An *eclectic* approach to the flavor (symmetry) problem

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Discrete 20–21
Bergen

December 2, 2021

From various collaborations with

M-C. Chen, V. Knapp-Pérez, M. Ramos-Hamud, M. Ratz & S. Shukla: 1909.06910 & 2108.02240

A. Baur, M. Kade, H.P. Nilles & P. Vaudrevange: 2001.01736, 2004.05200, 2008.07534, 2010.13798, 2104.03981

The flavor puzzle

and its potential solutions

Flavor puzzle

Despite the great success of the SM

- Need to explain
  - three flavors of SM particles
  - observed mass hierarchies
  - observed quark and lepton mixing *textures*
  - CP violation in CKM and PMNS
  - neutrino physics
  - ...

See Penedo's and Feruglio's talk

$$\begin{pmatrix} 0.974 & 0.224 & 0.0039 \\ 0.218 & 0.997 & 0.042 \\ 0.008 & 0.039 & 1.019 \end{pmatrix}_{CKM} , \qquad \begin{pmatrix} 0.829 & 0.539 & 0.147 \\ 0.493 & 0.584 & 0.645 \\ 0.262 & 0.607 & 0.75 \end{pmatrix}_{PMNS}$$

$m_{u_i} \sim 2.16, 1270, 172900$ MeV

$m_{d_i} \sim 4.67, 93, 4180$ MeV

$\Delta m^2_{21} = 7.4 \cdot 10^{-5}, \Delta m^2_{31(23)} \approx 2.5 \cdot 10^{-3}$ eV^2

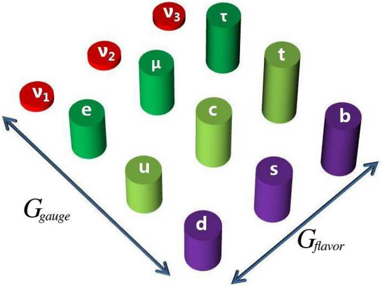
$m_{e_i} \sim 0.511, 105.7, 1776.9$ MeV

normal ordering

Approaches towards solving the flavor puzzle

Traditional: discrete non-Abelian flavor symmetries G_{flavor} lead to models for quarks and leptons with great fits, $\theta_{13} \neq 0$,... requiring careful choice of flavon sector and flavon vevs

see reviews by Ishimori, Kobayashi, Ohki, Okada, Shimizu, Tanimoto (2010); Feruglio, Romanino (2019)



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Modular: finite modular groups Γ_N, Γ'_N = modular flavor sym. $G_{modular}$:

$\Gamma_N \cong S_3, A_4, S_4, A_5, \quad \Gamma'_N \cong S_3, T', \mathrm{SL}(2,4), \mathrm{SL}(2,5)$ for $N = 2,3,4,5$

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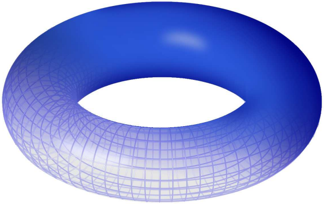

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In this talk

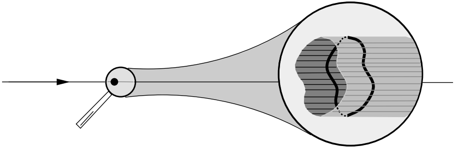
A possible solution inspired by string theory

A flavor of

strings

Stringy ingredients

particles $\longleftrightarrow$ strings



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- matter fields get all their properties from string features
- field couplings arise from string interactions

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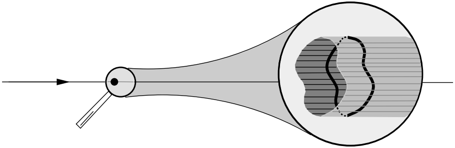

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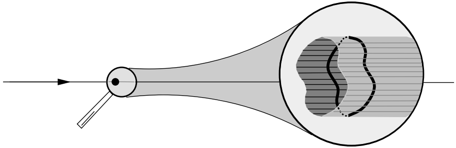

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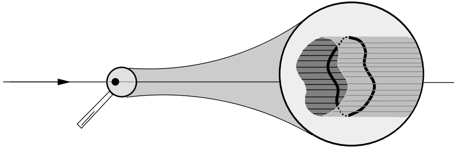

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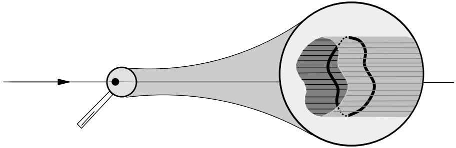

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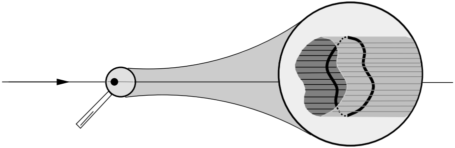

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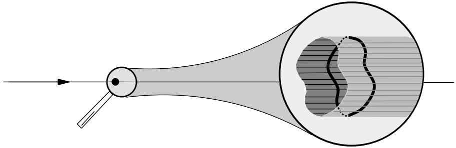

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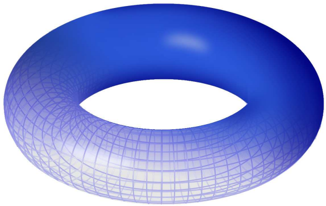
    $\rightarrow$ couplings are modular forms with fixed properties

Example: 2D toroidal orbifold $\mathbb{T}^2/\mathbb{Z}_3$



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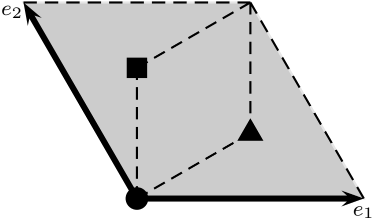

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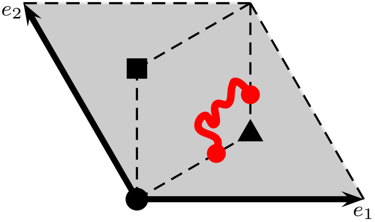
Matter at low energies arise from strings:

relevant matter fields are **fixed** *in compact space*



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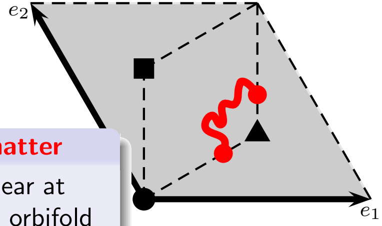

localized twisted matter

matter fields Φ_n appear at definite points in the orbifold

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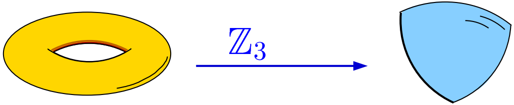
T^2:



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T'	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$	$\mathbf{2}'' \oplus \mathbf{1}$

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- Common origin of traditional and modular flavor symmetries

  $G_{flavor} \cup G_{modular} = \Delta(54) \cup T' \cong \Omega(1)$ with $T' \subset Out(\Delta(54))$

Baur, Nilles, Trautner, Vaudrevange (2019)

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Yukawa coupling coefficients $\hat{Y}$ are modular forms!

modular forms $\hat{Y}_{\boldsymbol{s}}^{(n_Y)}$	eclectic flavor group $\Omega(1)$: modular T' subgroup				eclectic flavor group $\Omega(1)$: traditional $\Delta(54)$ subgroup			
	irrep $\boldsymbol{s}$	$\rho_{\boldsymbol{s}}(\mathrm{S})$	$\rho_{\boldsymbol{s}}(\mathrm{T})$	n_Y	irrep $\boldsymbol{r}$	$\rho_{\boldsymbol{r}}(\mathrm{A})$	$\rho_{\boldsymbol{r}}(\mathrm{B})$	$\rho_{\boldsymbol{r}}(\mathrm{C})$
$\hat{Y}_{\mathbf{2}''}^{(1)}$	$\mathbf{2}''$	$\rho_{\mathbf{2}''}(\mathrm{S})$	$\rho_{\mathbf{2}''}(\mathrm{T})$	1	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{1}}^{(4)}$	$\mathbf{1}$	1	1	4	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{1}'}^{(4)}$	$\mathbf{1}'$	1	ω	4	$\mathbf{1}$	1	1	1
$\hat{Y}_{\mathbf{3}}^{(4)}$	$\mathbf{3}$	$\rho_{\mathbf{3}}(\mathrm{S})$	$\rho_{\mathbf{3}}(\mathrm{T})$	4	$\mathbf{1}$	1	1	1

$$\hat{Y}_{\mathbf{2}''}^{(1)} := \begin{pmatrix} \hat{Y}_1(T) \\ \hat{Y}_2(T) \end{pmatrix} = \begin{pmatrix} -3\sqrt{2} & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} \eta(3T)^3/\eta(T) \\ \eta(T/3)^3/\eta(T) \end{pmatrix}$$

No arbitrary modular weights n_Y nor representations $\boldsymbol{s}$! ☺

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$$G_{eclectic} = G_{modular} \cup G_{flavor} \quad \text{with} \quad G_{modular} \subset Out(G_{flavor})$$

You can also include a $\mathbb{Z}_2$ CP-like modular transformation!
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Interesting observation:

G_{flavor} does fix the kinetic terms of fields to their canonical form!

Nilles, SRS, Vaudrevange (2020); Chen, Knapp-Pérez, Ramos-Hamud, Ratz, Shukla (2021)

From top-down to bottom-up

eclectic flavor symmetries

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- $G_{eclectic} \cong$ multiplicative closure of G_{flavor} and $G_{modular}$
- Verify whether there is a third (class-inverting) outer automorphism that act as a $\mathbb{Z}_2$ CP-like transformation to further enhance the eclectic flavor symmetry

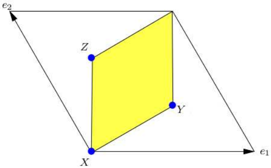
Eclectic flavor groups

flavor group $\mathcal{G}_{\text{fl}}$	GAP ID	$\text{Aut}(\mathcal{G}_{\text{fl}})$	finite modular groups		eclectic flavor group
Q_8	[8, 4]	S_4	without $\mathcal{CP}$	S_3	$\text{GL}(2,3)$
			with $\mathcal{CP}$	–	–
$\mathbb{Z}_3 \times \mathbb{Z}_3$	[9, 2]	$\text{GL}(2,3)$	without $\mathcal{CP}$	S_3	$\Delta(54)$
			with $\mathcal{CP}$	$S_3 \times \mathbb{Z}_2$	[108, 17]
A_4	[12, 3]	S_4	without $\mathcal{CP}$	S_3 S_4	S_4 S_4
			with $\mathcal{CP}$	–	–
T'	[24, 3]	S_4	without $\mathcal{CP}$	S_3	$\text{GL}(2,3)$
			with $\mathcal{CP}$	–	–
$\Delta(27)$	[27, 3]	[432, 734]	without $\mathcal{CP}$	S_3 T'	$\Delta(54)$ $\Omega(1)$
			with $\mathcal{CP}$	$S_3 \times \mathbb{Z}_2$ $\text{GL}(2,3)$	[108, 17] [1296, 2891]
$\Delta(54)$	[54, 8]	[432, 734]	without $\mathcal{CP}$	T'	$\Omega(1)$
			with $\mathcal{CP}$	$\text{GL}(2,3)$	[1296, 2891]

Nilles, SR-S, Vaudrevange (2001.01736)

Back in the $\mathbb{T}^2/\mathbb{Z}_3$ example

Restricted superpotential



$$\Rightarrow \mathcal{W} \quad \supset \quad c\left[\hat{Y}_2(T)\big(X_1\,X_2\,X_3 + Y_1\,Y_2\,Y_3 + Z_1\,Z_2\,Z_3\big) - \frac{\hat{Y}_1(T)}{\sqrt{2}}\big(X_1\,Y_2\,Z_3 + X_1\,Y_3\,Z_2 + X_2\,Y_1\,Z_3 + X_3\,Y_1\,Z_2 + X_2\,Y_3\,Z_1 + X_3\,Y_2\,Z_1\big)\right],$$

with $\quad \Phi^{i}_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

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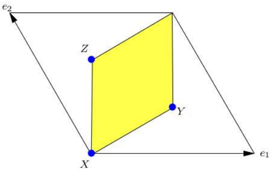

with $\quad \Phi^i_{-2/3} := (X_i, Y_i, Z_i)^T, \quad c \in \mathbb{R}$

More interestingly

$$K = -\log(-\mathrm{i}T + \mathrm{i}T) + \sum_i (-\mathrm{i}T + \mathrm{i}T)^{-2/3} |\Phi^i_{-2/3}|^2$$

Only canonical terms are allowed

$\rightarrow$ predictability of bottom-up models with Γ'_N recovered! ☺

Nilles, SRS, Vaudrevange (2004.05200)

Towards phenomenology

$G_{eclectic}$ is typically a very large group $\rightarrow$ it must be broken

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Successful fits can be obtained ☺

Baur, Nilles, SRS, Trautner, Vaudrevange (2201.xxxx)

In summary

Concluding remarks

- Traditional and finite modular flavor symmetries face some challenges

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- Toroidal orbifold compactifications of string theory reveal an

  eclectic flavor structure = traditional ∪ modular symmetries

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- In string models, more useful constraints:
  matter modular weights, representations and charges defined by compactification
- Superpotential and Kähler well restricted ☺

  → predictability of modular symmetries seems rescued

  (although flavons and moduli stabilization still needed; see Trautner's talk)

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To work on

- pheno from this $eclectic$ picture

  start with $\Delta(54)$, see Carballo, Peinado, SRS (2016)
- $\mathcal{CP}$ and $\mathcal{CP}$ violation ?

  Nilles, Ratz, Trautner, Vaudrevange (2018)
- eclectic flavor breakdown

  Baur, Nilles, SRS, Trautner, Vaudrevange (2112.xxxx)
- moduli stabilization
- non-supersymmetric constructions ?

Just in case...

Backup slides

Modular symmetries as flavor symmetries

Congruence modular subgroups: $\Gamma(N) \subset \mathrm{SL}(2,\mathbb{Z})$

$$\Gamma(N) = \{\gamma \in \mathrm{SL}(2,\mathbb{Z}) \,|\, \gamma = \mathbb{1} \mod N\}$$

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(Double-cover) finite modular subgroups: $\Gamma'_N \cong \mathrm{SL}(2,\mathbb{Z})/\Gamma(N)$

$$\Gamma'_N = \left\langle \mathrm{S}, \mathrm{T} \,|\, \mathrm{S}^4 = (\mathrm{ST})^3 = T^N = \mathbb{1}, \quad \mathrm{S}^2\mathrm{T} = \mathrm{TS}^2, \qquad N = 2,3,4,5 \right\rangle$$

$$\Gamma'_2 \cong S_3,\ \Gamma'_3 \cong T',\ \Gamma_4 \cong \mathrm{SL}(2,4),\ \Gamma_5 \cong \mathrm{SL}(2,5), \ldots$$

e.g. Liu, Ding (2019)

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Finite modular subgroups: $\Gamma_N \cong \mathrm{PSL}(2,\mathbb{Z})/\bar{\Gamma}(N)$ $(\mathrm{PSL}(2,\mathbb{Z}) \cong \mathrm{SL}(2,\mathbb{Z})/\{\pm\mathbb{1}\})$

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$$\Gamma_2 \cong S_3,\ \Gamma_3 \cong A_4,\ \Gamma_4 \cong S_4,\ \Gamma_5 \cong A_5, \ldots, \Gamma_7 \cong \Sigma(168), \ldots$$

e.g. de Adelhaart, Feruglio, Hagedorn (2011)

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Thus far, models with modular flavor symmetries are supersymmetric

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n_i: modular weight, $\rho(\gamma)$: matrix rep. of γ for Φ_{n_i}

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Couplings $\hat{Y}^{(n_Y)}(T)$ are $modular\ forms$

$$W \supset \sum \hat{Y}^{(n_Y)}(T)\Phi_{n_1}\Phi_{n_2}\Phi_{n_3}, \qquad \hat{Y}^{(n_Y)} \xrightarrow{\gamma} (cT+d)^{n_Y}\rho(\gamma)\hat{Y}^{(n_Y)}$$

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Admissible iff

$$W(\Phi_{n_1},\ldots) \xrightarrow{\gamma} (cT+d)^{-1}\,\mathbb{1}\,W(\Phi_{n_1},\ldots), \qquad \text{i.e. } {\scriptstyle n_Y+\sum n_i=-1}, \quad {\scriptstyle \prod \rho(\gamma)=\mathbb{1}}$$

Note the nontrivial $automorphy\ factor$ ${\scriptstyle (cT+d)^{-1}}$ $\rightarrow W$ covariant

How to proceed with *modular* flavor symmetries

- Take your favorite symmetry: $G_{mod} = \Gamma_N \in \{S_3, A_4, S_4, A_5, \ldots\}$
- Choose your favorite representations $\rho(\gamma)$ for quark and lepton fields

  e.g. quark doublets Q as $\mathbf{3}$ or $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$ of $\Gamma_3 \cong A_4, \ldots$

- Pick your favorite modular weights n_i and n_Y
- Write your G_{mod}-covariant superpotential W

$$\text{e.g. } W \supset \hat{Y}^u H_u Q \bar{u} + \hat{Y}^d H_d Q \bar{d} + \hat{Y}^e H_d L \bar{e} + \frac{\hat{Y}}{\Lambda} L H_u \, L H_u$$

- Take your favorite inv. Kähler potential K; typical choice $K = \sum |\Phi_{n_i}|^2$

  MANY other modular invariant K possible! - Chen, SR-S, Ratz (1909.06910)

- Choose a $\langle T \rangle \neq 0 \quad \rightarrow \quad$ nontrivial rep. of $\hat{Y}(\langle T \rangle)$ breaks G_{mod}
- EW breakdown with $\langle H_u \rangle, \langle H_d \rangle \neq 0$
- Diagonalize quark and lepton matrices to compute V_{CKM} and U_{PMNS} and adjust only $\langle T \rangle$ to data

Towards the *eclectic* flavor picture

Use Narain formalism: split string in independent components

$$X(\tau,\sigma) = X_R(\sigma-\tau) + X_L(\sigma+\tau)$$

Groot-Nibbelink, Vaudrevange (2017)

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Inspiration: C, P, T in SM are outer automorphisms of the Poincaré symmetry group

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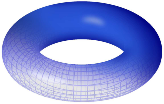
What are the outer automorphisms of $S_{Narain} = \{g\}$?

$$Out(S_{Narain}) = \left\{h = (\Sigma, t) \notin S_{Narain} \mid hgh^{-1} \in S_{Narain}\right\}$$

Rotations: $h_\Sigma = (\Sigma, 0) \to O(2,2;\mathbb{Z})$, Translations: $h_t = (\mathbb{1}_4, t)$

Towards the *eclectic* picture: what $Out(S_{Narain})$ is

String 2D toroidal compactifications have two moduli: T, U



$$G = \frac{\operatorname{Im} T}{\operatorname{Im} U} \begin{pmatrix} 1 & \operatorname{Re} U \\ \operatorname{Re} U & |U|^2 \end{pmatrix}, \qquad B = \operatorname{Re} T \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

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$h_\Sigma =$	S_U	T_U	S_T	T_T	M	K_*
$U \xrightarrow{h_\Sigma}$	$-1/U$	$U+1$	U	U	T	$-\bar{U}$
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Recall: in $\mathrm{SL}(2,\mathbb{Z})$ $\qquad T \xrightarrow{\mathrm{S}} -\frac{1}{T}, \qquad T \xrightarrow{\mathrm{T}} T+1$

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$$\mathrm{SL}(2,Z)_T = \langle \mathrm{S}_T, \mathrm{T}_T \rangle, \quad \mathrm{SL}(2,Z)_U = \langle \mathrm{S}_U, \mathrm{T}_U \rangle \qquad ☺$$

M: mirror symmetry, $\qquad$ K_*: $\mathcal{CP}$-like transformation ☺

Nilles, Ratz, Trautner, Vaudrevange (2018); Novichkov, Penedo, Petcov, Titov (2019)

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Further, $\{h_t\}$ don't change T, U, but do transform fields
$\rightarrow$ traditional flavor symmetry ☺

Common origin of modular and traditional flavor

Modular weights n_i, representations and couplings of Φ_{n_i} not *ad hoc*! ☺

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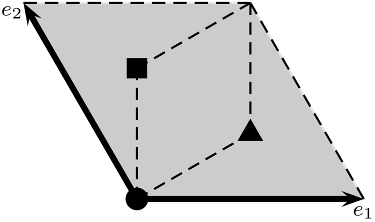
Example $\mathbb{T}^2/\mathbb{Z}_3$: must fix U to $\langle U\rangle = \omega = e^{2\pi\mathrm{i}/3} \to$ broken $\mathrm{SL}(2,\mathbb{Z})_U$



Lauer, Mas, Nilles (1989)

By using CFT formalism, inspect $\mathrm{SL}(2,\mathbb{Z})_T$ on the triplet of matter fields:

$$h_\Sigma : \rho(\mathrm{S}_T) = \frac{\mathrm{i}}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad \rho(\mathrm{T}_T) = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rho(\mathrm{S}_T)$ and $\rho(\mathrm{S}_T)$ build the reps. $\mathbf{2}' \oplus \mathbf{1}$ of modular group $\Gamma'_3 = T'$ ☺

$$\Phi_{n=-2/3,-5/3} \xrightarrow{\mathrm{S}_T} (-T)^n \rho(\mathrm{S}_T)\Phi_n, \qquad \Phi_n \xrightarrow{\mathrm{T}_T} \rho(\mathrm{T}_T)\Phi_n$$

Ibáñez, Lüst (1992)

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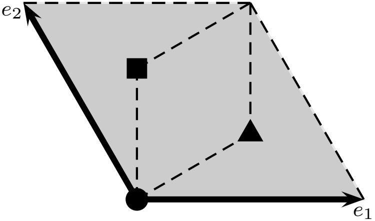

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$$h_t : \rho(\mathrm{A}) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},\ \rho(\mathrm{B}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},\ \rho(\mathrm{C}) = \rho(\mathrm{S}_T^2)$$

$\rho(\mathrm{A})$, $\rho(\mathrm{B})$ and $\rho(\mathrm{C})$ build the reps $\mathbf{3}_{2(1)}$ and $\mathbf{3}_{1(1)}$ of traditional flavor group $\Delta(54)$ for $\Phi_{-2/3}$ and $\Phi_{-5/3}$

cf. also in Kobayashi, Plöger, Nilles, Raby, Ratz (2006)

Common origin of modular and traditional flavor

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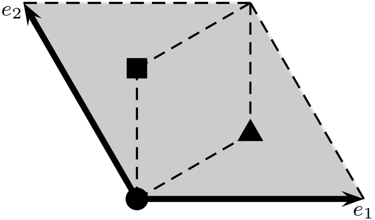

first eclectic flavor symmetry: modular + traditional flavor

$$\Delta(54) \cup T' \cong \Omega(1) = SG[648, 533]$$

with $\mathcal{CP}$: $\Delta(54) \cup T' \cup \mathbb{Z}_2^{\mathcal{CP}} \cong SG[1296, 2891]$

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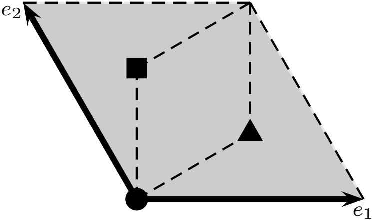

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Can we generalize this in a bottom-up fashion ?