Gravitational Imprints from Heavy Kaluza-Klein Resonances

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Based on: E.M., G.Nardini, M.Quirós, JHEP1809(2018) 095, PRD 102 (2020).
Other references: E.M., O.Pujolàs JHEP1408(2014) 081;
E.M., O.Pujolàs, M.Quirós, JHEP1605(2016) 137.

Issues

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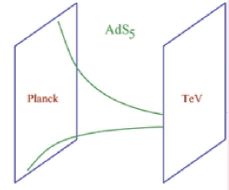
Introduction

Randall-Sundrum model in warped extra-dimension

- Proposed in 1999 by Randall and Sundrum (RS) **[PRL83, 3370 '99]**
- It was based on a 5D space with line element

$$ds^2 = e^{-2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu - dr^2 , \qquad A(r) = kr ,$$

- and two branes:



$$\mathrm{TeV} = e^{-kr_1} M_{\mathrm{Planck}} , \qquad kr_1 \sim 35 .$$

Higgs profile: $h(r) \propto e^{akr}$, $\qquad a > 2$.

AdS ⇔ CFT correspondence

- The Higgs is mainly localized toward the IR brane (composite).
- Heavy (light) fermions are mainly localized at the IR (UV) brane: composite (elementary).
- KK modes are mainly localized toward the IR brane (composite).

Introduction

Randall-Sundrum model in warped extra-dimension

- Zero mode gauge bosons are flat.
- Masses of KK modes: $m_{\mathrm{KK}} \sim \mathrm{TeV} \ll M_{\mathrm{Planck}} \longrightarrow$ Solve the hierarchy problem.
- In the RS model the brane distance has to be stabilized by a bulk scalar field ϕ breaking conformal invariance with bulk and brane potentials fixing its VEVs **[W. Goldberger, M. Wise, PRD60, 107505 '99].**
- It then appears a *"light state"*: the radion/dilaton with interesting Higgs-like phenomenology **[C.Csaki et al., PRD63, 065002 '01].**
- The radion experiences a first order confinement/deconfinement phase transition $\longrightarrow$ generates a *stochastic gravitational wave background (SGWB)* that could be detected at present and/or future interferometers.

Introduction

Randall-Sundrum model in warped extra-dimension

- The RS model has problems when confronting to electroweak precision measurements (oblique observables S, T, U too large).
- Possible way out:
  1. large backreaction on the metric ⟶ Deformed IR metric
     [J.Cabrer et al. JHEP 1105 ’11], [EM,G.Nardini,M.Quirós, JHEP 09 ’18]
     or
  2. Custodial symmetry in the bulk conserved in the IR
     [M.Carena,EM,M.Quirós,C.Wagner, JHEP 12 ’18]
     or
  3. KK resonances heavier than TeV: $m_{\rm KK} \sim \mathcal{O}(10 - 100\,\rm TeV)$.
- In this talk we will explore the 3rd possibility. This implies:
  - No need of solving the ”whole” hierarchy problem.
  - In accordance with ”no observation” of stable narrow resonances in current colliders.
  - Present and/or future gravitational wave interferometers could be sensitive to KK resonances heavier than TeV.

Issues

The Model

- Scalar-gravity system with UV and IR branes:

$$S = \int d^5x \sqrt{|\det g_{MN}|} \left[-\frac{1}{2\kappa^2} R + \frac{1}{2} g^{MN} (\partial_M \phi)(\partial_N \phi) - V(\phi) \right] - \sum_\alpha \int_{B_\alpha} d^4x \sqrt{|\det \bar{g}_{\mu\nu}|} \Lambda_\alpha(\phi) + S_{\text{GHY}}$$

- Metric: $ds^2 = g_{MN} dx^M dx^N \equiv \underbrace{e^{-2A(r)} \eta_{\mu\nu}}_{\bar{g}_{\mu\nu}} dx^\mu dx^\nu - dr^2$.
- $V(\phi)$ bulk potential.
- Λ_α $(\alpha = 0, 1) \equiv$ UV, IR 4-dim brane potentials at $(r(\phi_0), r(\phi_1))$.
- $S_{\text{GHY}} :=$ Gibbons-Hawking-York boundary term.
- Solve the hierarchy problem ⟶ Brane dynamics should fix (ϕ_0, ϕ_1) to get $A(\phi_1) - A(\phi_0) \approx 35 \Longrightarrow M_{\text{Planck}} \simeq 10^{15} M_{\text{TeV}}$.

The Effective Potential

- Using the equations of motion ⟶ Action:

$$S = S_{\rm bulk} + S_{\rm brane} + S_{\rm GHY} = -\int d^4x\, U_{\rm eff}\,.$$

- Effective Potential:

$$U_{\rm eff} = \left[e^{-4A} \left(W + \Lambda_1 \right) \right]_{r_1} + \left[e^{-4A} \left(-W + \Lambda_0 \right) \right]_{r_0} .$$

- $W \equiv$ superpotential.
- Fixing: $r_0 = 0$ and $A(0) = 0$ ⟶ r_1 is the brane distance, and

$$\kappa^2 M_P^2 = 2\ell \int_0^{\bar r_1} d\bar r\, e^{-2A(\bar r)}\,,$$

with $\bar r \equiv r/\ell$ and $M_P = 2.4 \times 10^{18}$ GeV.

$r_0 = 0$	$<$	r_1
(UV brane)		(IR brane)

The Effective Potential

A novel technique to compute the effective potential:

- The EoM for W

$$V(\phi)=\frac{1}{8}\left(\frac{\partial W}{\partial\phi}\right)^2-\frac{\kappa^2}{6}W^2(\phi)$$

admits an arbitrary integration constant $\equiv s$ → expand in s **[I. Papadimitriou '07], [EM,Pujolas '14]**:

$$W=\sum_{n=0}^{\infty}s^n W_n$$

- W_n can be computed iteratively from W_0. For $n=1$:

$$W_1(\phi)=\frac{1}{\ell\kappa^2}\exp\left(\frac{4\kappa^2}{3}\int^{\phi}\frac{W_0(\bar\phi)}{W_0'(\bar\phi)}d\bar\phi\right).$$

- Solution of EoM for ϕ and A:

$$\phi(r)=\phi_0(r)+s\,\phi_1(r)+\mathcal{O}(s^2)\,,\qquad A(r)=A_0(r)+s\,A_1(r)+\mathcal{O}(s^2)\,.$$

The Effective Potential

- Choose s to fulfill the BCs:

$$\phi(0) = v_0\,, \quad \phi(r_1) = v_1 \quad \longrightarrow \quad s(r_1) = \frac{v_1 - \phi_0(r_1)}{\phi_1[\phi_0(r_1)]}\,.$$

- The superpotential gets explicit dependence on the brane distance r_1:

$$W(v_\alpha) = W_0(v_\alpha) + s(r_1)W_1(v_\alpha) + \cdots\,, \qquad \alpha = 0, 1\,.$$

- Effective potential (up to first order in $s(r_1)$):

$$U_{\rm eff}(r_1) = [\Lambda_1 + W_0(v_1)]e^{-4A_0(r_1)}[1 - 4A_1(r_1)s(r_1)] + s(r_1)\left[e^{-4A_0(r_1)}W_1(v_1) - W_1(v_0)\right]\,.$$

Issues

The warped model

- Consider the warped phenomenological model:

$$W_0(\phi) = \frac{6}{\ell\kappa^2} + \frac{u}{\ell}\phi^2 .$$

- W_0 is an exact solution for bulk scalar potential:

$$V(\phi) = -\frac{1}{\ell^2}\left(\frac{6}{\kappa^2} + \frac{1}{2}u(4-u)\phi^2 + \frac{\kappa^2}{6}u^2\phi^4\right) .$$

- A solution is also $W(\phi) = W_0(\phi) + sW_1(\phi) + \cdots$, with

$$W_1(\phi) = \frac{1}{\ell\kappa^2}\left(\frac{\phi}{v_0}\right)^{4/u} e^{\kappa^2(\phi^2 - v_0^2)/3} .$$

- Scalar field and warp factor:

$$\bar{\phi}_0(r) = \bar{v}_0 e^{u\bar{r}} , \qquad A_0(r) = \bar{r} + \frac{\bar{v}_0^2}{12}\left(e^{2u\bar{r}} - 1\right) .$$

Dimensionless quantities: $\bar{v}_a \equiv \kappa v_a$, $\bar{\phi}(r) \equiv \kappa\phi(r)$, $\bar{r} \equiv r/\ell$.

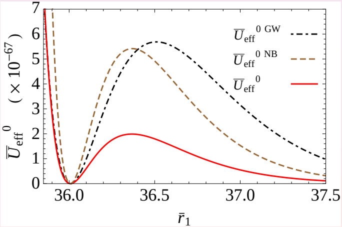
The tuned potential

- The tuned potential → $\Lambda_1 + W_0(v_1) = 0\,.$

$$\bar{U}^0_{\rm eff}(\bar{r}_1) \equiv \ell\kappa^2 U^0_{\rm eff}(r_1) = 2u^2\bar{v}_1^2(\bar{r}_1^0 - \bar{r}_1)\left[e^{4A_0(\bar{r}_1^0)-4A_0(\bar{r}_1)} - 1\right] e^{-4A_0(\bar{r}_1)}$$

- Goldberger-Wise potential **[W. Goldberger, M. Wise, PRL83 '99]**

$$\bar{U}^{0\,\rm GW}_{\rm eff}(\bar{r}_1) = 4\bar{v}_0^2\, e^{-4\bar{r}_1}\left(e^{u\bar{r}_1^0} - e^{-u\bar{r}_1}\right)^2 + \mathcal{O}(u)$$ → No backreaction



$s(r_1)W_1(v_1) \ll W_0(v_1)$ → $W(\phi) = W_0(\phi) + sW_1(\phi) + \cdots$ reliable.

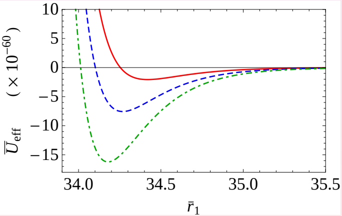
The detuned potential

- The detuned potential → $\Lambda_1 + W_0(v_1) \equiv \frac{6}{\ell\kappa^2}\lambda_1 \neq 0\,.$

$$\bar{U}_{\rm eff}(\bar{r}_1) \simeq \bar{U}^0_{\rm eff}(\bar{r}_1) + 6\lambda_1 e^{-4A_0(\bar{r}_1)}\,.$$

$\bar{r}_1^0$ → Position of the minimum of $\bar{U}^0_{\rm eff}(\bar{r}_1)$.
$\bar{r}_1^{\rm m}$ → " " " " " $\bar{U}_{\rm eff}(\bar{r}_1)$.

$$\delta = \bar{r}_1^{\rm m} - \bar{r}_1^0 = -\frac{1}{4}\mathcal{W}\left[-\frac{6\lambda_1}{u^2\bar{v}_1^2}\right] < 0\,, \qquad \mathcal{W} \equiv \text{Lambert function}$$



$\lambda_1 = -1, -2, -3.$

The radion field

- Scalar perturbation of the metric:

$$\begin{aligned} ds^2 &= -[1+2F(x,r)]^2 dr^2 + e^{-2[A+F(x,r)]}\bar{g}_{MN}dx^M dx^N \,, \\ \phi(x,r) &= \phi_{\rm Background}(x) + \varphi(x,r) \,. \end{aligned}$$

- Radion ansatz $F(x,r) = F(r)R(x)$.
- $F(r) \simeq e^{2A(r)}$ for *'light'* dilaton/radion.
- For the canonically normalized radion field $\chi(r)$:

$$\mathcal{L}_{\rm rad} = \frac{6\ell^3}{\kappa^2}\int d^4x \sqrt{|\det \bar{g}_{\mu\nu}|}\left(\frac{1}{2}(\partial_\mu \chi)^2 - \underbrace{\frac{1}{2}m^2_{\rm rad}\,\chi^2}_{V_{\rm rad}(\chi)}\right) ,$$

with

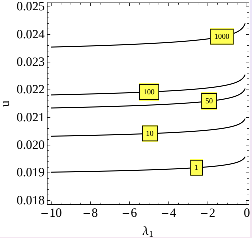
$$\chi(r) \simeq \frac{1}{\ell}e^{-A_0(r)} \,.$$

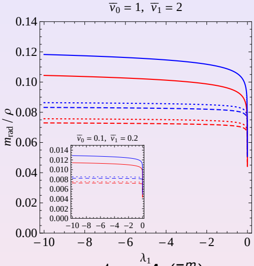
- Radion potential:

$$V_{\rm rad}(\chi) \equiv U_{\rm eff}[\bar{r}_1(\chi)] \quad \rightarrow \quad \text{minimum at} \quad \langle\chi\rangle = \rho \,.$$

The radion mass

ρ in TeV.





- Physically relevant parameter ➞ $\rho \equiv \frac{1}{\ell} e^{-A_0(\bar{r}_1^m)} \sim \mathrm{TeV}$ scale.
- Radion mass from mass formulas **[EM, Pujolàs, Quirós '16].**
- Results:
  - Radion mass ➞ $m_{\rm rad}/\rho \simeq 0.10$ for $\bar{v}_0 = 1$ and $\bar{v}_1 = 2$
    $m_{\rm rad}/\rho \simeq 0.010$ for $\bar{v}_0 = 0.1$ and $\bar{v}_1 = 0.2$.
  - Gauge bosons KK mass ➞ $m_{\rm KK}^{\rm gauge}/\rho \simeq 2.46$.
  - Graviton KK mass ➞ $m_{\rm KK}^{\rm grav}/\rho \simeq 3.88$.

$$m_{\rm rad} \ll m_{\rm KK}^{\rm gauge} \lesssim m_{\rm KK}^{\rm grav}$$

Issues

The effective potential at finite temperature

- Black-hole metric:

$$ds^2_{\rm BH} = -\frac{1}{h(r)}dr^2 + e^{-2A(r)}(h(r)dt^2 - d\vec{x}^{\,2})\,.$$

- 4 EoM with boundary conditions:

$$h(0) = 1\,, \quad h(r_h) = 0\,, \quad \phi(0) = v_0 \quad A(0) = 0\,.$$

- Solution of the EoM ($u \ll 1$ limit):

$$h(r) \simeq 1 - e^{4[A_0(r) - A_0(r_h)]}\,.$$

- Temperature and entropy of the BH:

$$T_h = \frac{1}{4\pi}e^{-A(r_h)}\left|h'(r)\right|_{r=r_h} \simeq \frac{1}{\ell\pi}e^{-A_0(r_h)}\,, \qquad S = \frac{4\pi}{\kappa^2}e^{-3A(r_h)}\,.$$

- Minimum of the free energy at $T_h = T$:

$$F^{\rm BH}_{\rm min}(T) \simeq \frac{\pi^4\ell^3}{\kappa^2}T^4\,.$$

The dilaton phase transition

- Phase transition starts when $F_{\rm deconfined} < F_{\rm confined}$:

$$F_d(T) = E_0 + F^{\rm BH}_{min} - \frac{\pi^2}{90} g_d^{eff} T^4 \,, \qquad F_c(T) = -\frac{\pi^2}{90} g_c^{eff} T^4 \,,$$

  where $E_0 = V_{eff}(\mu = 0) - V_{eff}(\mu = \langle\mu\rangle) > 0$.
- Critical temperature ⟶ $F_d(T_c) = F_c(T_c)$
- Action driven by thermal fluctuations is $O(3)$ sym (high T):

$$S_3 = 4\pi \int d\sigma \sigma^2 \frac{6\ell^3}{\kappa^2} \left(\frac{1}{2} \left(\frac{\partial\chi}{\partial\sigma} \right)^2 + V(\chi, T) \right) \,,$$

  with $\sigma \equiv \sqrt{\vec{x}^2}$ and $V(\chi, T) \equiv \frac{\kappa^2}{6\ell^3} \left(V_{\rm rad}(\chi) + \left| F^{\rm BH}_{\rm min}(T) \right| \right)$.
- Action driven by quantum fluctuations is $O(4)$ sym (low T):

$$S_4 = 2\pi^2 \int d\sigma \sigma^3 \frac{6\ell^3}{\kappa^2} \left(\frac{1}{2} \left(\frac{\partial\chi}{\partial\sigma} \right)^2 + V(\chi, T) \right) \,.$$

Bounce equation

- Solution of the bounce equation **[Coleman '77; Linde '81]**

$$\frac{\partial^2\chi}{\partial\rho^2} + \frac{2}{\rho}\frac{\partial\chi}{\partial\rho} - \frac{\partial V}{\partial\chi} = 0\,, \qquad \rho = \sqrt{\vec{x}^2}\,,$$

with BCs

$$\frac{3\ell^3}{\kappa^2}\left(\frac{\partial\chi}{\partial\sigma}\right)^2\Bigg|_{\chi=0} = \left|F^{\rm BH}_{\rm min}(T)\right|\,, \quad \chi(0) = \chi_0\,, \quad \frac{d\chi}{d\sigma}\Bigg|_{\sigma=0} = 0\,.$$

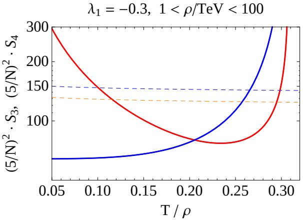
- Bubble nucleation rate from the false BH minimum to the true vacuum is given by

$$\Gamma/\mathcal{V} = \mathcal{A}\cdot e^{-S_E} \quad \simeq \quad \mathcal{A}\cdot\left(e^{-S_3/T} + e^{-S_4}\right)\,, \qquad \mathcal{A}\simeq T_c^4\,.$$

- Nucleation happens when

$$S_E(T_n) \lesssim 4\log\frac{M_P/\rho}{T_n/\rho}\;(\approx 140)\,.$$

The dilaton phase transition



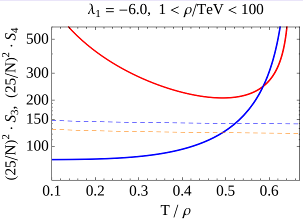


Figure : S_3/T (red) and S_4 (blue) as functions of T.
Horizontal lines: bounds for $\rho = 1\,\mathrm{TeV}$ (blue) and $\rho = 100\,\mathrm{TeV}$ (orange).

- Remark: In the holographic theory (AdS/CFT relation):

$$\frac{1}{N^2} = \frac{(M\ell)^{-3}}{16\pi^2} \quad \text{i.e.} \quad N^2 \simeq \frac{8\pi^2\ell^3}{\kappa^2}, \qquad \ell \equiv \quad \text{radius of AdS},$$

with $N \equiv$ number of colors in the dual theory.

The dilaton phase transition

- Inflation starts when E_0 dominates over thermal corrections:

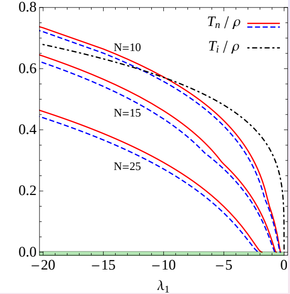
$$\text{Energy density:} \qquad \rho_d(T_i) = E_0 + \frac{3\pi^4 \ell^3}{\kappa^2} T_i^4 + \frac{\pi^2}{30} g_d^{eff} T_i^4 \simeq E_0 \,.$$

- Inflation finishes when bubbles percolate $\simeq T_n$.
- Number of e-folds of inflation is $N_e \approx \log(T_i/T_n)$.
- During the phase transition the energy density is approx. conserved. At the end of the phase transition the universe ends up in the confined phase at the reheating temperature

$$\rho_c(T_R) = \rho_d(T_n) \,, \qquad \text{with} \qquad \rho_c(T) = \frac{\pi^2}{30} g_c^{eff} T^4 \,.$$

- The reheating temperature is close to TeV.

The dilaton phase transition



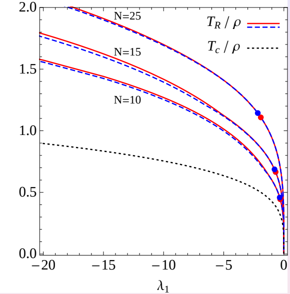


Figure : Left panel: Nucleation (T_n) and inflation (T_i) temperature vs λ_1.
Right panel: Reheating (T_R) and critical (T_c) temperature vs λ_1.
$\rho = 1\,\mathrm{TeV}$ (red lines) and $\rho = 100\,\mathrm{TeV}$ (blue lines).

Inflation: $T_i \simeq T_c \left[3 + 4g_d^{\mathrm{eff}}/15N^2\right]^{-1/4}$,

Reheating: $\dfrac{4}{15N^2} g_c^{\mathrm{eff}} T_R^4 = T_c^4 + \left(3 + \dfrac{4}{15N^2} g_d^{\mathrm{eff}}\right) T_n^4$.

Gravitational waves

- A cosmological first-order phase transition (PT) generates a *stochastic gravitational wave background (SGWB)*.
- Envelope approximation (plasma effects negligible)
  [Kosowsky, Tuner '93], $\cdots$

$$h^2\Omega_{\rm GW}^{\rm env}(f) \simeq h^2\overline{\Omega}_{\rm GW}^{\rm env}\, \frac{3.8(f/f_p)^{2.8}}{1+2.8(f/f_p)^{3.8}}\,,$$

with

$$h^2\overline{\Omega}_{\rm GW}^{\rm env} \simeq 0.80\times 10^{-4}\left(\frac{H_\star}{\beta}\,\frac{\alpha}{\alpha+1}\right)^2 \frac{\xi(v_w)}{\sqrt[3]{g_c(T_R)}}\,,$$

and

$$\alpha \simeq \frac{|F_d(T_n)-F_c(T_n)|}{\rho_d^*(T_n)} \qquad \text{where} \qquad \rho_d^*(T_n) = \rho_d(T_n) - E_0\,,$$

$$\frac{\beta}{H_\star} \simeq T_n \frac{dS_E}{dT}\bigg|_{T=T_n} \qquad \left(\beta \equiv -\frac{dS_E}{dt}\bigg|_{t=t_\star} \text{is the inverse time duration of the PT}\right)$$

Gravitational waves

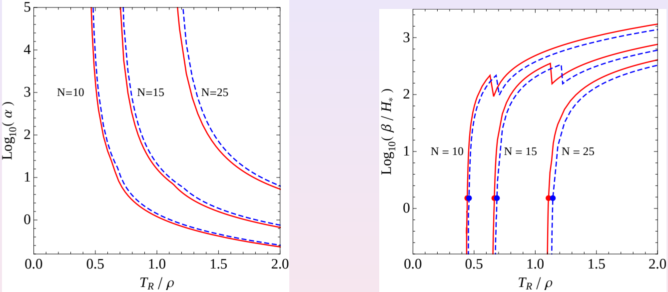


Figure : Left panel: α vs reheating temperature.
Right panel: $\beta/H_\star$ vs reheating temperature.
$\rho = 1\,\mathrm{TeV}$ (red lines) and $\rho = 100\,\mathrm{TeV}$ (blue lines).

$$\alpha \simeq \frac{|F_d(T_n) - F_c(T_n)|}{\rho_d^*(T_n)}\,, \qquad \frac{\beta}{H_\star} \simeq T_n \left.\frac{dS_E}{dT}\right|_{T=T_n} .$$

Gravitational waves

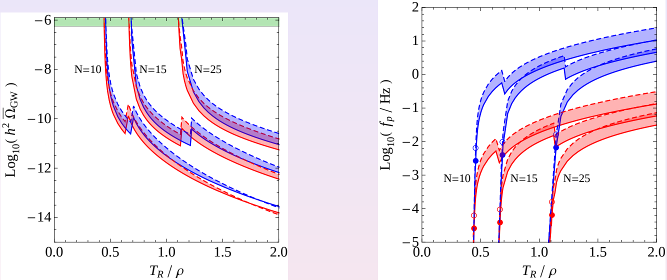


Figure : Left panel: $h^2\bar{\Omega}_{\rm GW}$ vs reheating temperature (T_R).
Right panel: peak frequency (f_p) vs reheating temperature (T_R).
$\rho = 1\,\mathrm{TeV}$ (red) and $\rho = 100\,\mathrm{TeV}$ (blue).

$$\underline{\text{Envelope:}}\quad \Omega_{\rm GW}^{\rm env}(f) \simeq \frac{3.8\,x^{2.8}}{1+2.8\,x^{3.8}}\,\overline{\Omega}_{\rm GW}^{\rm env}\,,$$

$$\underline{\text{Sound waves:}}\quad \Omega_{\rm GW}^{\rm sw}(f) \simeq x^3\left(\frac{7}{4+3\,x^2}\right)^{7/2}\overline{\Omega}_{\rm GW}^{\rm sw}\,.$$

Gravitational waves

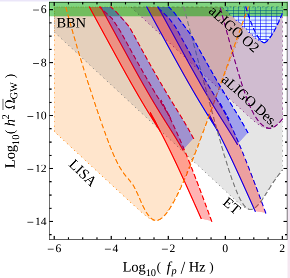


Figure : Parameter reach in the $\{h^2\overline{\Omega}_{\rm GW}, f_p\}$ plane for SGWBs in the regimes $\Omega_{\rm GW}^{\rm env}$ (regions inside dotted borders) and $\Omega_{\rm GW}^{\rm sw}$ (regions inside dashed borders). Diagonal strips are for $N = 10$ (red), 25 (blue) for $\rho = 1$ *TeV (left set)* and $\rho = 10^2$ *TeV (right set)*.

- Sensitivity regions for the next 3 years (LISA), 7 years (ET) and 8 years (aLIGO).
- Forecast: 2040s → interferometer network sensitive to $\rho \sim 10^9\mathrm{TeV}$.
- $m_{\rm KK} \sim \mathcal{O}(10^5) - \mathcal{O}(10^6)$ TeV is being cornered by current aLIGO O2 data.
- Forthcoming interferometers will probe resonances: $m_{\rm KK} \lesssim 10^5$ TeV (LISA), $10^2\ \mathrm{TeV} \lesssim m_{\rm KK} \lesssim 10^8$ TeV (aLIGO Design) and $m_{\rm KK} \lesssim 10^9$ TeV (ET).

Conclusions

- We have studied the cosmological phase transition in a 5D warped model. In particular:
  - Effective potential for the radion at zero temperature by using a novel technique → backreaction of the radion field on the gravitational metric.
  - Effective potential at finite temperature.
- We have studied the dilaton phase transition → First order confinement/deconfinement phase transition.
- Stochastic gravitational wave background → detectable by LISA, Einstein Telescope and LIGO.
  - Current aLIGO data already rule out vector boson KK resonances $m_{\rm KK} \sim (1-10) \times 10^5$ TeV.
  - Future gravitational wave experiments will be sensitive to vector boson resonances with masses $m_{\rm KK} \lesssim 10^5$ TeV (LISA), 10^8 TeV (aLIGO Design) and 10^9 TeV (ET).
- Future direction: Heavy radion detection at the LHC and future colliders. Preliminary analysis → possible future detection in pp collisions in *WW* and *ZZ* channels **[EM,Nardini,Quirós, JHEP '18]**.

Thank You!