SCREENING VS. CONFINEMENT IN 2-DIMENSIONAL QCD WITH ADJOINT FERMIONS

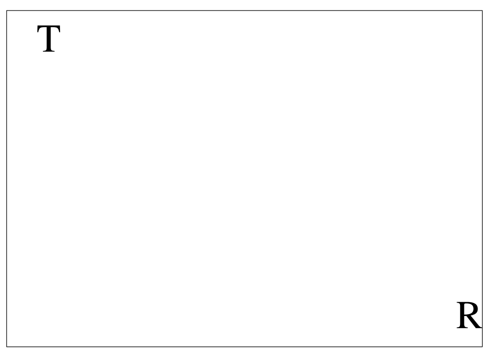
ICNFP-2020, Kolymbari, September 11

WELL-KNOWN FACTS

1. Pure Yang-Mills in 4 dimensions

Confinement of fundamental sources, i.e. linear growth of the potential and the area law for the Wilson loop:

$$\langle W(C)\rangle_{\rm vac} = \left\langle {\rm Tr}\left\{ e^{ig\int_C A_\mu^a(x) t^a dx^\mu} \right\}\right\rangle_{\rm vac}$$
$$= e^{-V(R)T} \sim e^{-\sigma RT} = e^{-\sigma A}.$$



2. QCD with dynamical quarks

Screening scenario:

- The string between the fundamental sources can be broken with formation of a pair of dynamical quarks.

- Perimeter law for the Wilson loop.
- Dynamical quarks screen fundamental heavy sources.
- Still no quark asymptotic states.

WHAT HAPPENS IN TWO DIMENSIONS?

$2d$ QCD as a model for $4d$ QCD.

$$\mathcal{L} \;=\; -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + i\bar{\psi}(\partial_\mu + igt^a A^a_\mu)\gamma_\mu\psi - m\bar{\psi}\psi\,.$$

Screening scenario; mesons as asymptotic states.

$2d$ QCD with adjoint fermions.

$$\mathcal{L} \;=\; -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{i}{2}\bar{\lambda}^a(\partial_\mu\delta^{ab} + gf^{abc}A^c_\mu)\gamma_\mu\lambda^b \qquad (-m\bar{\lambda}^a\lambda^a)\,,$$

where λ^a is a 2-component Majorana spinor.

- Adjoint heavy sources are screened. Perimeter law for the adjoint Wilson loop,

$$\langle W_{\rm adj}(C)\rangle_{\rm vac} \;=\; \left\langle {\rm Tr}\left\{e^{g\int_C A^a_\mu(x)f^{abc}dx^\mu}\right\}\right\rangle_{\rm vac}\,.$$

- Fundamental heavy sources are also screened in some cases.

Instantons in 2 dimensions

• 4*d* instantons — field configurations in R^4 representing a pure gauge at infinity,

$$\hat{A}_\mu(x) \stackrel{|x|\to\infty}{\to} ig^{-1}\partial_\mu g,$$

$g \in SU(2)$.

• The relevant topology: $\pi_3[SU(2)] = Z$.

• The same in two dimensions. The relevant topology: $\pi_1[G]$.

Schwinger model (2-dimensional QED)

• $G = U(1), \quad \pi_1[U(1)] = Z$.

• A topologically nontrivial configuration (instanton):

$$A_\mu = \frac{\epsilon_{\mu\nu} x_\nu}{x^2 + \rho^2}, \qquad \mu = 1, 2$$

• It has the topological charge

$$q = \frac{1}{4\pi} \int d^2x \, \epsilon_{\mu\nu} F_{\mu\nu} = 1.$$

• There are configurations with an arbitrary integer q.

• If the fermions are massless, $W(C) = 1$ in a topologically trivial sector.

• The contribution of topologically nontrivial sectors is suppressed due to the presence of zero modes of the Dirac operator there. (The factor $\det[\hat{\mathcal{D}}] = \det[\gamma_\mu(\partial_\mu - iA_\mu)] = \prod_n \mu_n = 0$ in the partition function.)

• Hence $\langle W(C)\rangle = 1$. No area law.

The massless Schwinger model exhibits screening of both integer and fractional heavy charges.

• If $m \neq 0$, the relevant determinant $\det[i\hat{\mathcal{D}} - m]$ does not vanish. For fractionally charged sources, $e_{\text{source}} = f e_{\text{dynamical}}$, one can derive

$$\langle W(C)\rangle \;=\; e^{-m\Sigma\mathcal{A}(1-\cos 2\pi f)},$$

where $\Sigma = \langle\bar{\psi}\psi\rangle_{\text{vac}}$ is the fermion condensate.

The massive Schwinger model exhibits confinement of fractional heavy charges. Integer charges are still screened.

2-dimensional QCD with fundamental fermions

- $\pi_1[SU(N)] = 0$. No nontrivial topology.

- $\langle W(C) \rangle = 1$. Heavy fundamental (as well as adjoint) sources are screened.

2-DIMENSIONAL QCD WITH ADJOINT FERMIONS

• The relevant group is $G = SU(N)/Z_N$.

The relevant topology is $\pi_1[SU(N)/Z_N] = Z_N$.
The presence of instantons *[Witten, 1979; A.S., 1994]*.

• The simplest case is $N = 2$. $SU(2)/Z_2 = SO(3)$.

The configuration

$$A_\mu^a = \delta^{a3} \frac{\epsilon_{\mu\nu} x_\nu}{x^2 + \rho^2}, \qquad \mu = 1, 2$$

is topologically nontrivial.

• Two fermion zero modes, a left one and a right one:

$$\lambda_L^{1+i2}(x) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \frac{1}{\sqrt{x^2 + \rho^2}},$$
$$\lambda_R^{1-i2}(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{1}{\sqrt{x^2 + \rho^2}},$$

• Note the different color structure.

• For massless λ^a, the fermion determinant in the topologically nontrivial sector $= 0$, the contribution of the top. nontrivial sectors is suppressed and $\langle W(C) \rangle = 0$. Screening.

[D. Gross, I. Klebanov, A. Matytsin and A.S., Nucl. Phys. B **461** *(1996) 109.]*

• When $m \neq 0$, the string tension appears:

$$\sigma \; = \; m|\langle \bar{\lambda}\lambda \rangle|.$$

Confinement

$SU(3)/Z_3$

• Two top. nontrivial sectors. Examples of configurations:

$$\hat{A}_\mu = \frac{1}{3}\mathrm{diag}(1,1,-2)\frac{\epsilon_{\mu\nu}x_\nu}{x^2+\rho^2} \quad \text{with} \quad W(C) = e^{2\pi i/3}\,, \tag{1}$$

and

$$\hat{A}_\mu = \frac{1}{3}\mathrm{diag}(2,-1,-1)\frac{\epsilon_{\mu\nu}x_\nu}{x^2+\rho^2} \quad \text{with} \quad W(C) = e^{-2\pi i/3}. \tag{2}$$

• Four fermion zero modes in each sector. For the field configuration (1), they are

$$\lambda_L^{4+i5}, \lambda_L^{6+i7}, \lambda_R^{4-i5}, \lambda_R^{6-i7}\,.$$

$\Rightarrow$ top. nontriv. sectors do not contribute in the massless case, $\langle W(C)\rangle = 1$, $\Rightarrow$ screening (?)

MOD. 2 INDEX

• Atiyah-Singer index

$$I_{AS} = n_L^{(0)} - n_R^{(0)} \tag{3}$$

is equal to zero in our case.

• The AS index is related to an integral invariant (like the magnetic flux) and the associated zero modes are robust.

Are our zero modes robust?

•

The claim of *[A. Cherman, T. Jacobson, Y. Tanizaki and M. Unsal, SciPost Phys.* **8** *(2020) 5, 072]:* In some cases they are and in some other cases they are not!

Spectrum of excited states of $\hat{\mathcal{D}}$

• For each eigenstate with positive μ, there is and eigenstate with negative μ.

• The states with both positive and negative μ are double degenerate $\Rightarrow$ quartic degeneracy of all excited levels in $\hat{H} = (\hat{\mathcal{D}})^2$.

• For $N = 2$, there are only two zero modes of $\hat{H}$. They cannot be lifted from zero under deformation due to the mathematical fact $2 \neq 4$.

• For $N = 3$ and for the Abelian top. nontrivial field configurations (1), (2) there are four zero modes of $\hat{H}$. They can in principle be shifted from zero under deformation.

• If so, in the $N = 2$ theory (generically, in the the $N = 2n$ theories) massless adjoint fermions do screen fundamental heavy sources and in the $N = 3$ theory (generically, in the the $N = 2n+1$ theories) they do not.

BREAKING NEWS
(not confirmed yet)

• In $N = 3$ theory, the zero modes are lifted under a non-Abelian perturbation of the Abelian background.

• They do so in the third order of perturbation theory. *[in preparation]*

• This confirms the mod. 2 index reasoning, but the claim of the **very** recent *[Z. Komargodski et al, arXiv:2008.07567]* is that, in spite of this fact, the theory exhibits screening for all values of N!

THANKS TO THE ORGANIZERS !!!

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THANKS TO THE OAC STUFF !!!