Gravitational Waves from Fundamental Axion Dynamics

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September 10, 2020

9th International Conference on New Frontiers in Physics (ICNFP 2020)
Workshop on New Physics Paradigms after Higgs and Gravitational Wave Discoveries

Talk mainly based on

- Salvio (Phys. Lett. B) arXiv:2003.10446
- Ghoshal, Salvio arXiv:2007.00005

Introduction: QCD and the axion

- QCD is perhaps the most satisfying building block of the Standard Model (SM): it is asymptotically free (AF) and agrees well with experiments

- It is surprising that the SM Yukawa interactions break CP while QCD doesn't. A possible explanation was proposed by Peccei and Quinn (PQ): an approximate global chiral and spontaneously broken U(1) symmetry was proposed. This $\mathrm{U}(1)_{\mathrm{PQ}}$ manifests at low energy with the axion

- Computable models introduce extra scalars, whose quartic typically suffer from Landau poles, which spoil asymptotic freedom

Building a fundamental field theory of the QCD axion

A way out: one can gauge the axion sector with a non-Abelian group.

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Minimal model

The axion sector features an SU(2) gauge group (henceforth SU$(2)_a$) and

- two extra Weyl fermions q and $\bar{q}$ with the same PQ charge and in the fundamental and antifundamental of SU$(3)_c$×SU$(2)_a$
- A *complex* scalar A in the adjoint of SU$(2)_a$ and neutral under SU$(3)_c$

such that we can write $\quad \mathscr{L}_y = -y\bar{q}Aq + \mathrm{H.c.}$

A features the generic potential

$$V_A = -m^2 \mathrm{Tr}(A^\dagger A) + \lambda_1 \mathrm{Tr}^2(A^\dagger A) + \lambda_2 |\mathrm{Tr}(AA)|^2$$

The λ_i have LPs unless y acquires a non-vanishing IR attractive value and $\lambda_i \propto g_s^2$ in the UV, with specific proportionality factors

Building a fundamental field theory of the QCD axion

Renormalization group analysis of the minimal model

$$\frac{dg^2}{dt} = -bg^4 \qquad b_a = \frac{14}{3}, \qquad b_s = \frac{29}{3} - \Delta$$

where $t \equiv \ln(\mu^2/\mu_0^2)/(4\pi)^2$ and Δ is the positive extra contribution due to the fermions and scalars in the SM sector. The λ_i have to have the UV behavior $\lambda_i = \tilde{\lambda}_i/t$ with the following values of $(\tilde{\lambda}_1, \tilde{\lambda}_2)$

Δ	unstable vacuum	stable vacuum
28/3	$(0.183, -3.23)$	$(1.68, -0.951)$
26/3	$(0.149, -1.05)$	$(0.575, -0.343)$
8	$(0.145, -0.598)$	$(0.349, -0.231)$

The unstable vacua have to be excluded so the λ_i are predicted at low energies like y

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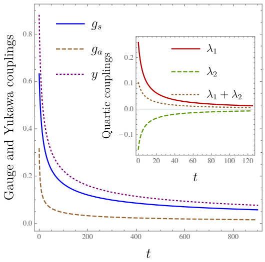
Fixing g_s and g_a at low energies we then get a totally asymptotically free (TAF) axion model
(In the plot we set $\Delta = 28/3$)

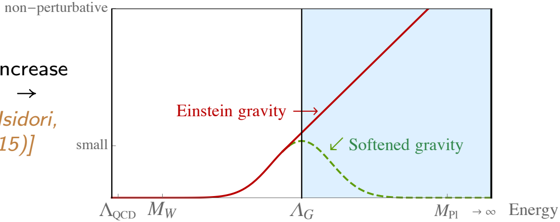


We assume that gravity is “softened” in the UV

(Einstein) gravitational interactions increase with energy

Idea (*softened gravity*):

consider theories where the increase of the gravitational coupling $\rightarrow$ *stops at some* $\Lambda_G \ll M_{\rm Pl}$. *[Isidori, Giudice, Salvio, Strumia (2015)]*



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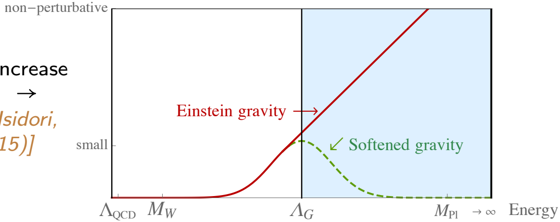

We will also require that the PQ symmetry breaking scale f_a satisfies

$$f_a \lesssim \Lambda_G$$

This ensures that gravity, as far as the production of gravitational waves is concerned, is well-described by Einstein's theory

Quantum generation of the Peccei-Quinn (PQ) scale

We can obtain an even more predictive model by breaking the PQ symmetry through the Coleman-Weinberg (CW) mechanism: in that case there is one less adjustable parameter

$$V_A = \cancel{-m^2 \mathrm{Tr}(A^\dagger A)} + \lambda_1 \mathrm{Tr}^2(A^\dagger A) + \lambda_2 |\mathrm{Tr}(AA)|^2$$

At a scale μ_{PQ} where $\lambda \equiv \lambda_1 + \lambda_2 = 0$ the effective potential develops a flat direction ($A = A^\dagger$). More generically we assume $m \ll \mu_{\mathrm{PQ}}$.

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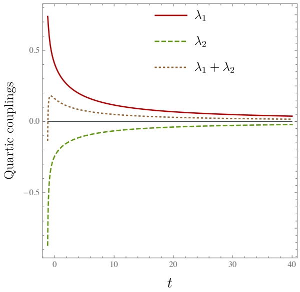
In the plot we show how eventually λ crosses zero at low energies:



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$$V_{\mathrm{CW}}(\phi) = \frac{\bar{\beta}}{4}\left(\ln\left(\frac{\phi}{f_a}\right) - \frac{1}{4}\right)\phi^4, \qquad \text{where} \quad \bar{\beta} \equiv \left[\mu \frac{d\lambda}{d\mu}\right]_{\mu=\mu_{\mathrm{PQ}}}$$

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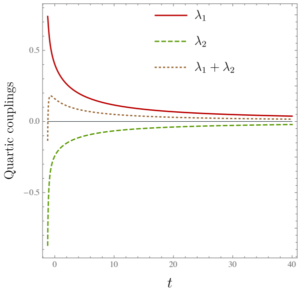

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At a scale $\mu_{\rm PQ}$ where $\lambda \equiv \lambda_1 + \lambda_2 = 0$ the effective potential develops a flat direction ($A = A^\dagger$). More generically we assume $m \ll \mu_{\rm PQ}$. This leads to

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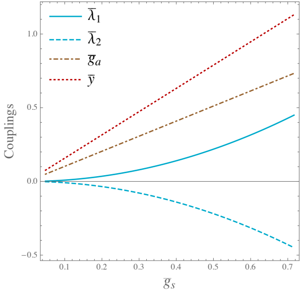
The TAF axion sector features only one dimensionless parameter, which can be taken to be $\bar{g}_s \equiv g_s(t_{\rm PQ})$:

once the gauge couplings are chosen at $\mu = \mu_{\rm PQ}$ the other couplings $\bar{y}$, $\bar{\lambda}_1$ and $\bar{\lambda}_2$ are predicted and one must consider a particular IR value of one of the gauge couplings, say $\bar{g}_a$, to enforce $\bar{\lambda}_1 + \bar{\lambda}_2 = 0$



Peccei-Quinn phase transition

We consider the one-loop effective potential

$$V_{\text{eff}}(\phi, T) \equiv V_{\text{CW}}(\phi) + V_T(\phi) + \Lambda_0,$$

where V_T is the thermal part and Λ_0 accounts for the observed value of the cosmological constant

The CW mechanism leads to a strongly first-order phase transition

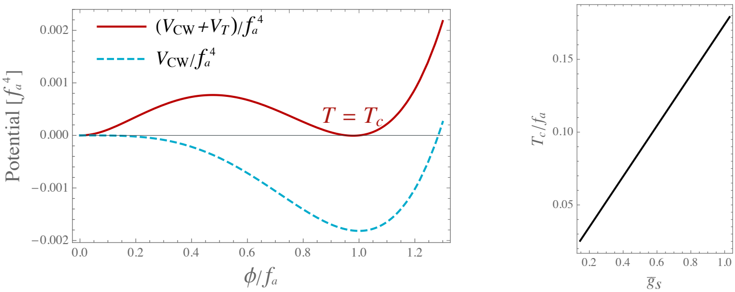


Figure: *In the left plot the example $\bar{g}_s \approx 0.91$ has been considered and a constant has been added such that $V_{\text{eff}}(0, T) = 0$*

(previously observed in effective models with CW symmetry breaking: *[Witten (1981); delle Rose et al (2019); von Harling et al (2019)]*)

Peccei-Quinn phase transition: supercooling

Decay rate per unit volume Γ of the false vacuum $\phi = 0$ into the true vacuum $\phi \neq 0$

Nucleation temperature T_n

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For $T < T_c$ we have *[Coleman (1977); Callan, Coleman (1980); Linde (1981); Linde (1983)]*

$$\Gamma \approx \max\left(T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T},\ \frac{1}{R_4^4}\left(\frac{S_4}{2\pi}\right)^2 e^{-S_4} \right)$$

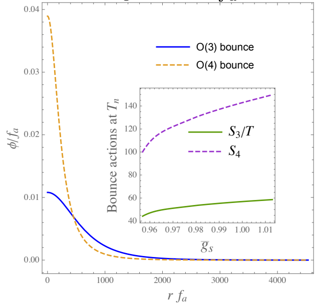
S_d is the action

$$S_d = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int_0^\infty dr\, r^{d-1} \left(\frac{1}{2}\phi'^2 + V_{\rm eff}(\phi, T) \right)$$

evaluated at the O(d) bounce:

$$\phi'' + \frac{d-1}{r}\phi' = \frac{dV_{\rm eff}}{d\phi}, \quad \phi'(0) = 0, \quad \lim_{r\to\infty} \phi(r) = 0$$

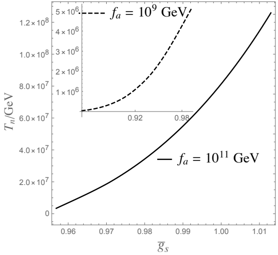
$\bar{g}_s = 0.91$ and
$T \approx 1\times10^{-2}\, T_c \approx 2\times10^{-3}\, f_a$



Nucleation temperature T_n

ϕ is trapped ($\phi = 0$) until $T \ll T_c$; the universe features a phase of strong supercooling and the universe inflates with Hubble rate $H_I = \sqrt{\bar{\beta}} f_a^2/(4\sqrt{3}\bar{M}_{\rm Pl})$.
T_n corresponds to $\Gamma/H_I^4 \sim 1$ or

$$\frac{S_3}{T_n} - \frac{3}{2}\ln\left(\frac{S_3/T_n}{2\pi}\right) = 4\ln\left(\frac{T_n}{H_I}\right)$$



Peccei-Quinn phase transition: reheating and duration

Reheating

Duration of the phase transition

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Reheating

At the end of supercooling the universe should be reheated. This occurs thanks to the unavoidable coupling between the axion and the SM sectors due to gluons, leading to

$$\Gamma_{\phi\to gg} \sim \frac{\bar{y}^2\bar{g}_s^4 M_\phi^3}{(4\pi)^5 M_Q^2} \qquad \rightarrow \qquad T_{\rm RH} = \left(\min\left(\frac{45\Gamma_{\phi\to gg}^2\bar{M}_{\rm Pl}^2}{4\pi^3 g_*}, \frac{15\bar{\beta} f_a^4}{8\pi^2 g_*}\right)\right)^{1/4}$$

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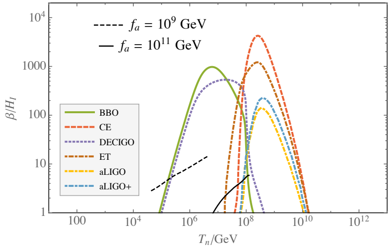
Duration of the phase transition

The inverse of the duration of the phase transition is defined by

$$\beta \equiv \left[\frac{1}{\Gamma}\frac{d\Gamma}{dt}\right]_{T_n}$$

This quantity, for fast reheating, can be computed with the formula

$$\frac{\beta}{H_I} = \left[T\frac{d}{dT}(S_3/T) - 4\right]_{T=T_n}$$



Monopole dilution

Strong supercooling and the corresponding inflation dilute the density $n(T)$ of monopoles due to $\mathrm{SU}(2)_a \to \mathrm{U}(1)_a$

In a strong first-order phase transition monopoles may be created by bubble collisions and well-known estimates *[Preskill (1984)]* lead to

$$\frac{n(T_n)}{T_n^3} \gtrsim p \left(\frac{T_n}{C M_P} \right)^3 , \tag{1}$$

where

- p is the probability that the scalar field configuration is topologically non trivial,
- $C = 0.6/\sqrt{g_*(T_n)}$

Even for $p \approx 1$, setting $g_*(T_n)$ of order 10^2 (a realistic setup given the existing TAF SM sectors) the theoretical bound in (1) is amply compatible with the bound coming from the fact that the mass density of monopoles must not exceed the limit on the total mass density imposed by the observed Hubble constant and deceleration parameter. Indeed, the latter bound is around $n(T_0)/T_0^3 \lesssim 10$ eV$/M_m$, where T_0 is today's temperature, M_m is the monopole mass and $M_m \sim 4\pi f_a/\bar{g}_a$, $n(T_0)/T_0^3 \lesssim n(T_n)/T_n^3$ and the window 10^8 GeV $\lesssim f_a \lesssim 10^{12}$ GeV have been used.

Gravitational waves

Because of supercooling and inflation the main source of GWs are vacuum bubble collisions *[Caprini et al (2015)]*

$$h^2\Omega_{\rm GW}(f) \approx 1.29\times 10^{-6}\left(\frac{H(T_{\rm RH})}{\beta}\right)^2\left(\frac{100}{g_*(T_{\rm RH})}\right)^{1/3}\frac{3.8(f/f_{\rm peak})^{2.8}}{1+2.8(f/f_{\rm peak})^{3.8}},$$

where

$$f_{\rm peak} \approx 3.79\times 10^2\,\frac{\beta}{H(T_{\rm RH})}\,\frac{T_{\rm RH}}{10^{10}{\rm GeV}}\left(\frac{g_*(T_{\rm RH})}{100}\right)^{1/6}\ {\rm Hz}$$

$\Omega_{\rm GW}$ is subject to a big-bang nucleaosynthesis (BBN) bound, which depends on the effective number of neutrinos $N_{\rm eff}$

$$h^2\bar{\Omega}_{\rm GW} \equiv \int_{f_{\rm BBN}}^{f_{\rm UV}}\frac{df}{f}h^2\Omega_{\rm GW}(f) < 1.3\times 10^{-6}\,\frac{N_{\rm eff}-3.046}{0.234}$$

where $f_{\rm BBN} \sim 10^{-11}$Hz and $f_{\rm UV}$ is some UV cutoff, which in our case can be conservatively taken to be Λ_G

Gravitational waves

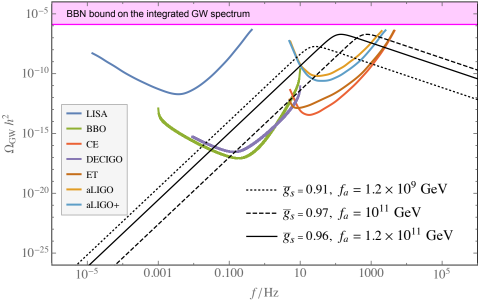


Figure: *The areas above the colored lines correspond to the (projected) sensitivities for various GW observatories. The colored region represents the BBN bound on the integrated GW spectrum* $\bar{\Omega}_{\rm GW} h^2$*, taking as reference value* $N_{\rm eff} = 3.28$*, which corresponds to an upper experimental bound on* $N_{\rm eff}$ *at 95% c.l..*

Conclusions

- We have studied the PQ phase transition and the corresponding spectrum of GWs in a *fundamental* QCD axion model
- All couplings flow to zero in the infinite energy limit (TAF property) and f_a is generated through the CW mechanism, leading to a very predictive model
- This model features a very strong first-order phase transition, characterized by a period of supercooling
- We have compared the predicted theoretical GW spectrum with the sensitivities of several future detectors such as ET, CE, DECIGO, BBO and advanced LIGO, finding that these experiments will be able to test the fundamental QCD axion model

Thank you very much for your attention!