-12}$	$B^+ \to K^+ \nu\bar{\nu}$	$(4.45 \pm 0.62) \times 10^{-6}$
		$B^0 \to K^{0*} \nu\bar{\nu}$	$(9.70 \pm 0.92) \times 10^{-6}$

Eliminating $|V_{cb}|$ dependence:
most accurate estimates to date!

$\mathrm{Br}(B_s \to \mu\mu)$ and $|V_{cb}|$ independent estimates

Correlations between $\mathrm{Br}(B_s \to \mu\mu)^{1.4}$ and $\mathrm{Br}(K^+ \to \pi^+\nu\nu)$ [R_1]



Linear correlation varying $|V_{cb}|$ (slope from γ)

SM $|V_{cb}|$ independent predictions

Test of the SM

$\mathrm{Br}(B_s \to \mu\mu)$ measurements (LHCb, CMS, ATLAS) to be improved by LHC, γ by LHCb and Belle II, $\mathrm{Br}(K^+ \to \pi^+\nu\nu)$ by NA62

Conclusions

$|V_{cb}|$ independent analysis

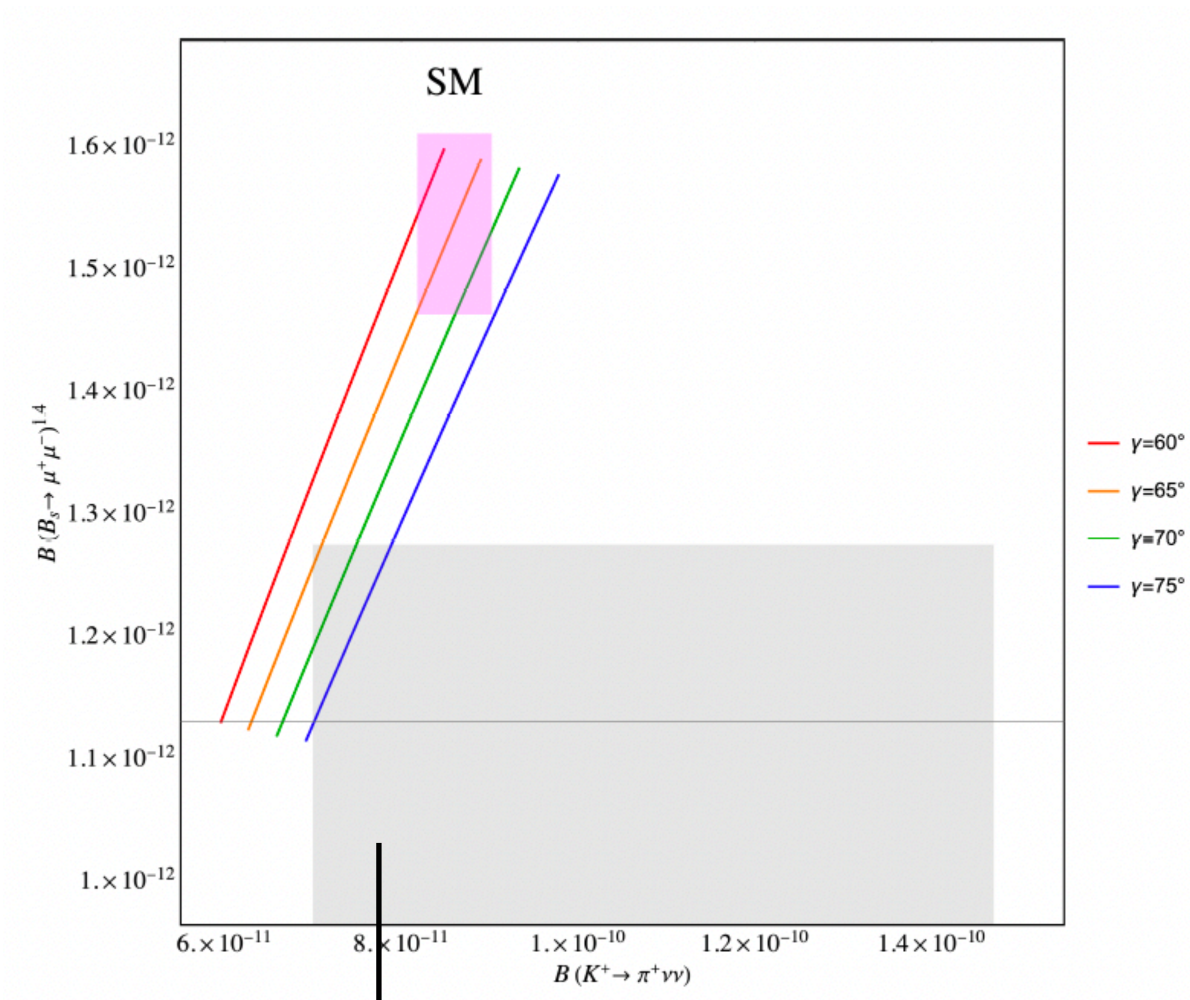
- $|V_{cb}|$ independent ratios of observables in the SM
- $|V_{cb}|$ independent SM predictions for rare K and B decays

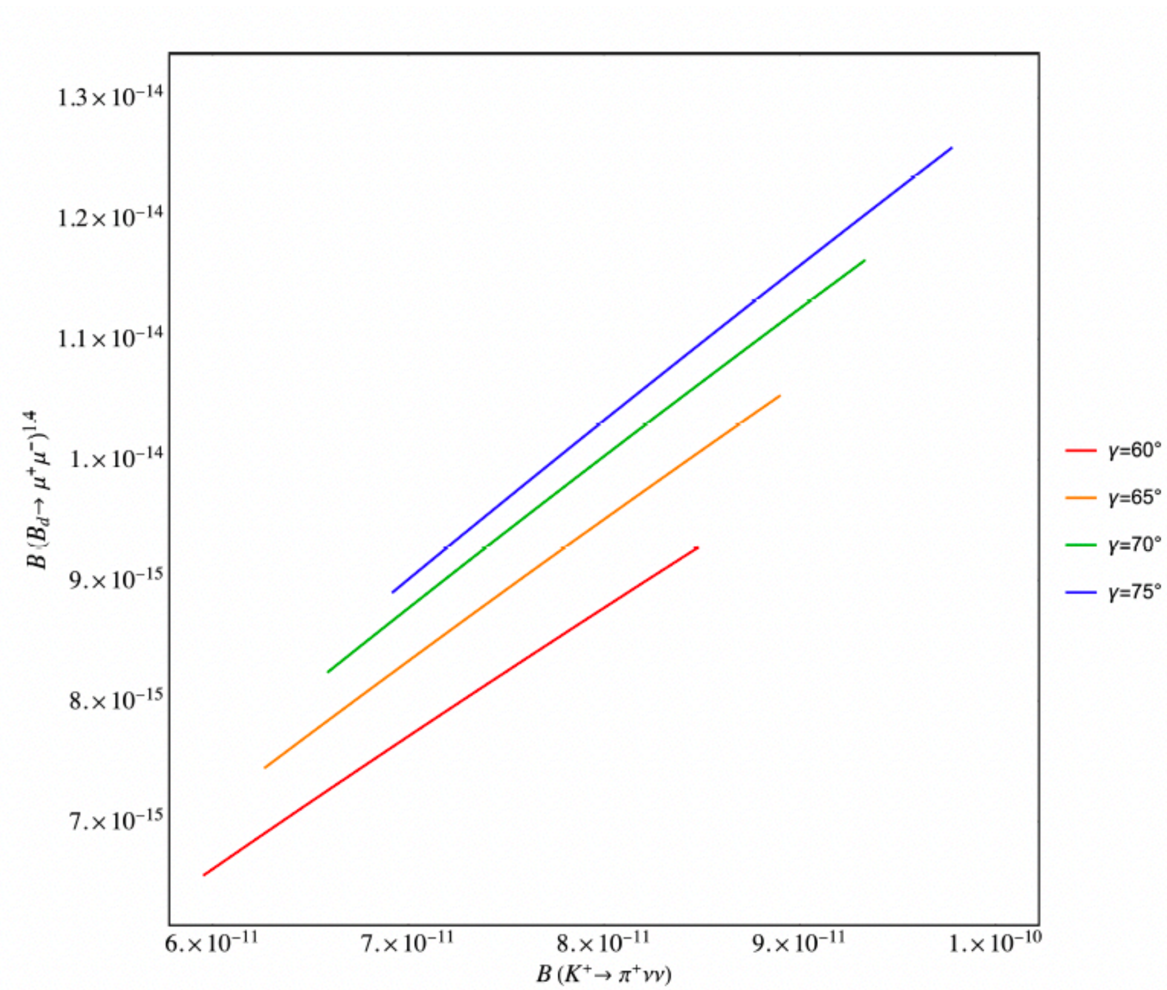
Thank you

Backup

$\mathbf{Br(B_{s,d} \to \mu\mu)}$

4. **Correlations between $\mathrm{Br}(\mathrm{B_{s,d}} \to \mu\mu)^{1.4}$ and $\mathrm{Br}(\mathrm{K}^+ \to \pi^+\nu\nu)$**





Linear correlation varying $|\mathrm{V_{cb}}|$

$\mathrm{Br}(\mathrm{B_s} \to \mu\mu)$ **measurements (LHCb, CMS, ATLAS) to be improved by LHC,**

γ **by LHCb and Belle II,**

$\mathrm{Br}(\mathrm{K}^+ \to \pi^+\nu\nu)$ **by NA62**

Test of the SM