# Quantum Information and Entanglement with Top Quarks at the LHC

ICNFP 2020

Based on: 2003.02280

Y. Afik, J. R. M. De Nova

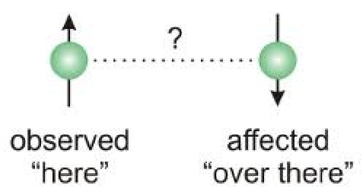
05.09.2020

# Motivation

- The Standard Model is a Quantum Field Theory: Special Relativity and Quantum Mechanics.
- Fundamental properties of Quantum Mechanics can be tested via the Standard Model.
- Entanglement is one of the most genuine features of Quantum Mechanics.
- First study of entanglement between a pair of quarks.
- Applying quantum tomography, which is a genuine Quantum Information technique.
- Quantum Information $\rightarrow$ High Energy physics.

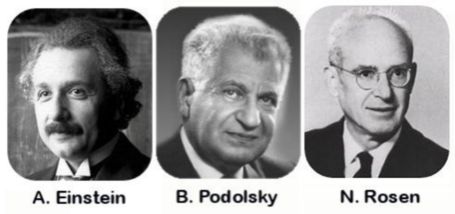
# What is Quantum Entanglement?

- Quantum state of one particle cannot be described independently from another particle.
- ⇒ **Correlations** of observed physical properties of both systems.
- ⇒ **Measurement** performed on one system seems to be instantaneously influencing other systems entangled with it.




- Observed in photons, atoms, superconductors, mesons, analog Hawking radiation, nitrogen-vacancy centers in diamond and even macroscopic diamond.

# EPR Paradox




MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

**Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?**

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

# EPR Paradox

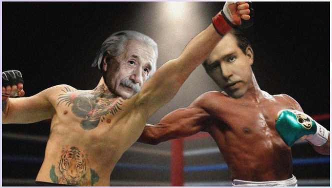
- Entanglement: "spooky action at a distance" (A. Einstein).

- Assuming two particles with spacial distance.
- When a measurement is done on one of the particles, the other one "knows" about it immediately.
- Information travel faster than light?
- Contradicts the theory of relativity.
- **Conclusion:** the theory of Quantum Mechanics is incomplete.

# Hidden Variables

- By EPR, each particle "carries" variables that knows the state before the measurement.
- $\Rightarrow$ There are some hidden variables that are missing in order to have a full theory.
- The Copenhagen Interpretation: superposition of states until a measurement was done.
- Bohr Vs. Einstein.

"God does not play at dice with the universe".



"Quit telling God what to do!"

- Who is right?

# Bell's Inequality

III.5 ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

JOHN S. BELL†

- If local hidden variables holds, they should satisfy some inequality.
- $C(x, y)$ are the correlations between different measurements at different detectors.
- The parameters a,b,c are different directions for the measurement.
- Original form: $1 + C(b, c) \geq |C(a, b) - C(a, c)|$.

# Quantum State

- **Pure state:** can be described by wave-functions $\sum_i \alpha_i \cdot |\psi_i\rangle$.

# Quantum State

- **Pure state:** can be described by wave-functions $\sum_i \alpha_i \cdot |\psi_i\rangle$.

- **Mixed state:** can be described by a density matrix: $\rho = \sum_i p_i \cdot |\psi_i\rangle \langle\psi_i|$.
  - Example: at the LHC we cannot control the initial state.

- Some inequalities can be measured related to $\rho$, providing an entanglement criterion.

# Mathematical Formalism

- Two different systems A and B: $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$.
- Separable: $\rho = \sum_n p_n \rho_n^a \otimes \rho_n^b$.
- $\rho_n^{a,b}$ are quantum states in $A, B$, $\sum_n p_n = 1,\ p_n \geq 0$
- Classically correlated state in $\mathcal{H} \rightarrow$ can be written in this form.

# Mathematical Formalism

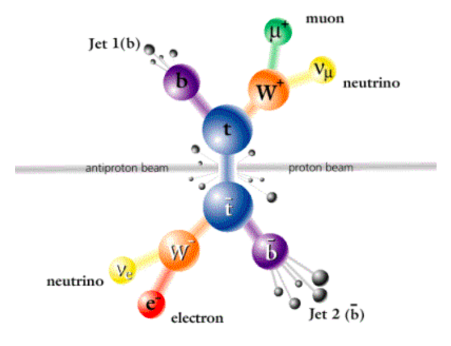
- Two different systems A and B: $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$.
- Separable: $\rho = \sum_n p_n \rho_n^a \otimes \rho_n^b$.
- $\rho_n^{a,b}$ are quantum states in $A, B$, $\sum_n p_n = 1,\ p_n \geq 0$
- Classically correlated state in $\mathcal{H} \rightarrow$ can be written in this form.
- Non-separable state is called entangled and hence, it is a non-classical state.


Separable

Non-Separable

# Top-Quark

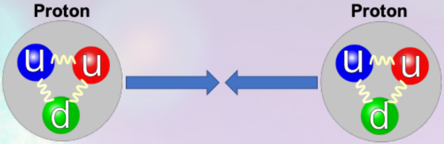
- **General:**
  - Hadronisation: $\sim 10^{-23}\mathrm{s}$.
  - Spin-decorrelation: $\sim 10^{-21}\mathrm{s}$.
- **Top quark:**
  - Lifetime: $\sim 10^{-25}\mathrm{s}$.
- Spin information $\rightarrow$ decay products.
- Spin-correlations between a pair of top-quarks can be measured.
- Considering leptonic decays.



# Spin-Correlations between Top-Quark Pairs

- Studied extensively theoreticaly.
- Measured by the D0, CDF, ATLAS and CMS collaborations.
- No link between spin-correlations and quantum entanglement so far.
- **Spin-Correlations $\neq$ Quantum Entanglement!**
  However, Quantum Entanglement $\subset$ Spin-Correlations.

# Collisions at the LHC




- At the LHC, protons are being collided at high energies.
- Proton: quarks and gluons (partons).
- Parton distribution function (PDF): the density of each parton in the proton.

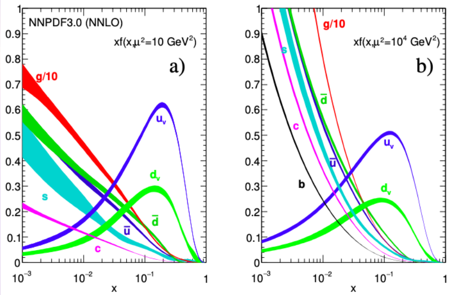
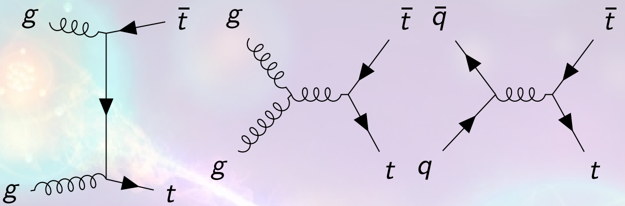



Figure: Parton density at the proton.
Figure is from JHEP 2015, 40 (2015).

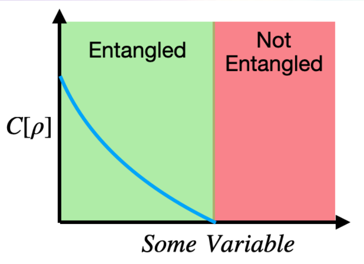
# LO Analytical Calculation




- Analytical calculation at leading-order. The system is defined by:
  - $M_{t\bar{t}}$: the invariant mass of the top pair.
  - $\hat{k}$: the direction of the top with respect to the beam axis.
- Initial states: $q\bar{q}$ and $gg$.
- Each one $I = q\bar{q}, gg$ gives rise to $\rho^I(M_{t\bar{t}}, \hat{k})$ with probability $w_I(M_{t\bar{t}}, \hat{k})$, which is PDF dependent.
- The spin density matrix: $\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}}, \hat{k})\rho^I(M_{t\bar{t}}, \hat{k})$.
- The total quantum state:
  $\rho(M_{t\bar{t}}) \equiv \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \int \mathrm{d}\Omega \; p(M, \hat{k})\rho(M, \hat{k}) = \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \; p(M)\rho_\Omega(M)$

# Entanglement Criterion

- **Concurrence** $C[\rho]$: Quantitative measurement of entanglement.
- $0 \leq C[\rho] \leq 1$, $C[\rho] \neq 0$ iff the state is entangled.
- Here, $C[\rho] = \max(\Delta, 0)/2$. Entanglement equivalent to $\Delta = -\mathrm{tr}[\mathbf{C}] - 1 > 0$.
- $D = \frac{\mathrm{tr}[\mathbf{C}]}{3} = -\frac{1+\Delta}{3}$ provides an experimental entanglement marker.



# Entanglement Before Integration

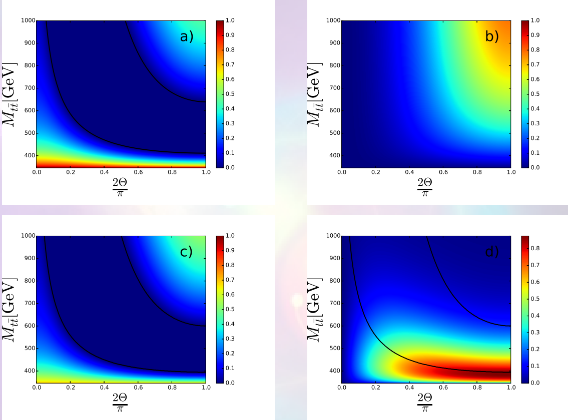
a) $gg \to t\bar{t}$ Concurrence.

b) $q\bar{q} \to t\bar{t}$ Concurrence.

c) Full $\rho(M_{t\bar{t}}, \hat{k})$ Concurrence.

d) Differential cross-section $\frac{d\sigma}{dM_{t\bar{t}}d\Theta} = 2\pi \sin\Theta \frac{d\sigma}{dM_{t\bar{t}}d\Omega}$ in units of $\mathrm{pb/GeV\ rad}$.

- Motivates integration only in part of the parameter space.



# Measurable Entanglement Marker

- Plots are shown with integration only for $[2m_t, M_{t\bar{t}}]$.
- In particular: $\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$ where $\varphi$ is the angle between the lepton directions in each one of the parent top and antitop rest frames.
- $\Delta > 0 \rightarrow D < -1/3$.
- $|\mathrm{tr}[\mathbf{C}]| = |\langle\sigma\cdot\bar{\sigma}\rangle| > 1$ $\rightarrow$ **violation of a Cauchy-Schwarz inequality.**

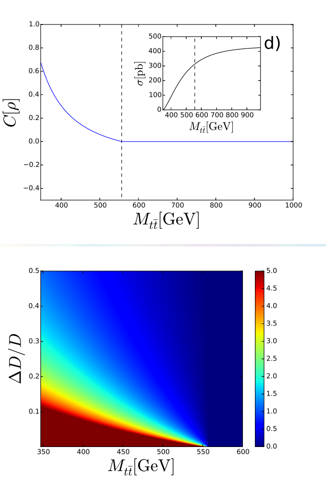



Figure: Up: concurrence; bottom: statistical deviation from the null hypothesis ($D = -1/3$).

# Recent Related Measurement

- Recently, $D$ was measured with no selection on $M_{t\bar{t}}$ by the CMS collaboration.
- CMS: $D = -0.237 \pm 0.011 > -1/3$; $\Delta D/D = 4.6\%$.

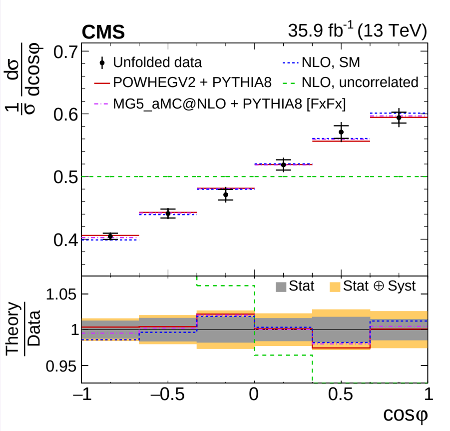



Figure: Distribution of $cos\varphi$. Figure is from Phys. Rev. D 100, 072002.

# Quantum Tomography

- Quantum tomography=Reconstruction of the quantum state of the system.
- Only need to measure 4 parameters (transverse and longitudinal spin correlations and the longitudinal polarizations).
- Test theoretical predictions for the $t\bar{t}$ quantum state.

# Summary

- First study of entanglement between quarks.
- Quantum Information study in a relativistic system.
- Although the calculation is analytical at leading order, the conclusion still holds at higher orders.
- Quantum tomography: new platform to test new theories and new physics effects.
- Interdisciplinary measurement: propagate Quantum Information theory into High Energy physics.

# Summary

- First study of entanglement between quarks.
- Quantum Information study in a relativistic system.
- Although the calculation is analytical at leading order, the conclusion still holds at higher orders.
- Quantum tomography: new platform to test new theories and new physics effects.
- Interdisciplinary measurement: propagate Quantum Information theory into High Energy physics.
- **Can be detected at the LHC with currently recorded data!**

# Thank You

# Backup

# NLO Corrections

- Leading-order: analytical calculation.
- NLO: numerical calculation by using Monte Carlo simulation.
- MadGraph and MadSpin are used.
- Good agreement is observed.

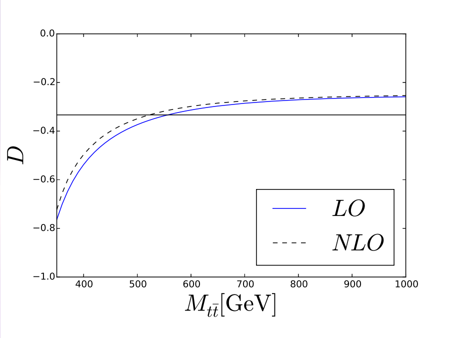


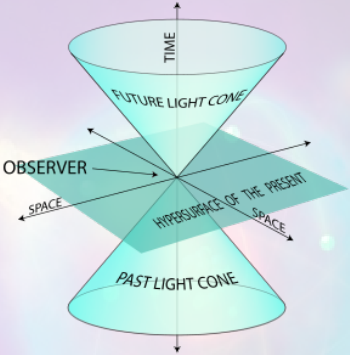

Figure: The value of $D$ within the mass window $[2m_t, M_{t\bar{t}}]$. The horizontal line represents the critical value $D = -1/3$.

# Mathematical Formalism

- Two different systems A and B: $H_A \otimes H_B$.
- The state of the composite system: $|\psi\rangle_A \otimes |\phi\rangle_B$.
- A common state for $H_A \otimes H_B$: $\sum_{i,j} c_{ij} |i\rangle_A \otimes |j\rangle_B$
- The state is separable if for any basis $[c_i^A], [c_j^B]$ we can write $c_{ij} = c_i^A \cdot c_j^B$.
- The state is Entangled if for any basis we have at least one pair of coordinates in which: $c_{ij} \neq c_i^A \cdot c_j^B$.
- Example: two basis vectors $|0\rangle_A, |1\rangle_A$ and $|0\rangle_B, |1\rangle_B$, the following is an entangled state: $\frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$.

# Local Realism

- Locality: physical influences do not propagate faster than light.
- Realism: physical properties are defined before, and independent of observation.
- Both of the assumptions (together, not separately) are in tension with Quantum Mechanics.



# Top-Quark Pair Spin Density Matrix

- General form:

$$\rho = \frac{I_4 + \sum_i \left( B_i^+ \sigma^i + B_i^- \bar{\sigma}^i \right) + \sum_{i,j} C_{ij} \sigma^i \bar{\sigma}^j}{4}$$

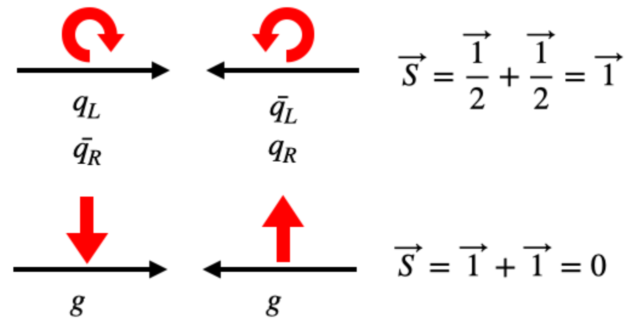
- $\sigma^i/2, \bar{\sigma}^i/2$ - spin operators of the top, antitop.
- $B_i^+,\ B_i^-$ characterize the spin polarizations, $B_i^+ = \langle \sigma^i \rangle,\ B_i^- = \langle \bar{\sigma}^i \rangle$.
- At leading-order $B_i^\pm = 0$.
- $C_{ij}$ the $t\bar{t}$ spin correlations, $C_{ij} = \langle \sigma^i \bar{\sigma}^j \rangle$.

# High Energy Physics Example

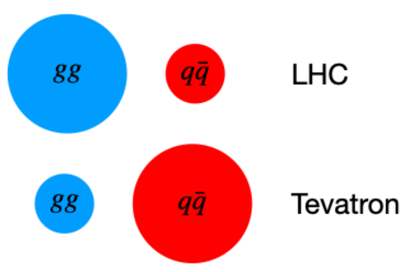
- At B-Factories, $e^+e^-$ collisions can be properly adjusted in order to create $\Upsilon(4S)(b\bar{b})$.
- $\Upsilon(4S)(b\bar{b})$ decays to $B^0 + \bar{B}^0$, where we have $|B^0\rangle = |\bar{b}d\rangle, |\bar{B}^0\rangle = |b\bar{d}\rangle$.
- We get an entangled state: $\frac{1}{\sqrt{2}}(|B^0\rangle|\bar{B}^0\rangle - |\bar{B}^0\rangle|B^0\rangle)$.

# Intuition: Spin States at Threshold

- The state is determined by the initial spins.
- $q\bar{q}$: $\rho^{q\bar{q}} = \left(\left|\uparrow_{\hat{p}}\uparrow_{\hat{p}}\right\rangle\left\langle\uparrow_{\hat{p}}\uparrow_{\hat{p}}\right| + \left|\downarrow_{\hat{p}}\downarrow_{\hat{p}}\right\rangle\left\langle\downarrow_{\hat{p}}\downarrow_{\hat{p}}\right|\right)/2.$
- $gg$: $\rho^{gg} = \left|\Psi_0\right\rangle\left\langle\Psi_0\right|$, with $\left|\Psi_0\right\rangle = \left(\left|\uparrow_{\hat{p}}\downarrow_{\hat{p}}\right\rangle - \left|\downarrow_{\hat{p}}\uparrow_{\hat{p}}\right\rangle\right)/\sqrt{2}.$
- $q\bar{q} \to$ correlated, not entangled; $gg \to$ correlated, entangled.



# LO Analytical Calculation




- Analytical calculation at leading-order.
- Initial states: $q\bar{q}$ and $gg$.
- Each one $I = q\bar{q}, gg$ gives rise to $\rho^I(M_{t\bar{t}}, \hat{k})$ with probability $w_I(M_{t\bar{t}}, \hat{k})$, which is PDF dependent.
- The spin density matrix: $\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}}, \hat{k})\rho^I(M_{t\bar{t}}, \hat{k})$.
- The total quantum state:
  $\rho(M_{t\bar{t}}) \equiv \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \int \mathrm{d}\Omega \; p(M, \hat{k})\rho(M, \hat{k}) = \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \; p(M)\rho_\Omega(M)$

# Basis Selection

- Helicity basis: $\{\hat{k}, \hat{r}, \hat{n}\}$:
  - $\hat{k}$ - direction of the top in the $t\bar{t}$ CM frame.
  - $\hat{p}$ - direction of the beam.
  - $\cos\Theta = \hat{k} \cdot \hat{p}$.
  - $\hat{r} = (\hat{p} - \cos\Theta\hat{k}) / \sin\Theta$.
  - $\hat{n} = \hat{r} \times \hat{k}$.
  - Describe each individual process with a **fixed direction**.
- Beam basis: $\{\hat{x}, \hat{y}, \hat{z}\}$:
  - $\hat{z}$ along the beam axis.
  - $\hat{x}, \hat{y}$ transverse directions to the beam.
  - After averaging: $C_x = C_y = C_\perp$.
  - Studying the **total quantum state**.

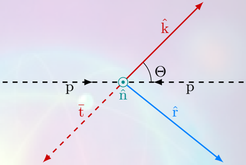


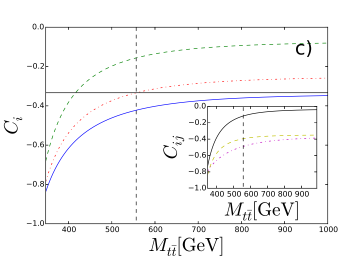

Figure: Helicity basis. Figure is from Phys. Rev. D 100, 072002.

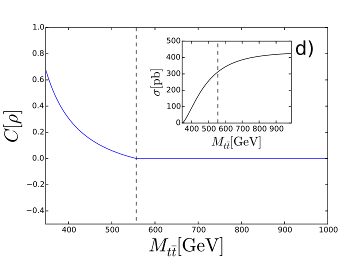
# Spin-Correlations - $M_{t\bar{t}}$ Dependence

- Plots are shown with integration only for $[2m_t, M_{t\bar{t}}]$.

c) Spin-correlations after $[2m_t, M_{t\bar{t}}]$ integration.
Main: $C_\perp$, $C_z$, $D$ (quantum tomography).
Inset: $C_{rr}$, $C_{nn}$, $C_{kk}$.

d) Main plot: Concurrence.
Inset: integrated cross-section.



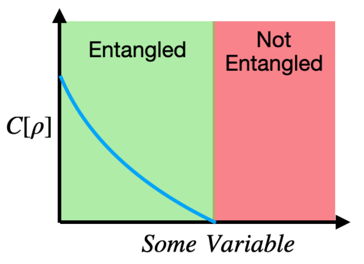



# Entanglement Criterion - Concurrence

- Concurrence:

$$C[\rho] \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

- $\lambda_i$ are the eigenvalues of the Concurrence matrix $\mathcal{C}(\rho)$.
- In our case $\mathcal{C}(\rho) = \rho$.
- $0 \leq C[\rho] \leq 1$, vanishing if and only if the state is separable.
- Compute the eigenvalues of $\rho$ - apply a criterion for entanglement.



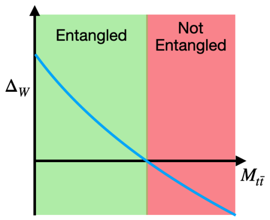
# Entanglement Criterion - Peres-Horodecki

- Partial transpose in one subsystem. Example:

$$\rho^{T_B} = \sum_n p_n \rho_n^a \otimes (\rho_n^b)^T$$

- If $\rho$ is separable, all of the eigenvalues of $\rho^{T_B}$ are non-negative.
- Reduces to the condition $\Delta > 0$, with $\Delta \equiv -C_{nn} + |C_{kk} + C_{rr}| - 1$.

- Total quantum state: can use any orthonormal basis to characterize entanglement.
- Link to concurrence: $C = max(\Delta, 0)/2$.
- We also depict: $D = \frac{\mathrm{tr}[\mathbf{C}]}{3} = -\frac{1+\Delta}{3}$.



# Expected Statistics

- Selection applied: $m_{t\bar{t}} < 450$ GeV.
- Integrated luminosity: $36 fb^{-1}$.
- Full LHC Run-II dataset $(139 fb^{-1}) \rightarrow \sim 50k$ events, accounting for selection efficiency and detector acceptance.
- Good statistics is expected.

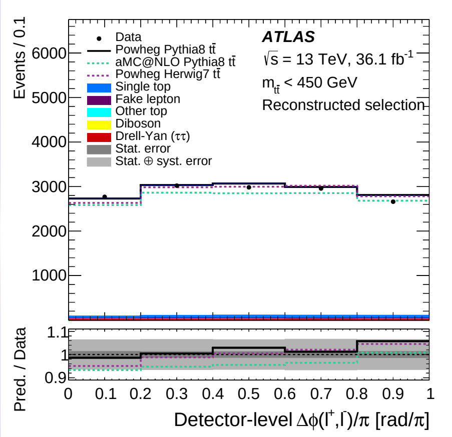



Figure: Angular separation between both leptons in the lab-frame transverse plane. Figure is from 1903.07570.