

Multiple modular symmetries as the origin of flavour

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You e.g. TALK BY J. PENEIRO @ DISCRETE (29/11/2021)

See e.g. <https://arxiv.org/pdf/1706.08749.pdf> Fermilab

Consider a complex field τ , ($\Im \tau > 0$)
a Symmetry Γ with elements γ acts

$$\gamma : \tau \rightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d},$$

It is convenient to represent as 2×2

$$\Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / (\pm 1), \ a, b, c, d \in \mathbb{Z}, \ ad - bc = 1 \right\}.$$

(2)

$$\overline{\Gamma} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / (\pm 1), \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1 \right\}. \quad (2)$$

The group has 2 gens

$$S_\tau^2 = (S_\tau T_\tau)^3 = 1.$$

$$S_\tau : \tau \rightarrow -\frac{1}{\tau}, \quad T_\tau : \tau \rightarrow \tau + 1,$$

$$S_\tau = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T_\tau = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \quad (3)$$

A subgroup $\overline{\Gamma}(N)$

$$a = k_a N + 1, \quad d = k_d N + 1, \quad b = k_b N, \quad c = k_c N,$$

$$\overline{\Gamma}(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2, \mathbb{Z}), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}.$$

Make factor group
 $\overline{\Gamma}/\overline{\Gamma}(N) \sim \Gamma_N$

$$\Gamma_2 \sim S_2, \quad \Gamma_3 \sim A_4, \quad \Gamma_4 \sim S_4, \quad \Gamma_5 \sim A_5 \quad (4)$$

Remark : Factor Group

G wth Inv. subgroup H ; also cosets

$H, g_1 H, \dots$ Then for - group
where multiplication \rightarrow

$$g_i H \bullet g_j H = (g_i \cdot g_j) H$$

$$G = S_3 : \{e, (12), (23), (31), (123), (121)\}$$

$$Z_3 : \{e, (123), (321)\} \subset H \quad S_3/Z_3 \cong Z_2$$
$$\{eH, (12)H\} \cong Z_2$$

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In a \mathbb{Z}_N invariant theory,
chiral superfield ϕ

$$\phi_i(\tau) \rightarrow \phi_i(\gamma\tau) = (c\tau + d)^{-2k_i} \rho_{I_i}(\gamma) \phi_i(\tau),$$

$SUSY$ action

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta} K(\phi_i, \bar{\phi}_i; \tau, \bar{\tau}) + \left[\int d^4x d^2\theta W(\phi_i; \tau) + \text{h.c.} \right],$$

$$K(\phi_i, \bar{\phi}_i; \tau, \bar{\tau}) \rightarrow K(\phi_i, \bar{\phi}_i; \tau, \bar{\tau}) + f(\phi_i, \tau) + \bar{f}(\bar{\phi}_i, \bar{\tau}),$$

$$W(\phi_i; \tau) \rightarrow W(\phi_i; \tau). \quad \text{invariant w}$$

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$$W(\phi_i; \tau) = \sum_n \sum_{\{i_1, \dots, i_n\}} \sum_{I_Y} (Y_{I_Y} \phi_{i_1} \cdots \phi_{i_n})_1 \cdot$$

Coefficients

Complicated way of writing a general form

Coefficients transform

$$Y_{I_Y}(\tau) \rightarrow Y_{I_Y}(\gamma\tau) = (c\tau + d)^{2k_Y} \rho_{I_Y}(\gamma) Y_{I_Y}(\tau)$$

(7)

Stabilisers

$$\boxed{\gamma \cdot z_i = z_i}$$

$$\rho_I(\gamma) Y_I(\tau_\gamma) = (c\tau_\gamma + d)^{-2k} Y_I(\tau_\gamma). \quad (31)$$

This equation lead us to the following important properties for the stabiliser and the modular form:

- A modular form at a stabiliser $Y_I(\tau_\gamma)$ is an eigenvector of the representation matrix $\rho_I(\gamma)$ with respective eigenvalue $(c\tau_\gamma + d)^{-2k}$.
- The stabiliser τ_γ satisfies $|c\tau_\gamma + d| = 1$ since $(c\tau_\gamma + d)^{-2k}$ is an eigenvalue of a unitary matrix.

A special case is that when $(c\tau_\gamma + d)^{-2k} = 1$ is satisfied, $\boxed{\rho_I(\gamma) Y_I(\tau_\gamma) = Y_I(\tau_\gamma)}$ and we recover the residual flavour symmetry generated by γ . In general, the eigenvalue does not need to be fixed at 1 in the framework of modular symmetry.

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Symmetries and stabilisers in modular invariant flavour models

Ivo de Medeiros Varzielas, Miguel Levy, Ye-Ling Zhou

The idea of modular invariance provides a novel explanation of flavour mixing. Within the context of finite modular symmetries Γ_N and for a given element $\gamma \in \Gamma_N$, we present an algorithm for finding stabilisers (specific values for moduli fields τ_γ which remain unchanged under the action associated to γ). We then employ this algorithm to find all stabilisers for each element of finite modular groups for $N = 2$ to 5 , namely, $\Gamma_2 \simeq S_3$, $\Gamma_3 \simeq A_4$, $\Gamma_4 \simeq S_4$ and $\Gamma_5 \simeq A_5$. These stabilisers then leave preserved a specific cyclic subgroup of Γ_N . This is of interest to build models of fermionic mixing where each fermionic sector preserves a separate residual symmetry.

Comments: 18 pages, 5 figures, 4 tables, accepted for publication in JHEP

Subjects: High Energy Physics - Phenomenology (hep-ph); High Energy Physics - Theory (hep-th)

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(9)

E.g. $\tau = i$ is a stabiliser for S_2

$$S_2[i] = -\frac{1}{i} = i$$

$\tau = i\infty$ is a stabiliser for T_2

$$T_2[i\infty] = 1 + i\infty = i\infty$$

Stabilisers usually depend on N

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We know what
 τ stabilises



Method

1. Take $\tau = \tau_i$, where $\tau_i = \gamma_i \tau, i = 1, \dots, 4$ is a stabiliser of \mathcal{D} ;
2. Act γ on τ : $\tau' = \gamma\tau$. Compute γ^{-1} ; → Go from τ to τ' with γ
3. The element that stabilises τ' is given by $\gamma^{-1}\gamma_i\gamma$.

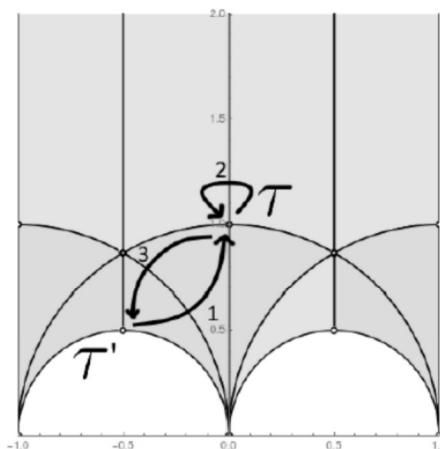


Figure 1: An example of the applied methodology to find the stabilisers of Γ_N . The example shown is for Γ_2 , where the arrows denote the actions of different elements, $\gamma^{-1}, \gamma_i, \gamma$, for 1,2,3 respectively, following the convention of the text.

(11)

$\Gamma_2 \sim S_3$

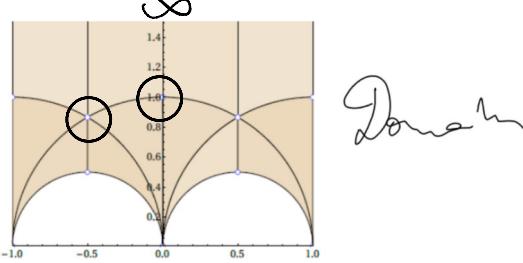


Figure 2: The fundamental domain $D(2)$ of $\bar{\Gamma}(2)$ (i.e., the full target space of $\Gamma_2 \simeq S_3$) with the stabilisers of modular transformations of Γ_2 denoted as dots.

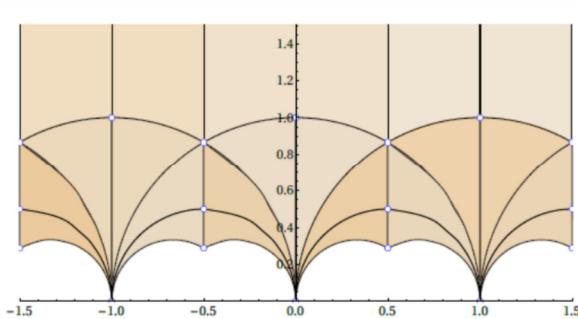
	γ	τ_γ
\mathcal{C}_2	$T_\tau C_\tau$	$0, 1+i$
	T_τ	$i\infty, \frac{1}{2} + \frac{i}{2}$
	S_τ	$i, 1$
\mathcal{C}_3	$T_\tau S_\tau$	$-\frac{1}{2} + \frac{i\sqrt{3}}{2}, \frac{1}{2} + \frac{i\sqrt{3}}{2}$
	C_τ	$-\frac{1}{2} + \frac{i\sqrt{3}}{2}, \frac{1}{2} + \frac{i\sqrt{3}}{2}$

Table 1: The non-identity elements of Γ_2 and respective stabilisers.

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$\Gamma_3 \sim A_4$

Habitus



	γ	τ_γ
\mathcal{C}_2	C_τ^2	$-\frac{1}{2} + \frac{i\sqrt{3}}{2}, 1$
	T_τ^2	$i\infty, \frac{3}{2} + \frac{i}{2\sqrt{3}}$
	$T_\tau C_\tau$	$0, \frac{3}{2} + \frac{i\sqrt{3}}{2}$
	$C_\tau T_\tau$	$-1, \frac{1}{2} + \frac{i\sqrt{3}}{2}$
\mathcal{C}_3	C_τ	$-\frac{1}{2} + \frac{i\sqrt{3}}{2}, 1$
	T_τ	$i\infty, \frac{3}{2} + \frac{i}{2\sqrt{3}}$
	$C_\tau S_\tau$	$0, \frac{3}{2} + \frac{i\sqrt{3}}{2}$
	$T_\tau S_\tau$	$-1, \frac{1}{2} + \frac{i\sqrt{3}}{2}$
\mathcal{C}_4	$T_\tau^2 C_\tau$	$-1 + i, \frac{1}{2} + \frac{i}{2}$
	S_τ	$i, \frac{3}{2} + \frac{i}{2}$
	$T_\tau C_\tau T_\tau$	$-\frac{1}{2} + \frac{i}{2}, 1 + i$

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Multiples

Multiple modular symmetries as the origin of flavour

Ivo de Medeiros Varzielas, Stephen F. King, Ye-Ling Zhou

We develop a general formalism for multiple moduli and their associated modular symmetries. We apply this formalism to an example based on three moduli with finite modular symmetries S_4^A , S_4^B and S_4^C , associated with two right-handed neutrinos and the charged lepton sector, respectively. The symmetry is broken by two bi-triplet scalars to the diagonal S_4 subgroup. The low energy effective theory involves the three independent moduli fields τ_A , τ_B and τ_C , which preserve the residual modular subgroups Z_3^A , Z_2^B and Z_3^C , in their respective sectors, leading to trimaximal TM_1 lepton mixing, consistent with current data, without flavons.

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Two A4 modular symmetries for Tri-Maximal 2 mixing

Ivo de Medeiros Varzielas, João Lourenço

We construct lepton flavour models based on two A_4 modular symmetries. The two A_4 are broken by a bi-triplet field to the diagonal A_4 subgroup, resulting in an effective modular A_4 flavour symmetry with two moduli. We employ these moduli as stabilisers, that preserve distinct residual symmetries, enabling us to obtain Tri-Maximal 2 (TM2) mixing with a minimal field content (without flavons).

Comments: 17 pages, 2 figures

Subjects: High Energy Physics - Phenomenology (hep-ph)

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(14)

Multifl Moduli $\gamma_J : \tau_J \rightarrow \gamma_J \tau_J = \frac{a_J \tau_J + b_J}{c_J \tau_J + d_J}$.

$\Gamma_{N_1}^1 \times \Gamma_{N_2}^2 \times \cdots \times \Gamma_{N_M}^M$, *Chiral Superfield*

$$\begin{aligned} \phi_i(\tau_1, \dots, \tau_M) &\rightarrow \phi_i(\gamma_1 \tau_1, \dots, \gamma_M \tau_M) \\ &= \prod_{J=1, \dots, M} (c_J \tau_J + d_J)^{-2k_{i,J}} \bigotimes_{J=1, \dots, M} \rho_{I_{i,J}}(\gamma_J) \phi_i(\tau_1, \tau_2, \dots, \tau_M), \end{aligned}$$

SUSY Action :

$$S = \int d^4x d^2\theta d^2\bar{\theta} K(\phi_i, \bar{\phi}_i; \tau_1, \dots, \tau_M, \bar{\tau}_1, \dots, \bar{\tau}_M) + \int d^4x d^2\theta W(\phi_i; \tau_1, \dots, \tau_M) + \text{h.c.},$$

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$$W(\phi_i; \tau_1, \dots, \tau_M) = \sum_n \sum_{\{i_1, \dots, i_n\}} (Y_{(I_{Y,1}, \dots, I_{Y,M})} \phi_{i_1} \cdots \phi_{i_n})_{\mathbf{1}},$$

where the "coefficients" are
modular forms . . .

$$\begin{aligned} Y_{(I_{Y,1}, \dots, I_{Y,M})}(\tau_1, \dots, \tau_M) &\rightarrow Y_{(I_{Y,1}, \dots, I_{Y,M})}(\gamma_1 \tau_1, \dots, \gamma_M \tau_M) \\ &= \prod_{J=1, \dots, M} (c_J \tau_J + d_J)^{2k_{Y,J}} \bigotimes_{J=1, \dots, M} \rho_{I_{Y,J}}(\gamma_J) Y_{(I_{Y,1}, \dots, I_{Y,M})}(\tau_1, \dots, \tau_M). \end{aligned}$$

*Complicated way of writing a
general form!*

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Example with S_4

$$S = T_\tau^2, \quad T = S_\tau T_\tau, \quad U = T_\tau S_\tau T_\tau^2 S_\tau.$$

$$\hookrightarrow P_I(S) = \begin{pmatrix} -1 & 2 & 2 \\ 2 & \ddots & 2 \\ 2 & 2 & -1 \end{pmatrix}/3 ; \quad \boxed{\rho_I(\gamma)Y_I(\tau_\gamma) = Y_I(\tau_\gamma)} \\ Y_A(\zeta_A) = \begin{pmatrix} -1 \\ 2\omega \\ 2\omega^2 \end{pmatrix}$$

$$\left\{ \begin{array}{c} \zeta_A \\ S = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \\ \zeta_B \\ \zeta_C \end{array} \right. \quad (17)$$

$$P_I(\tau) = \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \omega & \\ & & & \omega^2 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$S_4^A, \tau_A \quad S_4^B, \tau_B$$

$$\Phi_{AC}, \quad \Phi_{BC}$$

$$S_4^C, \tau_C$$

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WVX UC(1)Y



Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
L	1	1	3	0	0	0
e^c	1	1	1	0	0	-6
μ^c	1	1	1	0	0	-4
τ^c	1	1	1	0	0	-2
N_A^c	1	1	1	-6	0	0
N_B^c	1	1	1	0	-4	0
Φ_{AC}	3	1	3	0	0	0
Φ_{BC}	1	3	3	0	0	0

Yuk/Mass	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
$Y_e(\tau_C)$	1	1	3	0	0	6
$Y_\mu(\tau_C)$	1	1	3	0	0	4
$Y_\tau(\tau_C)$	1	1	3	0	0	2
$Y_A(\tau_A)$	3	1	1	6	0	0
$Y_B(\tau_B)$	1	3	1	0	4	0
$M_A(\tau_A)$	1	1	1	12	0	0
$M_B(\tau_B)$	1	1	1	0	8	0
$M_{AB}(\tau_A, \tau_B)$	1	1	1	6	4	0

\oint are here - Tmbls

$$w_\ell = \frac{1}{\Lambda} [L\Phi_{AC}Y_A(\tau_A)N_A^c + L\Phi_{BC}Y_B(\tau_B)N_B^c] H_u \\ + [LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c] H_d$$

$$\begin{aligned} \Lambda &= [LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c] H_d \\ &+ \frac{1}{2} M_A(\tau_A) N_A^c N_A^c + \frac{1}{2} M_B(\tau_B) N_B^c N_B^c + M_{AB}(\tau_A, \tau_B) N_A^c N_B^c, \end{aligned}$$

constant ahead

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$$\langle \Phi_{AC} \rangle_{i\alpha} = v_{AC}(P_{23})_{i\alpha}, \quad \langle \Phi_{BC} \rangle_{m\alpha} = v_{BC}(P_{23})_{m\alpha}.$$

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(the one from the potential)

$$\begin{aligned} L\Phi_{AC}Y_A(\tau_A)N_A^c &= L_1 [(\Phi_{AC})_{11}(Y_A)_1 + (\Phi_{AC})_{21}(Y_A)_3 + (\Phi_{AC})_{31}(Y_A)_2] N_A^c \\ &+ L_2 [(\Phi_{AC})_{13}(Y_A)_1 + (\Phi_{AC})_{23}(Y_A)_3 + (\Phi_{AC})_{33}(Y_A)_2] N_A^c \\ &+ L_3 [(\Phi_{AC})_{12}(Y_A)_1 + (\Phi_{AC})_{22}(Y_A)_3 + (\Phi_{AC})_{32}(Y_A)_2] N_A^c, \\ &= (L_1, L_2, L_3) P_{23} \begin{pmatrix} (\Phi_{AC})_{11} & (\Phi_{AC})_{12} & (\Phi_{AC})_{13} \\ (\Phi_{AC})_{21} & (\Phi_{AC})_{22} & (\Phi_{AC})_{23} \\ (\Phi_{AC})_{31} & (\Phi_{AC})_{32} & (\Phi_{AC})_{33} \end{pmatrix}^T P_{23} \begin{pmatrix} (Y_A)_1 \\ (Y_A)_2 \\ (Y_A)_3 \end{pmatrix} N_A^c, \end{aligned}$$

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$$S_4^A \times S_4^B \times \circlearrowleft S_4^C \rightarrow S_4^D,$$

$$\begin{aligned} w_\ell^{\text{eff}} &= \left[\frac{v_{AC}}{\Lambda} LY_A(\tau_A) N_A^c + \frac{v_{BC}}{\Lambda} LY_B(\tau_B) N_B^c \right] H_u \\ &+ [LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c] H_d \\ &+ \frac{1}{2} M_A(\tau_A) N_A^c N_A^c + \frac{1}{2} M_B(\tau_B) N_B^c N_B^c + M_{AB}(\tau_A, \tau_B) N_A^c N_B^c, \end{aligned}$$

$$\text{where } LY_A(\tau_A) N_A^c = [L_1(Y_A)_1 + L_2(Y_A)_3 + L_3(Y_A)_2] N_A^c,$$

$$Y_e(\langle \tau_C \rangle) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad Y_\mu(\langle \tau_C \rangle) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad Y_\tau(\langle \tau_C \rangle) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

$$Y_A(\langle \tau_A \rangle) = \begin{pmatrix} -1 \\ 2\omega \\ 2\omega^2 \end{pmatrix}, \quad Y_B(\langle \tau_B \rangle) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix},$$

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$$M_N = \begin{pmatrix} M_A & M_{AB} \\ M_{AB} & M_B \end{pmatrix}, \quad V = e^{i\alpha_3} \begin{pmatrix} \hat{c}_R & \hat{s}_R^* \\ -\hat{s}_R & \hat{c}_R^* \end{pmatrix},$$

comes from \mathfrak{I}

$$M_\nu = (\mu_1 \hat{c}_R^2 + \mu_2 \hat{s}_R^{*2}) \begin{pmatrix} 1 & -2\omega^2 & -2\omega \\ -2\omega^2 & 4\omega & 4 \\ -2\omega & 4 & 4\omega^2 \end{pmatrix} + (\mu_1 \hat{s}_R^2 + \mu_2 \hat{c}_R^{*2}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \\ + (\mu_1 \hat{c}_R \hat{s}_R - \mu_2 \hat{c}_R^* \hat{s}_R^*) \begin{pmatrix} 0 & -1 & 1 \\ -1 & 4\omega^2 & 2i\sqrt{3} \\ 1 & 2i\sqrt{3} & -4\omega \end{pmatrix}, \rightarrow \begin{matrix} P_{MN} \\ TM_1 \end{matrix}$$

$$U_{TM_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \end{pmatrix}. \quad (22)$$

Good Fit

BF	Para.	χ^2	α_1	α_2	θ_R	μ_1	μ_2
		0.74	64.53°	20.38°	43.01°	0.00633 eV	0.0114 eV
Obs.		θ_{12} 34.33°	θ_{13} 8.61°	θ_{23} 49.6°	δ 290°	m_2 0.00860 eV	m_3 0.0502 eV
						m_{ee} 0.00206 eV	

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See also talks by

J. PENEZO 29/11



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02/12



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Conclusions

- Modular Symmetries are broken

Conclusions

- Modular Symmetries are found as the origin of Flavours
- Stabilisers play a key role
- Multiple Modular Symmetries have specific advantages

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Thanks

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