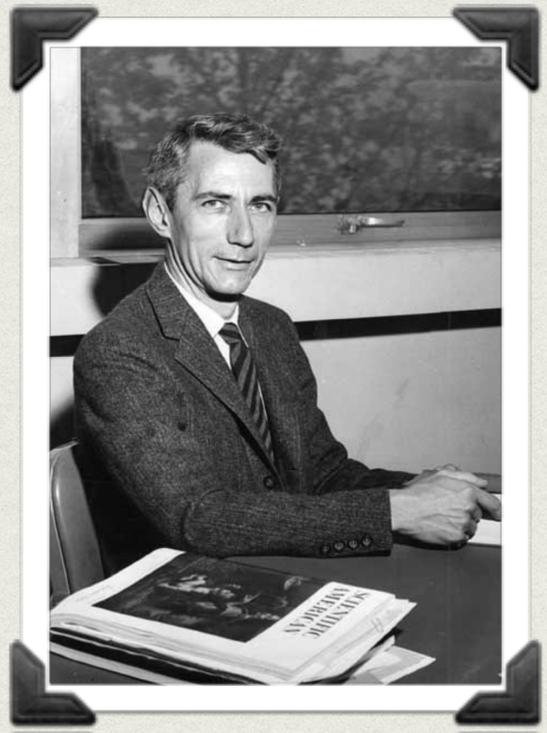
9th International Conference on
New Frontiers in Physics (ICNFP 2020)

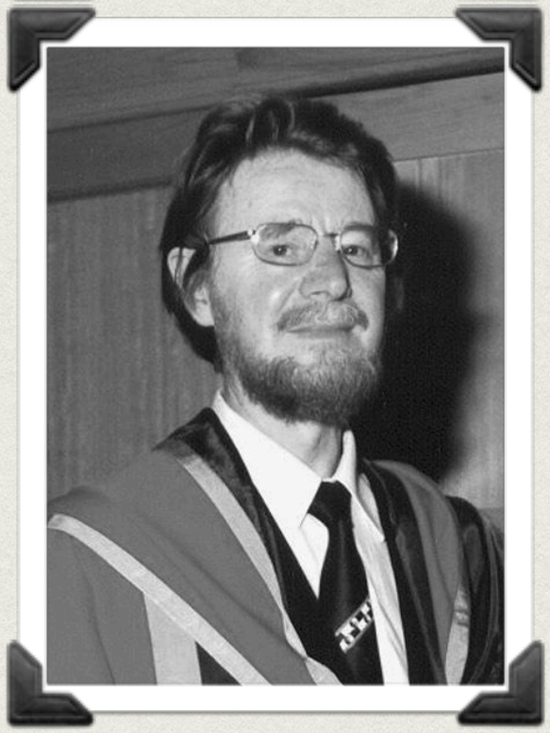
A WHOLE WORLD LIES BETWEEN QUANTUM NON-LOCALITY AND COMMUNICATION

Claude Crépeau

McGill

- Information theoretical model.
- What is Signalling?
- What is No-Signalling?
- Quantum Mechanics
- Special Relativity
- Future work

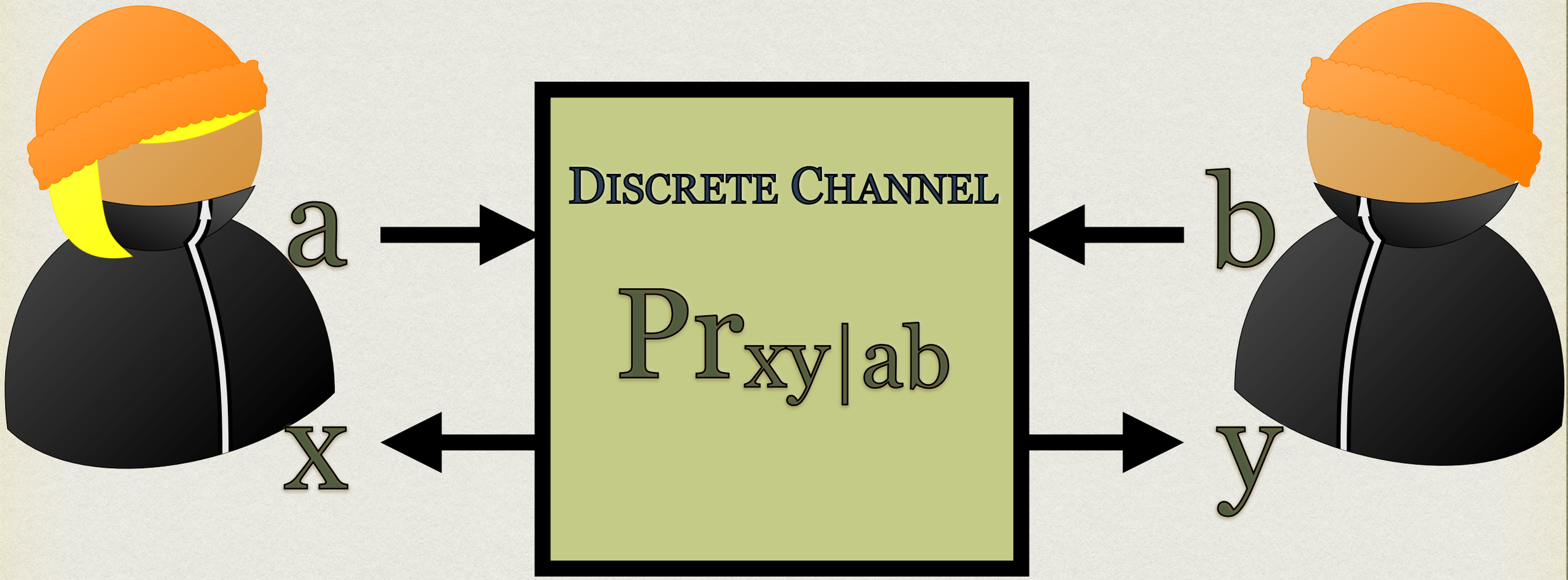




INTRODUCTION

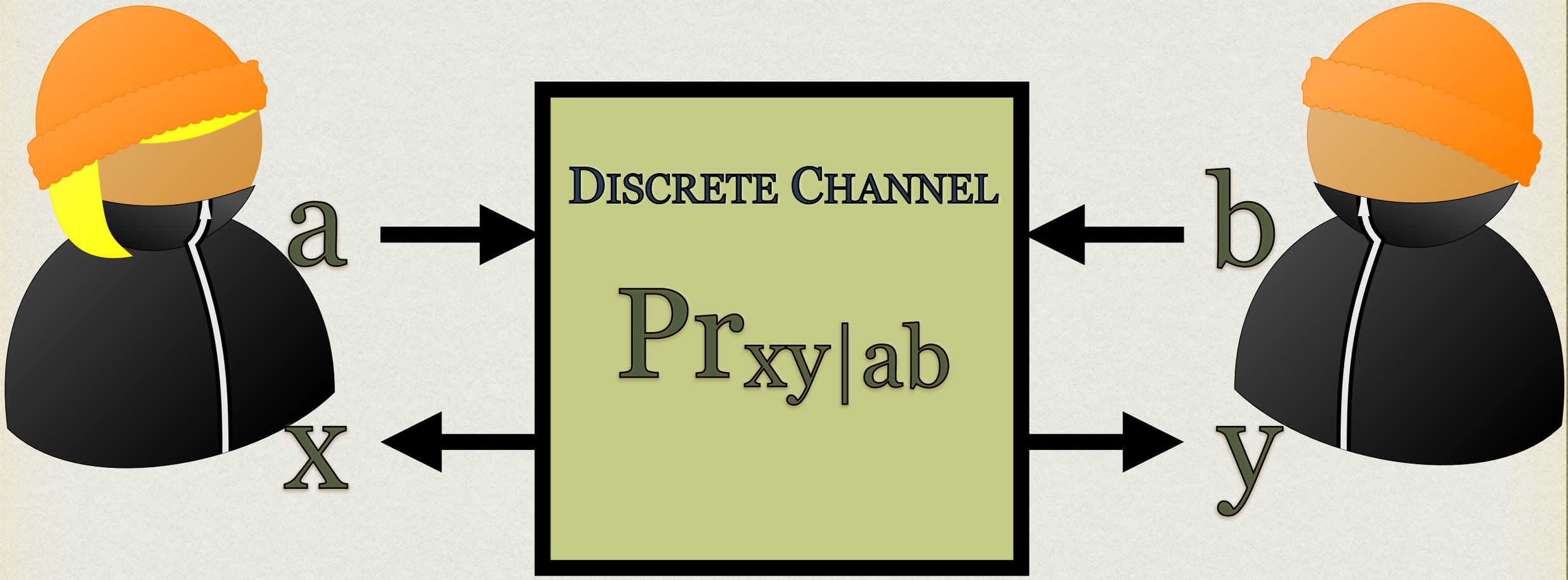
DISCRETE CHANNELS

DISCRETE CHANNELS



DISCRETE CHANNELS

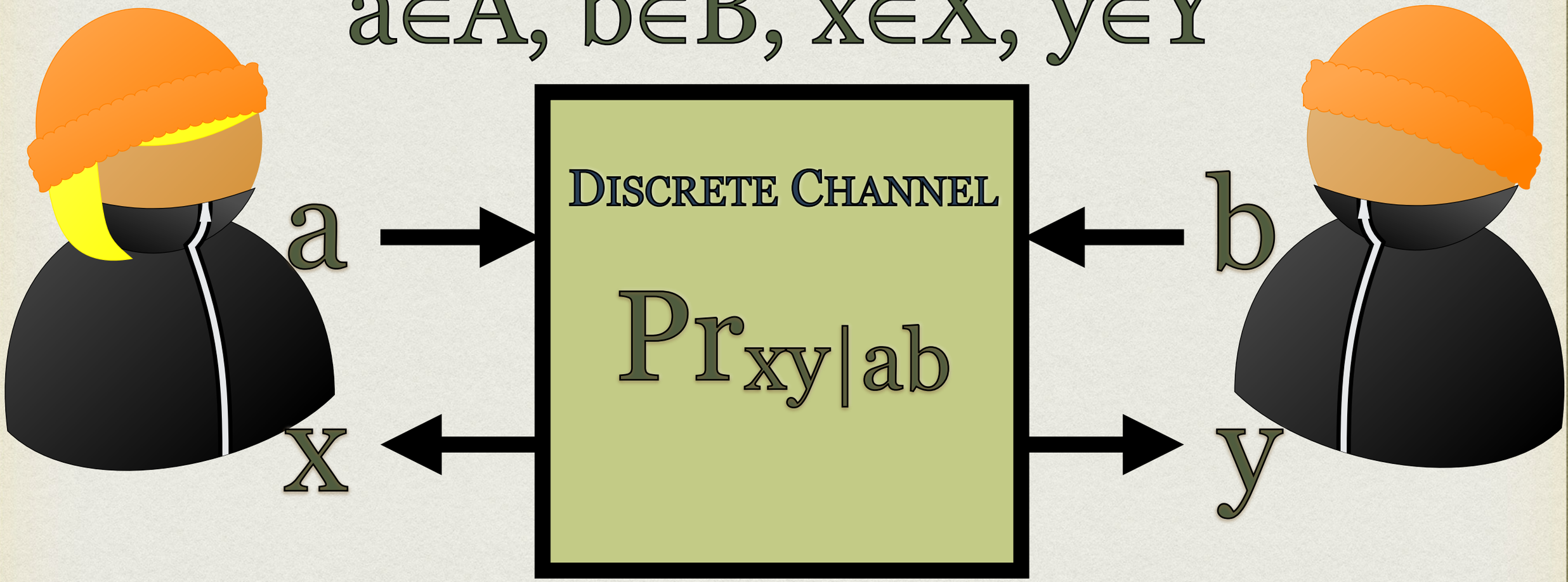
A,B,X,Y: finite sets



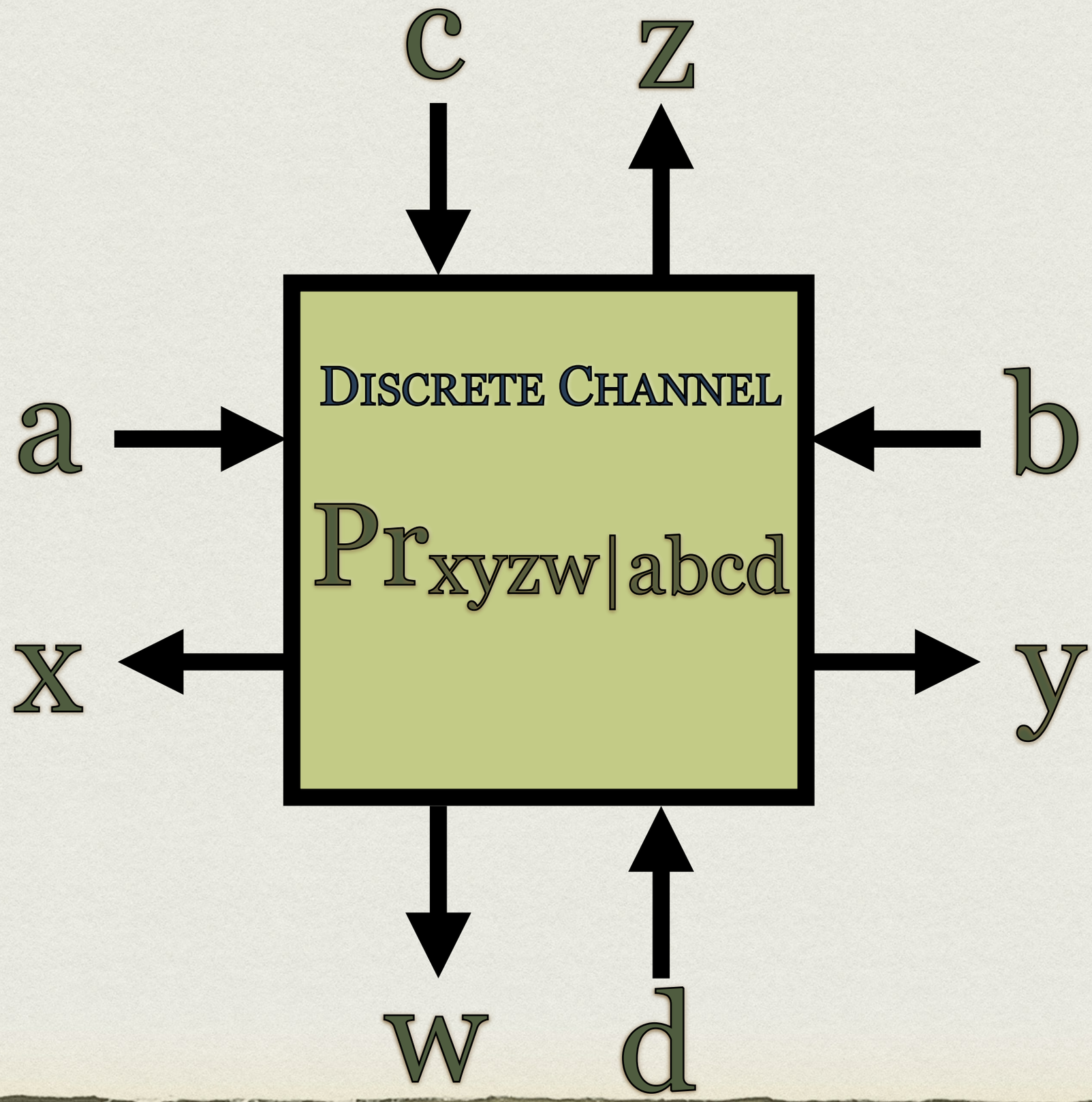
DISCRETE CHANNELS

A, B, X, Y: finite sets

$a \in A,\ b \in B,\ x \in X,\ y \in Y$



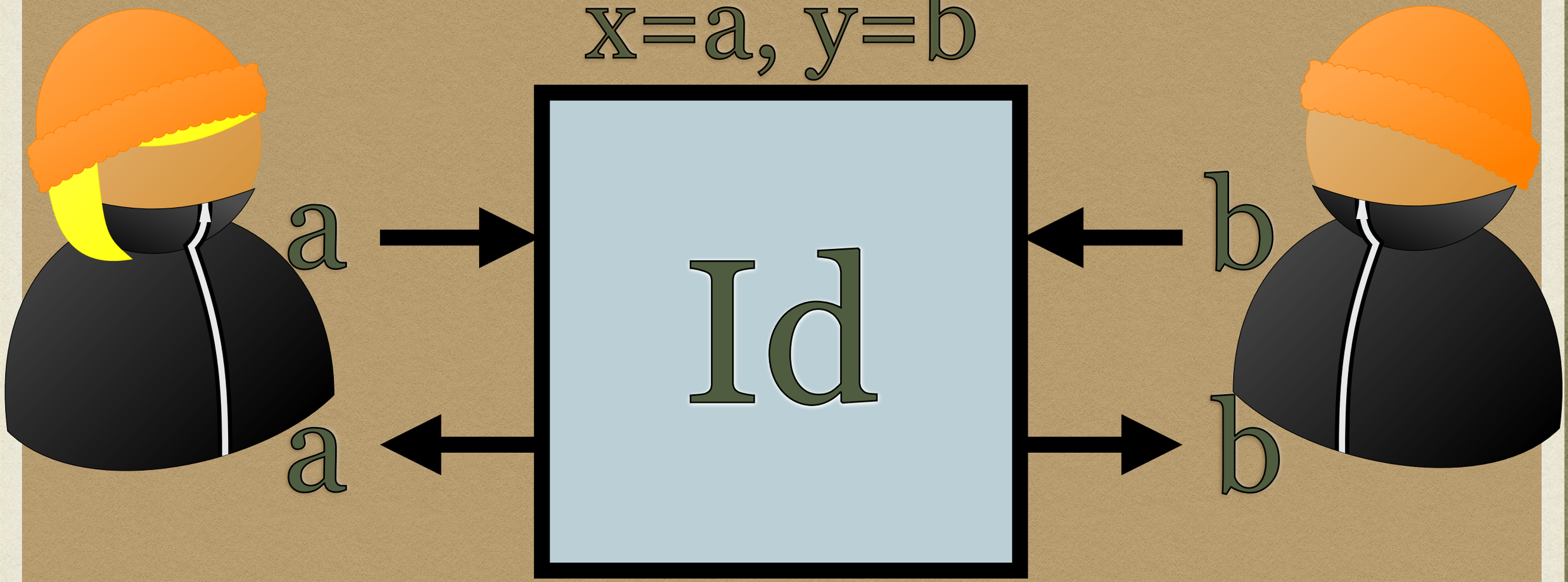
GENERAL CHANNELS



DISCRETE CHANNELS

X=A, Y=B

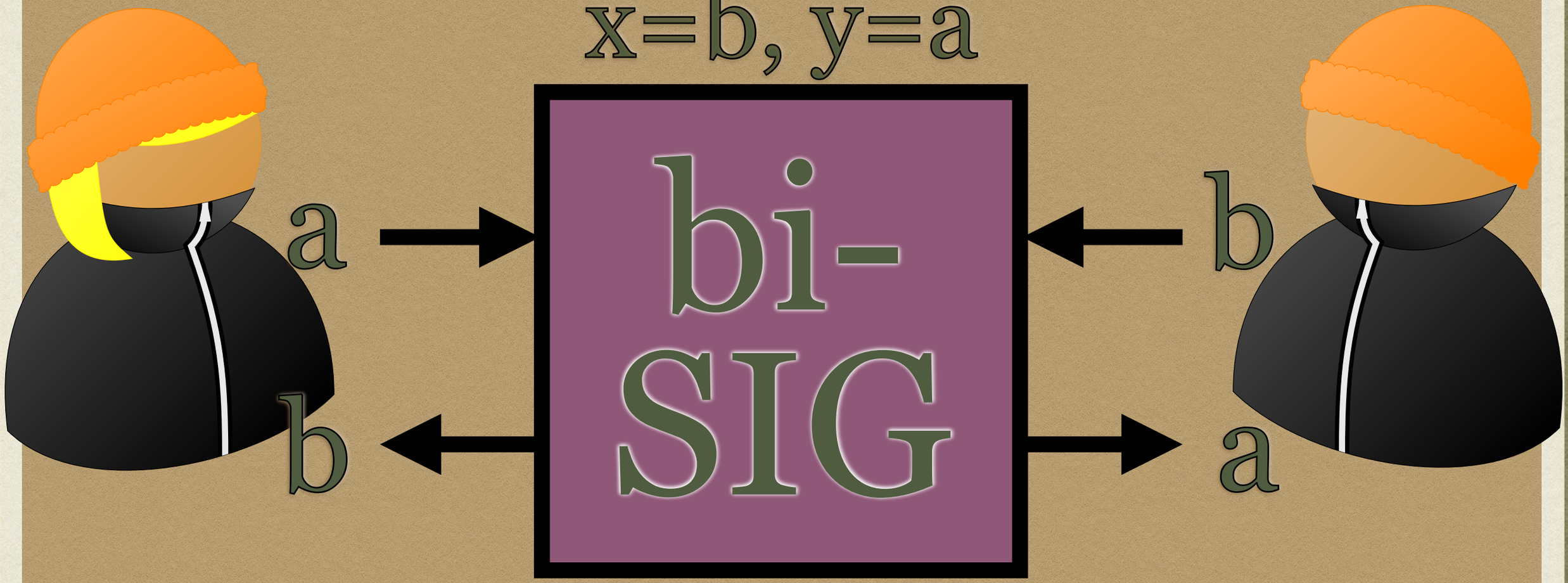
x=a, y=b



DISCRETE CHANNELS

X=B, Y=A

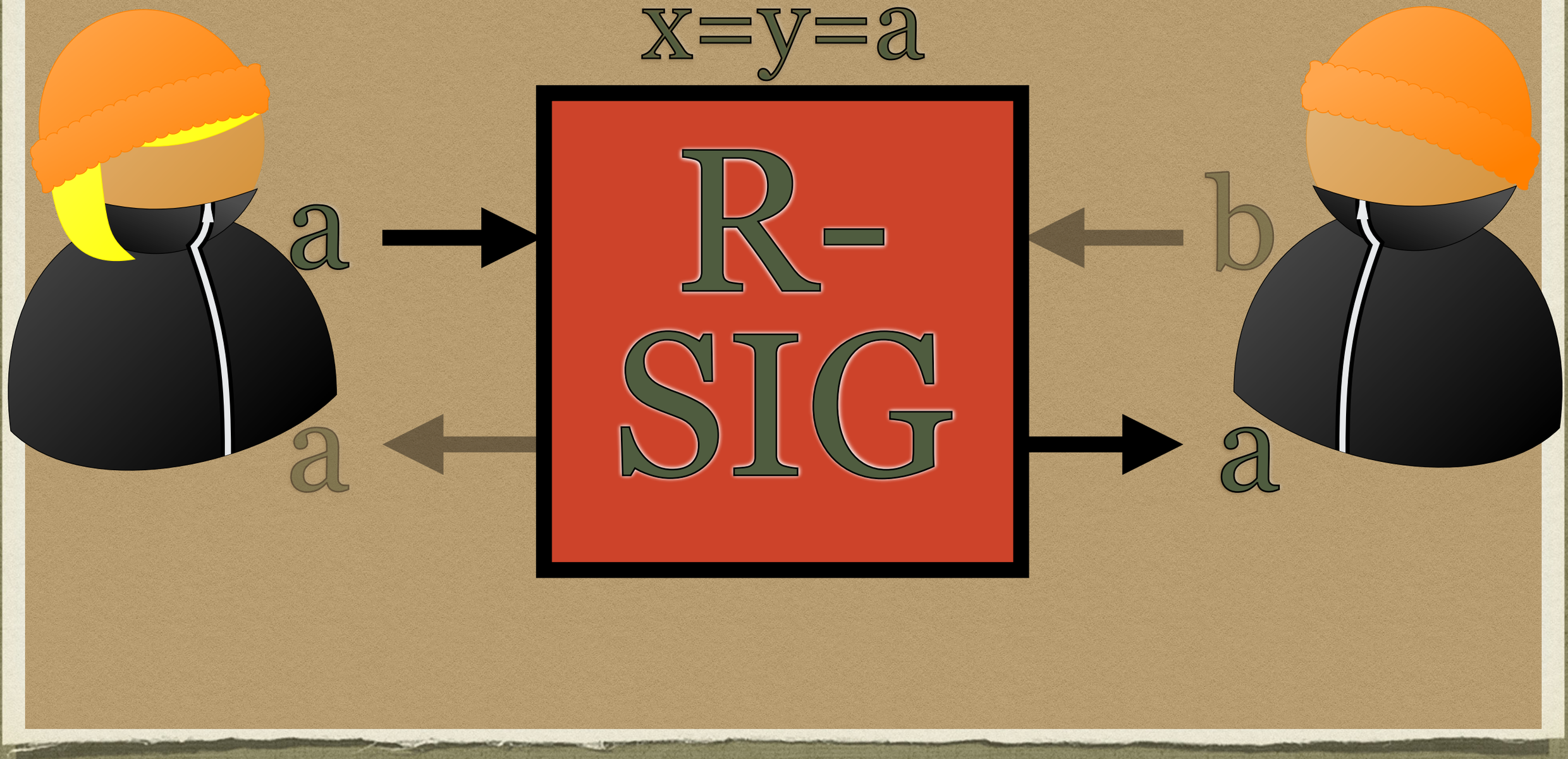
x=b, y=a



DISCRETE CHANNELS

X=Y=A

x=y=a



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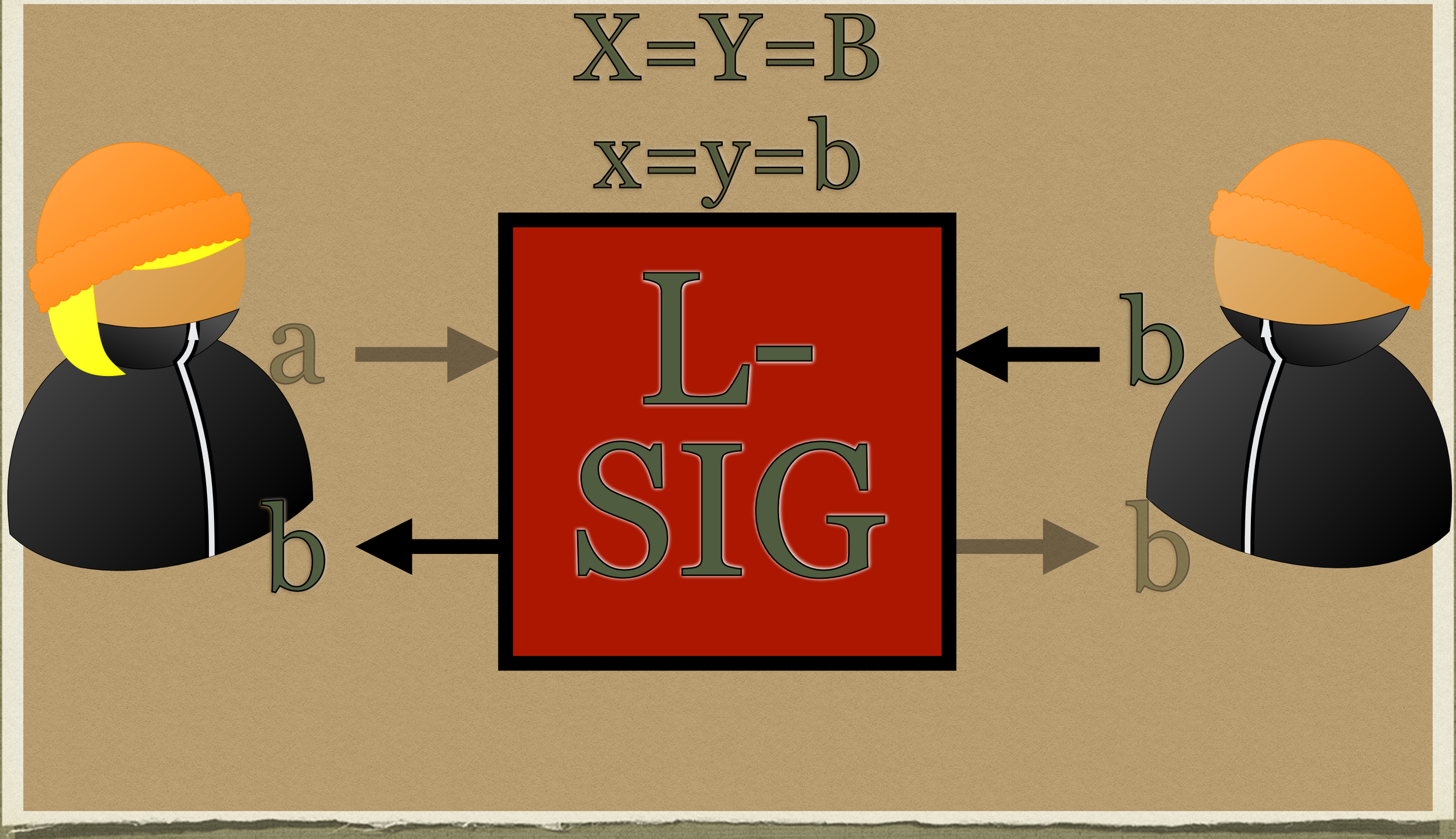
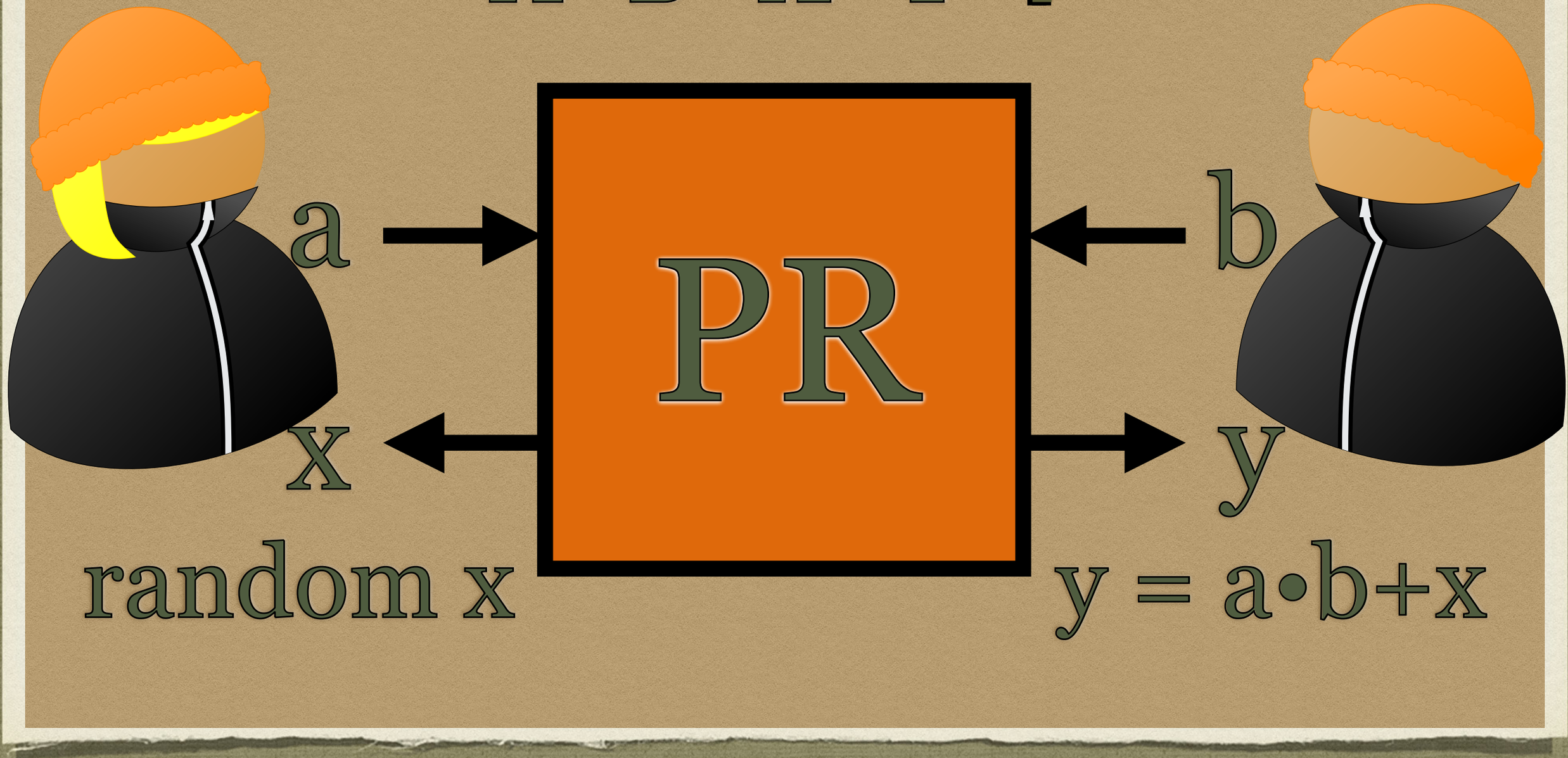


DISCRETE CHANNELS

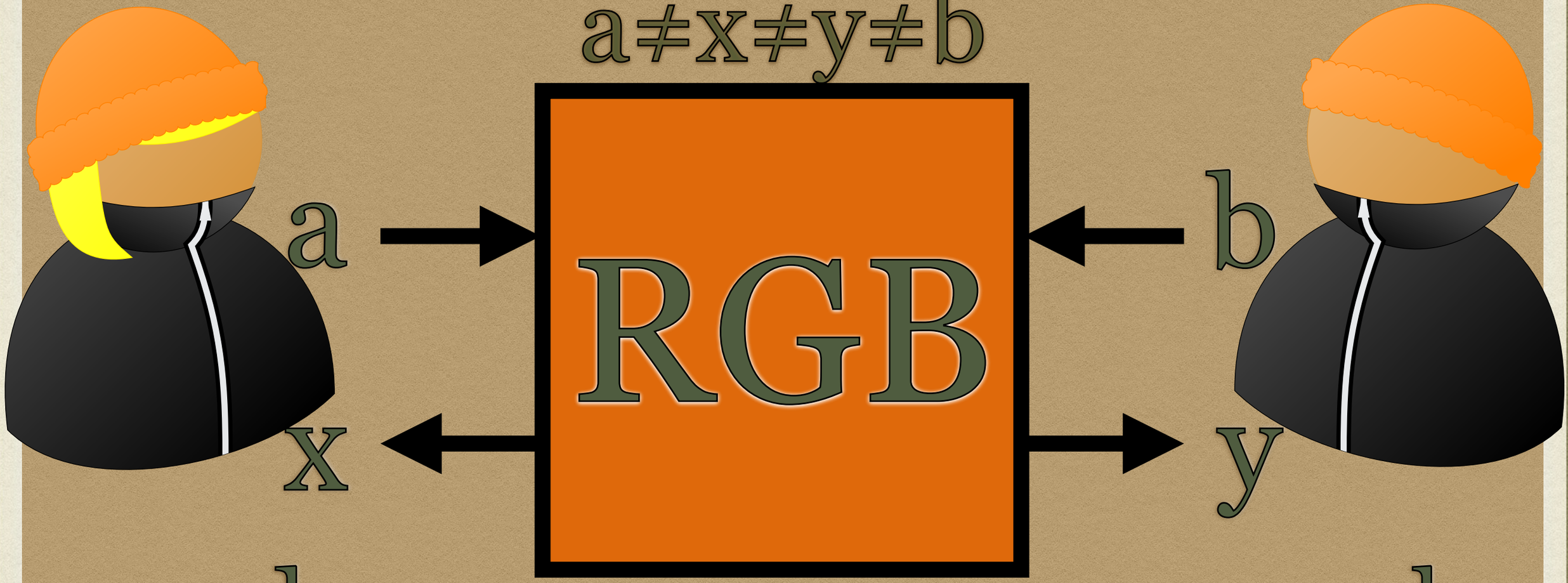
$$A=B=X=Y=\mathbb{F}$$



DISCRETE CHANNELS

$A=B=X=Y=\{●,●,●\}$

$a \neq x \neq y \neq b$



WHAT CONSTITUTES A SIGNALLING CHANNEL?

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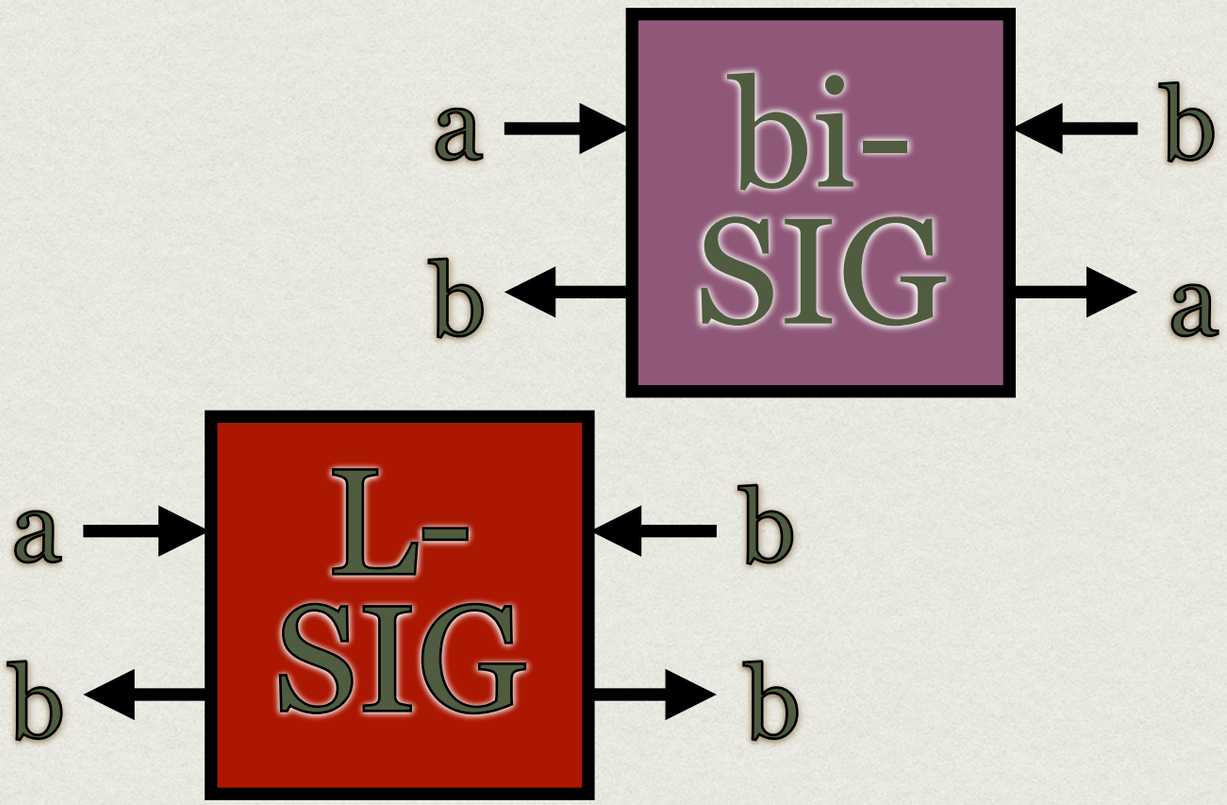

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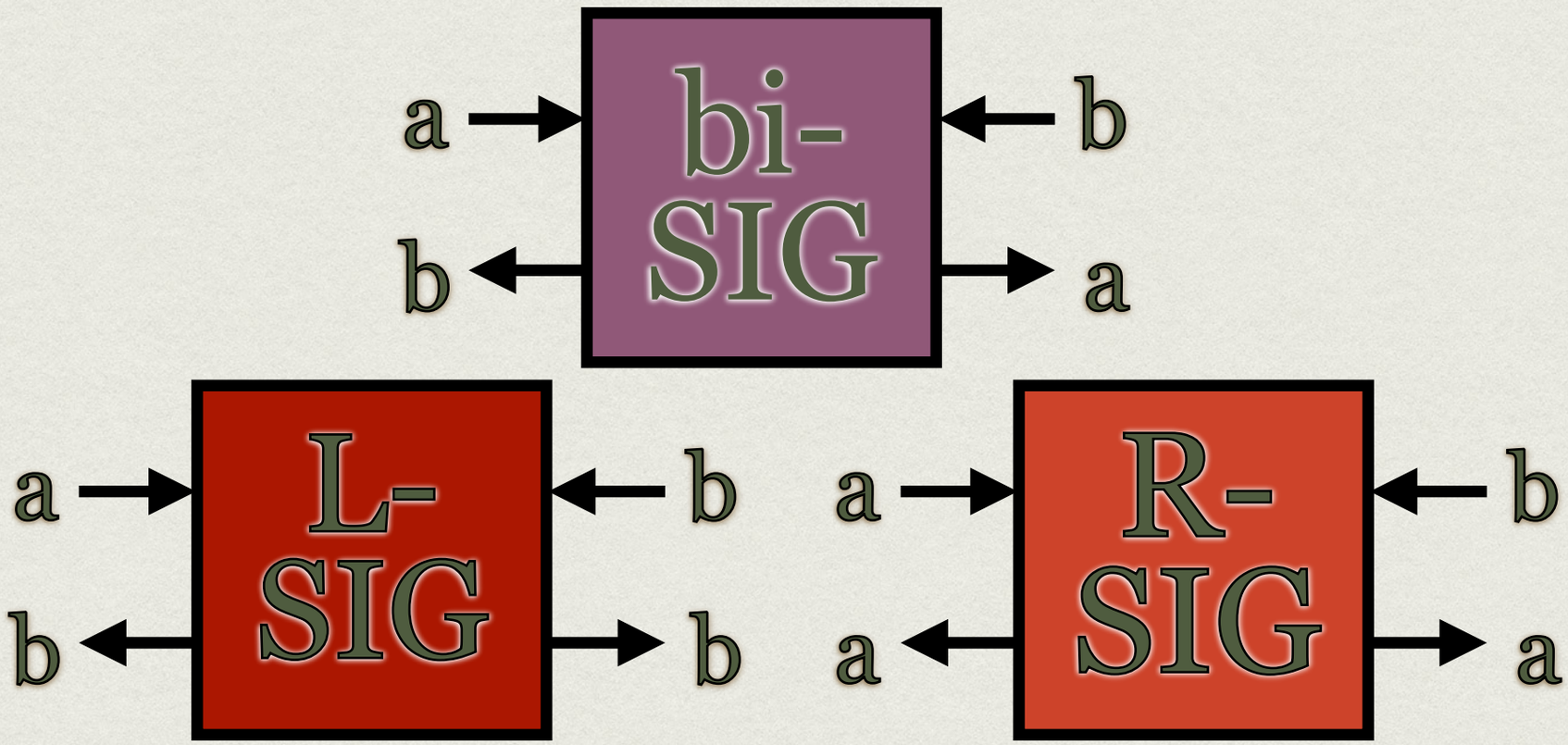

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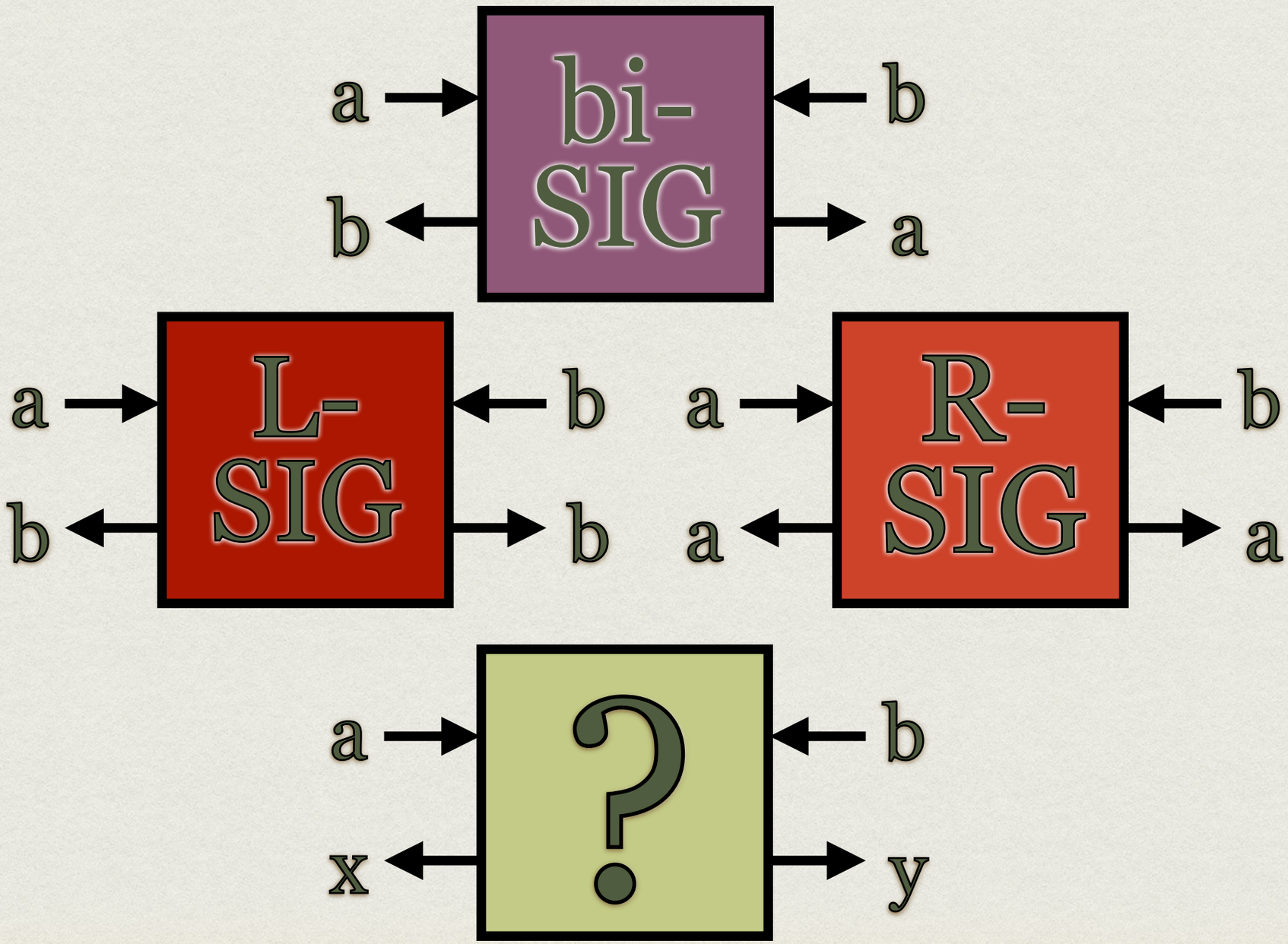

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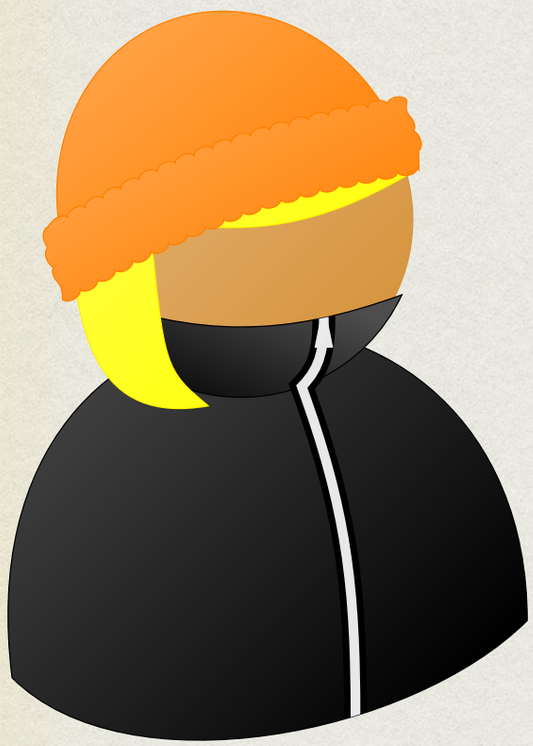

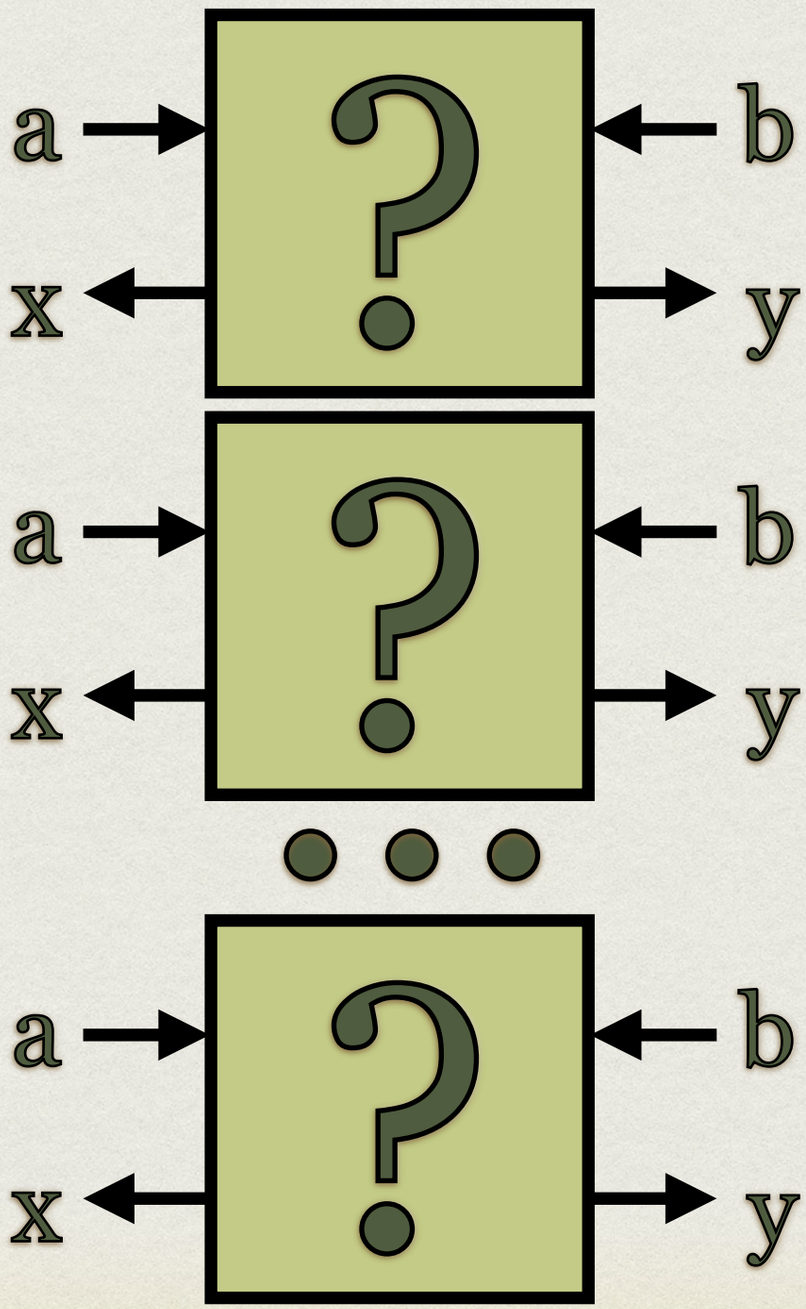

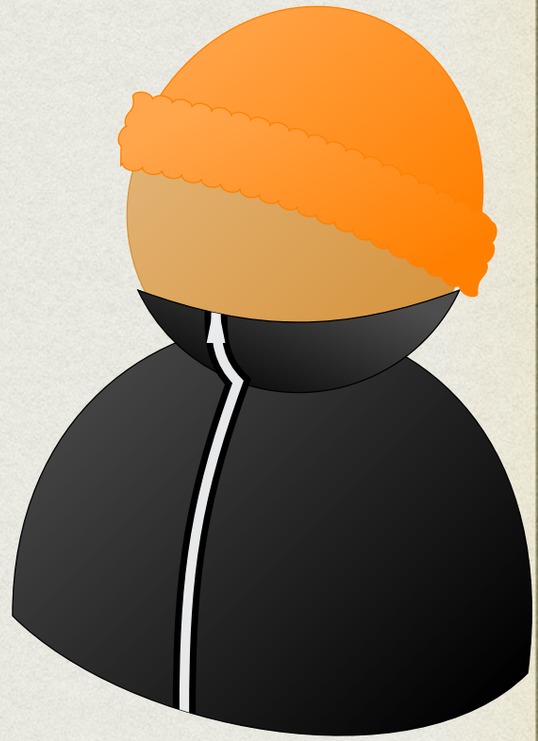

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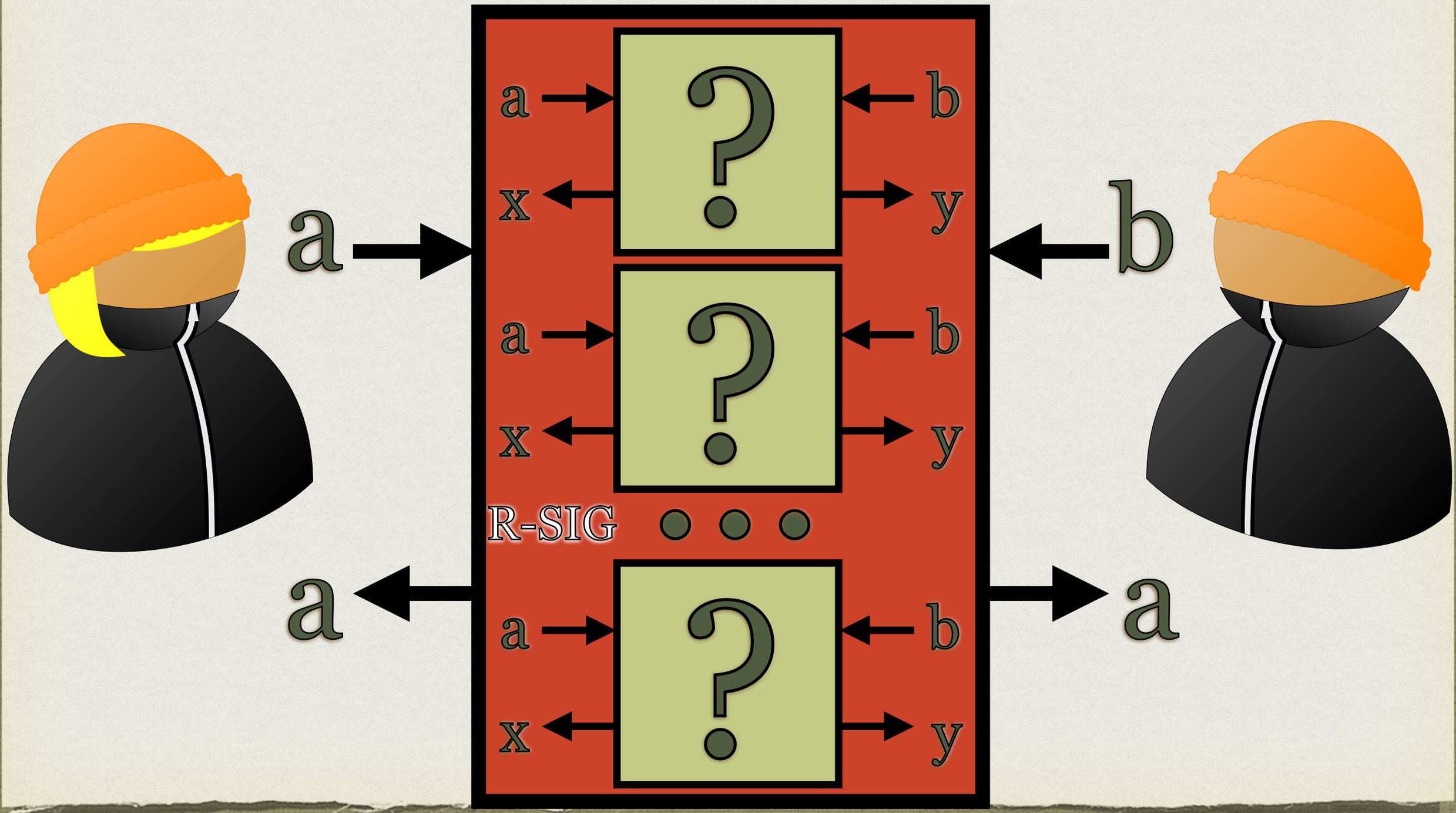

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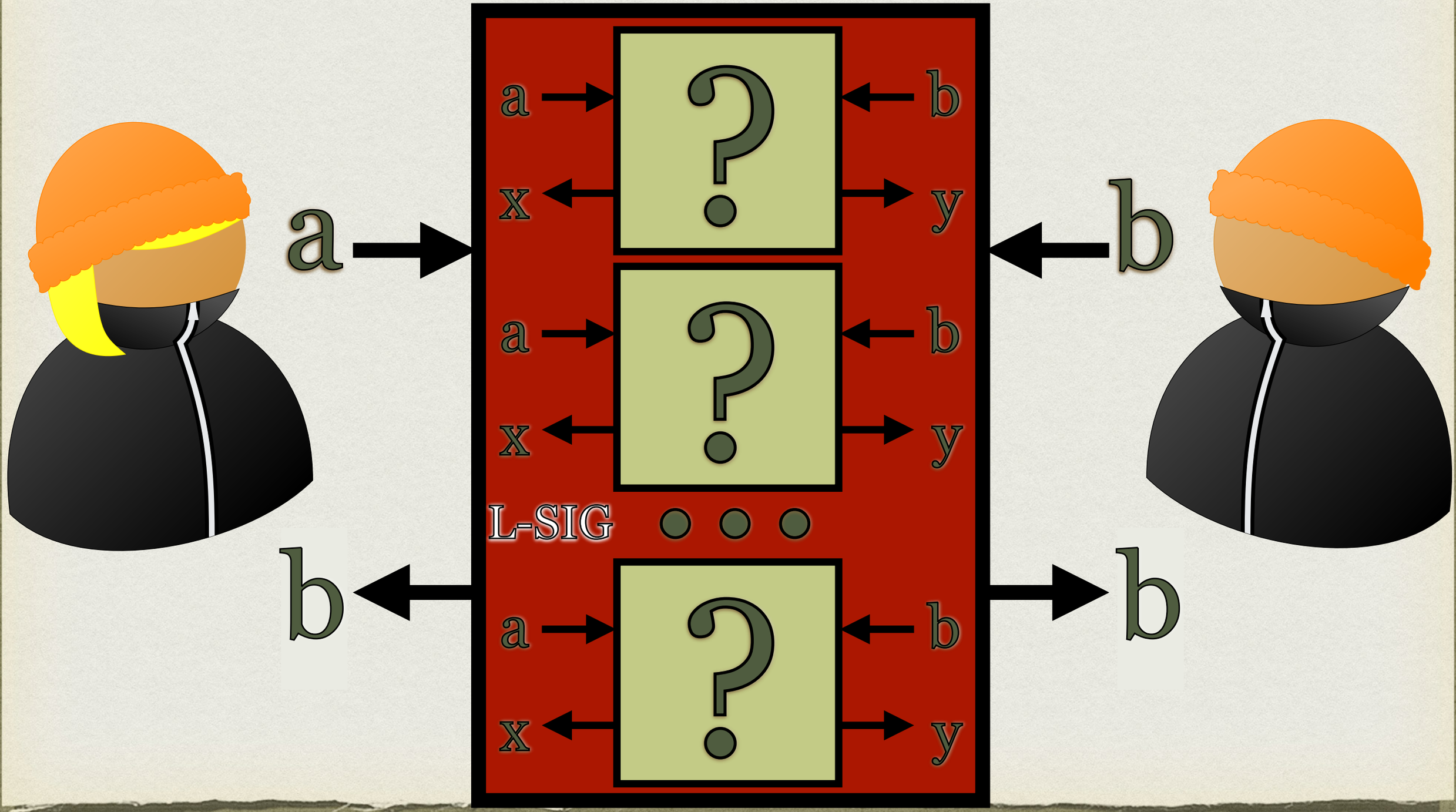

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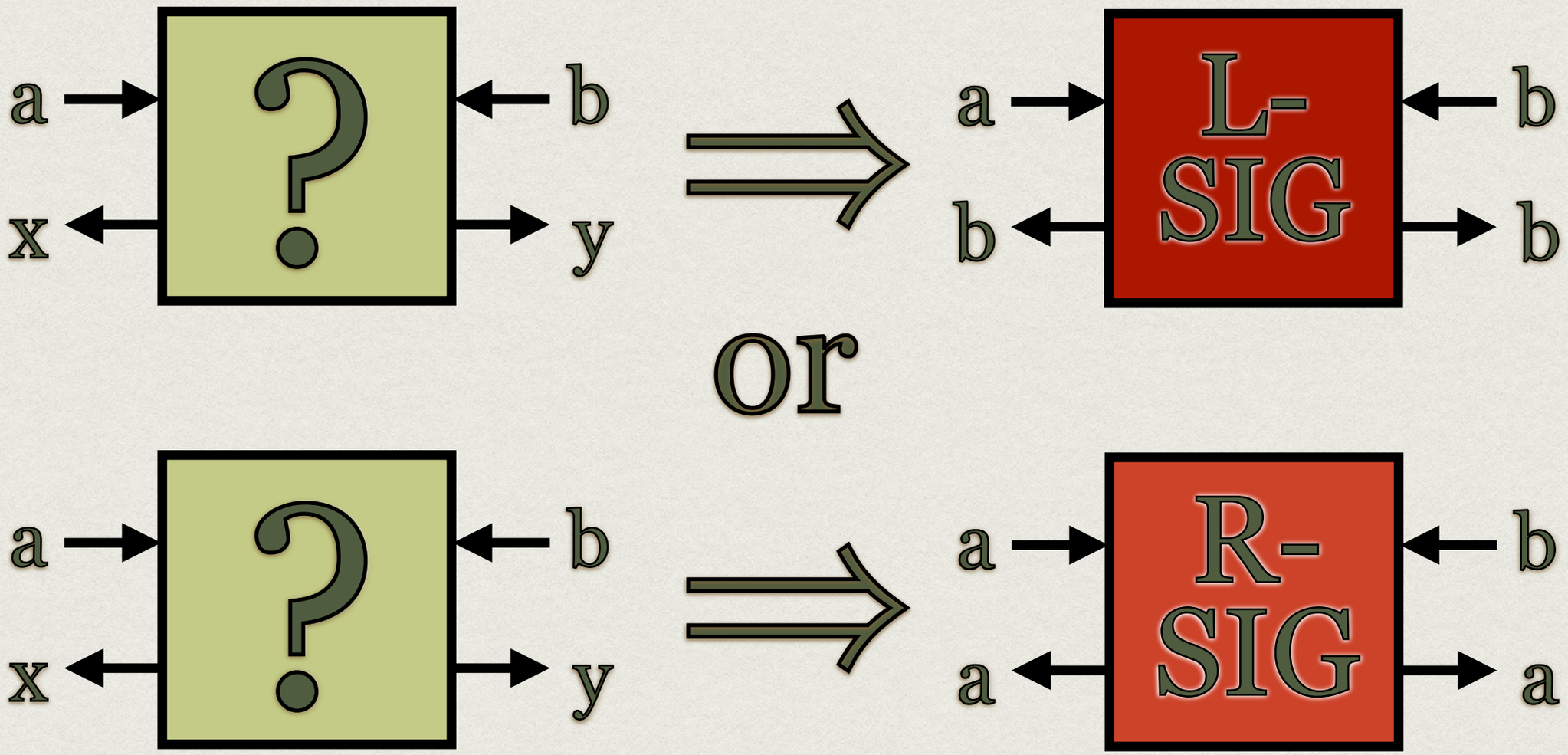

or

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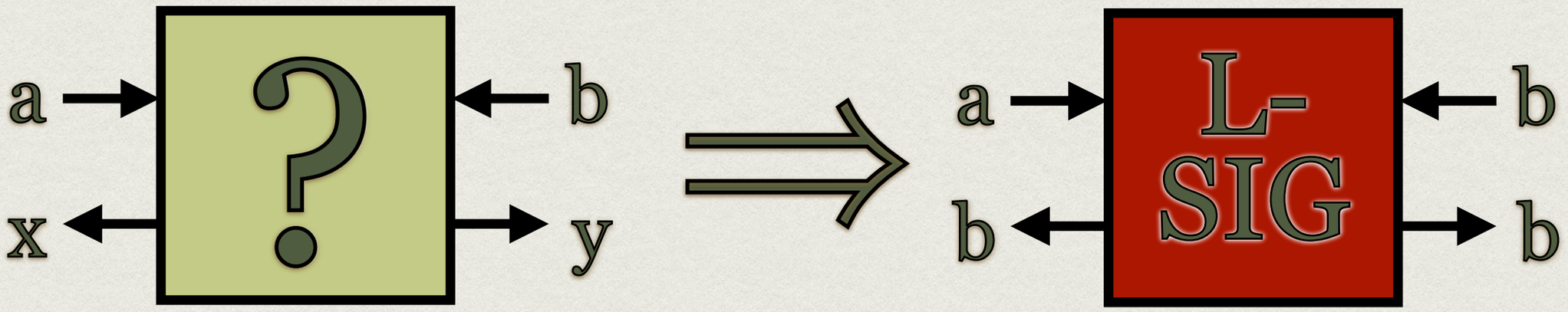

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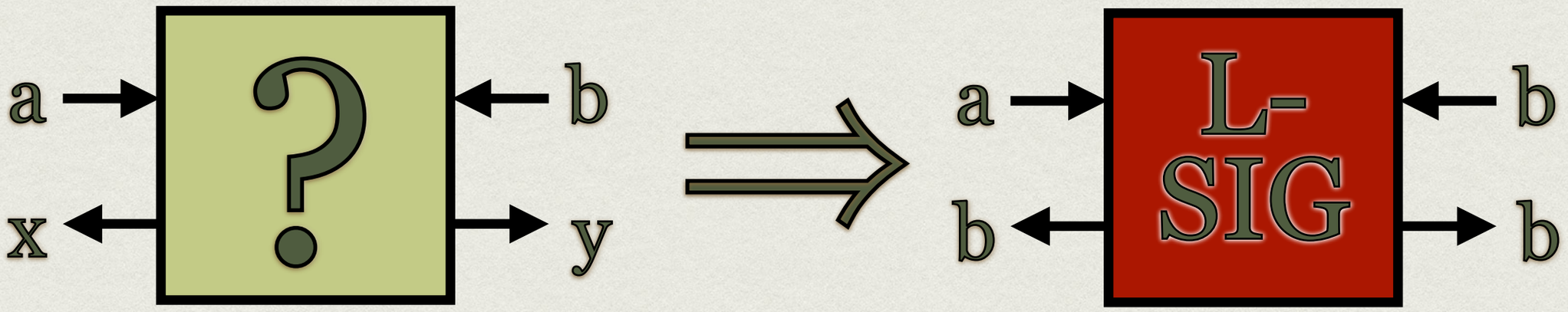

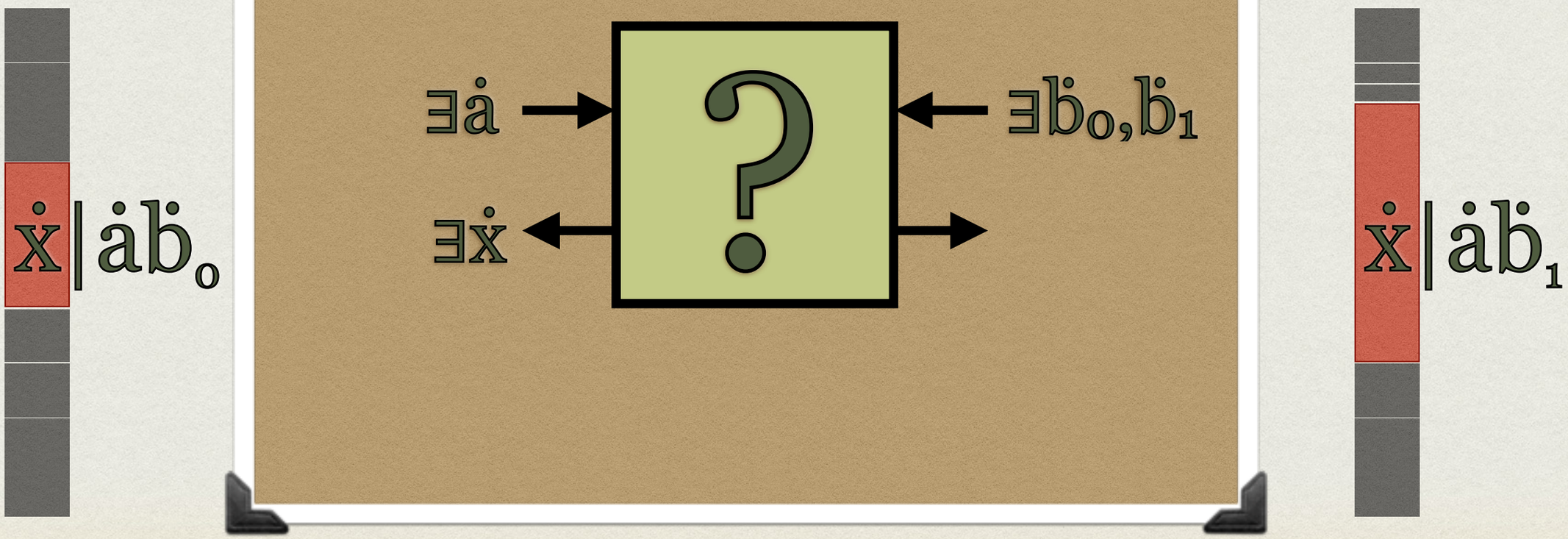

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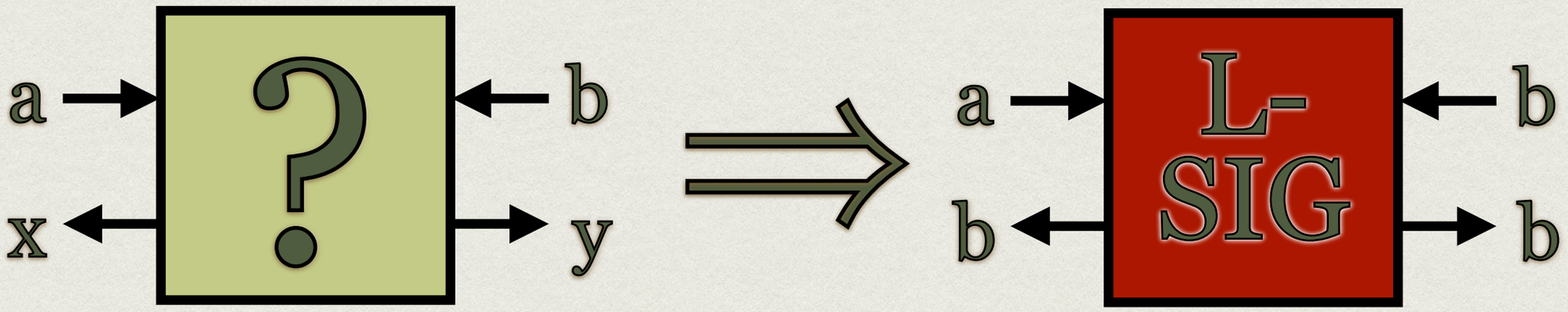
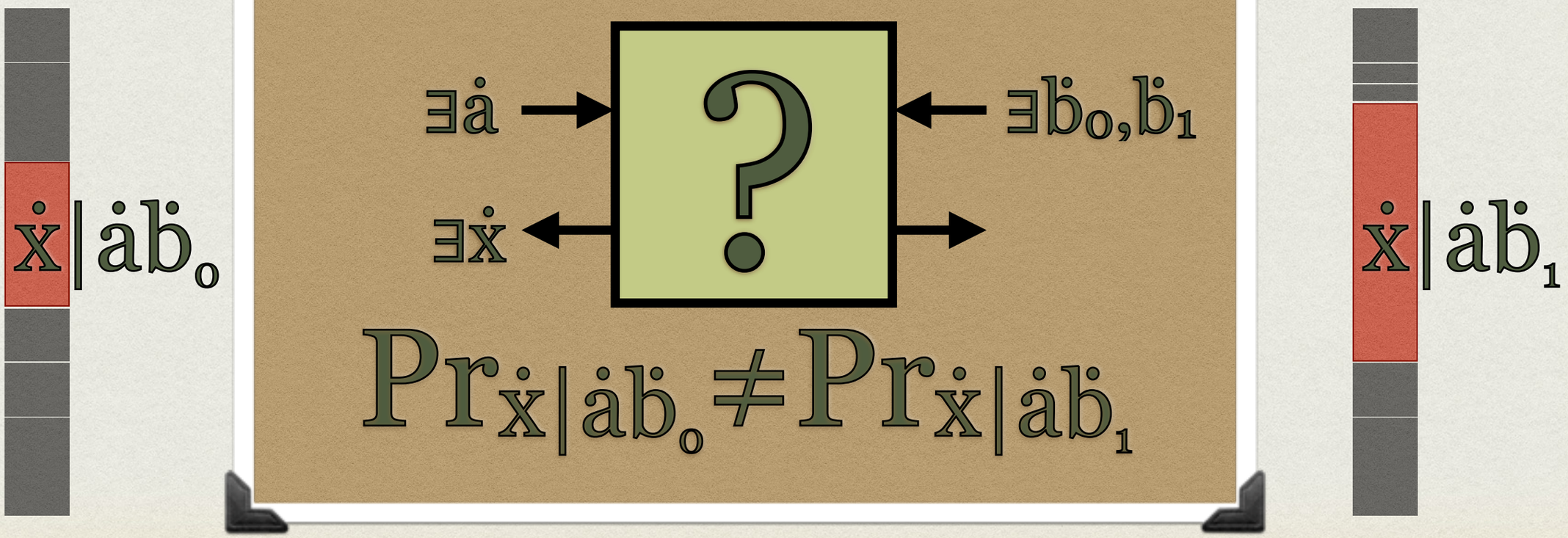
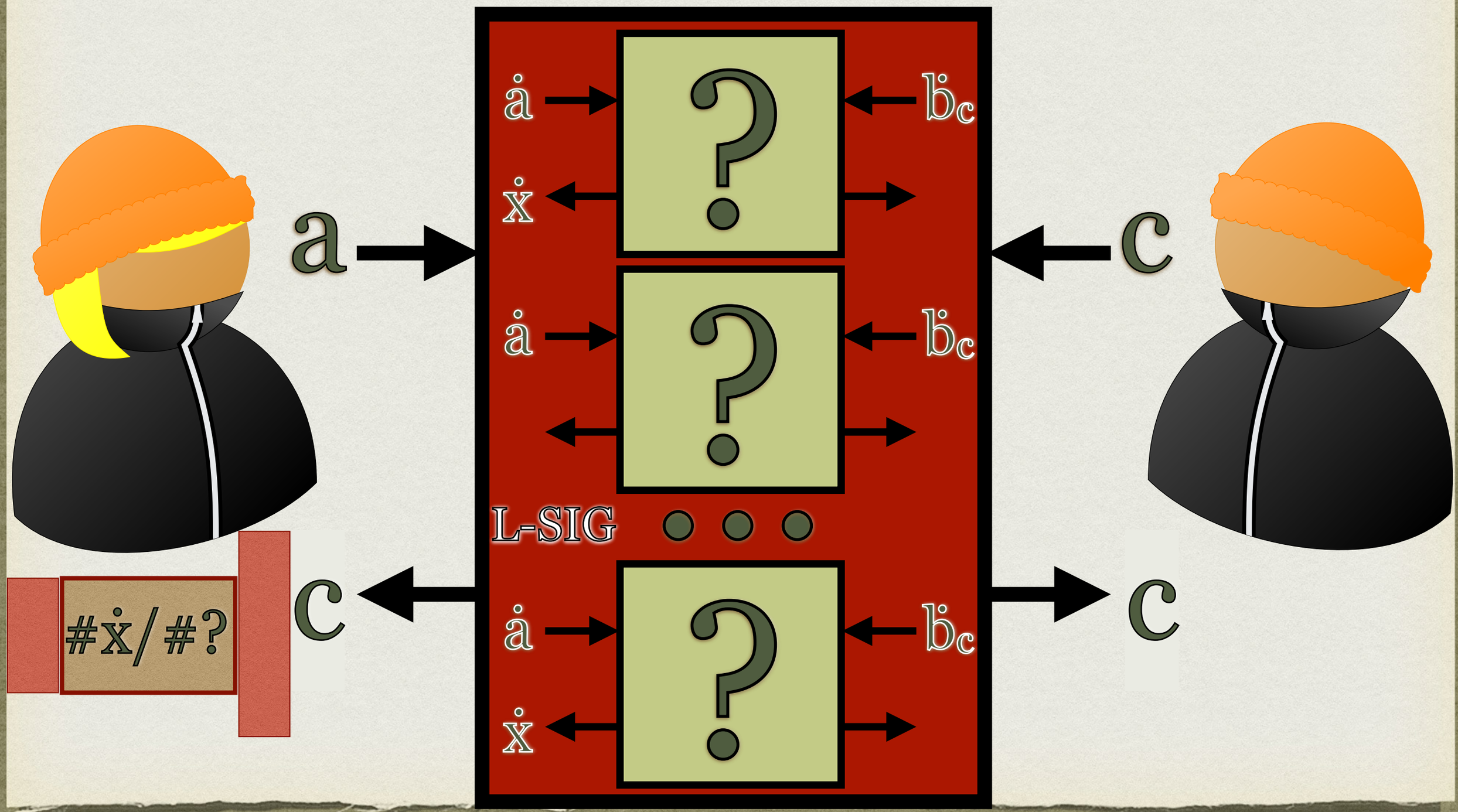


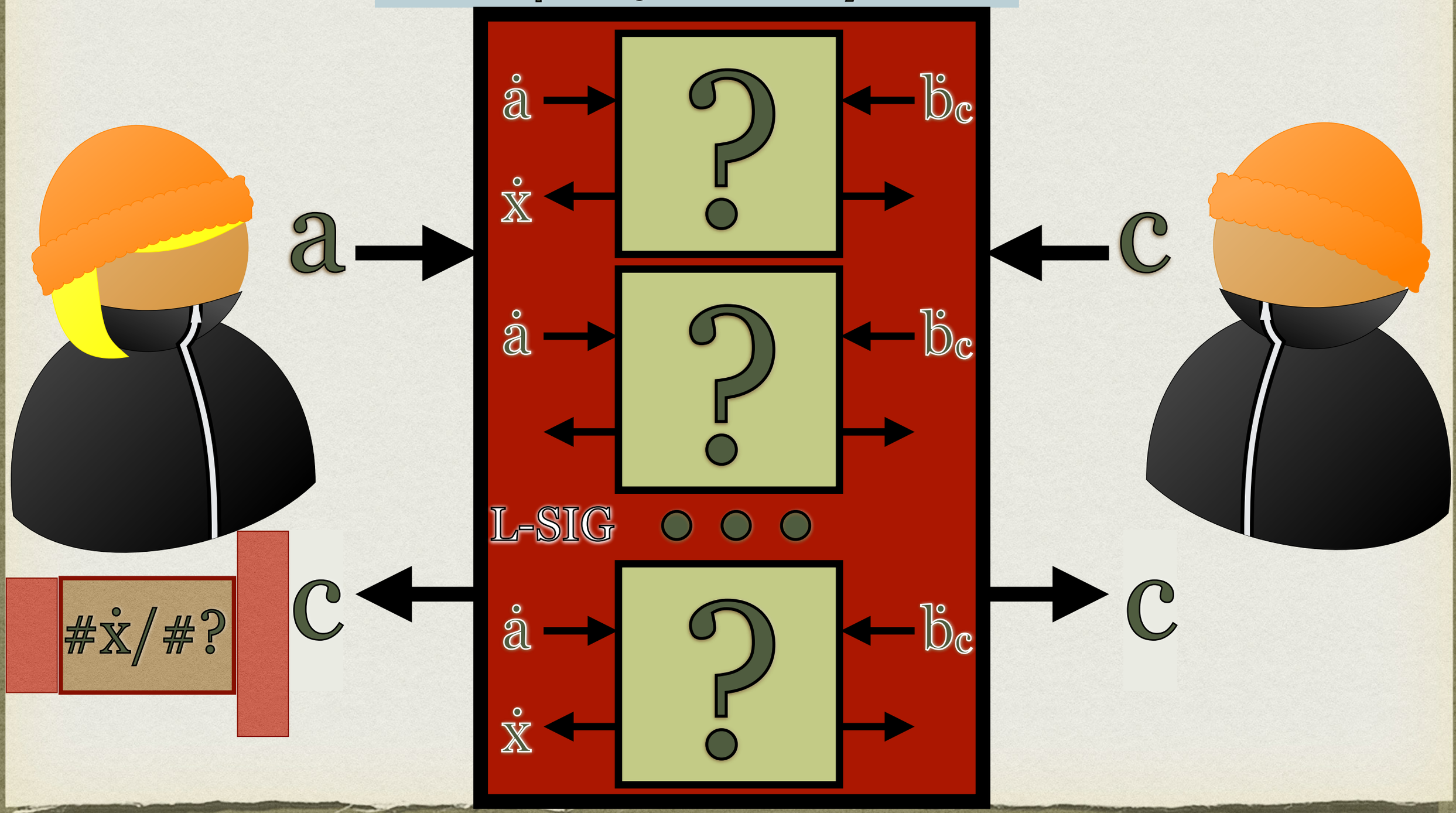


$$\exists \dot{a}\ \exists \ddot{b}_0,\ddot{b}_1\ \exists \dot{x}\ \Pr_{\dot{x}|\dot{a}\ddot{b}_0} \neq \Pr_{\dot{x}|\dot{a}\ddot{b}_1}$$



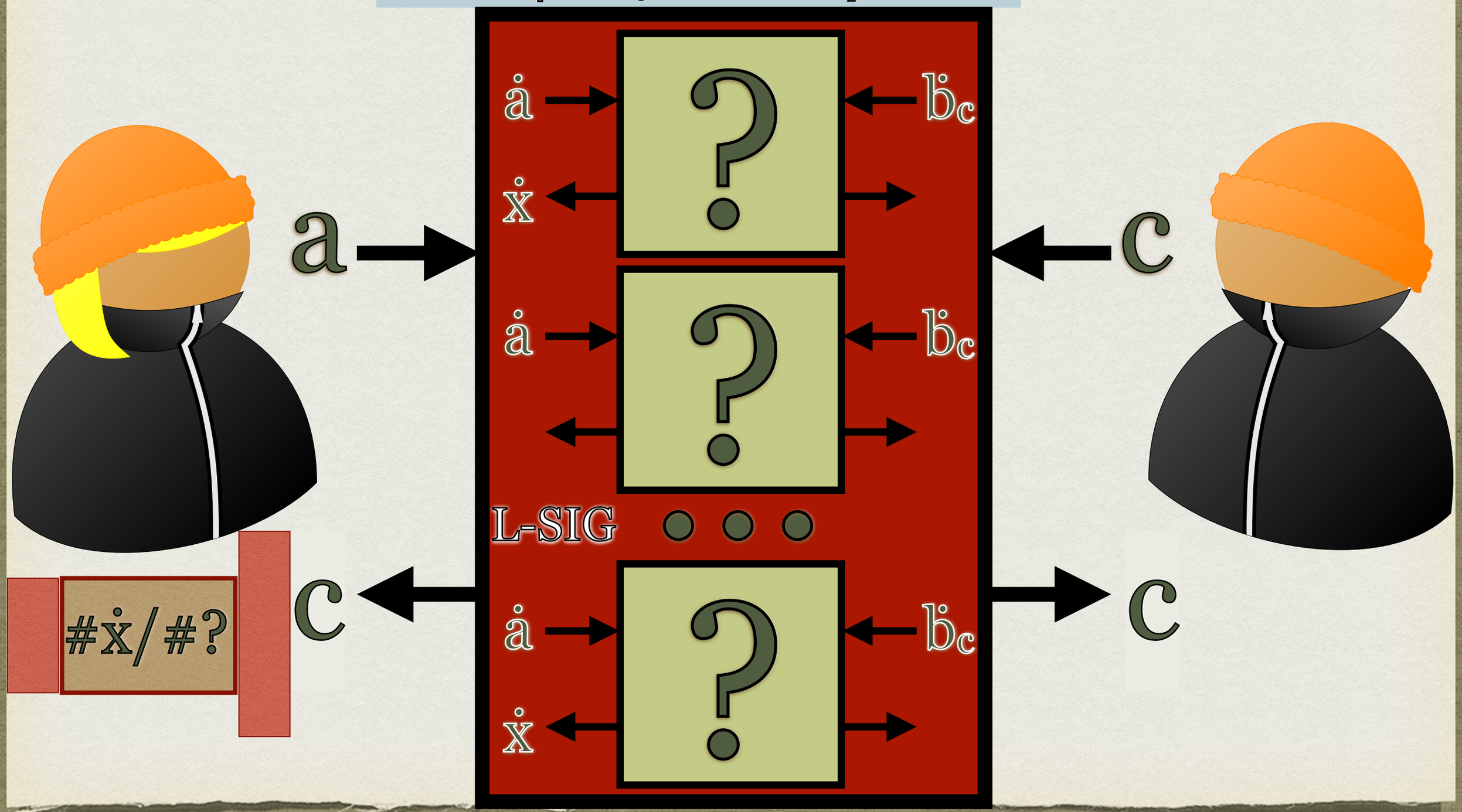
$$\exists \dot{a}\ \exists \ddot{b}_0,\ddot{b}_1\ \exists \dot{x}\ Pr_{\dot{x}|\dot{a}\ddot{b}_0} \neq Pr_{\dot{x}|\dot{a}\ddot{b}_1}$$

$$Pr_{\dot{x}|\dot{a}\ddot{b}_c} \approx \#\dot{x} / \#?$$



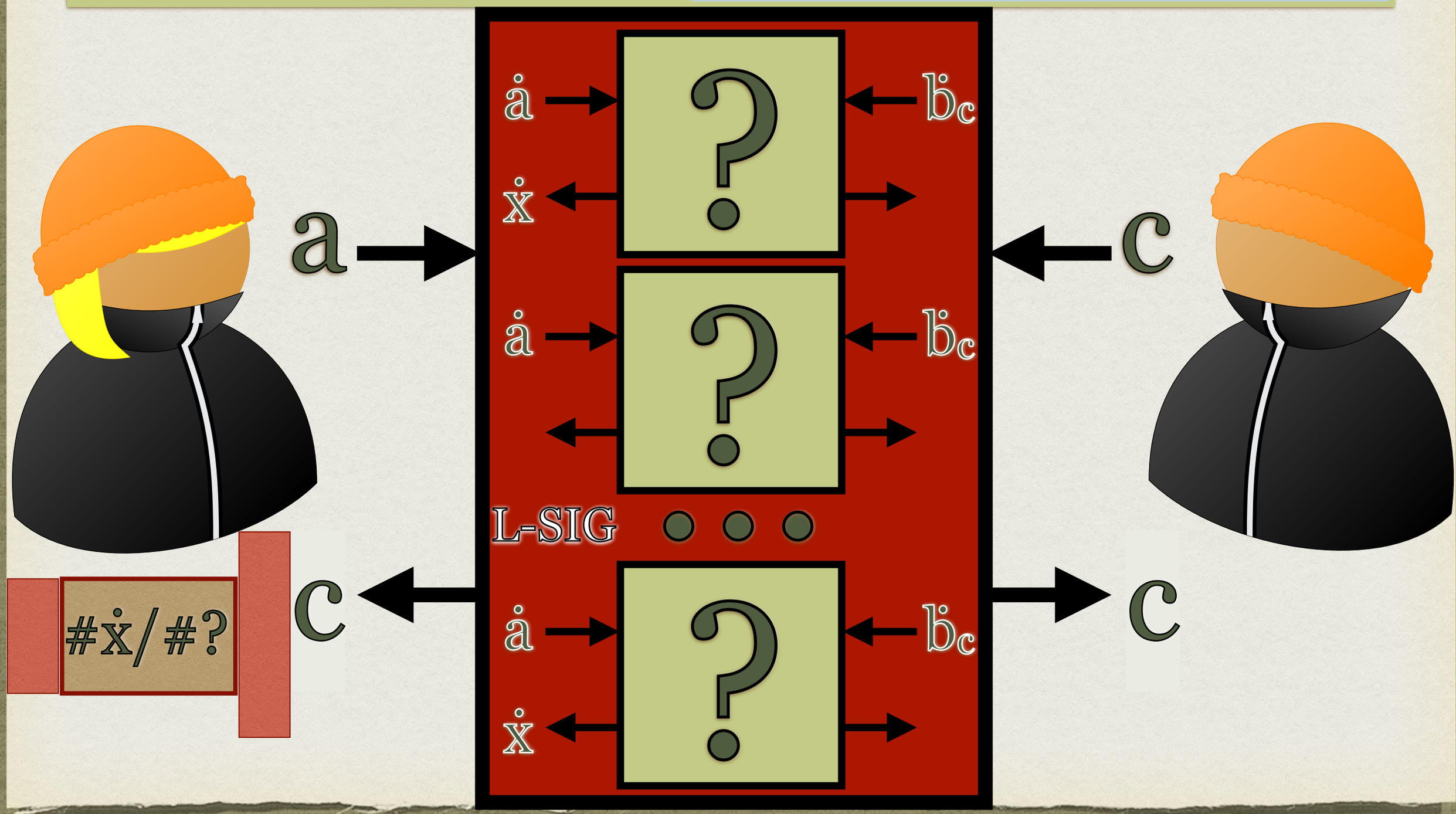
$$c{=}0 : Pr_{\dot{x}|\ddot{a}\ddot{b}_o} \simeq \#\dot{x}/\#? \not\simeq Pr_{\dot{x}|\ddot{a}\ddot{b}_1}$$

$$Pr_{\dot{x}|\ddot{a}\ddot{b}_c} \simeq \#\dot{x}/\#?$$

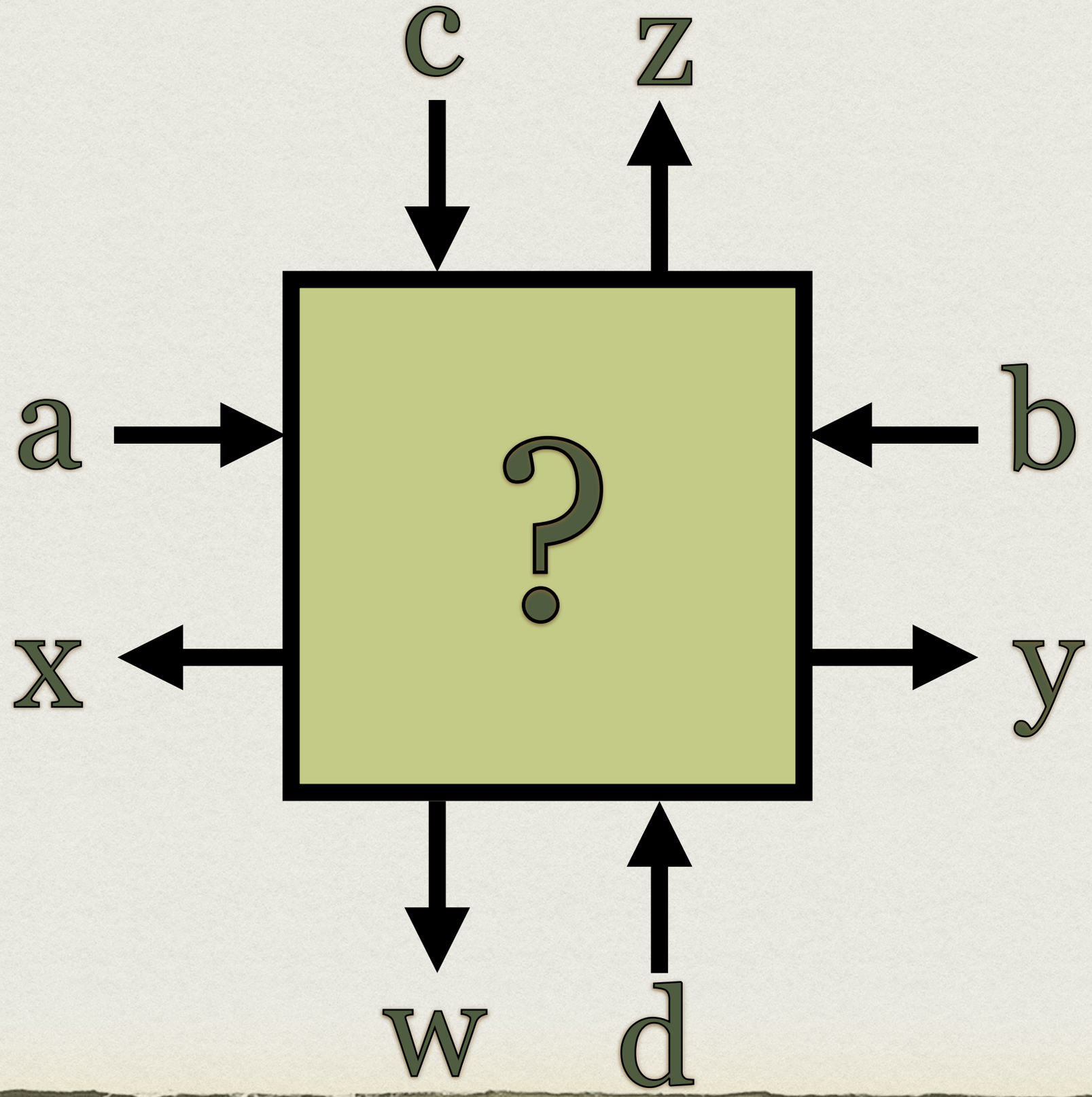


$$c{=}0 : Pr_{\dot{x}|\ddot{a}\ddot{b}_o} \simeq \#\dot{x}/\#? \not\simeq Pr_{\dot{x}|\ddot{a}\ddot{b}_1}$$

$$c{=}1 : Pr_{\dot{x}|\ddot{a}\ddot{b}_o} \not\simeq \#\dot{x}/\#? \simeq Pr_{\dot{x}|\ddot{a}\ddot{b}_1}$$



GENERAL SIGNALLING



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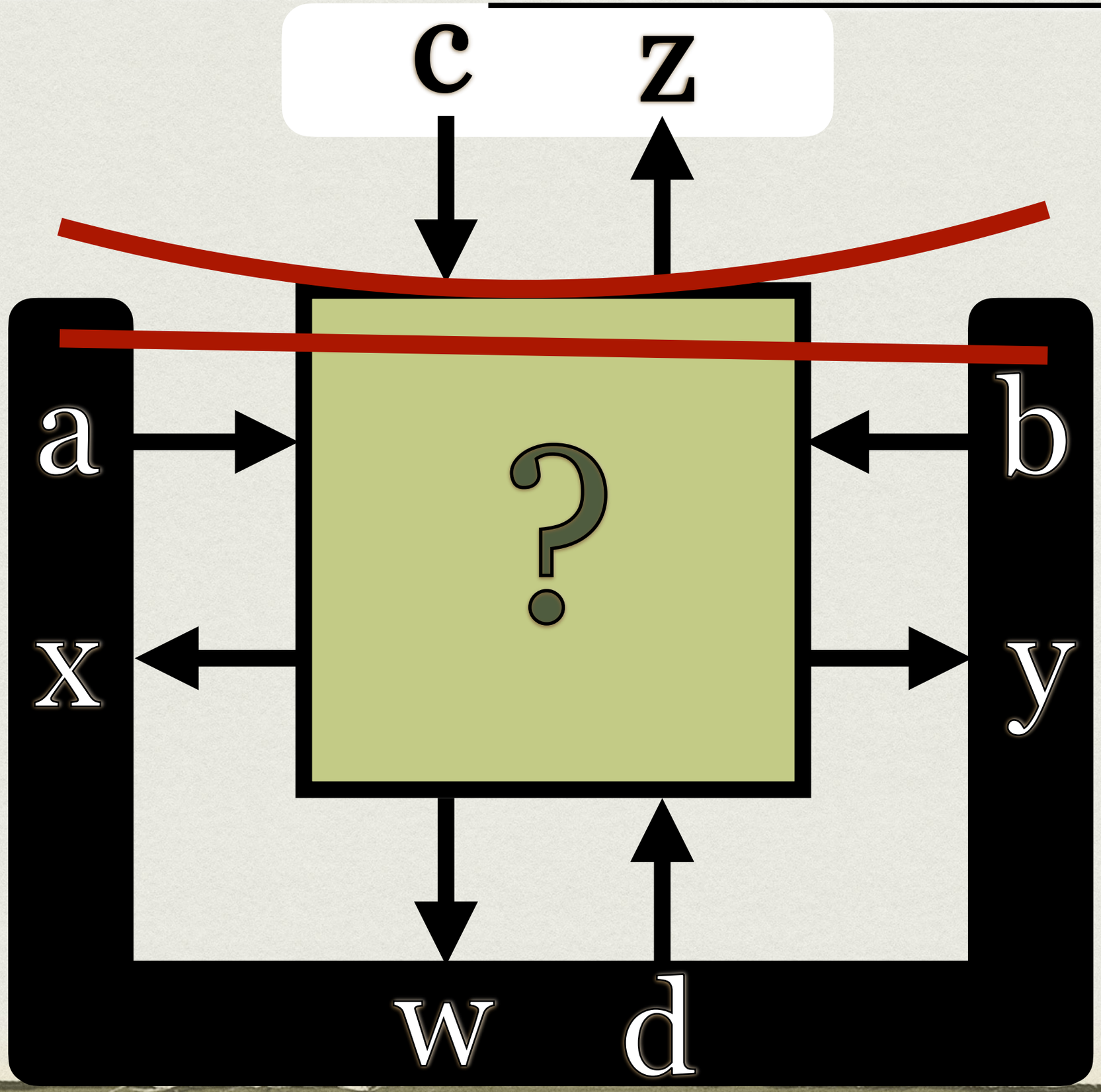

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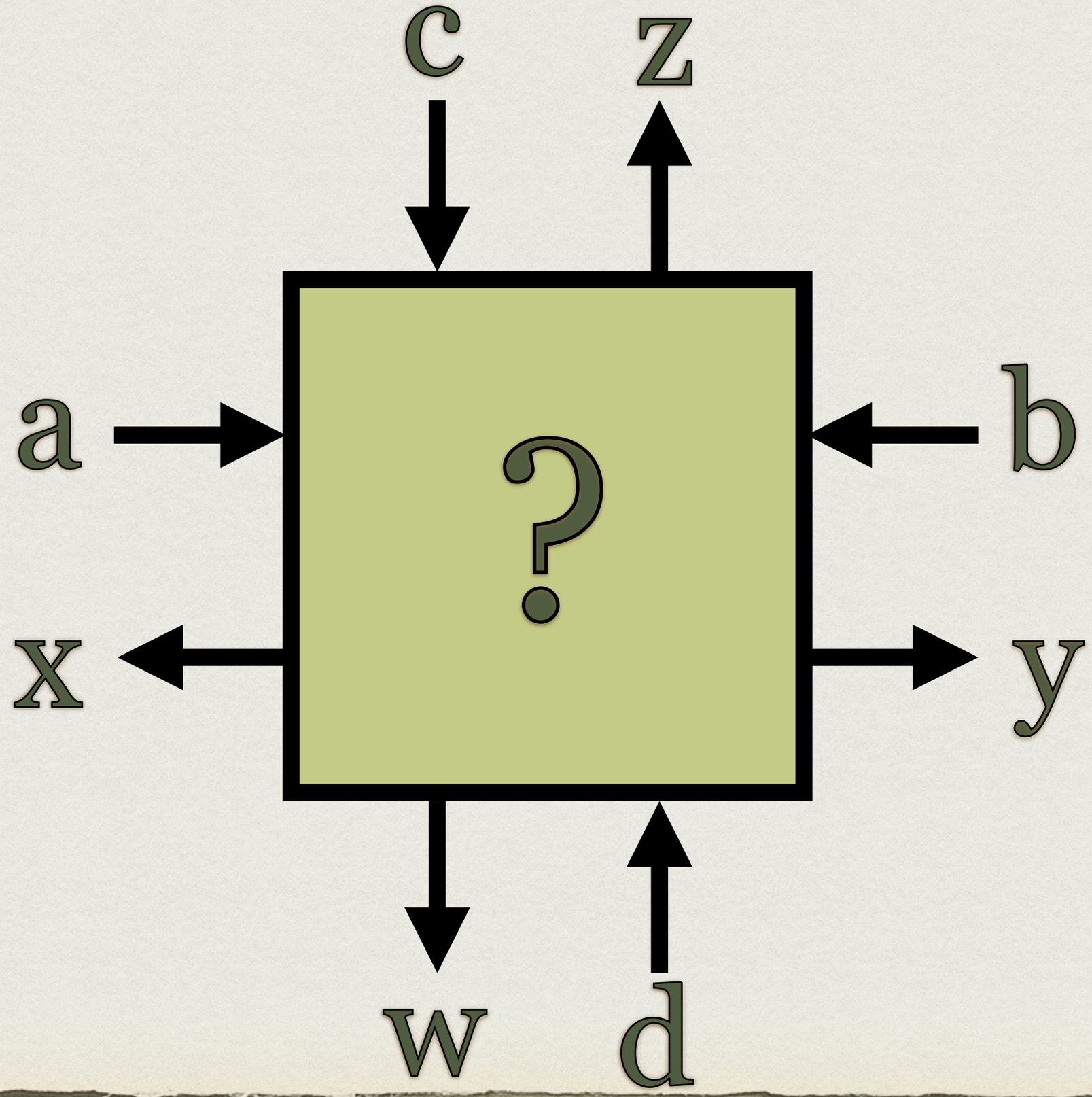

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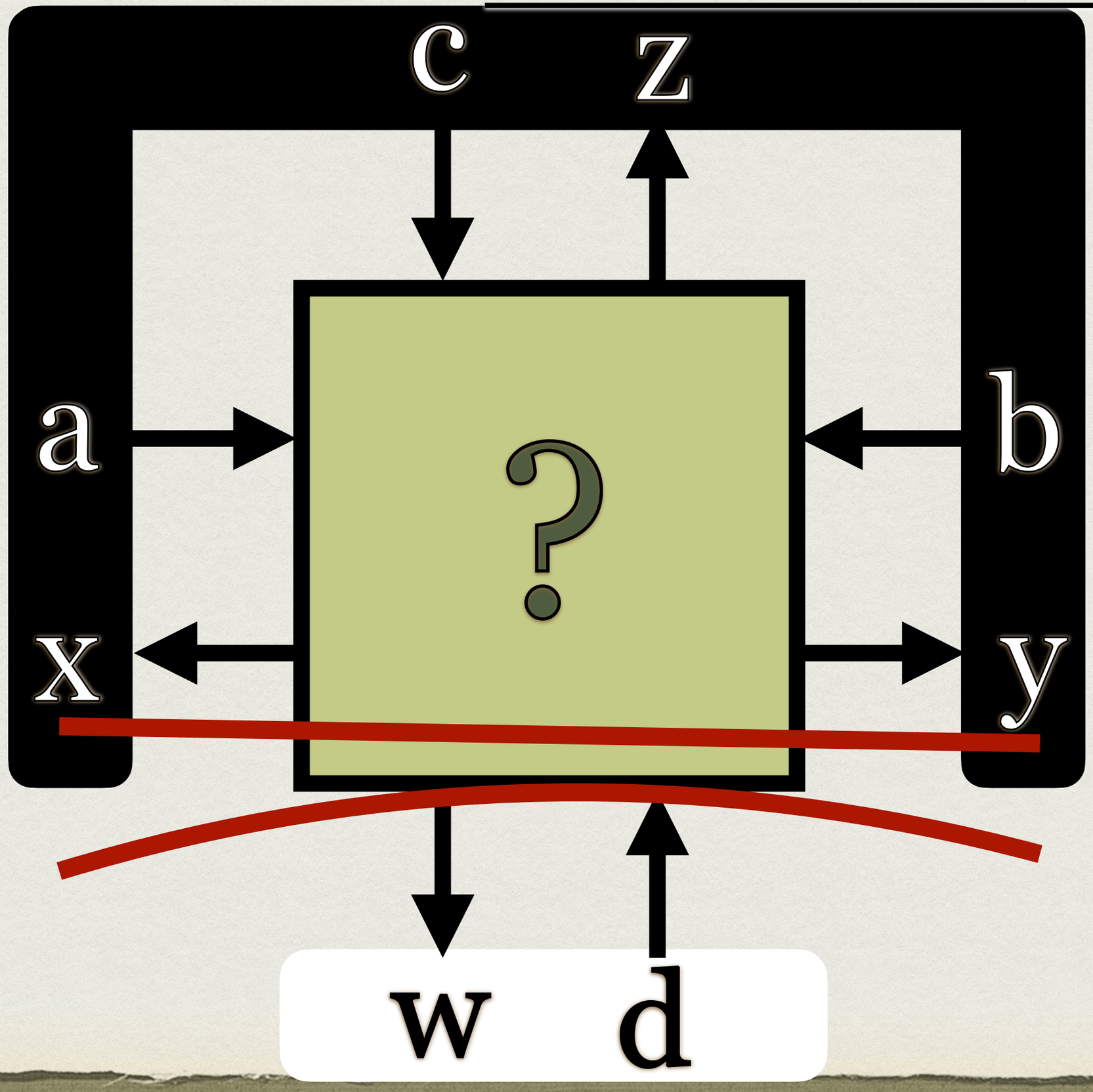

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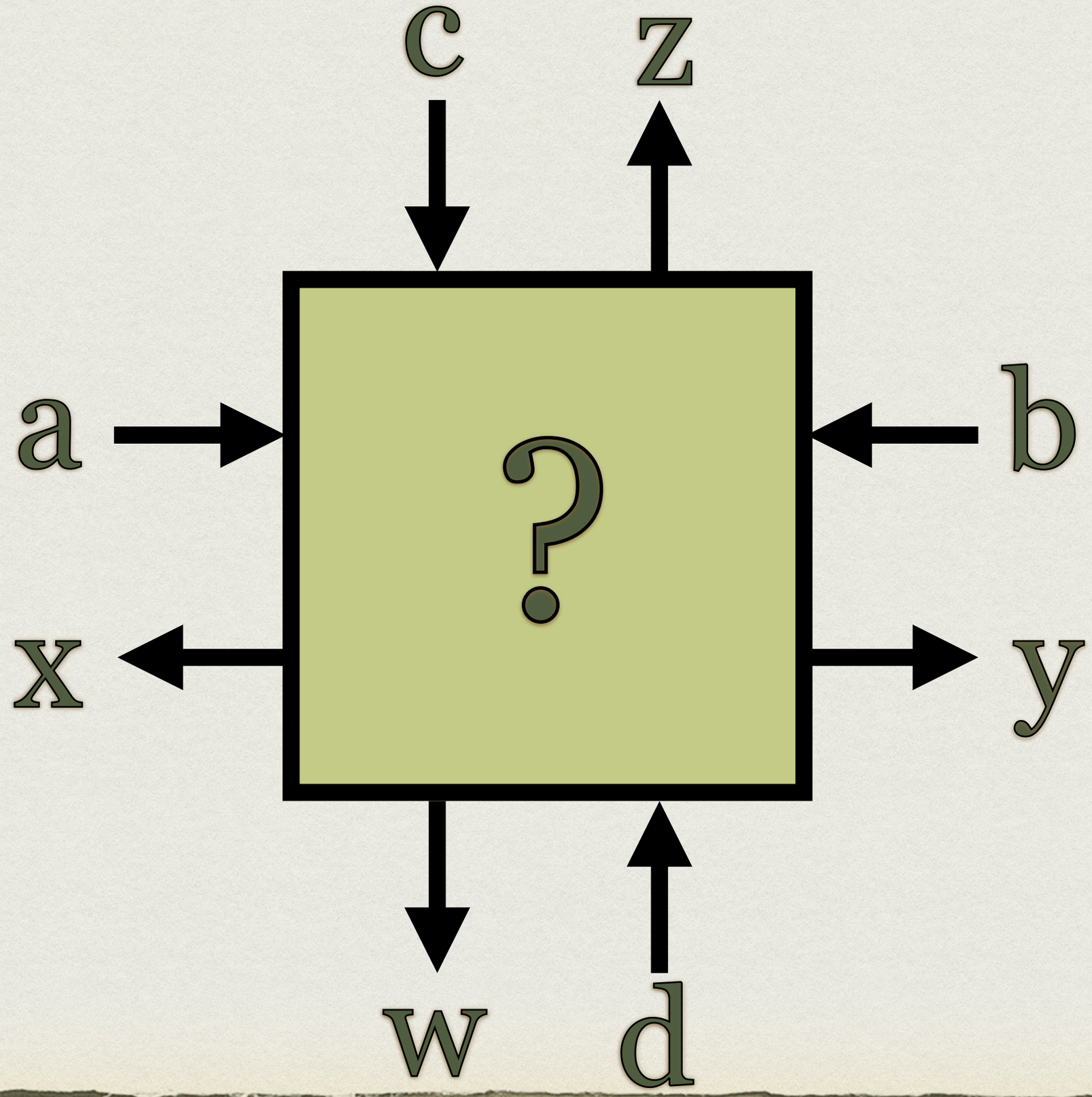

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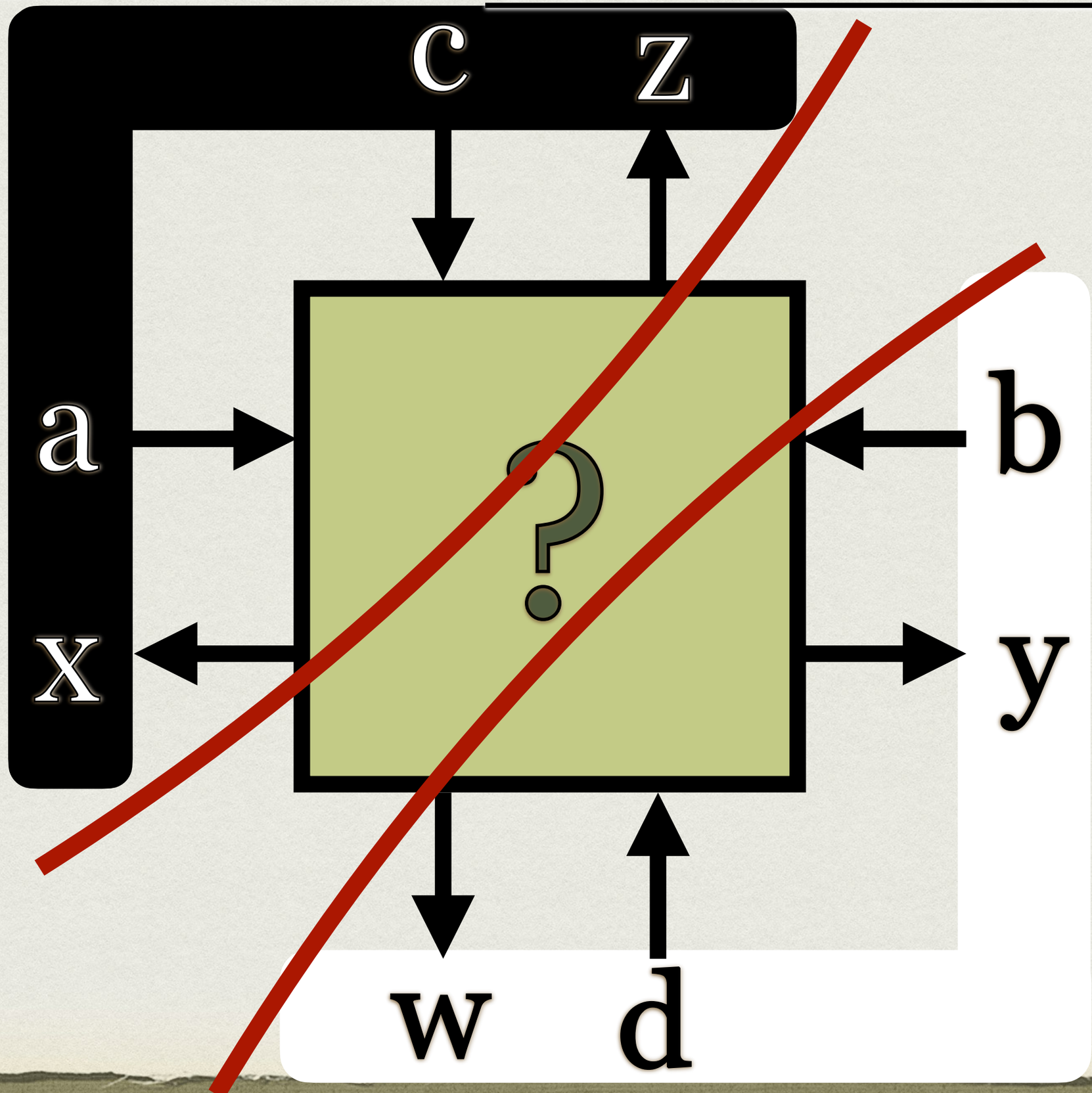
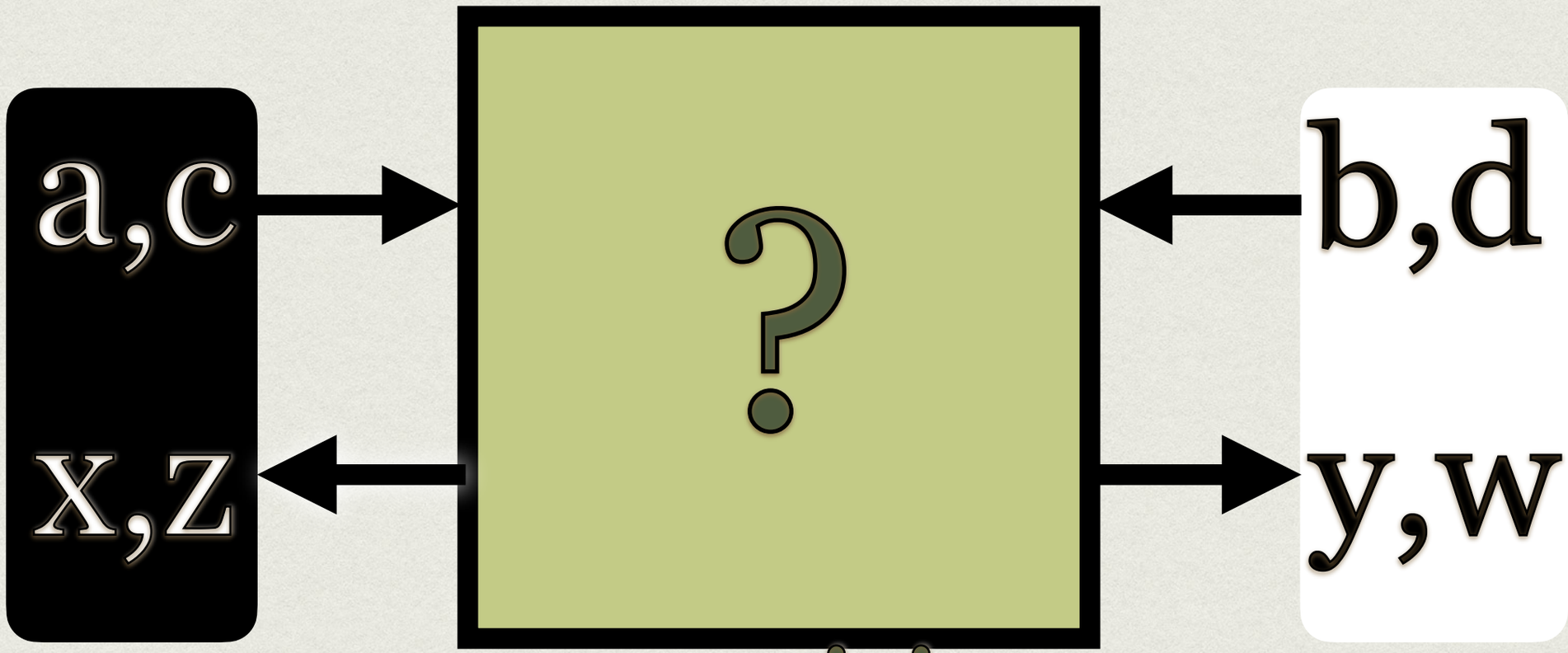


WHAT CONSTITUTES A SIGNALLING CHANNEL?



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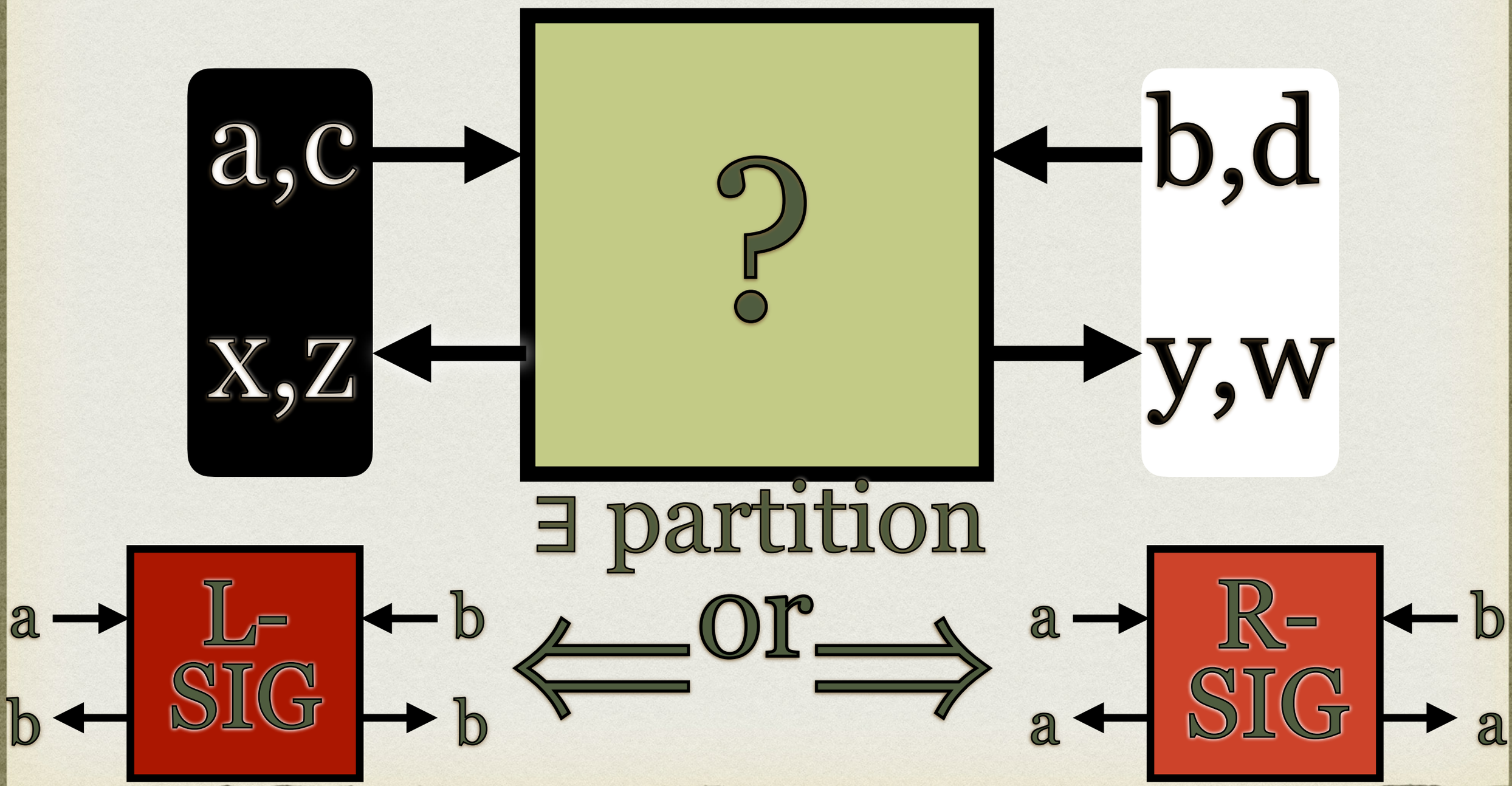
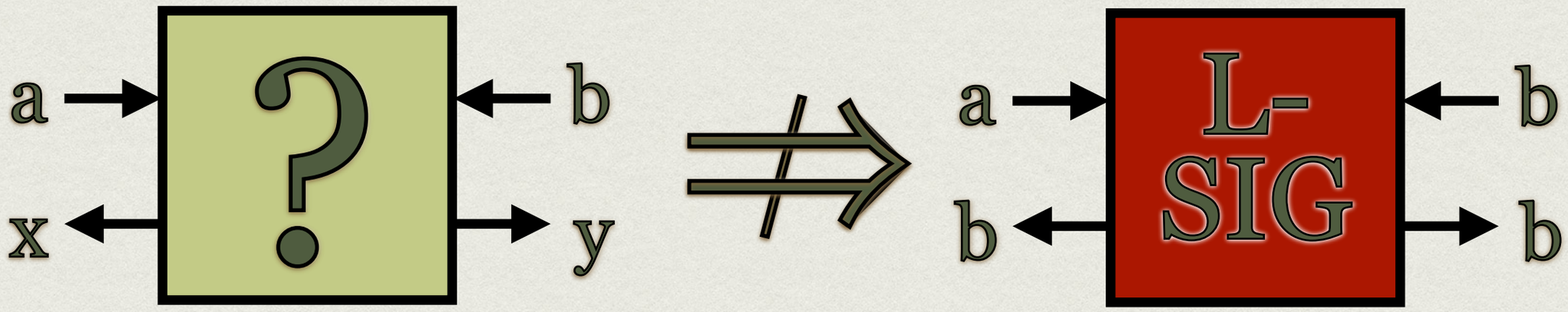


WHAT CONSTITUTES A NO-SIGNALLING CHANNEL?

WHAT CONSTITUTES A NO-SIGNALLING CHANNEL?



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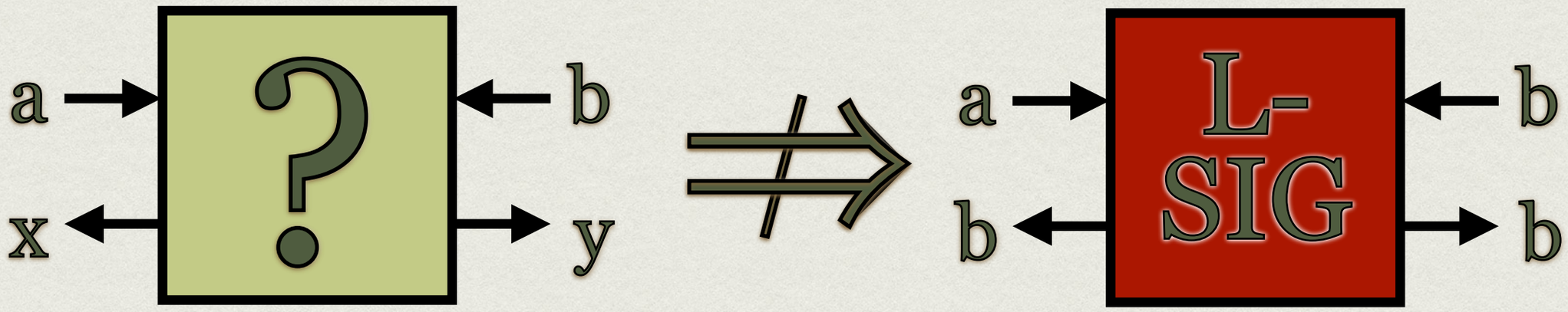
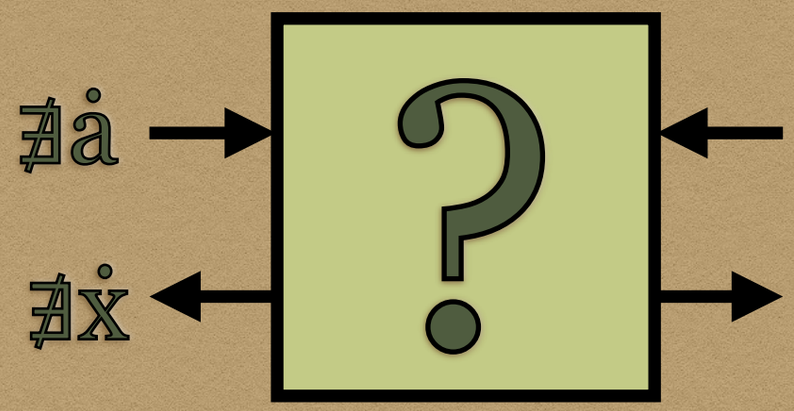
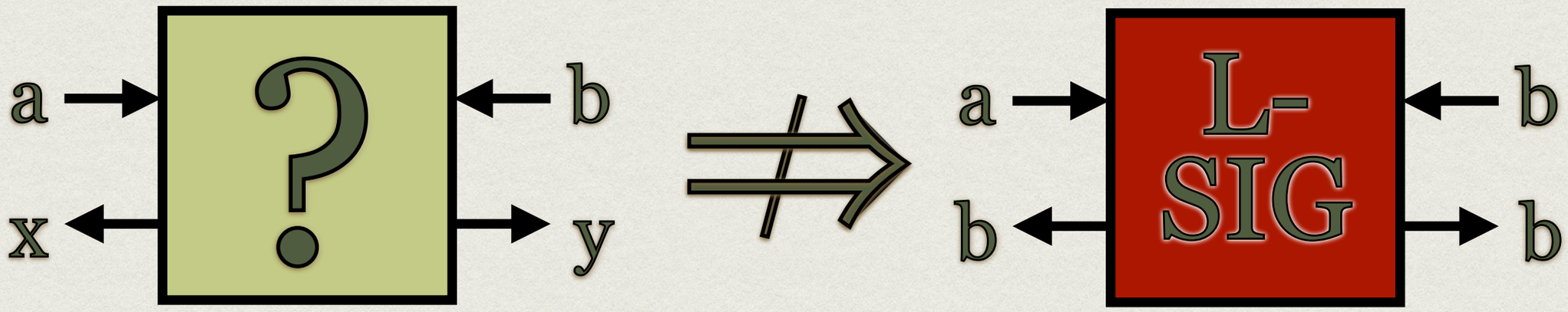




$$Pr_{\dot{x}|\dot{a}\dot{b}_0} \neq Pr_{\dot{x}|\dot{a}\dot{b}_1}$$

WHAT CONSTITUTES A NO-SIGNALLING CHANNEL?



$$\forall \dot{a} \quad \forall \dot{x} \quad ? \quad \forall \ddot{b}_0, \ddot{b}_1$$

$$Pr_{\dot{x}|\dot{a}\ddot{b}_0} = Pr_{\dot{x}|\dot{a}\ddot{b}_1}$$

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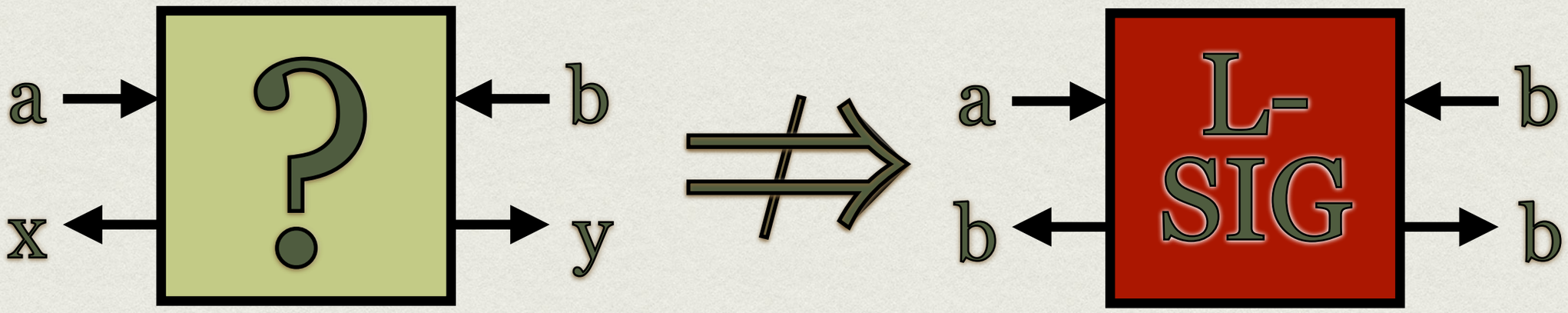

$\dot{x}|\dot{a}\ddot{b}_0$

$\forall\dot{a}$
$\forall\dot{x}$
?
$\forall\ddot{b}_0,\ddot{b}_1$

$$Pr_{\dot{x}|\dot{a}\ddot{b}_0} = Pr_{\dot{x}|\dot{a}\ddot{b}_1}$$

$\dot{x}|\dot{a}\ddot{b}_1$

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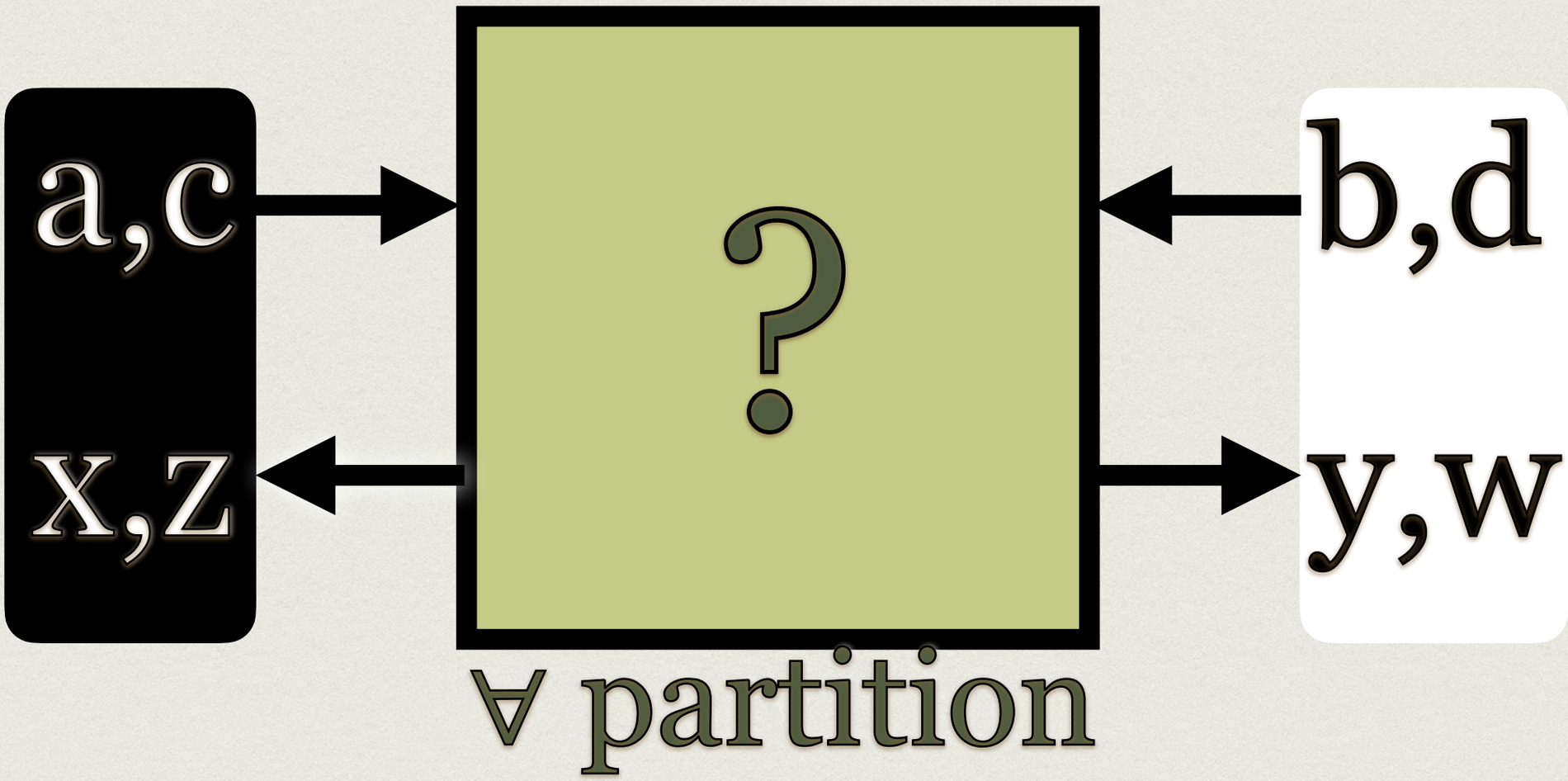

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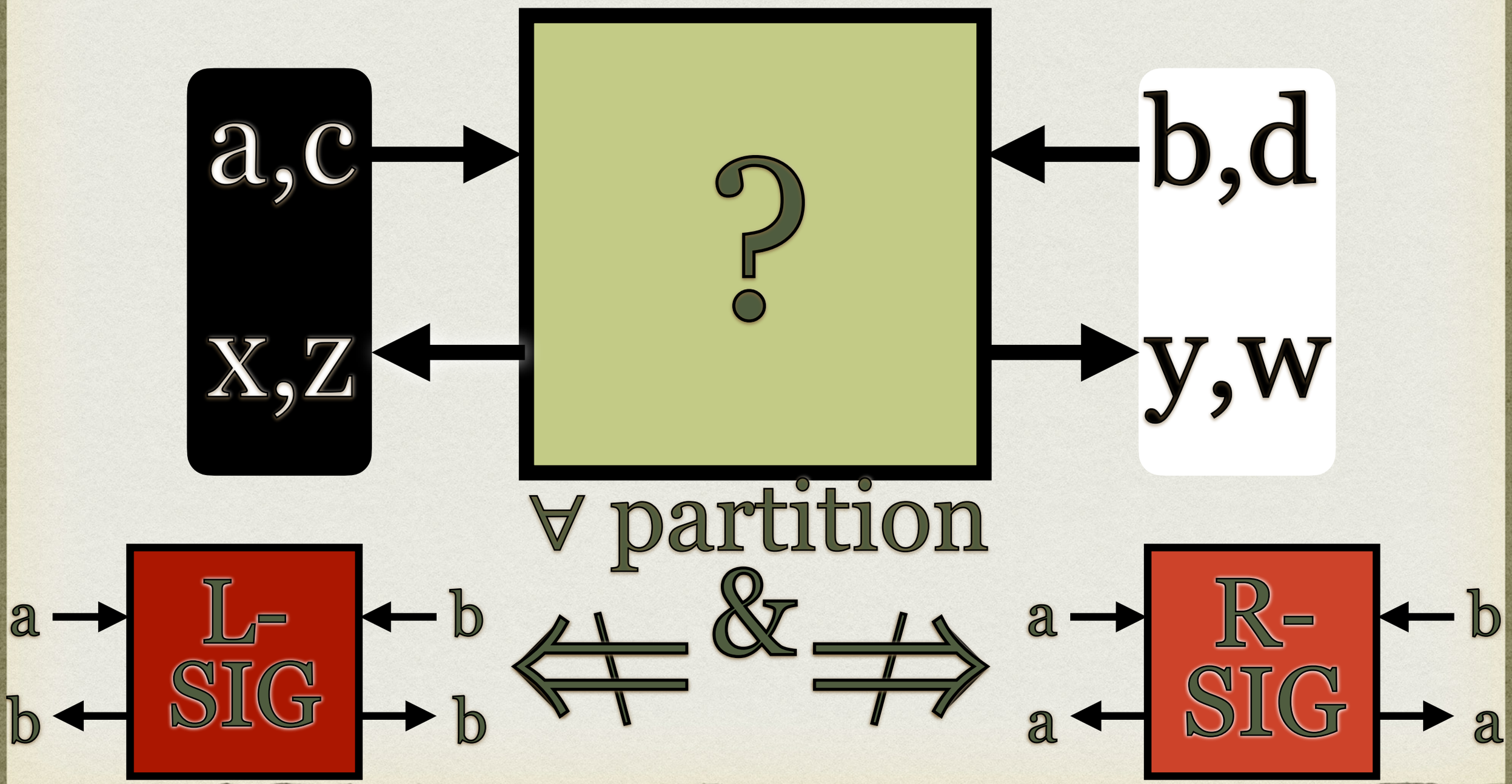


NO-SIGNALLING CHANNELS

A=B=X=Y={●,●,●}

a≠x≠y≠b

a

x

RGB

b

y

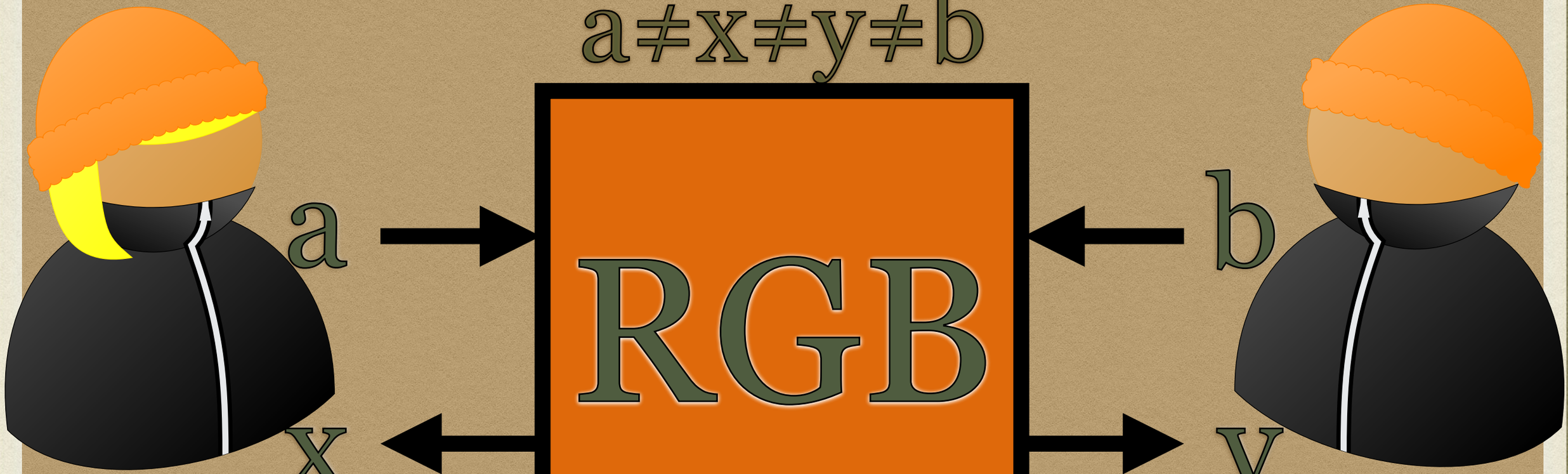
random x≠a

random y≠b≠x

NO-SIGNALLING CHANNELS

$A=B=X=Y=\{●,●,●\}$

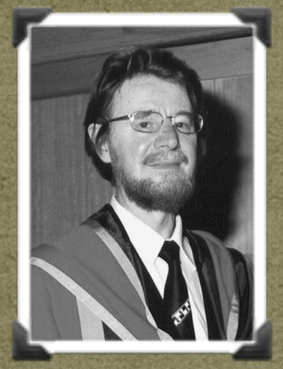
$a \neq x \neq y \neq b$



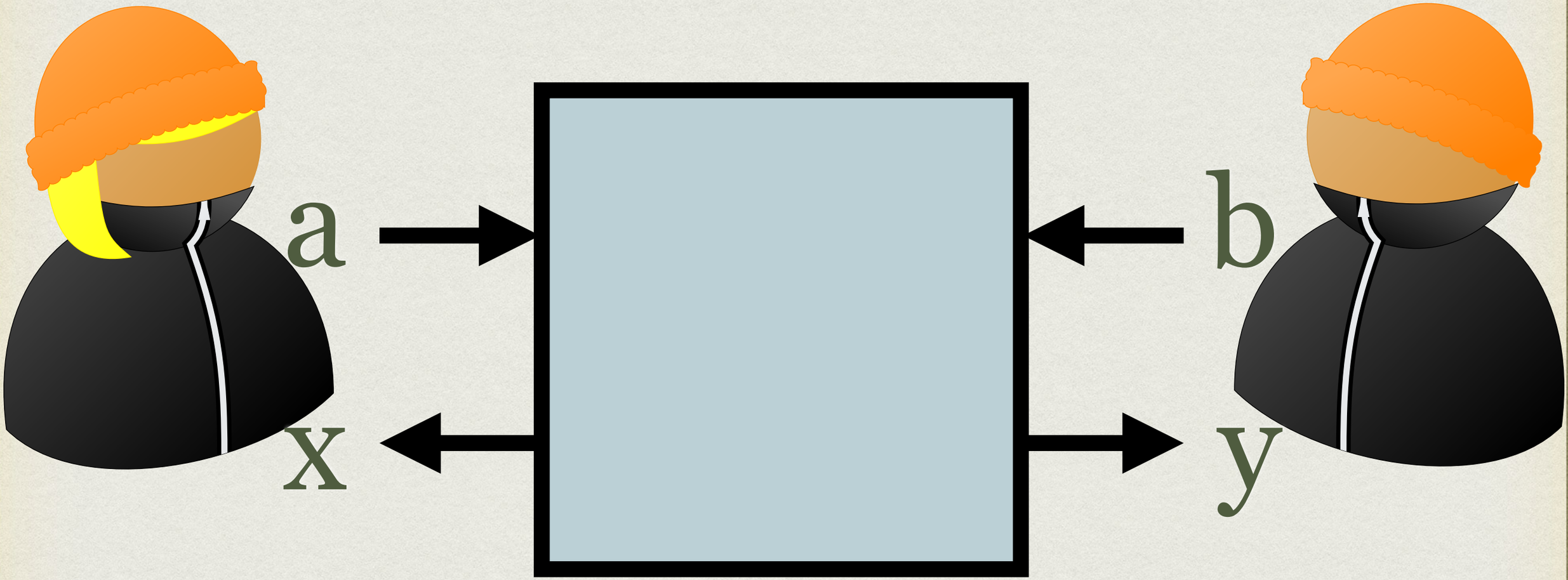
random $x \neq a \neq y$

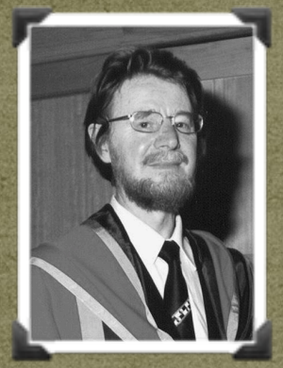
random $y \neq b$

NON-LOCALITY HIERARCHY



LOCALITY





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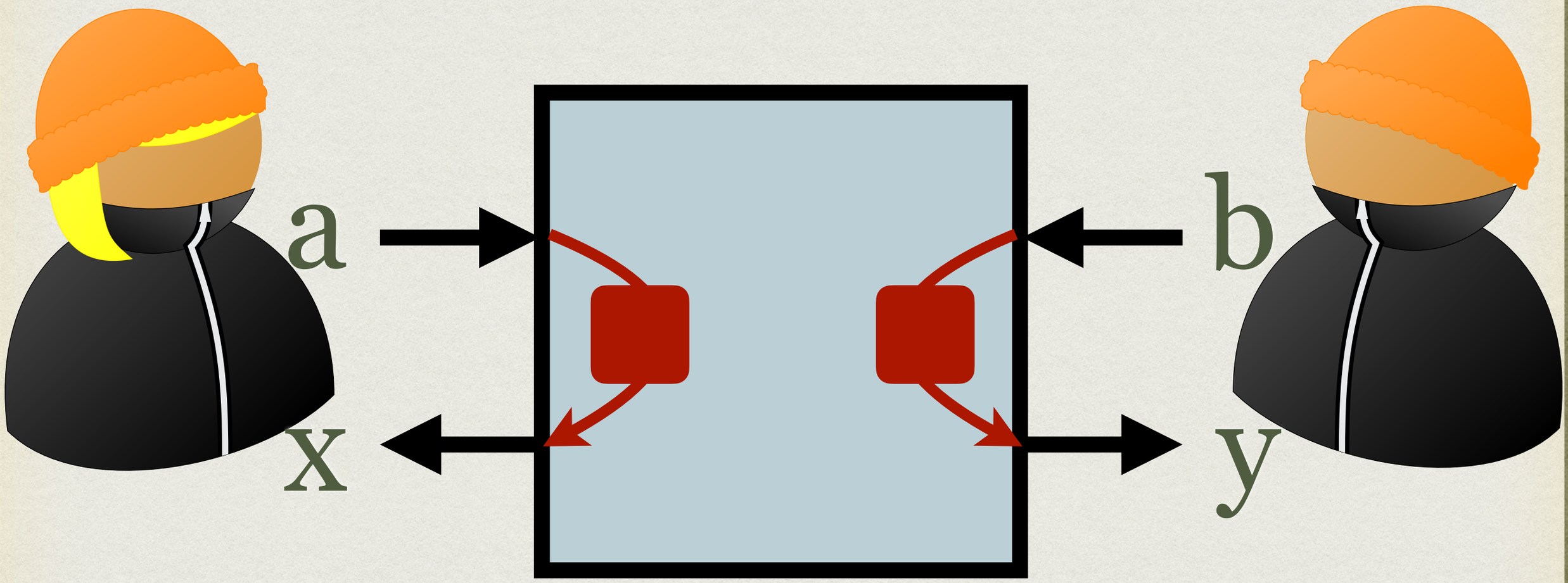

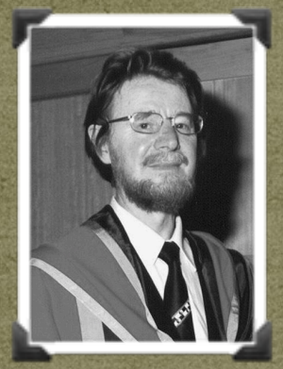

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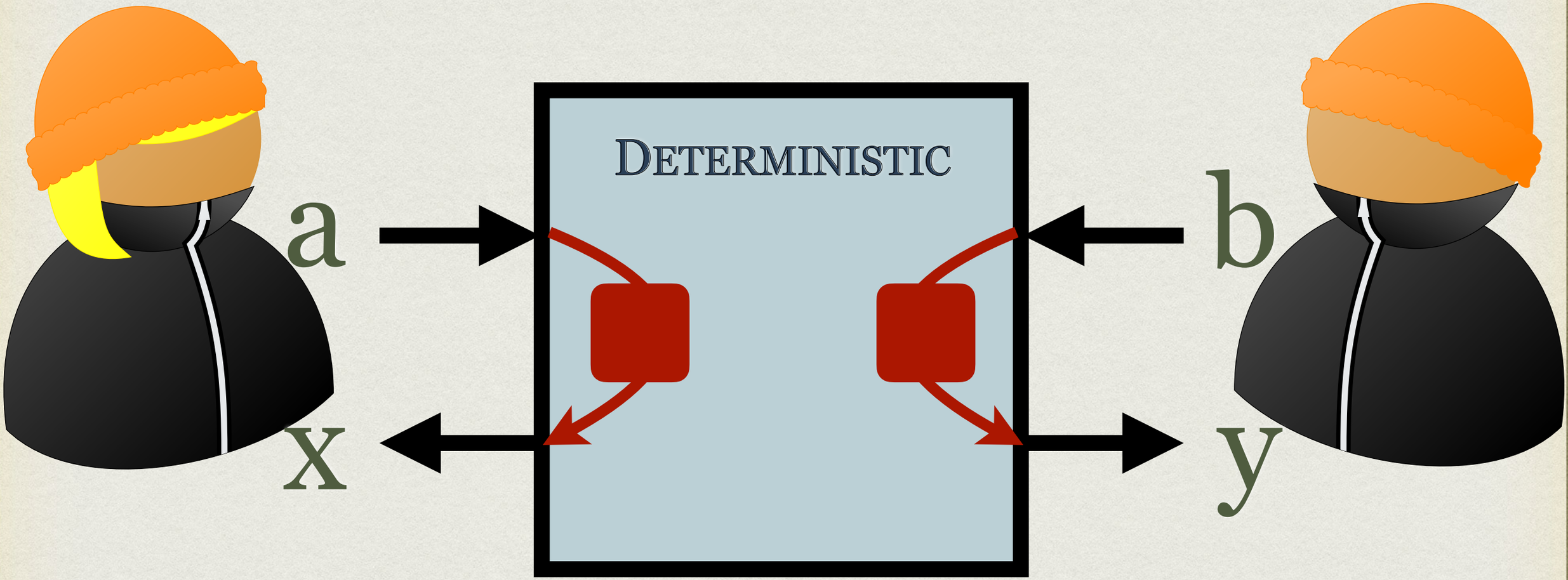

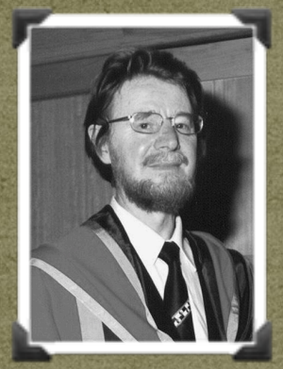

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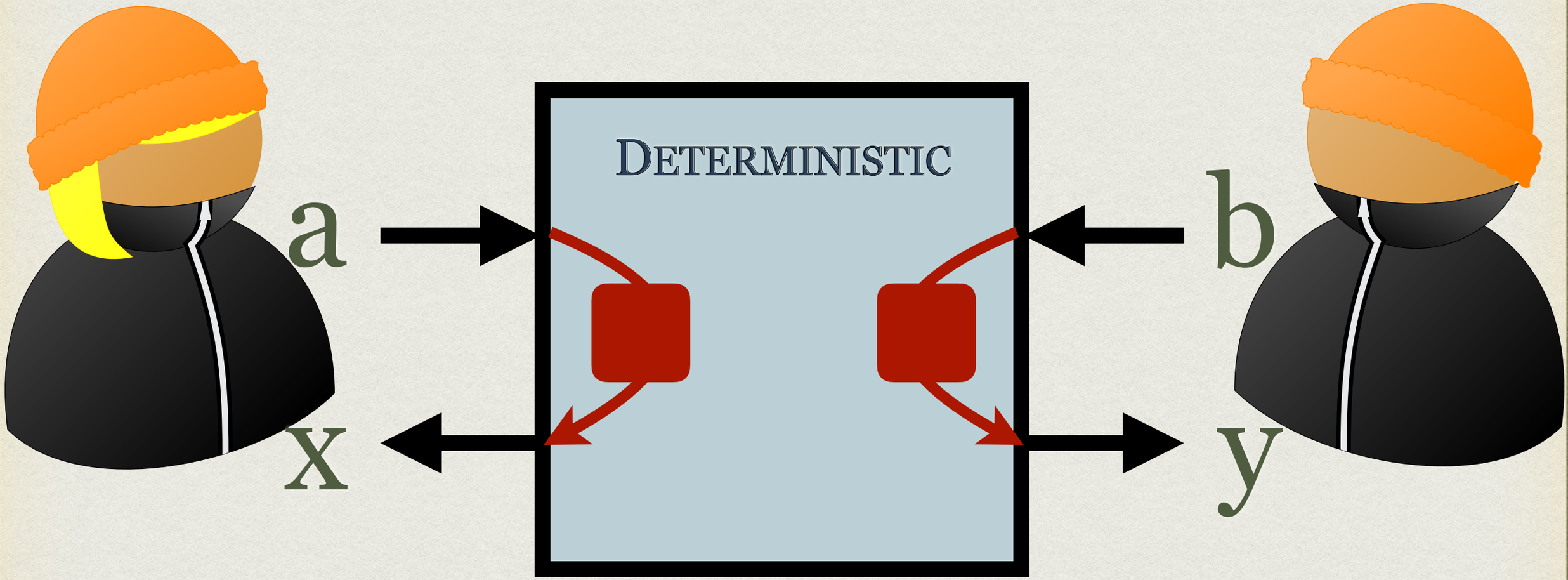

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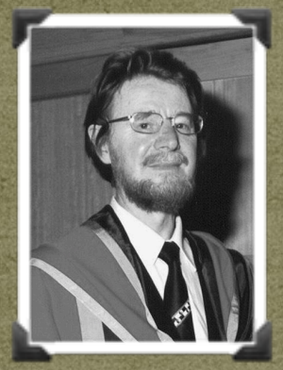

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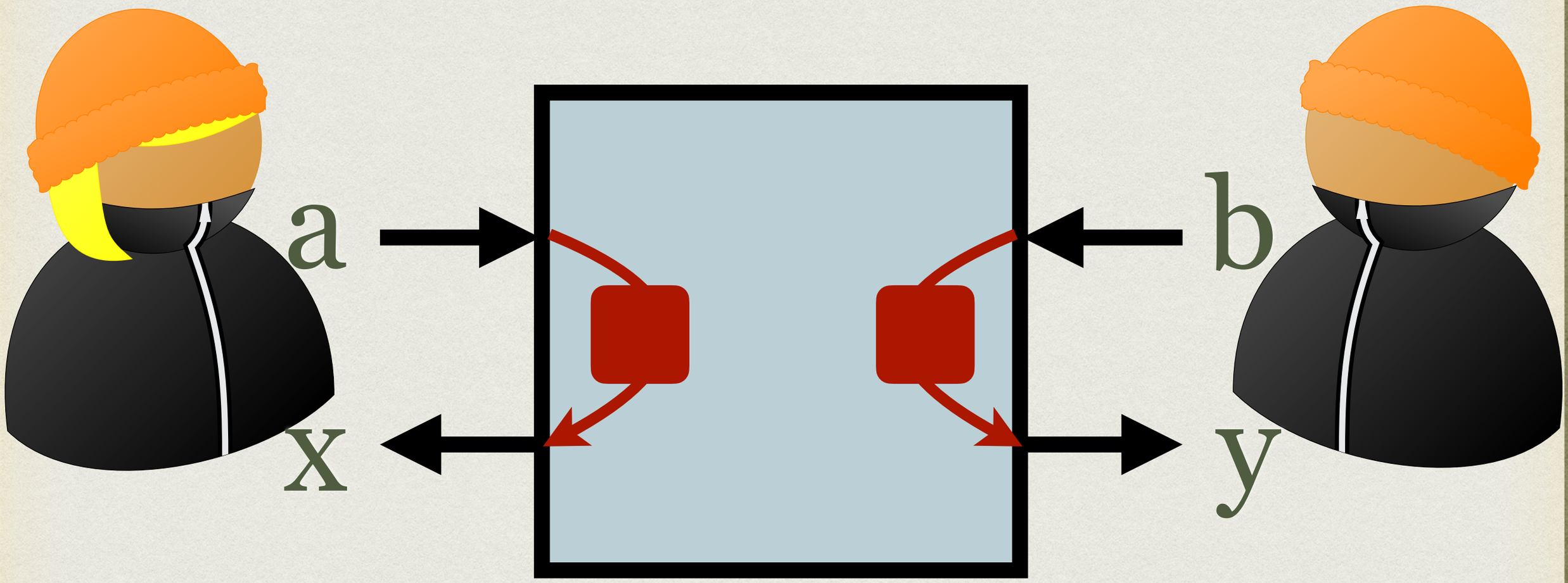

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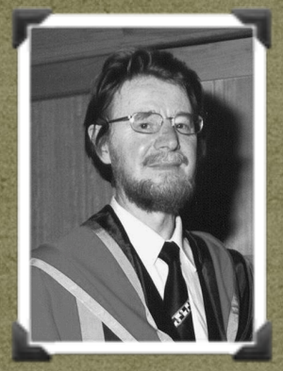

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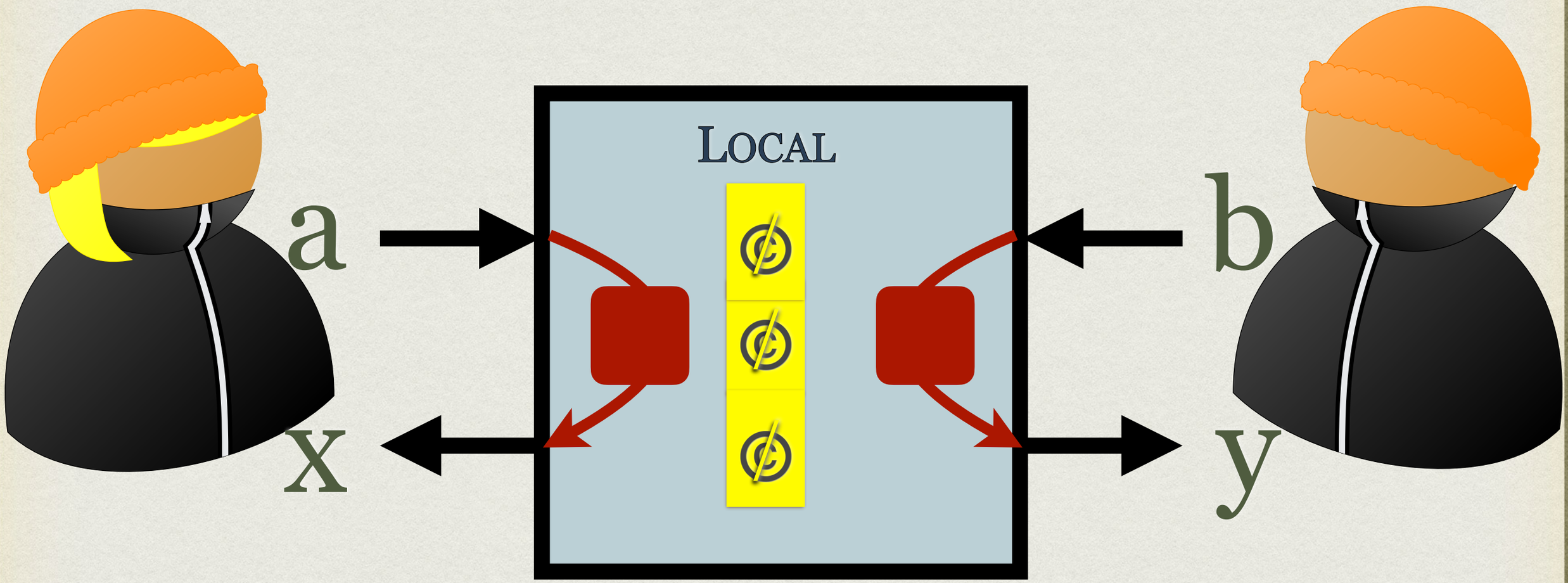

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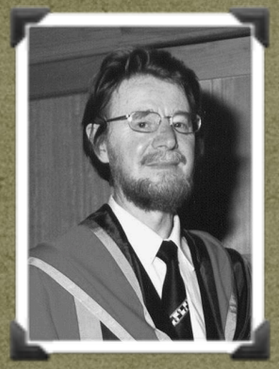

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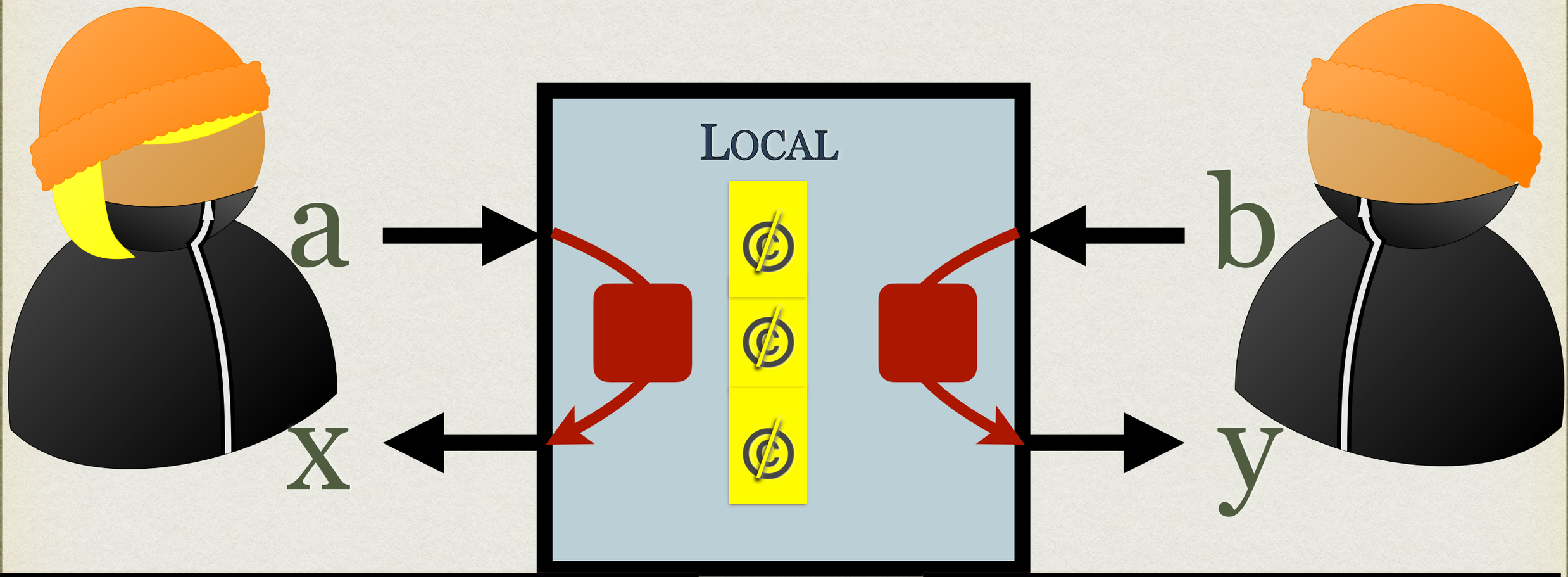
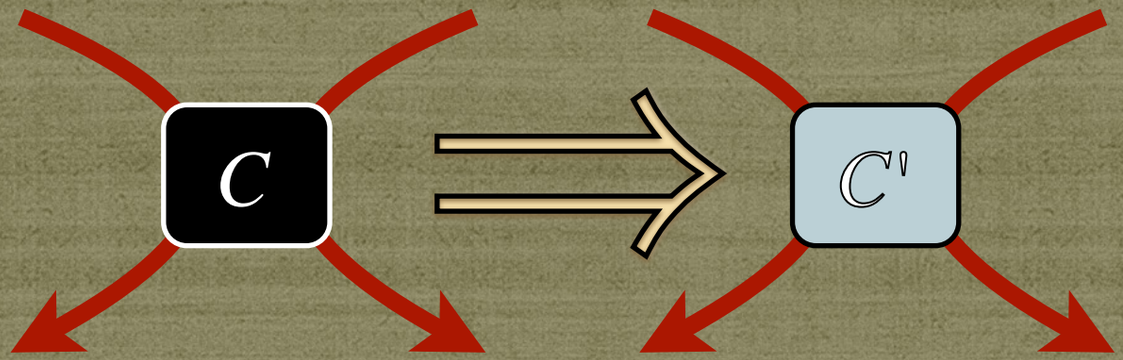


$x=f(a,©¢¢¢)$

$y=g(b,¢¢¢)$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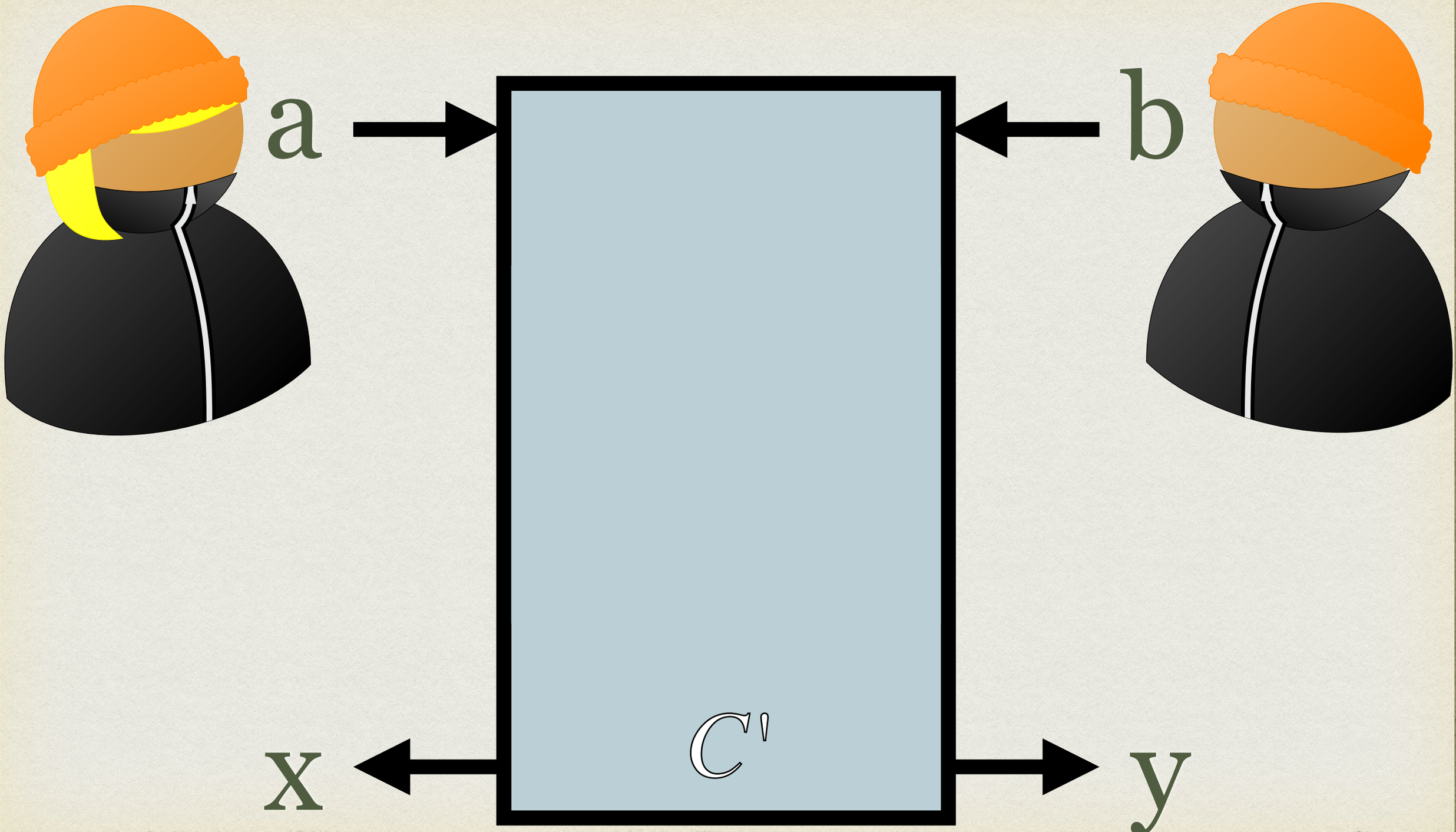
a
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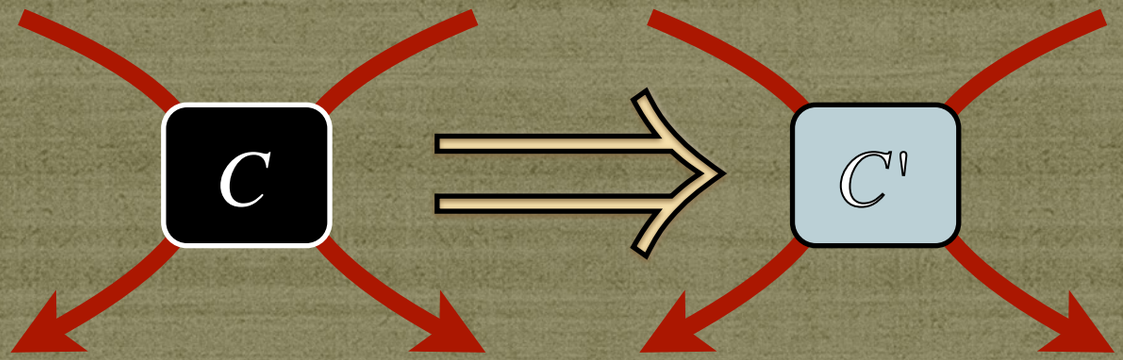
C'
x
y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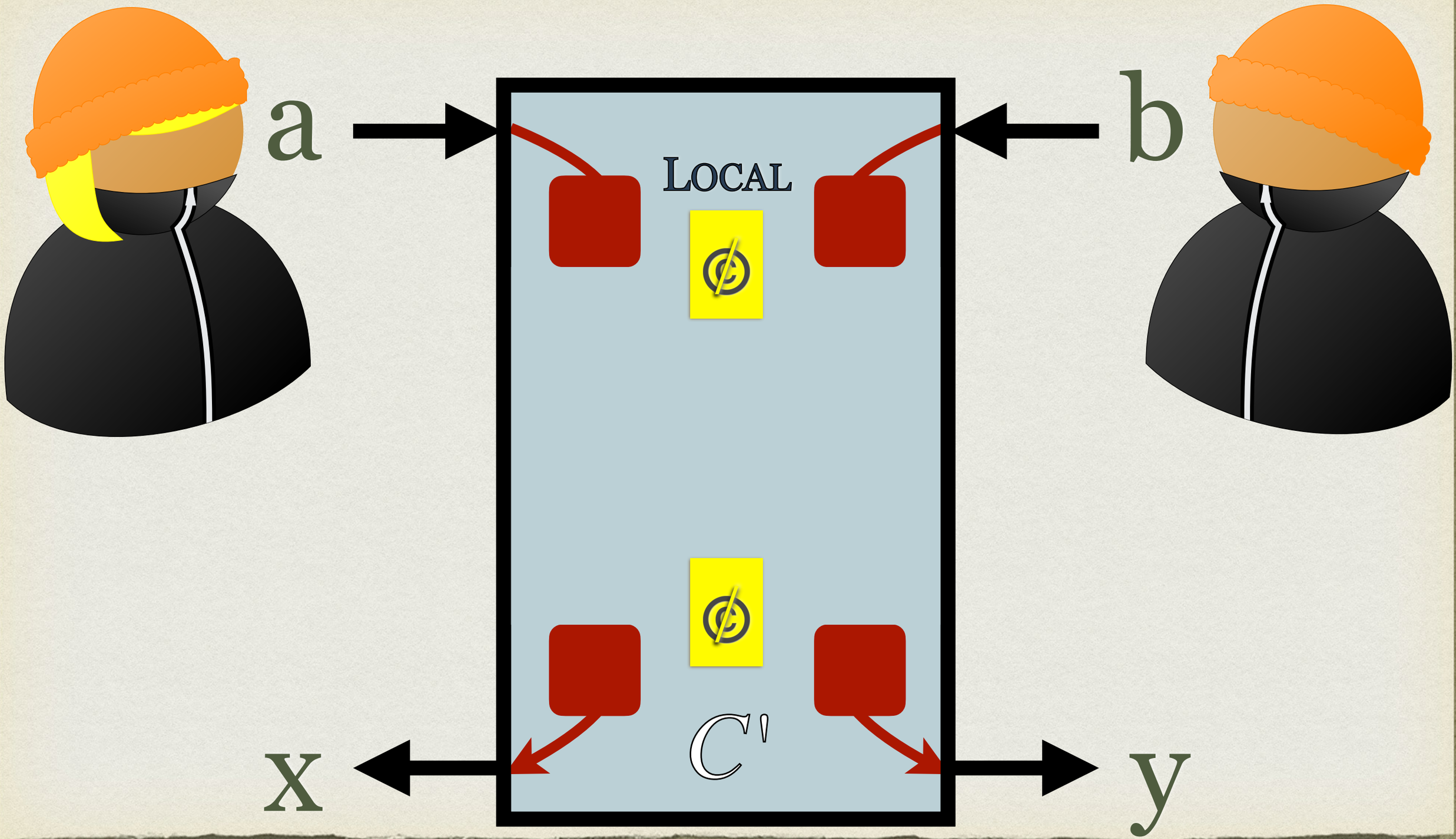
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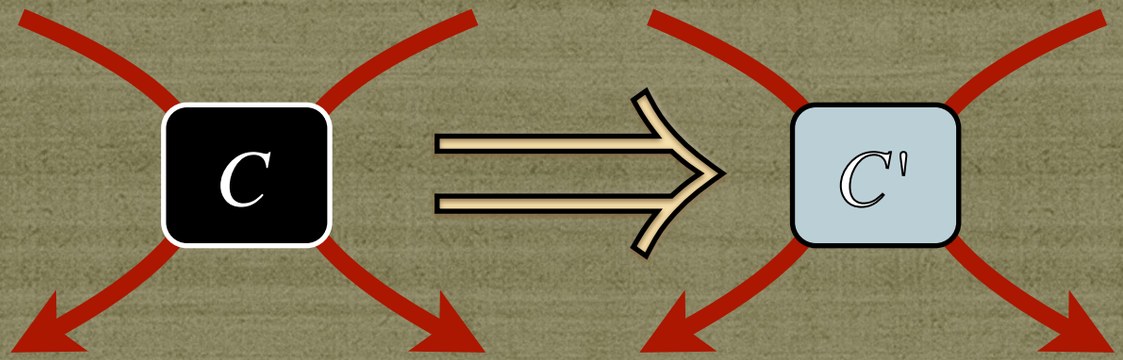
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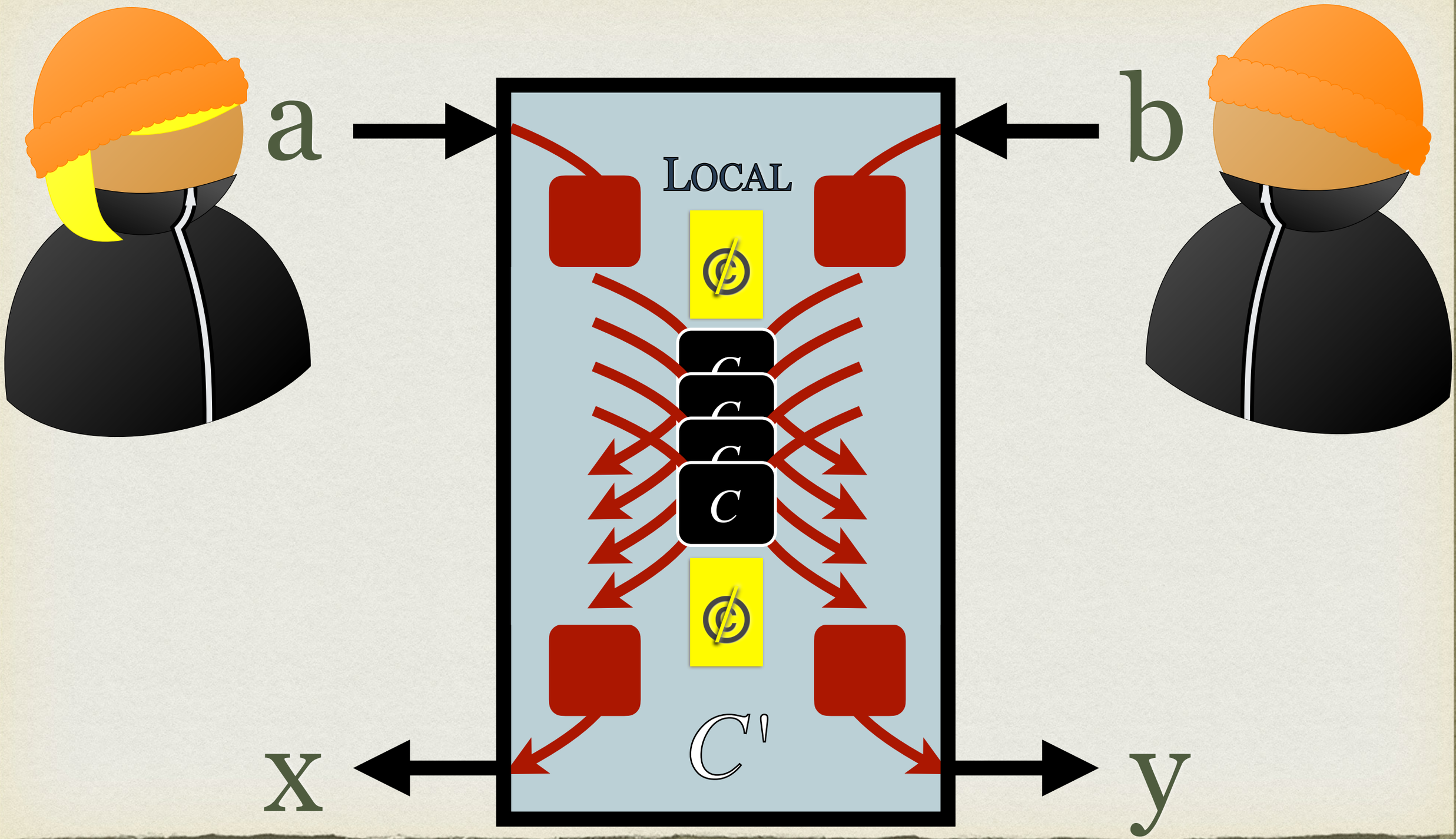
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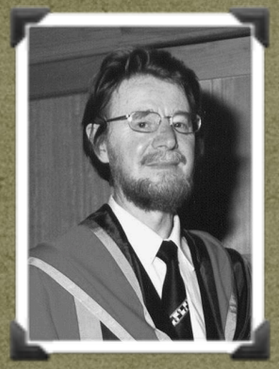

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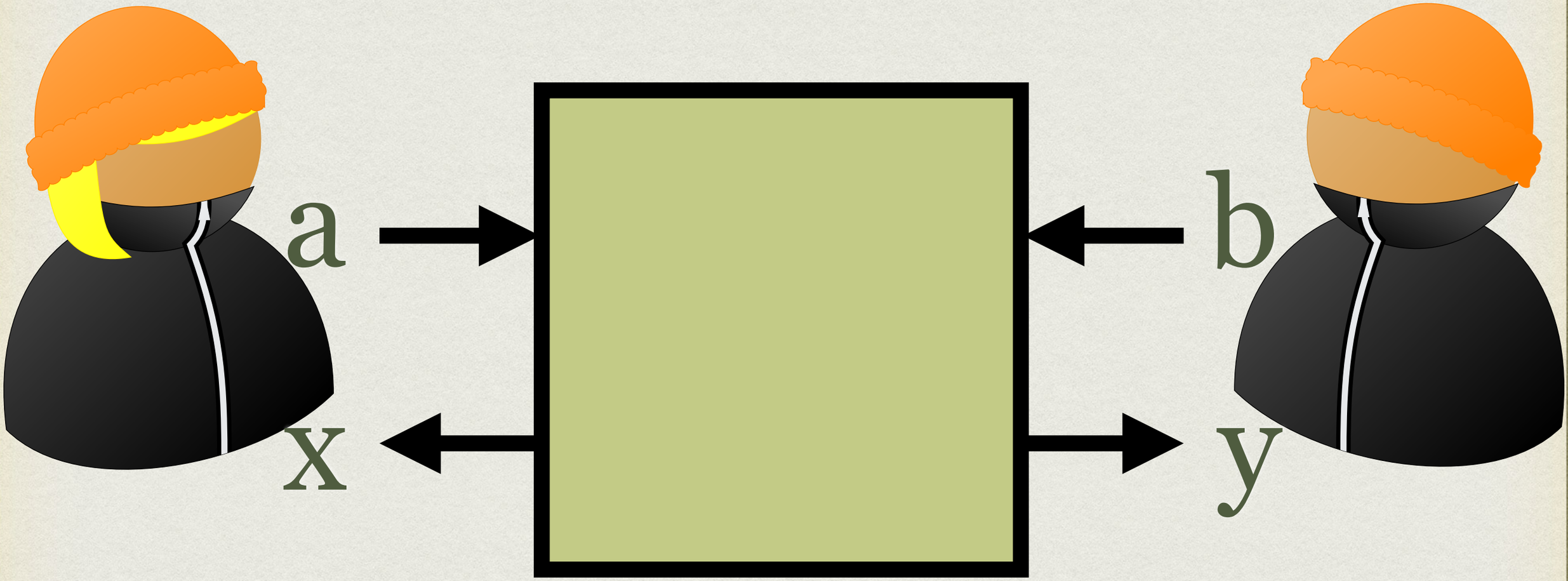

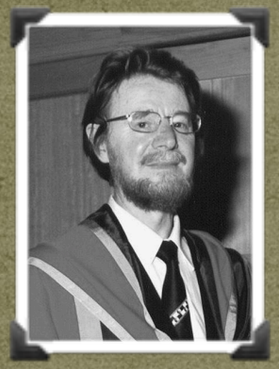

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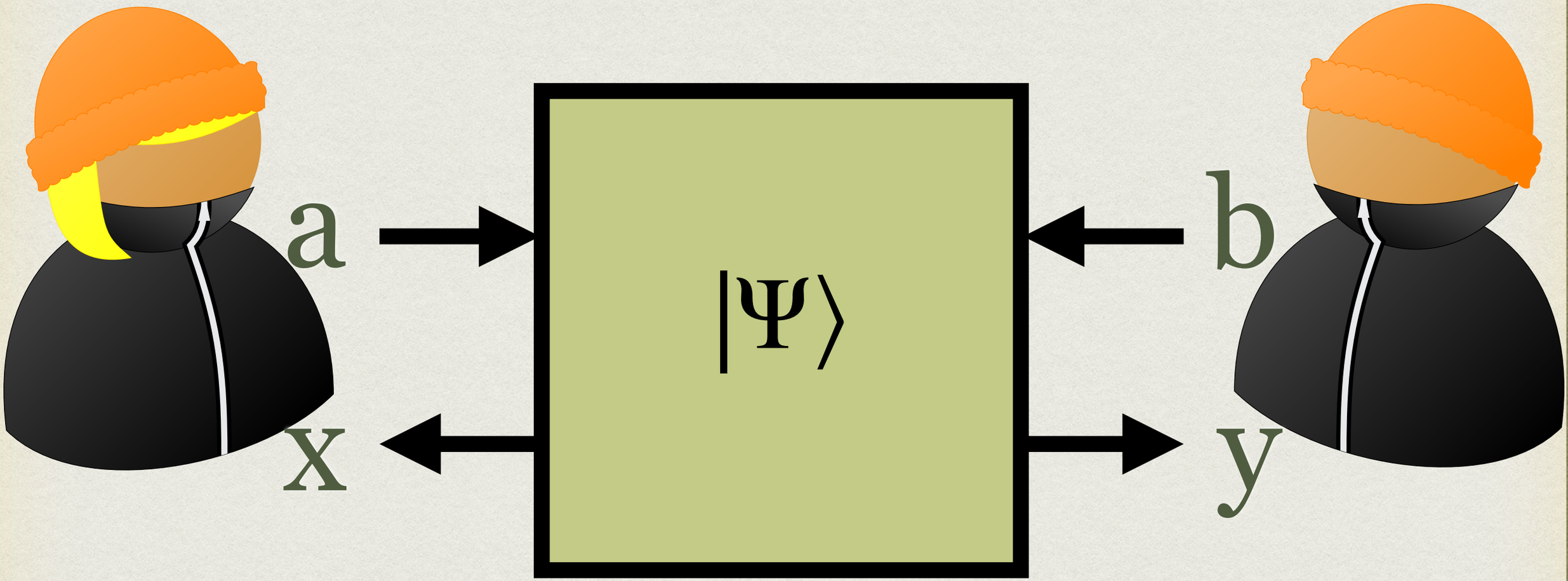

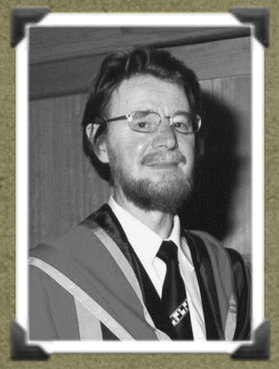

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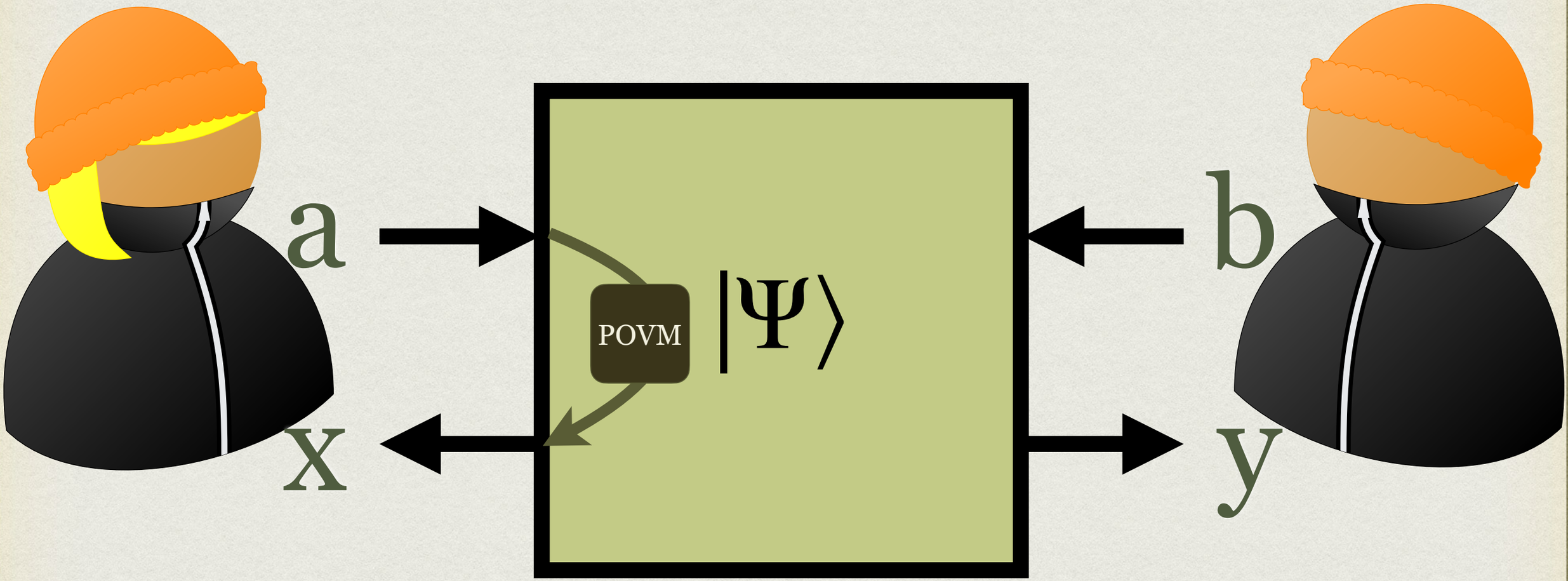

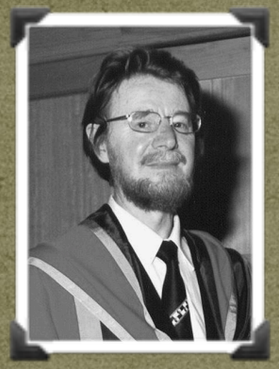

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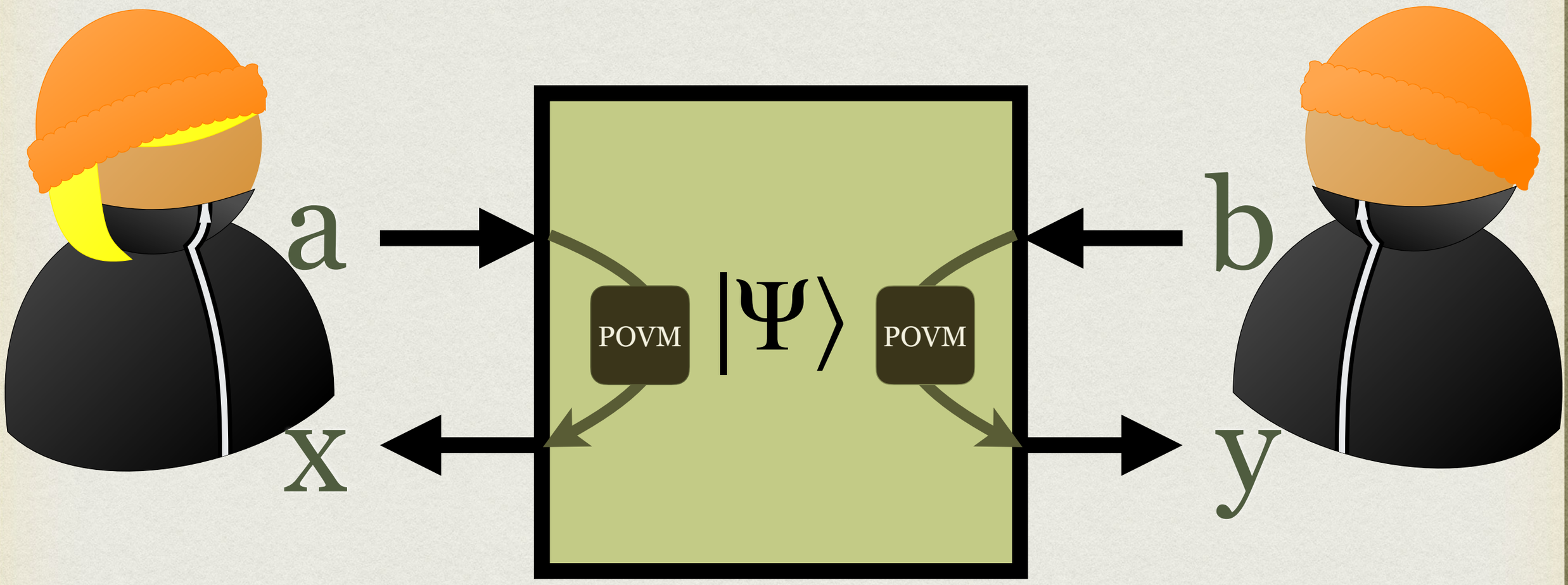
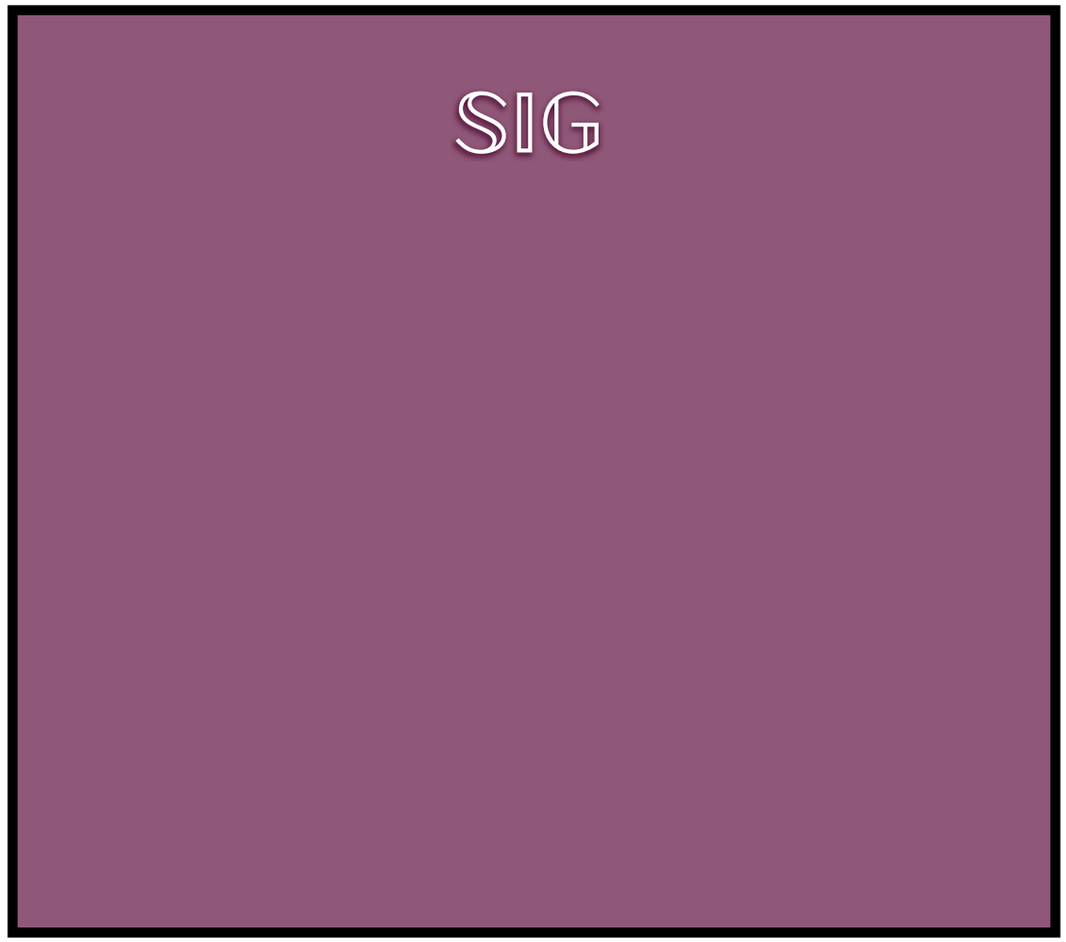
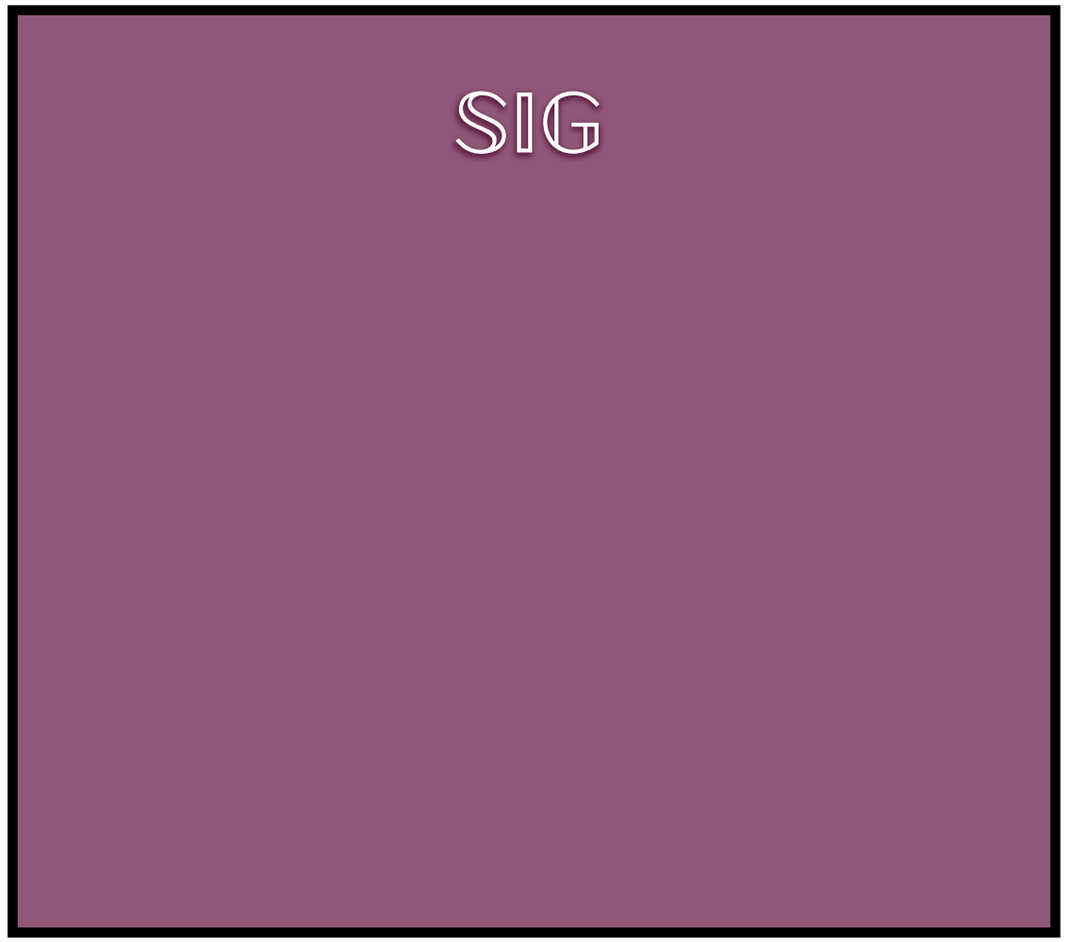


NON-LOCALITY HIERARCHY



NON-LOCALITY HIERARCHY

SIG = all two-party correlations



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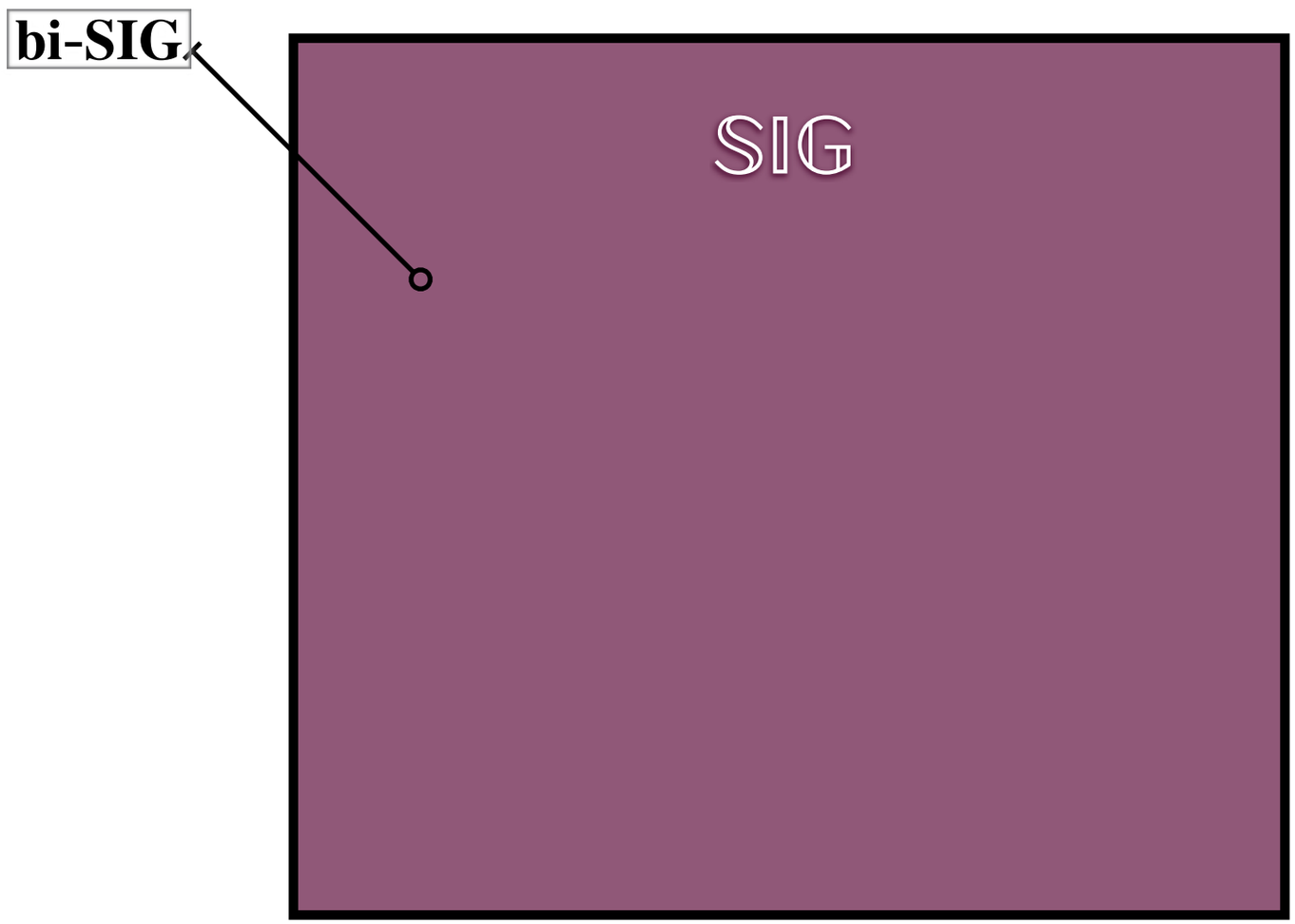

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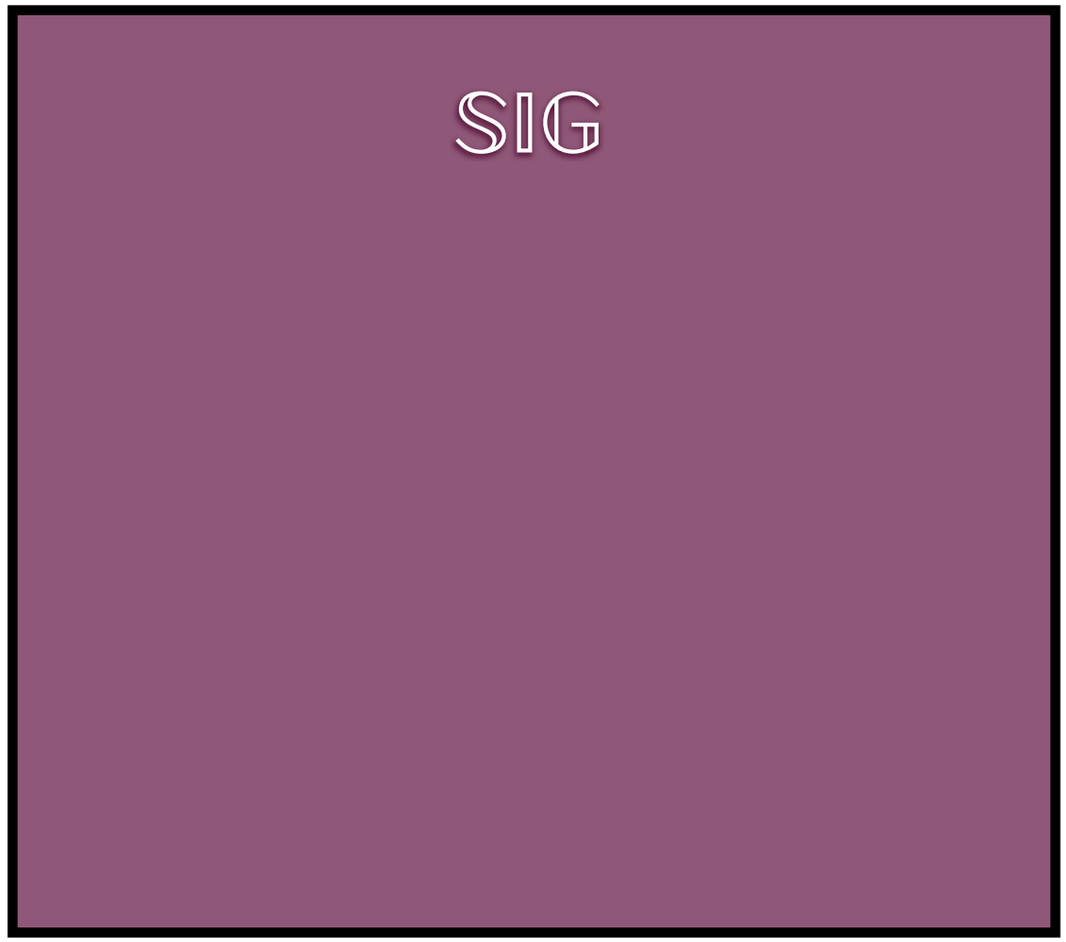

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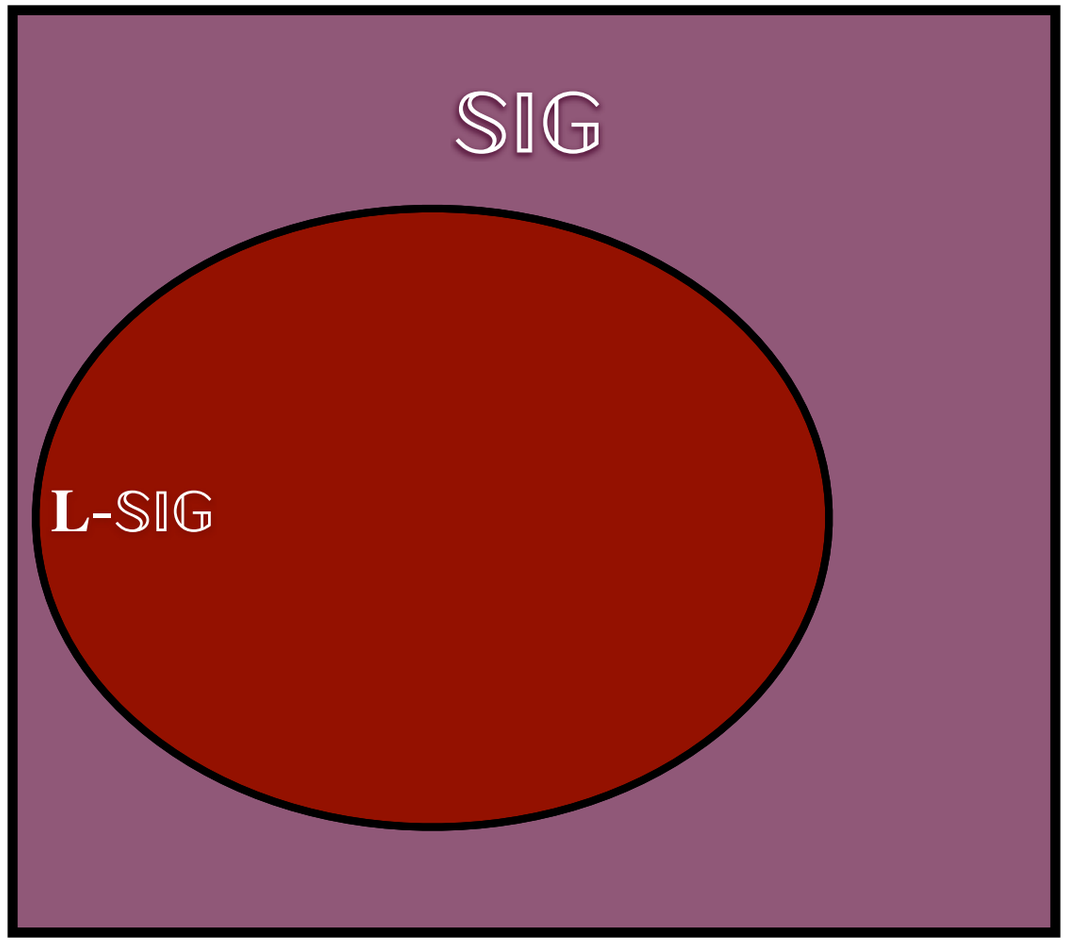

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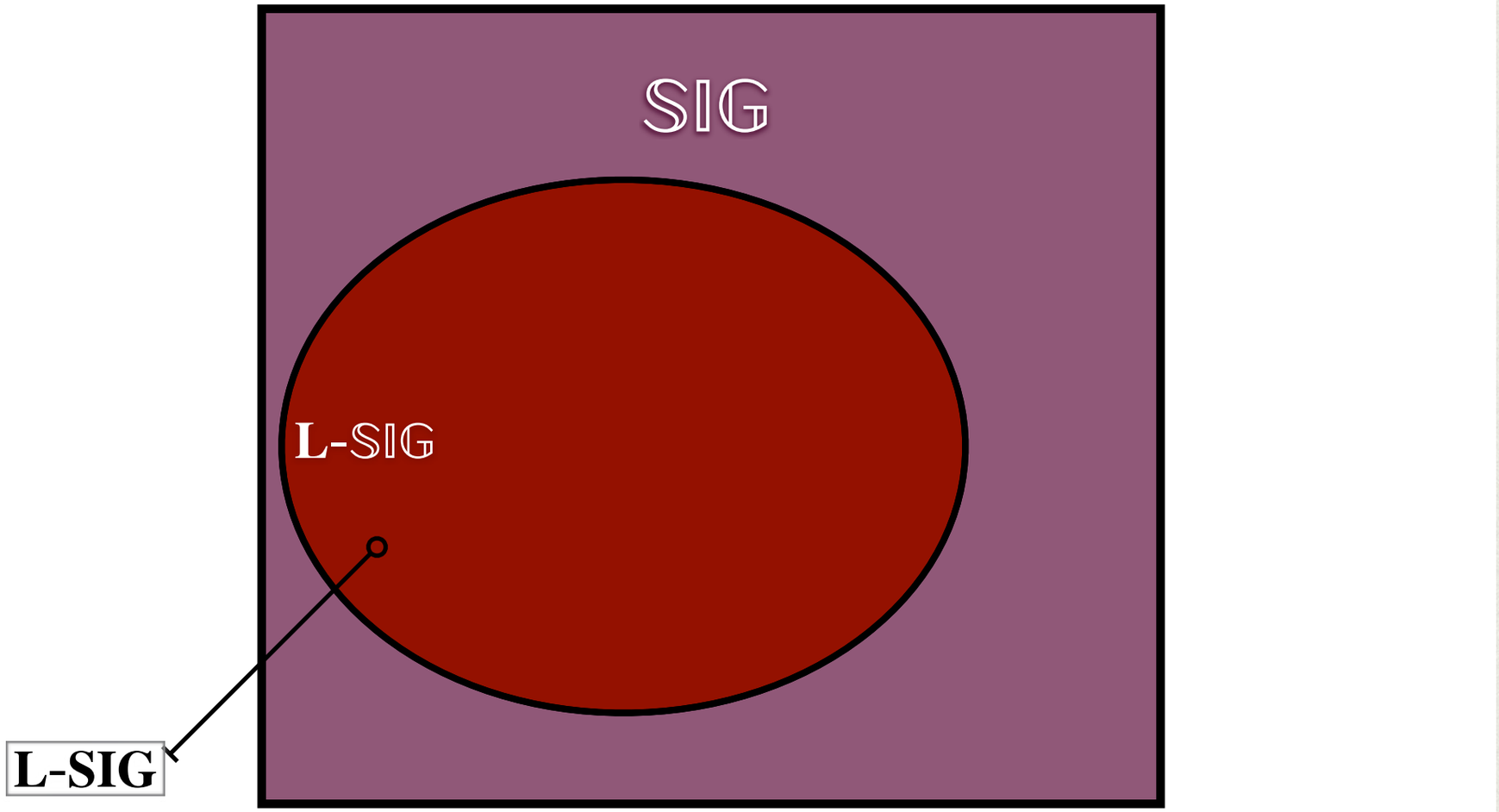

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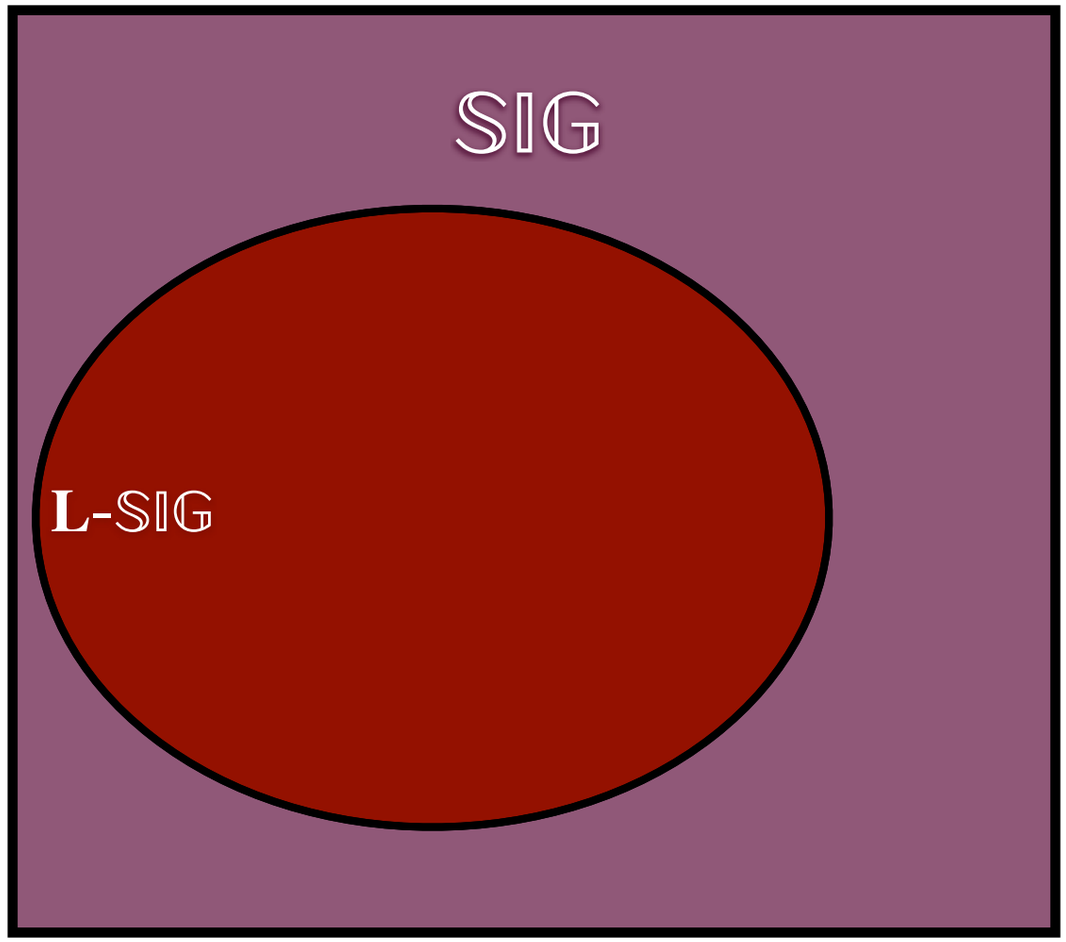

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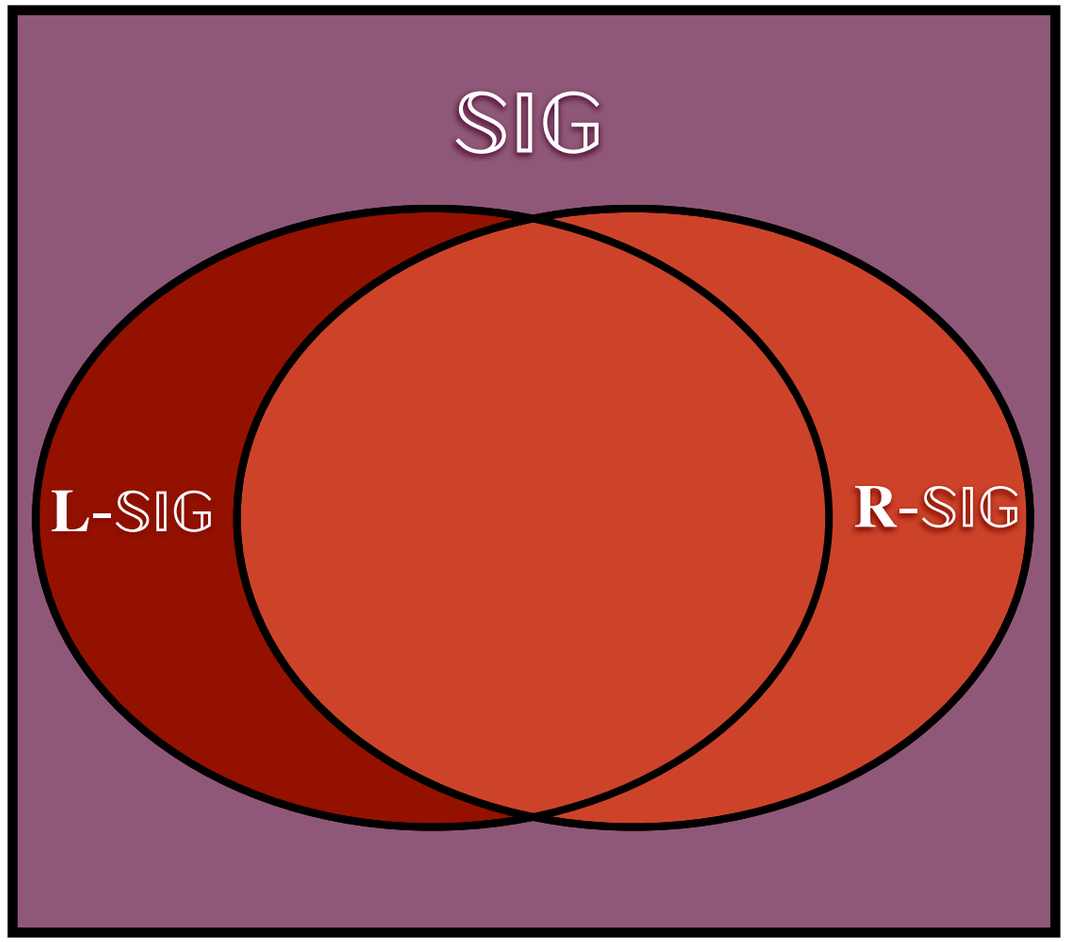

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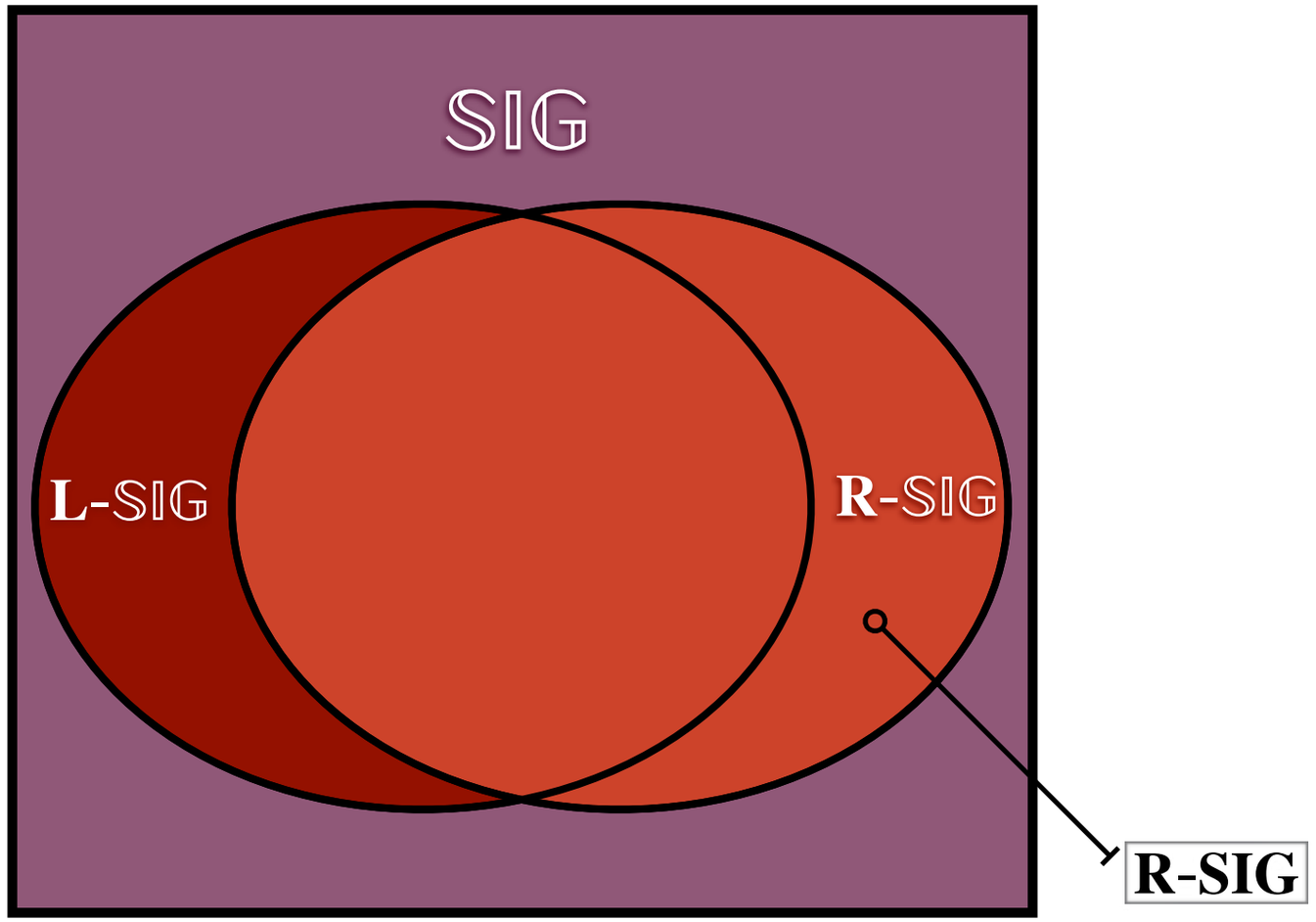

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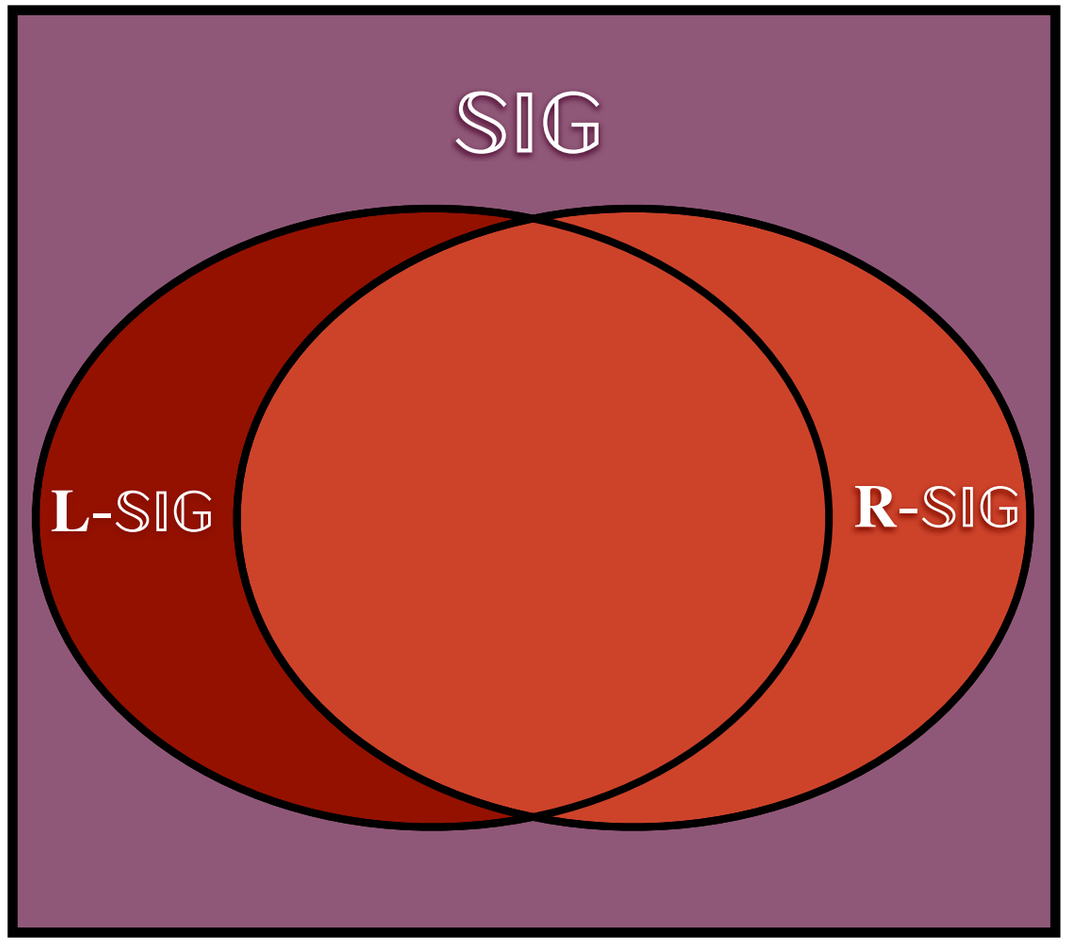

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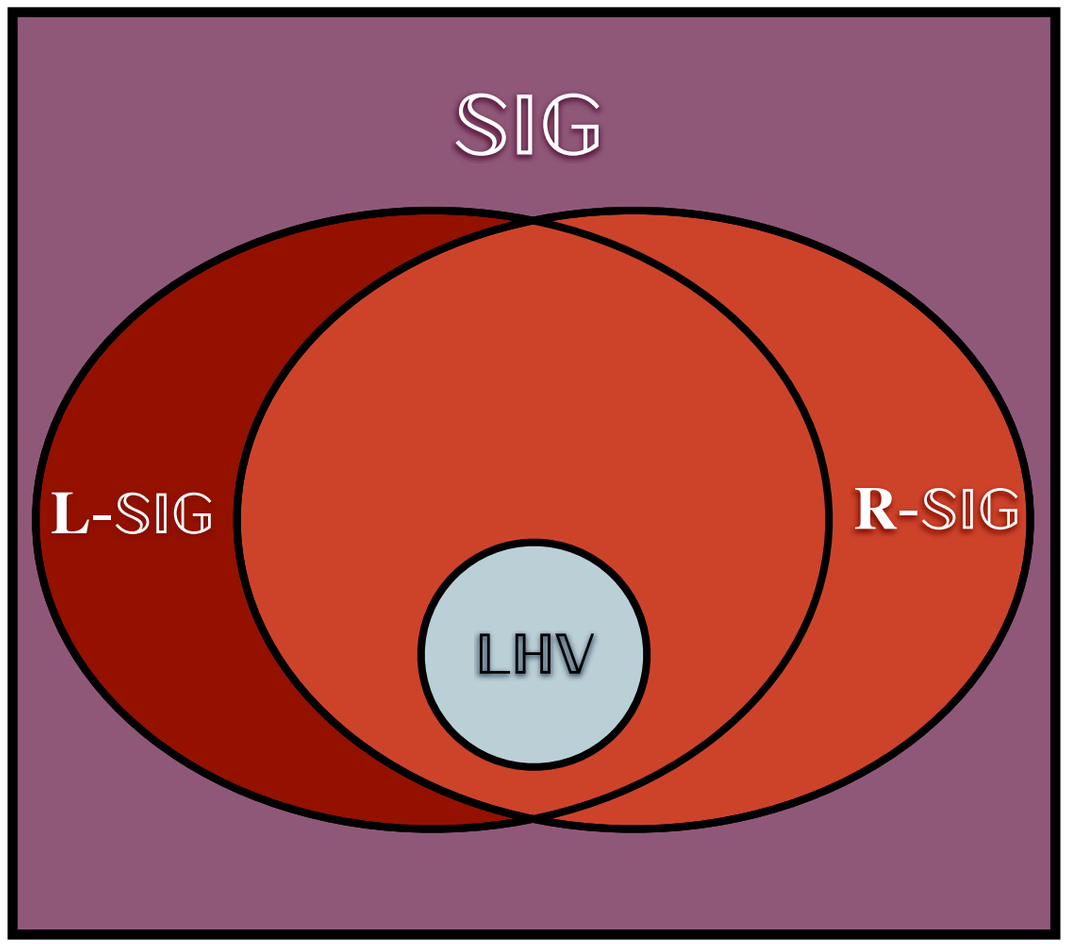

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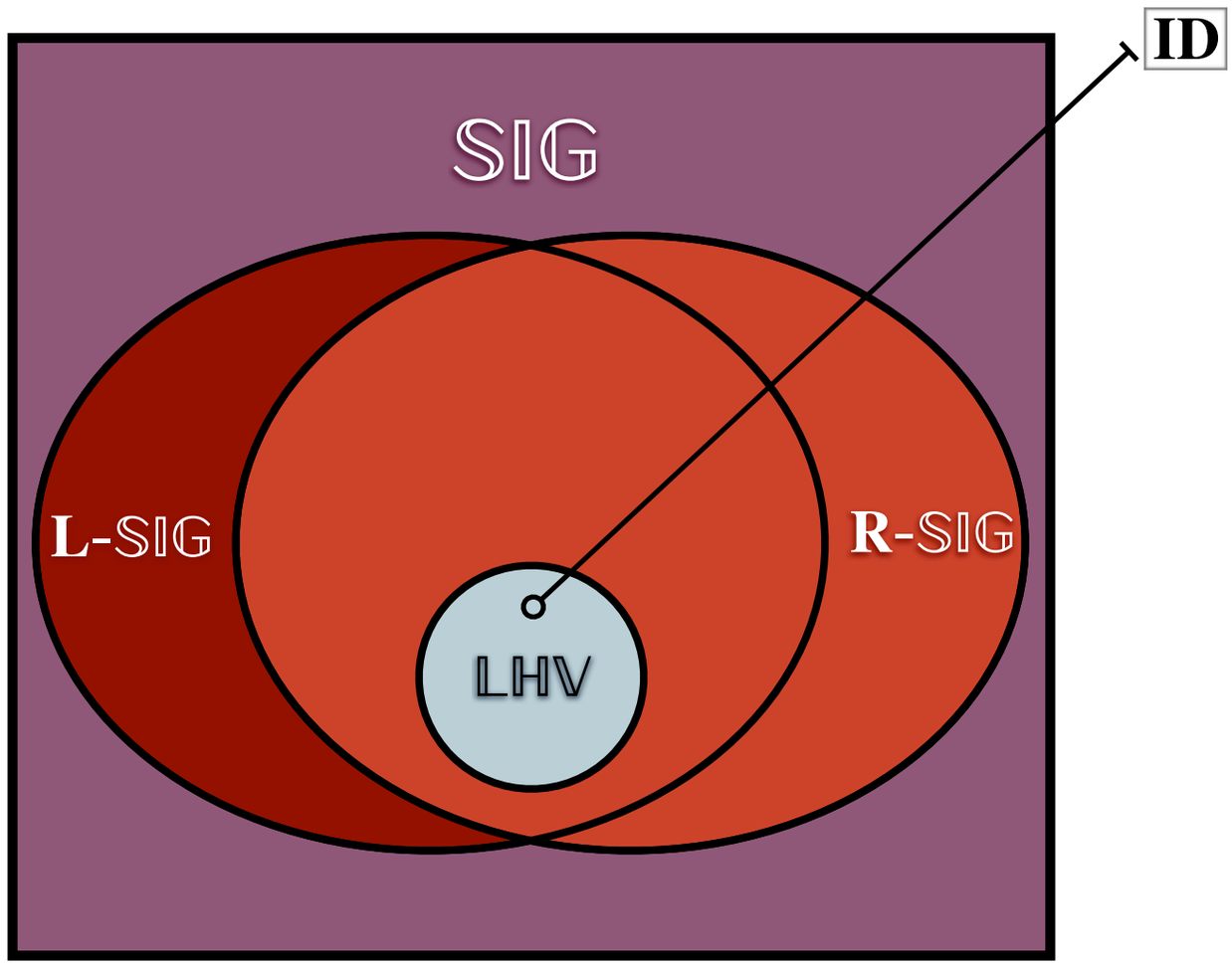

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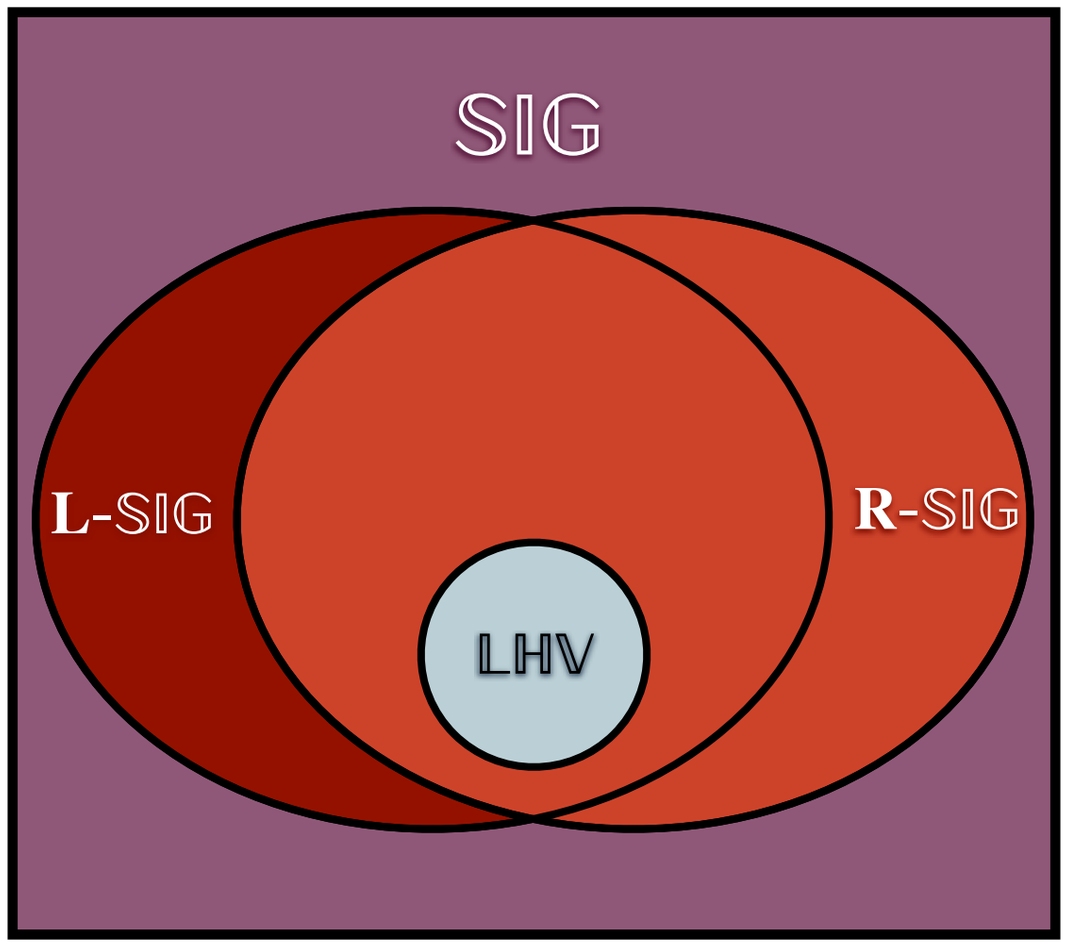

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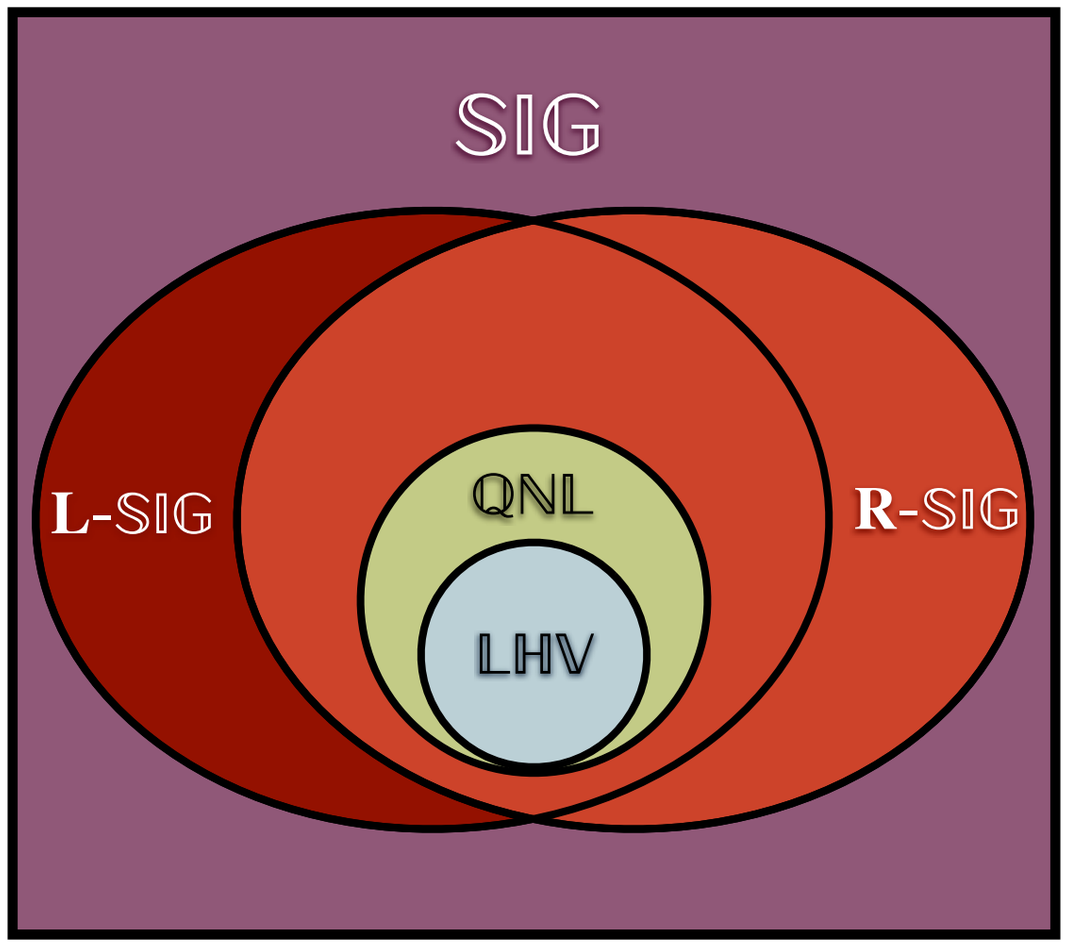

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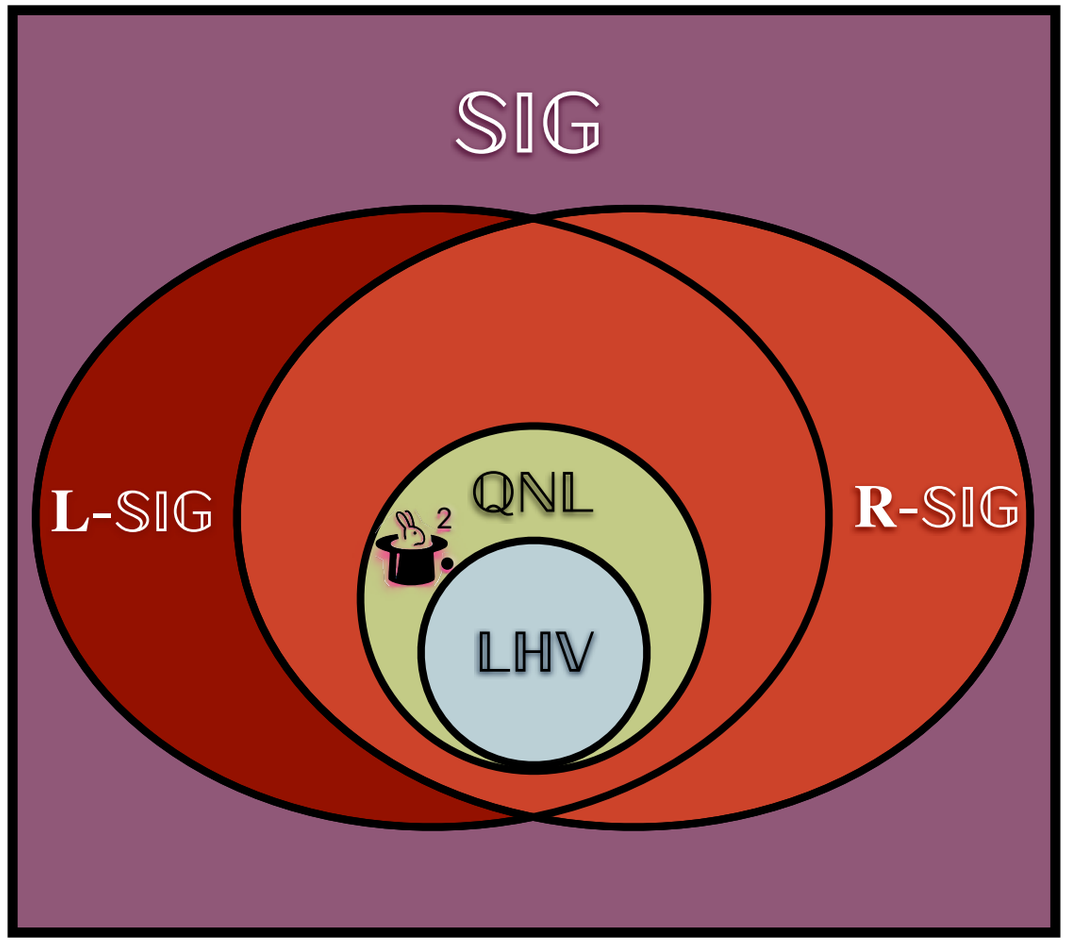

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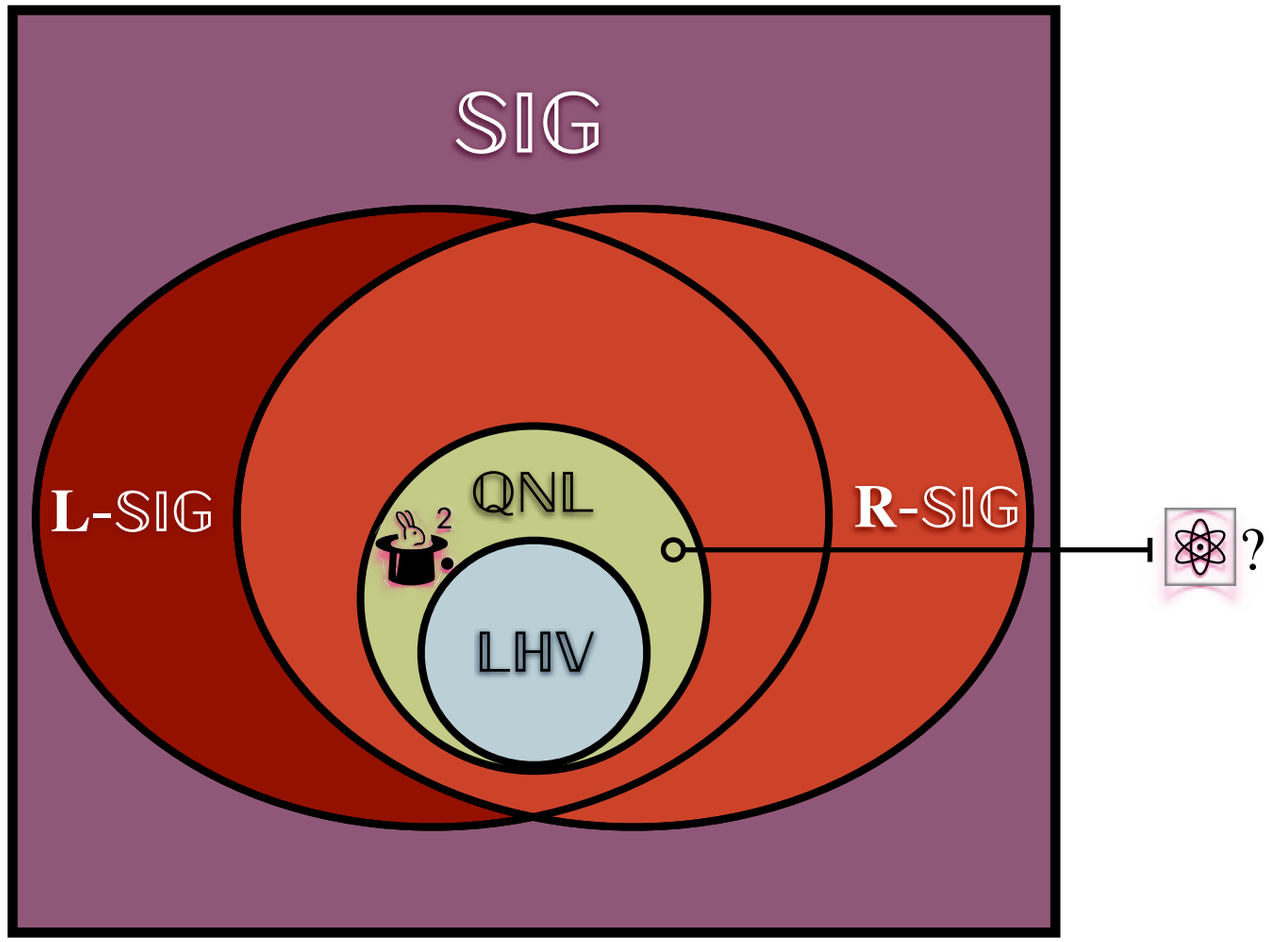

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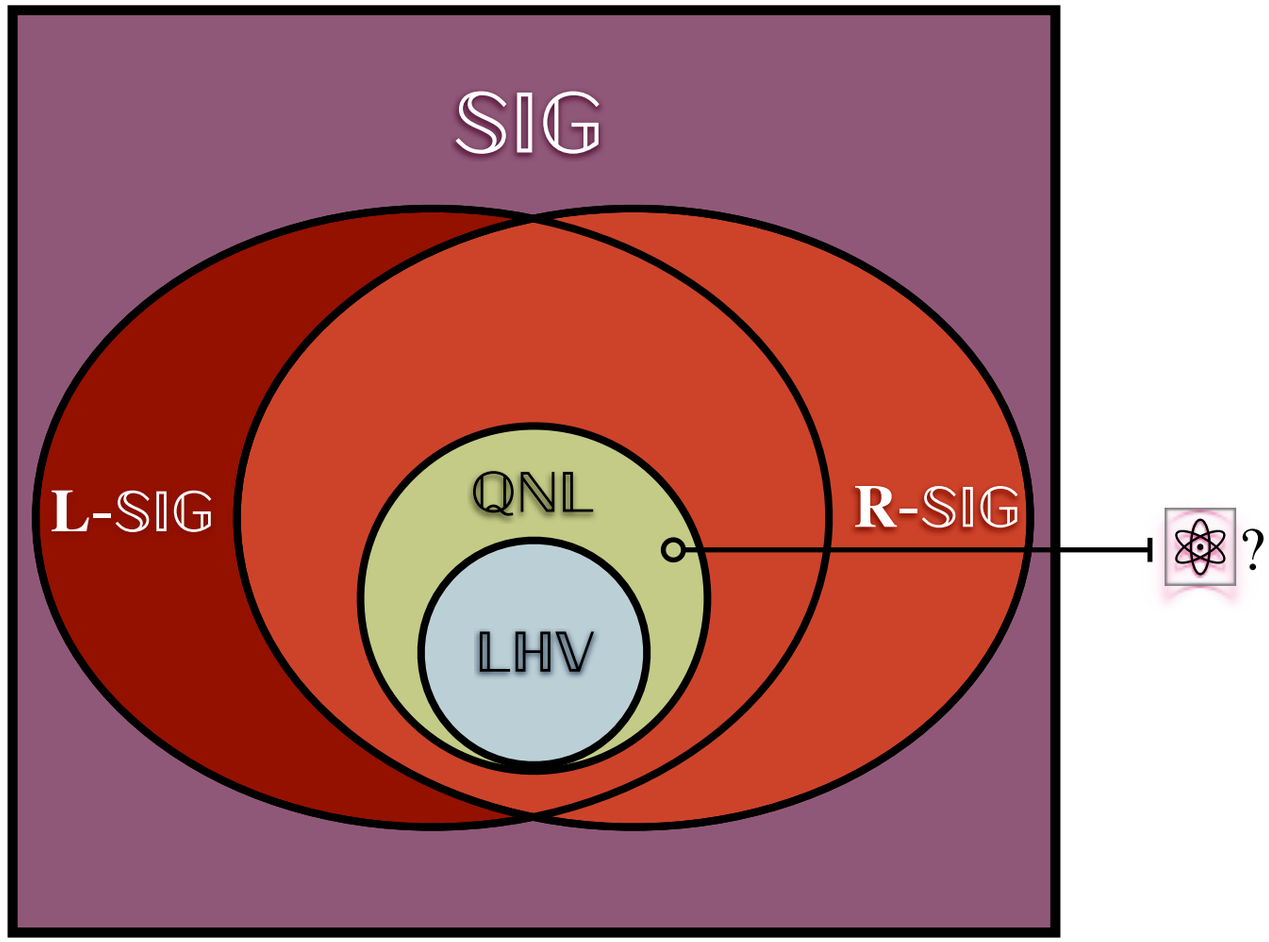

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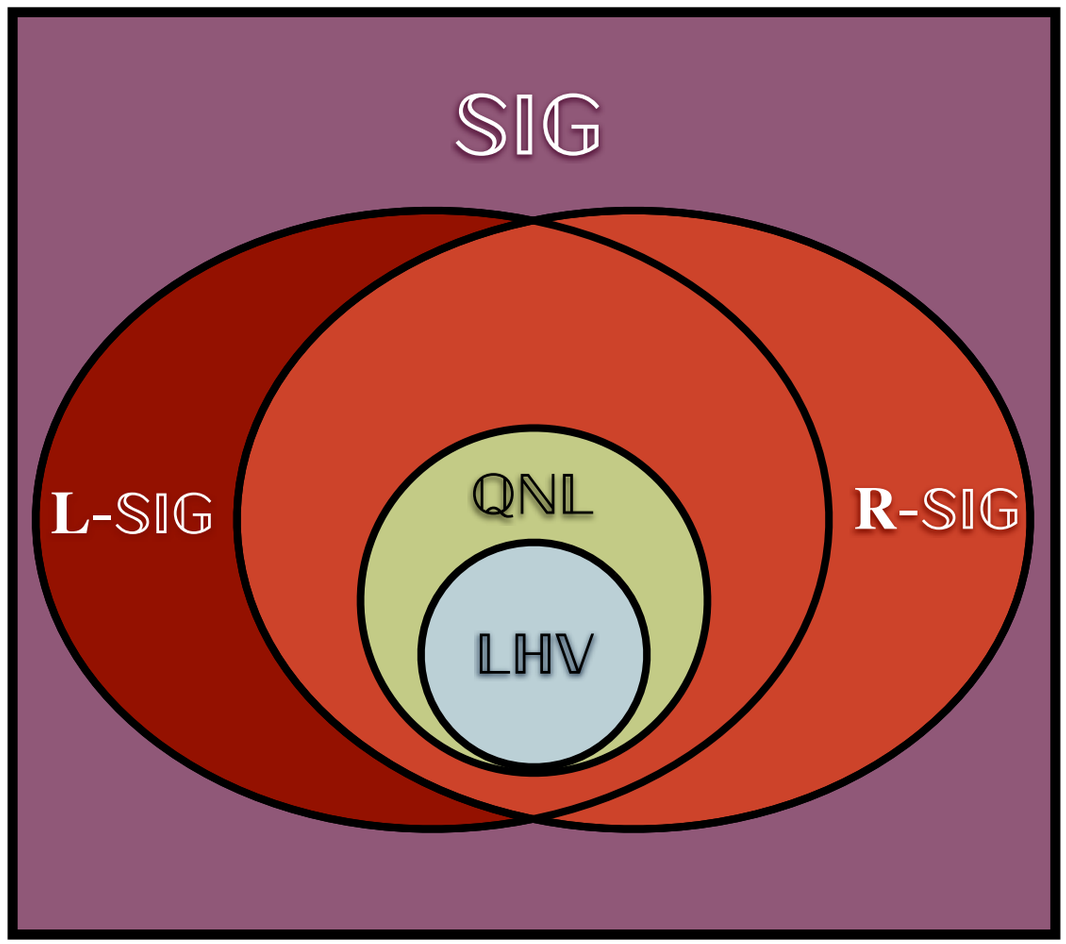

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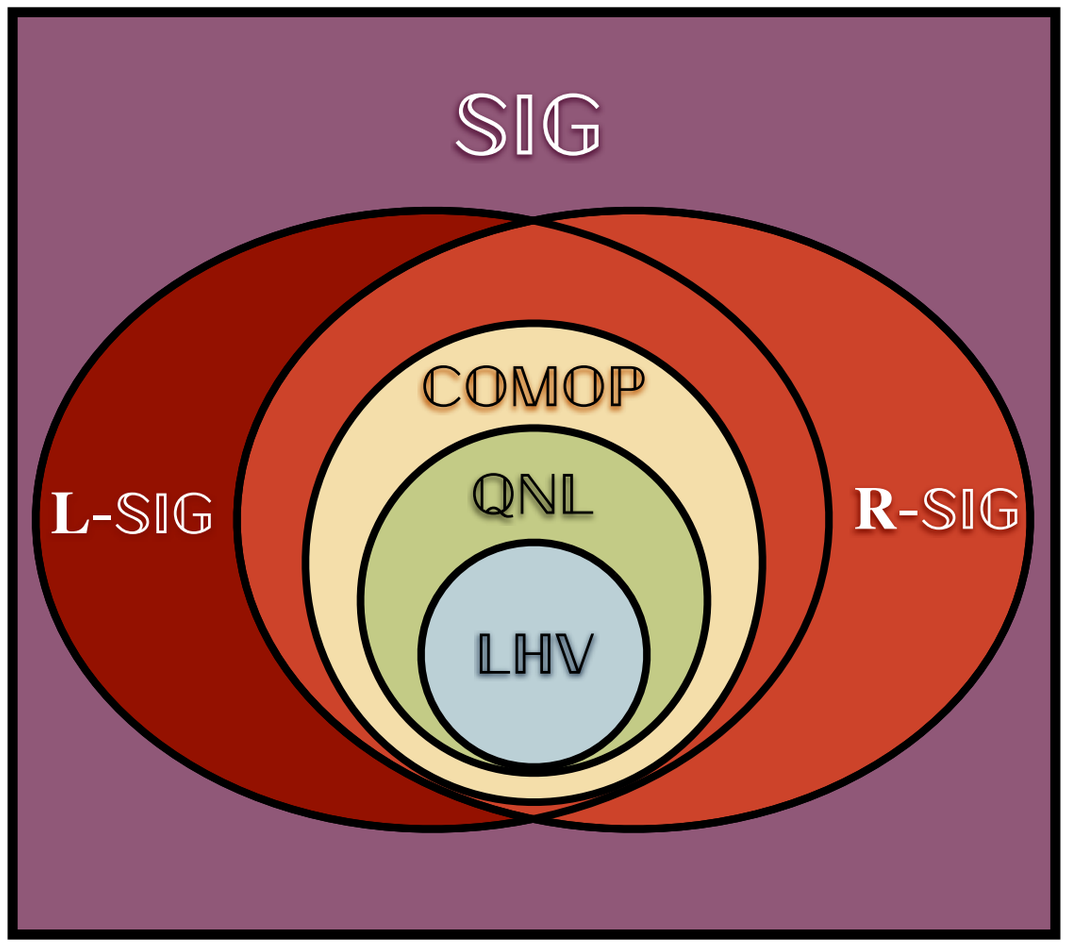

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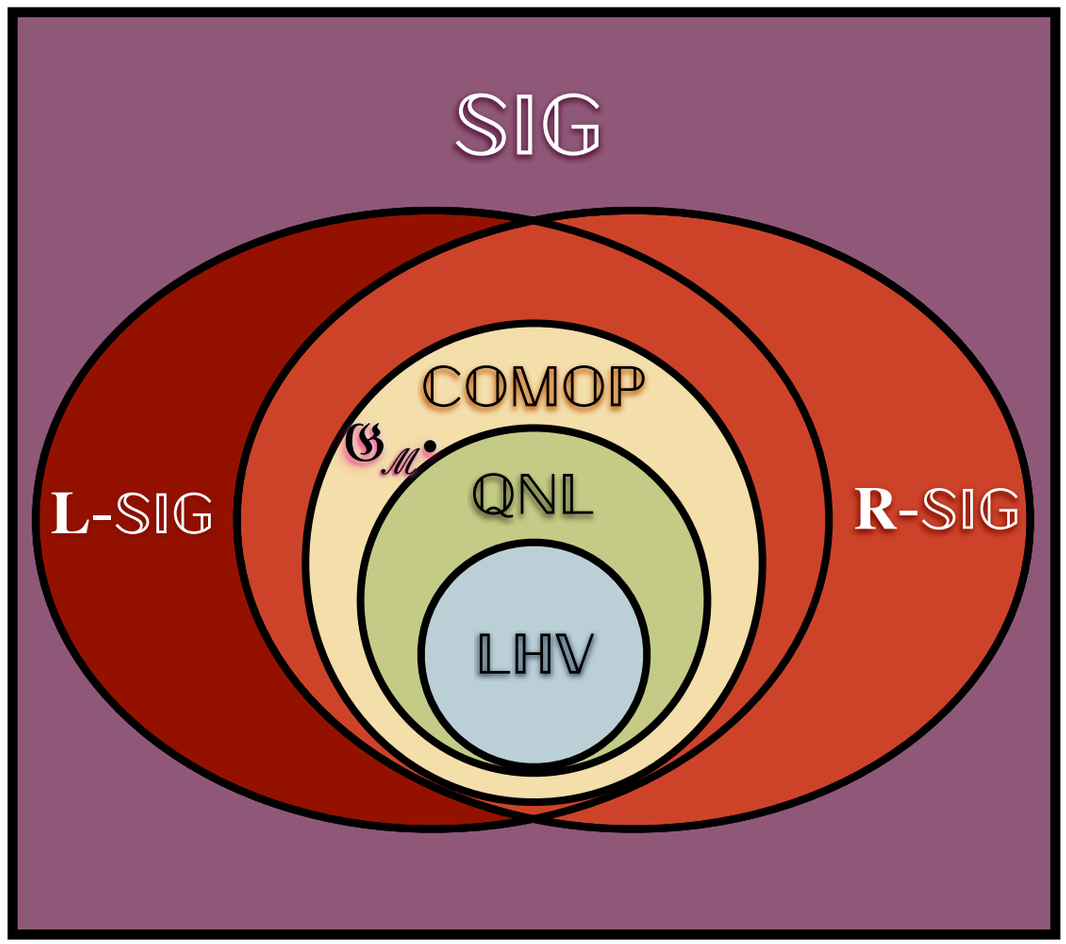
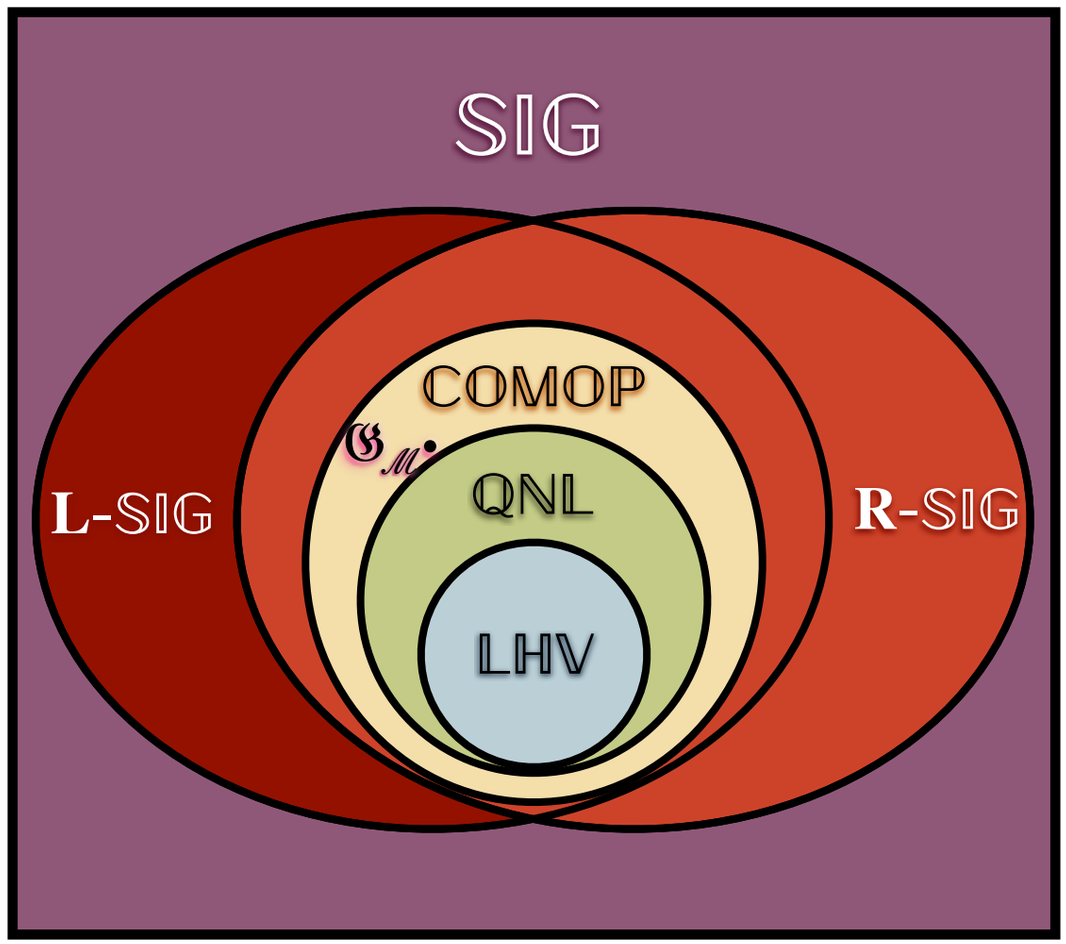


NON-LOCALITY HIERARCHY

Note: $\mathfrak{G}_{\mathcal{M}}$ is over infinite A,B



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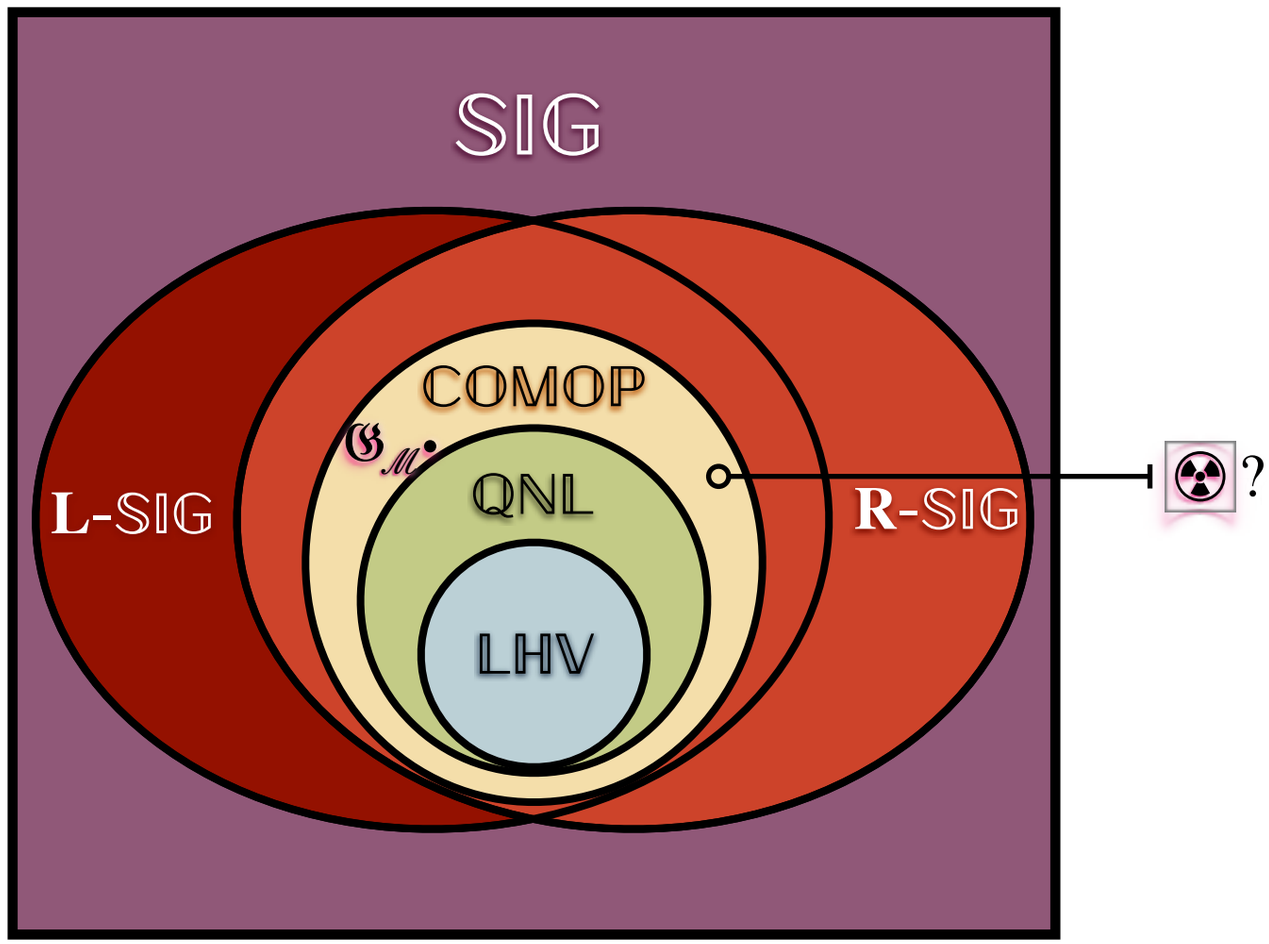
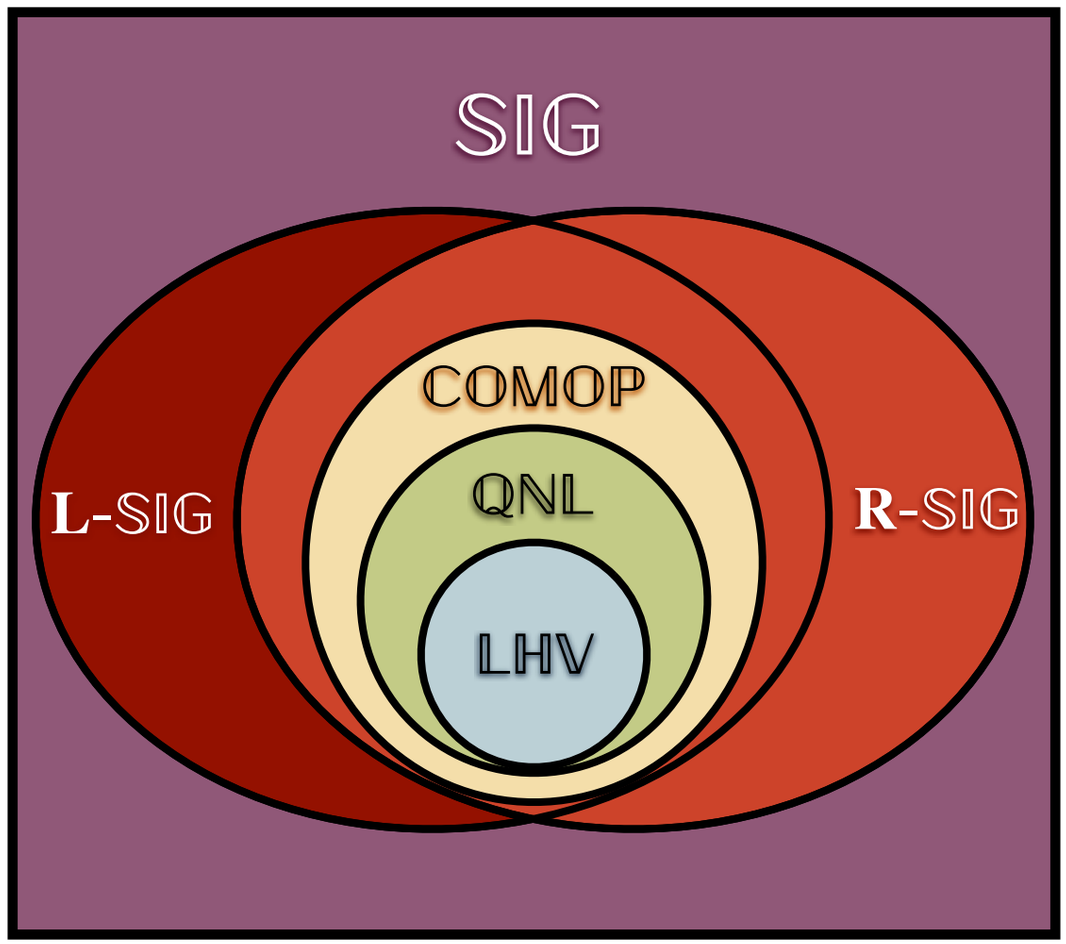


NON-LOCALITY HIERARCHY



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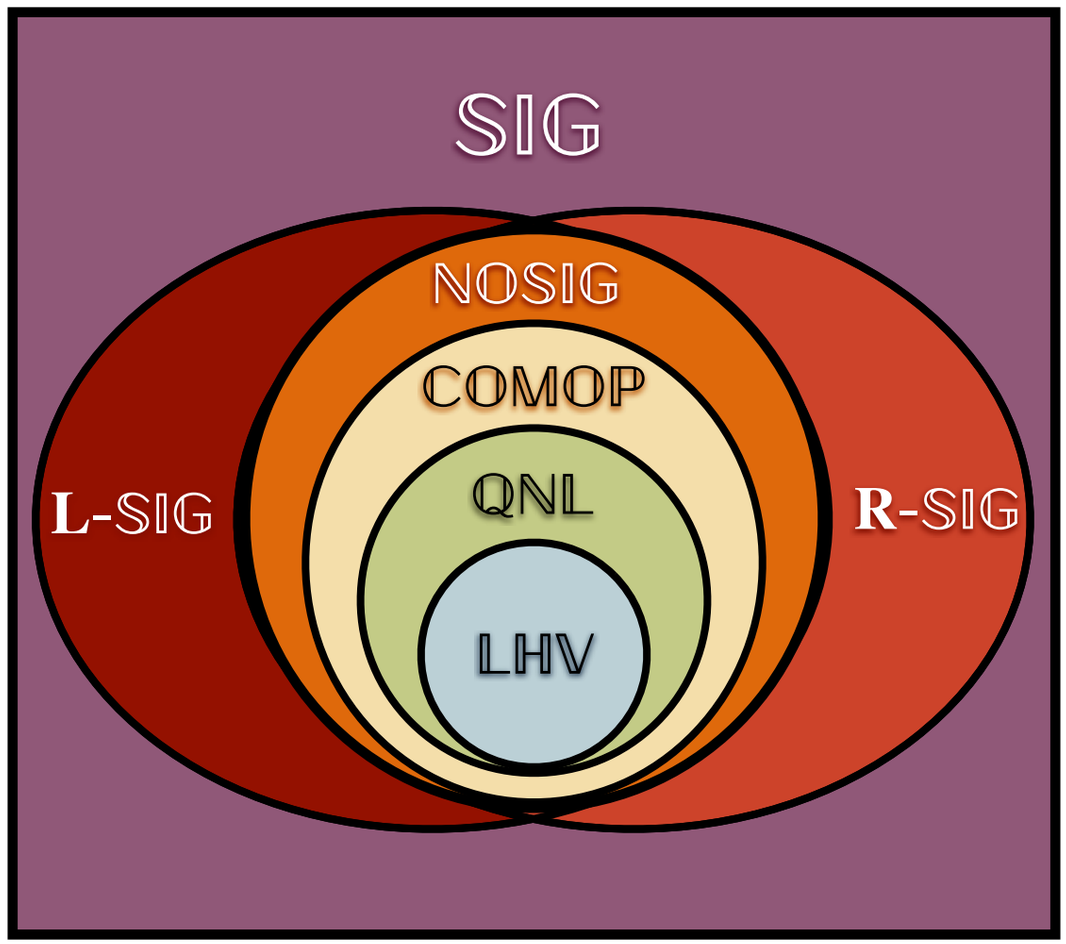

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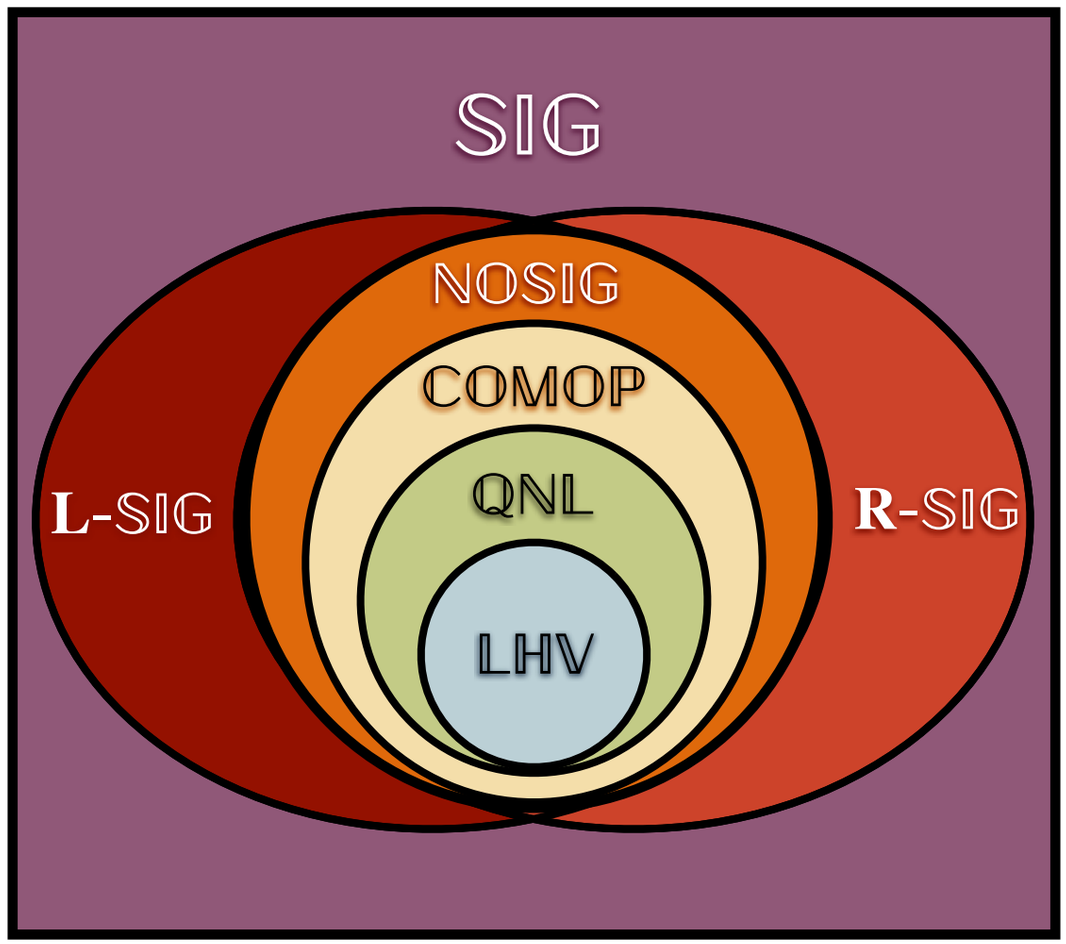
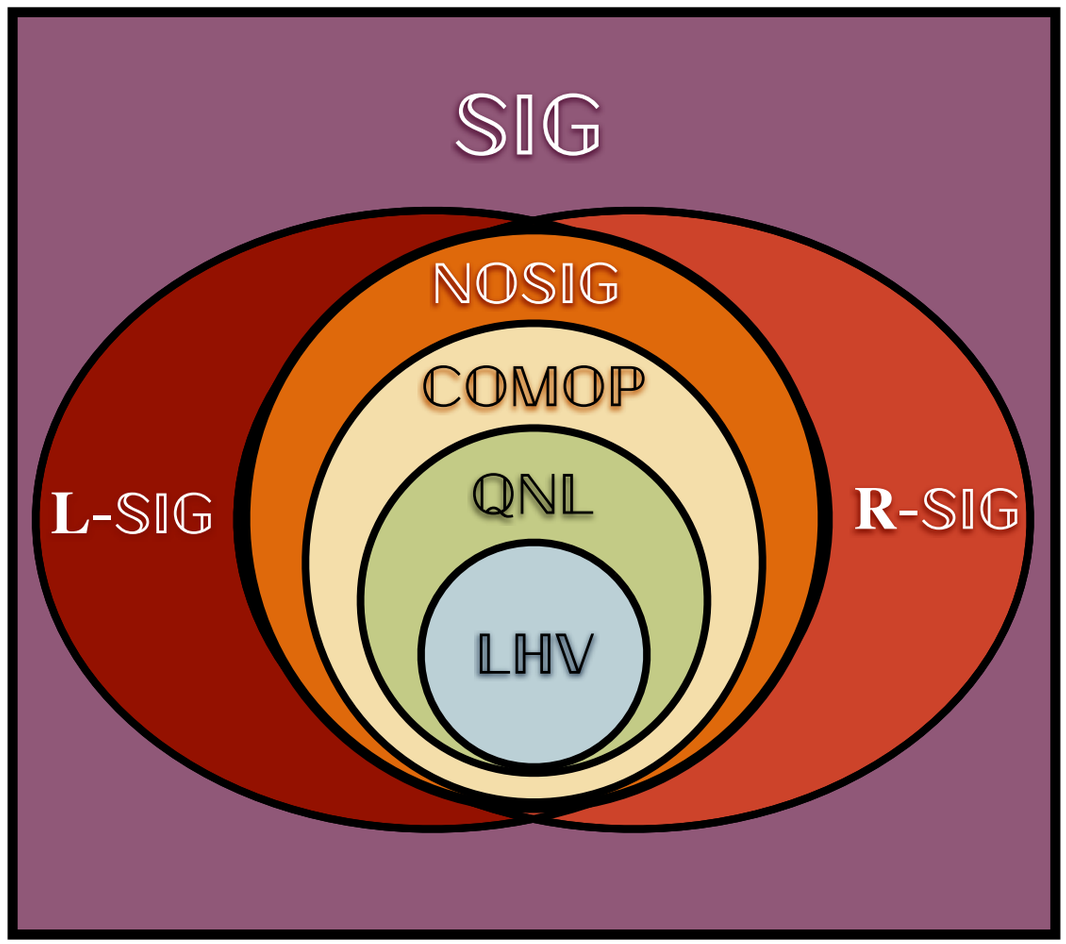
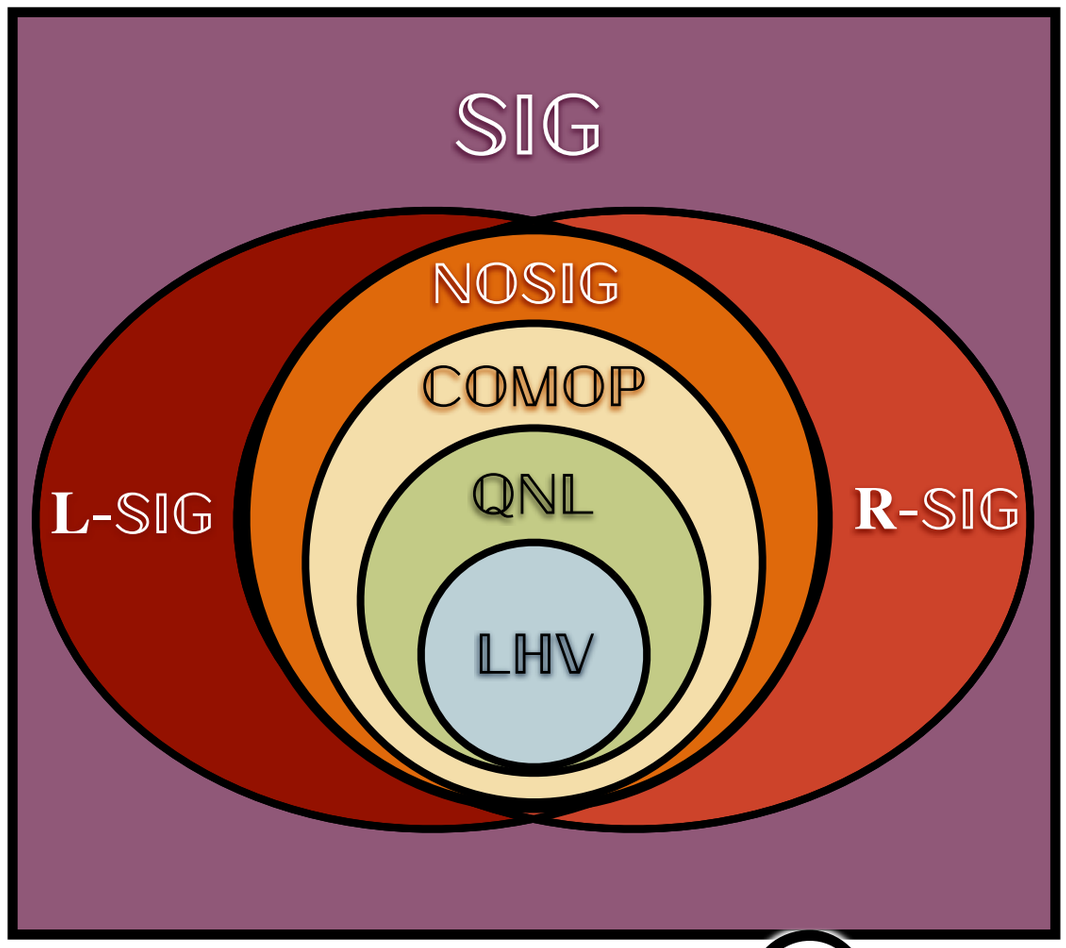


2-party NOSIG = **L**-SIG ∩ **R**-SIG

NON-LOCALITY HIERARCHY

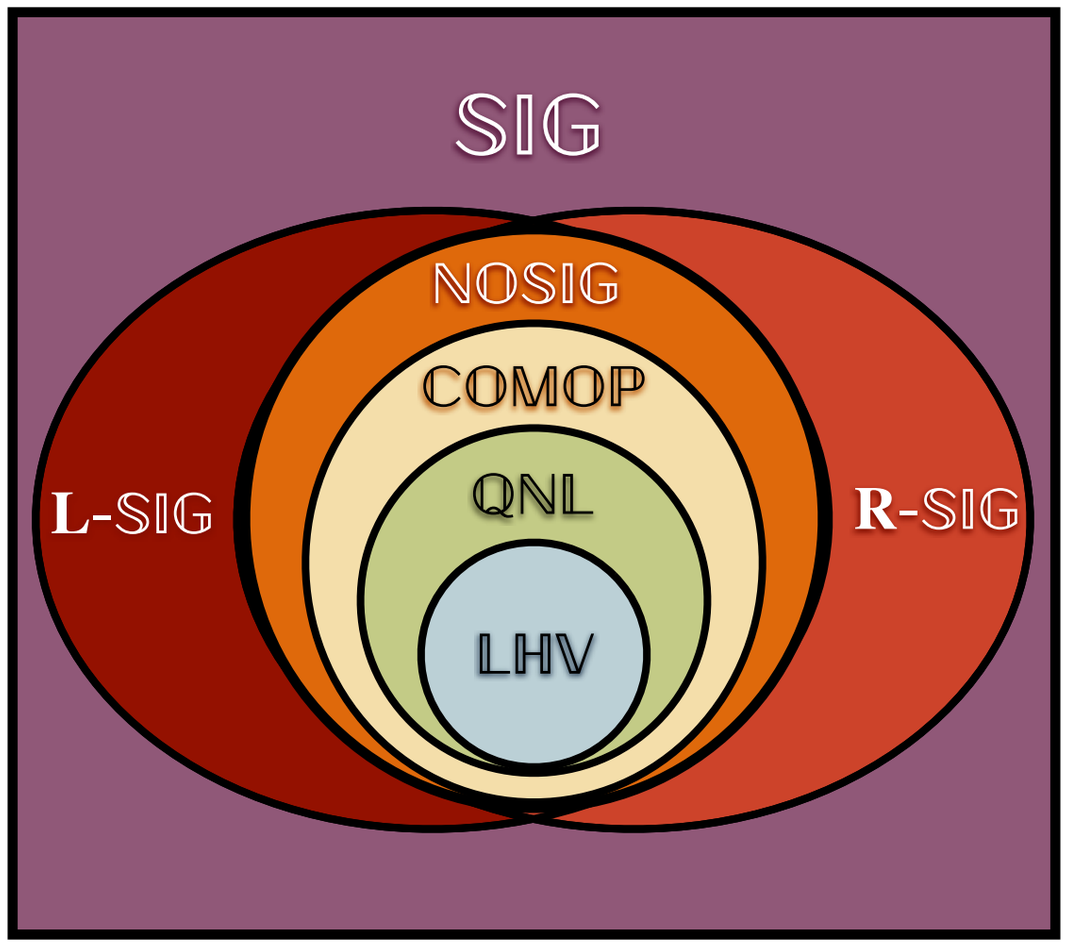


NON-LOCALITY HIERARCHY



$$\text{N-party NOSIG} = \bigcap_{\pi} \pi\text{-SIG}$$

NON-LOCALITY HIERARCHY



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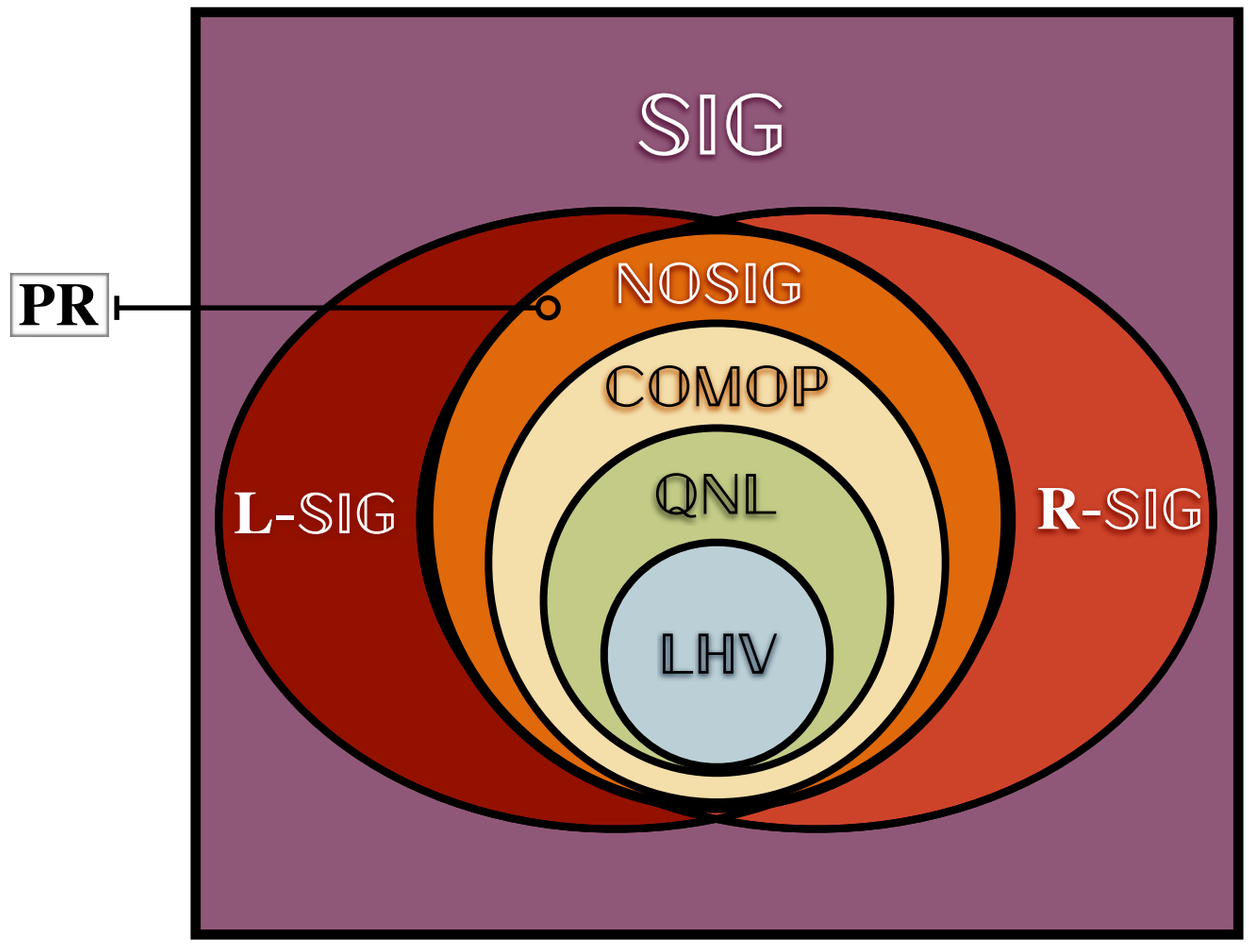

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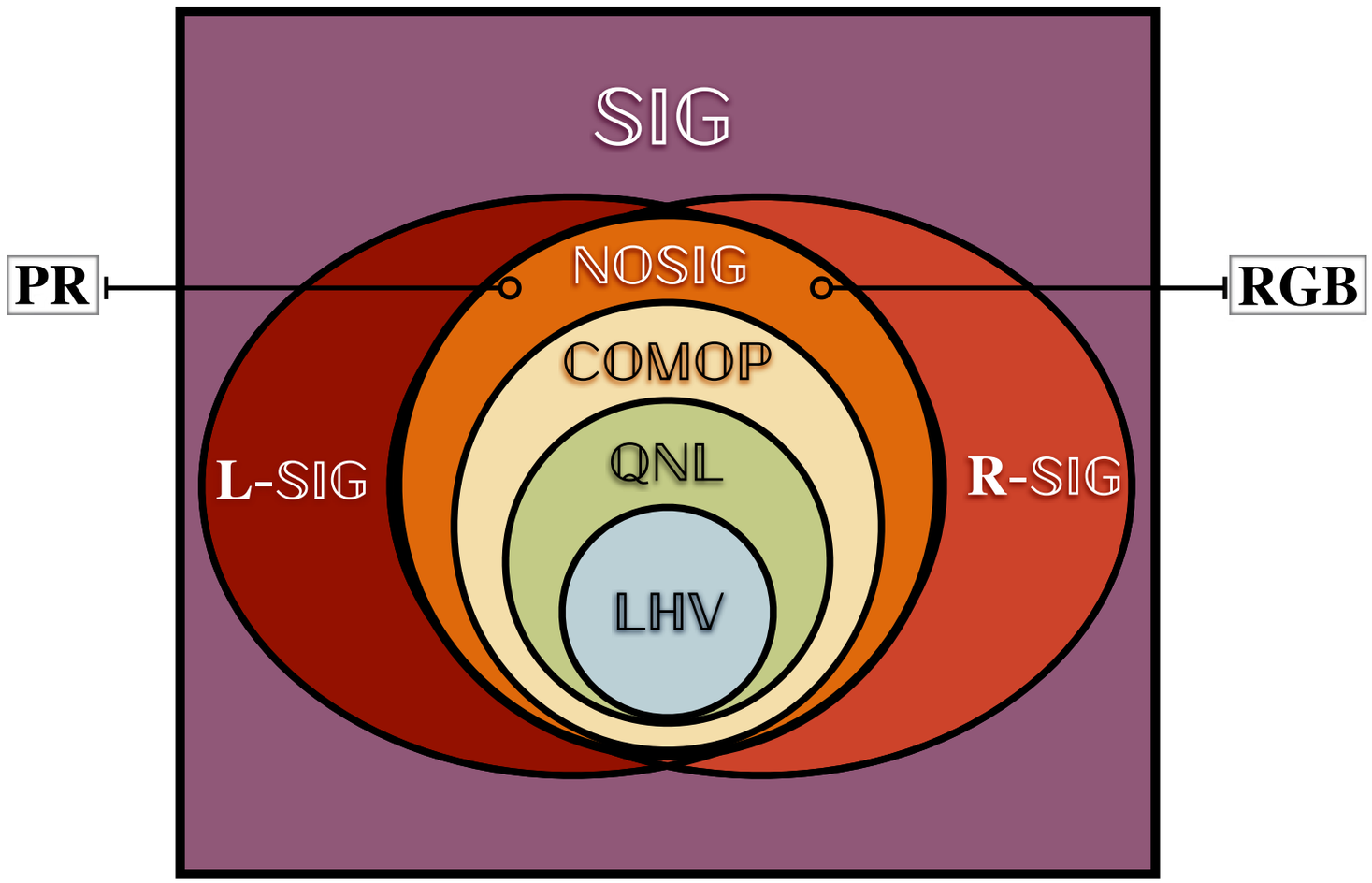

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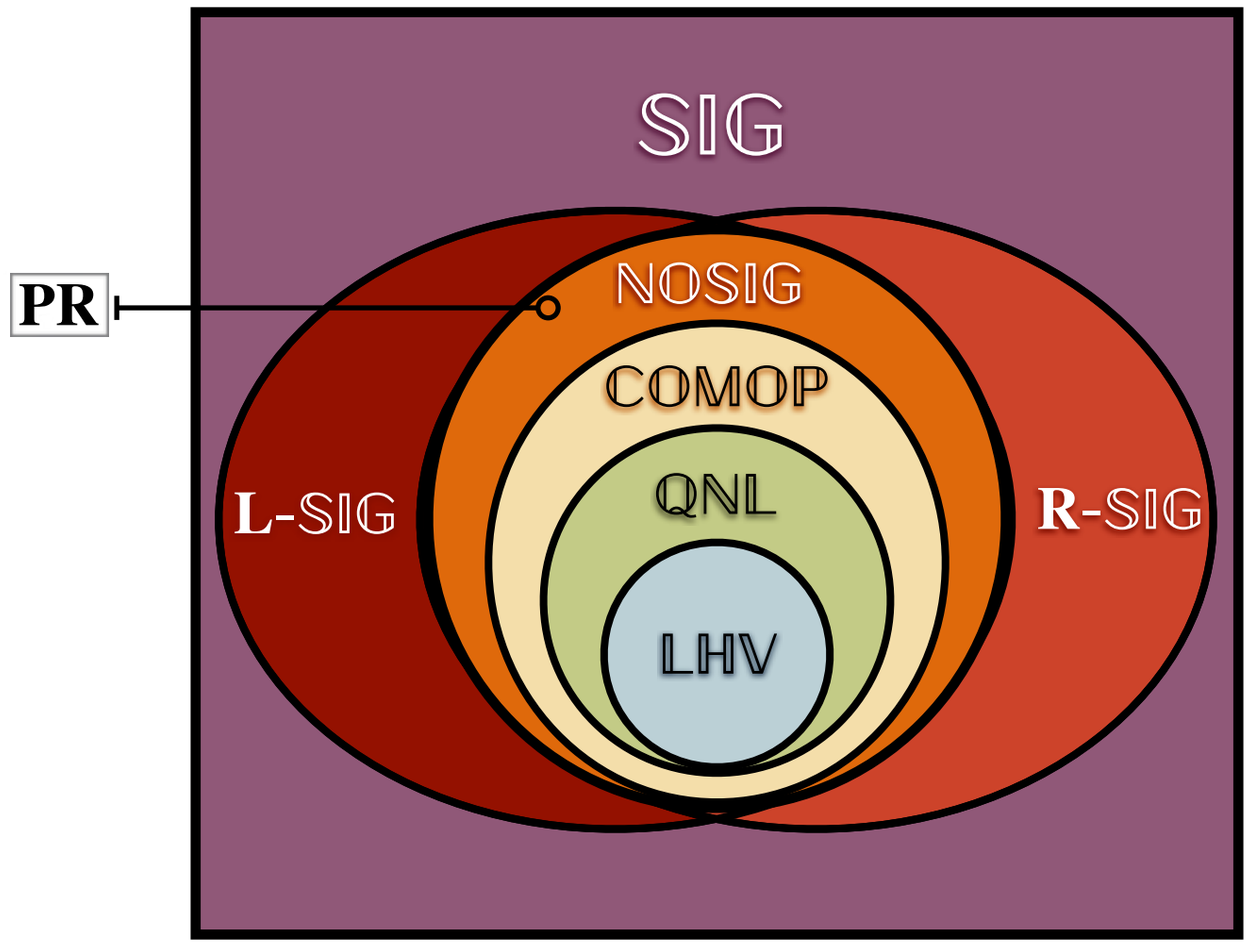

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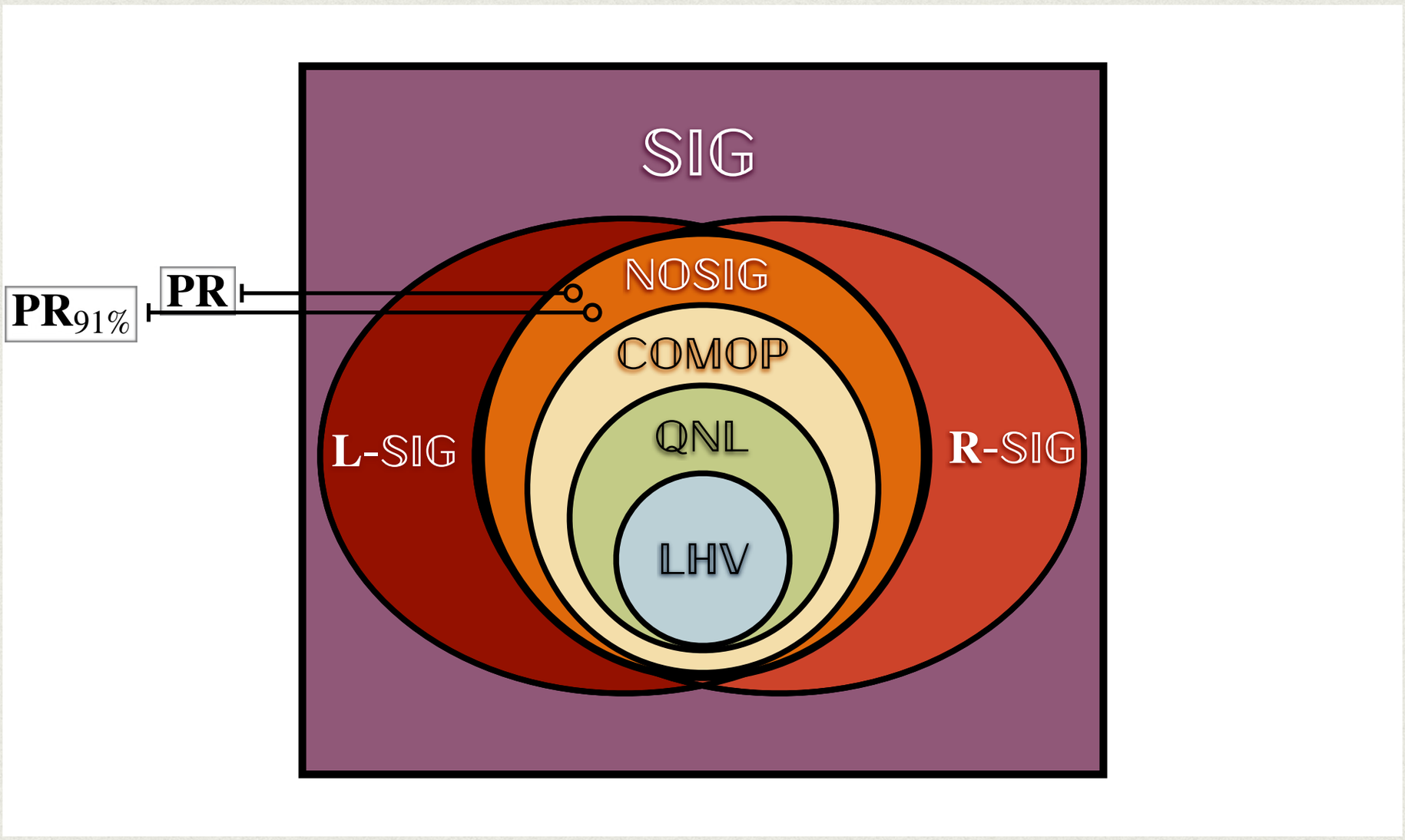

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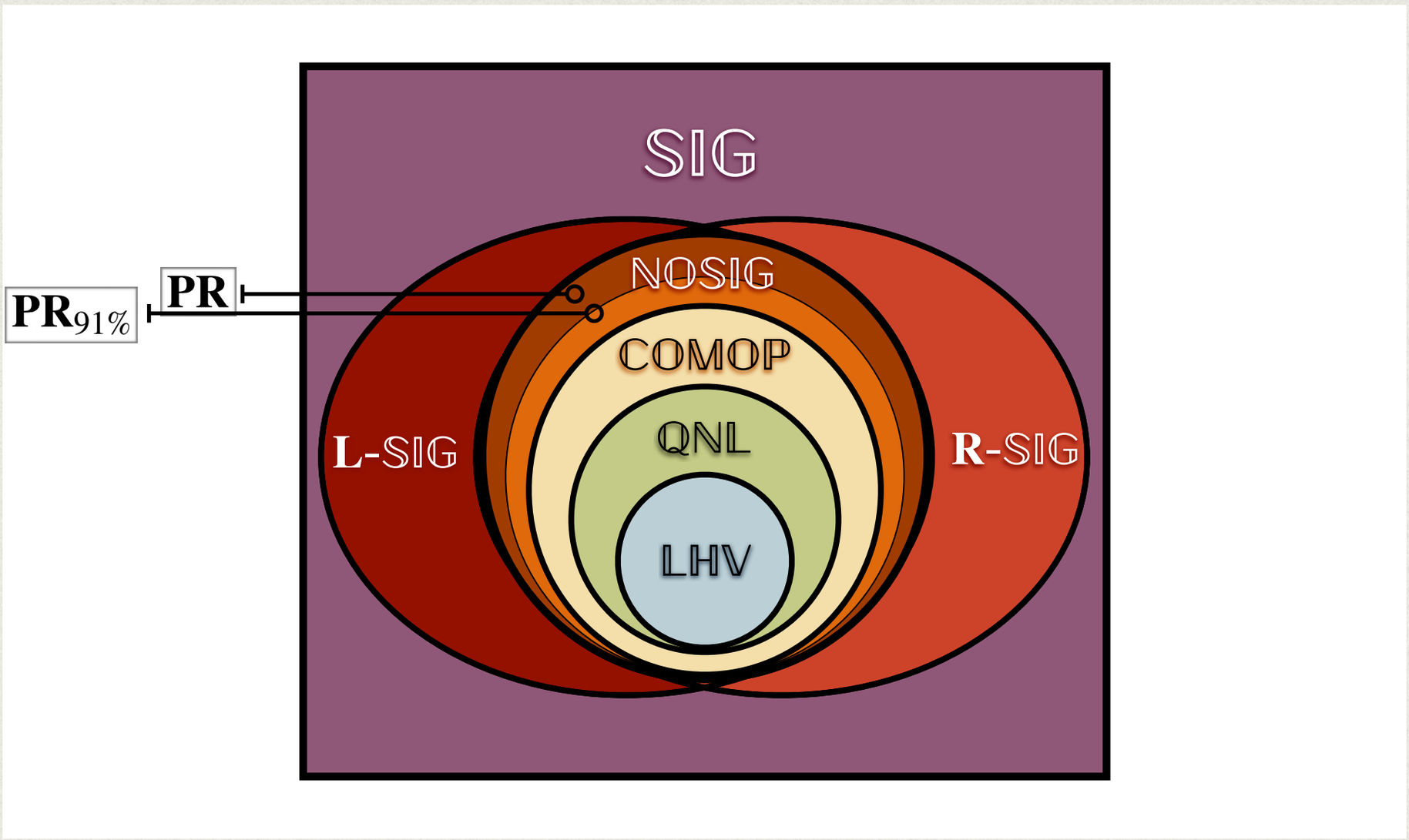

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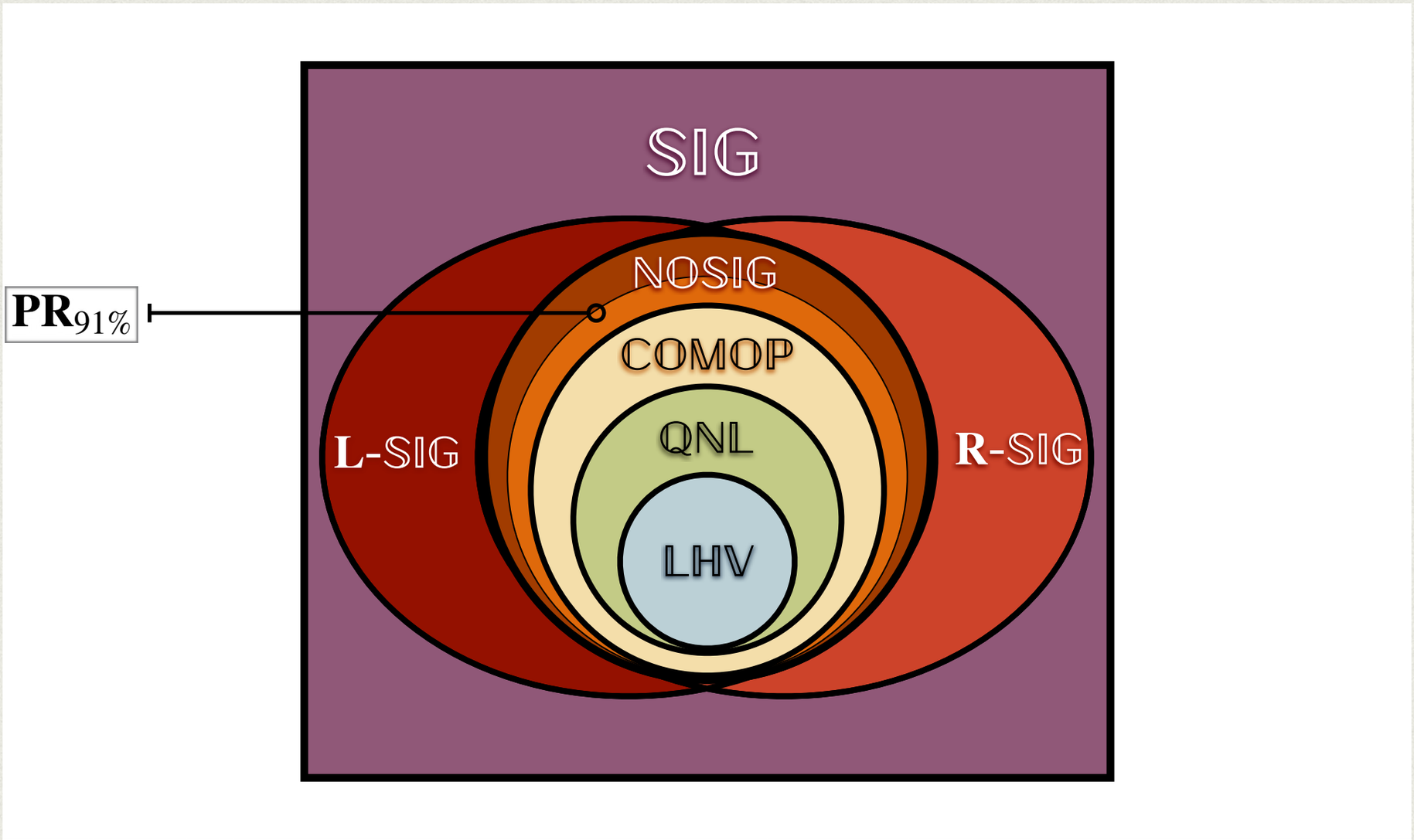

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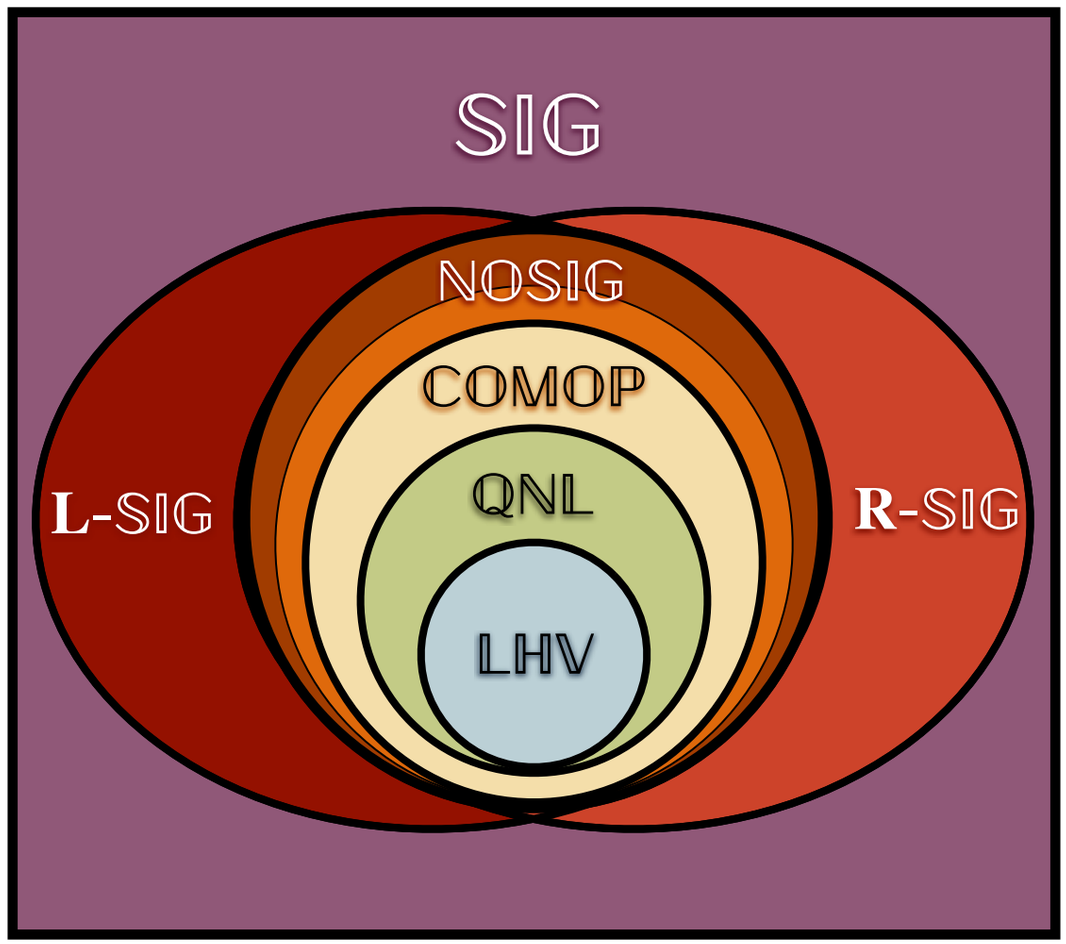

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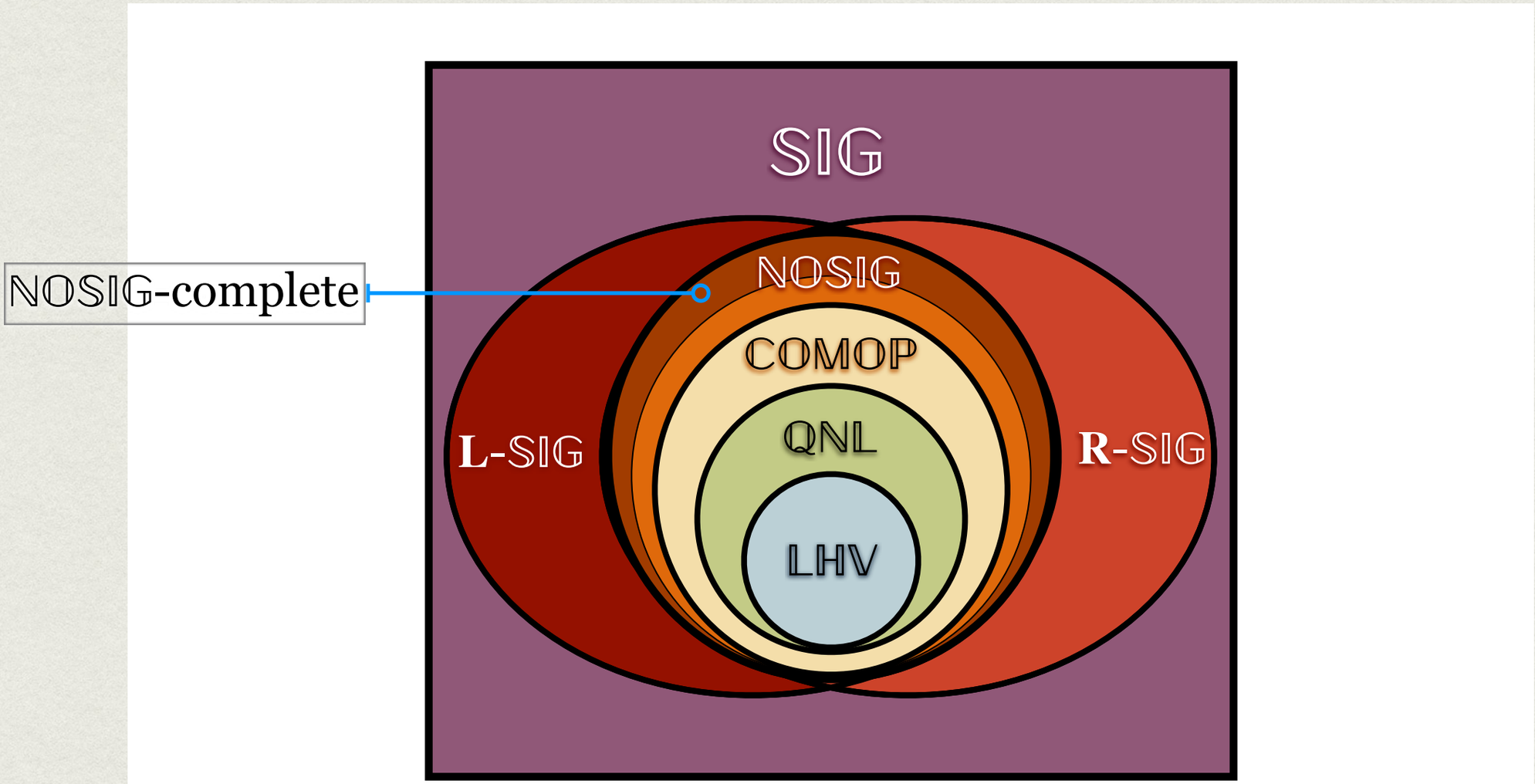

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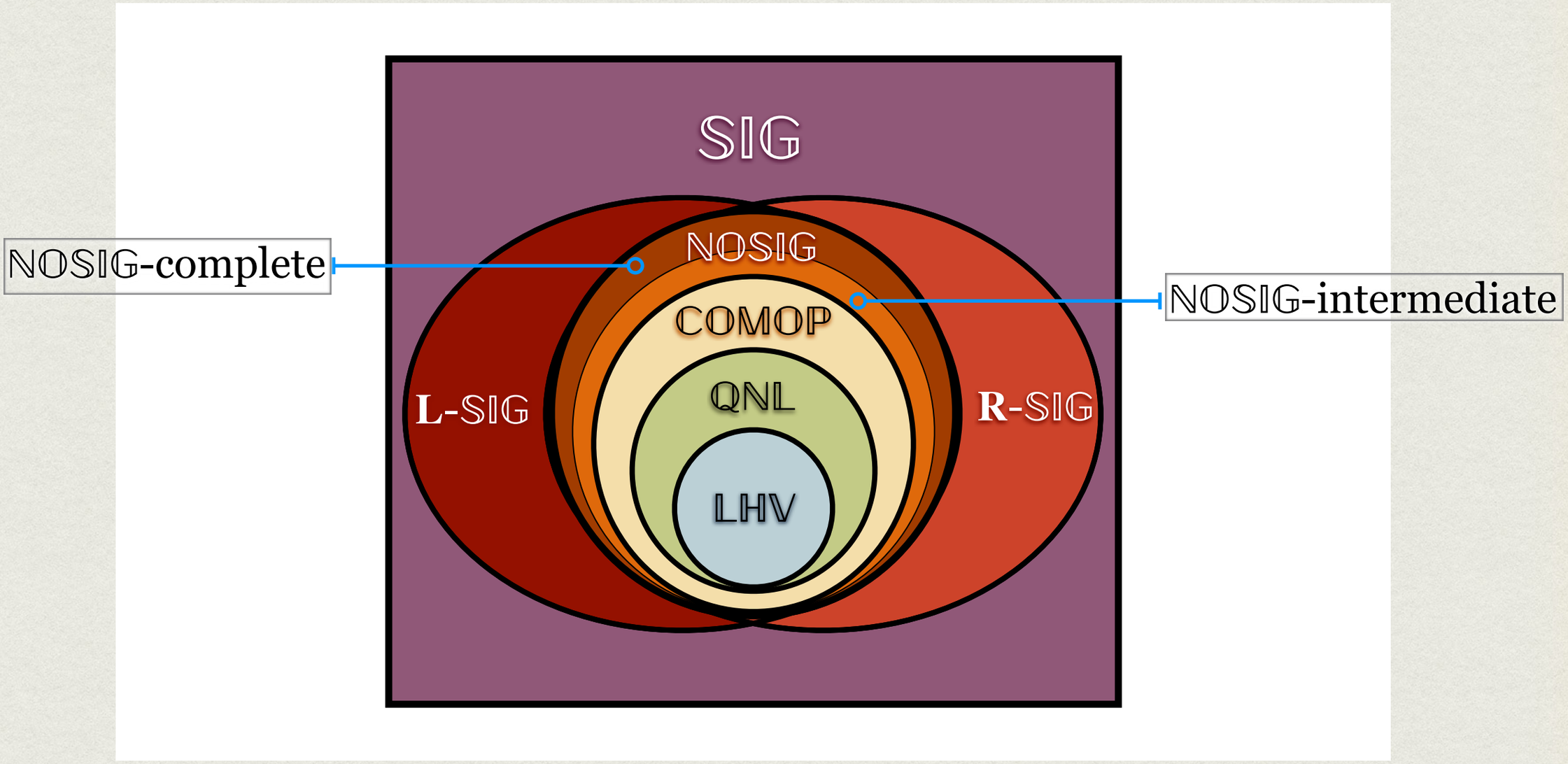
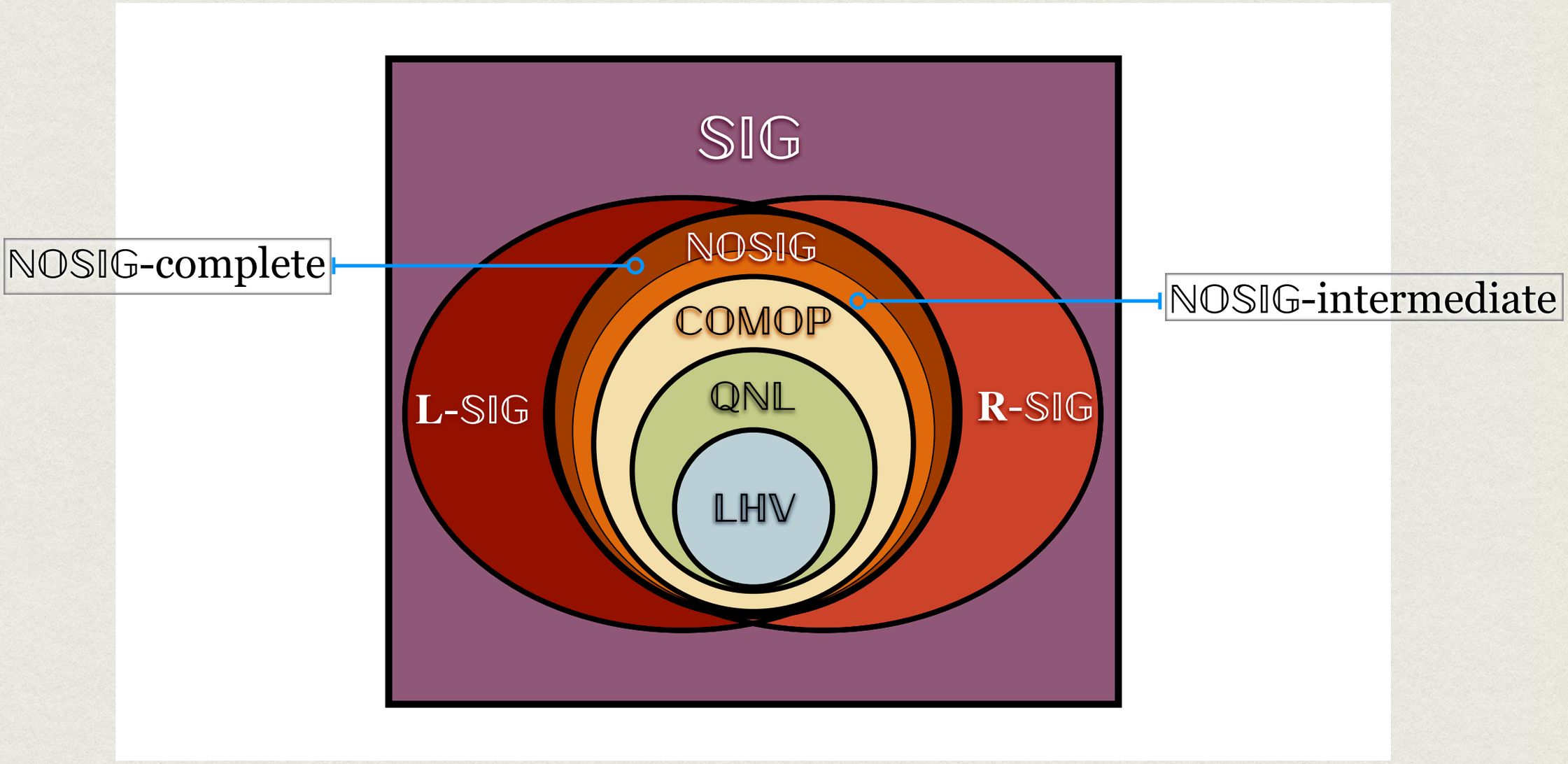


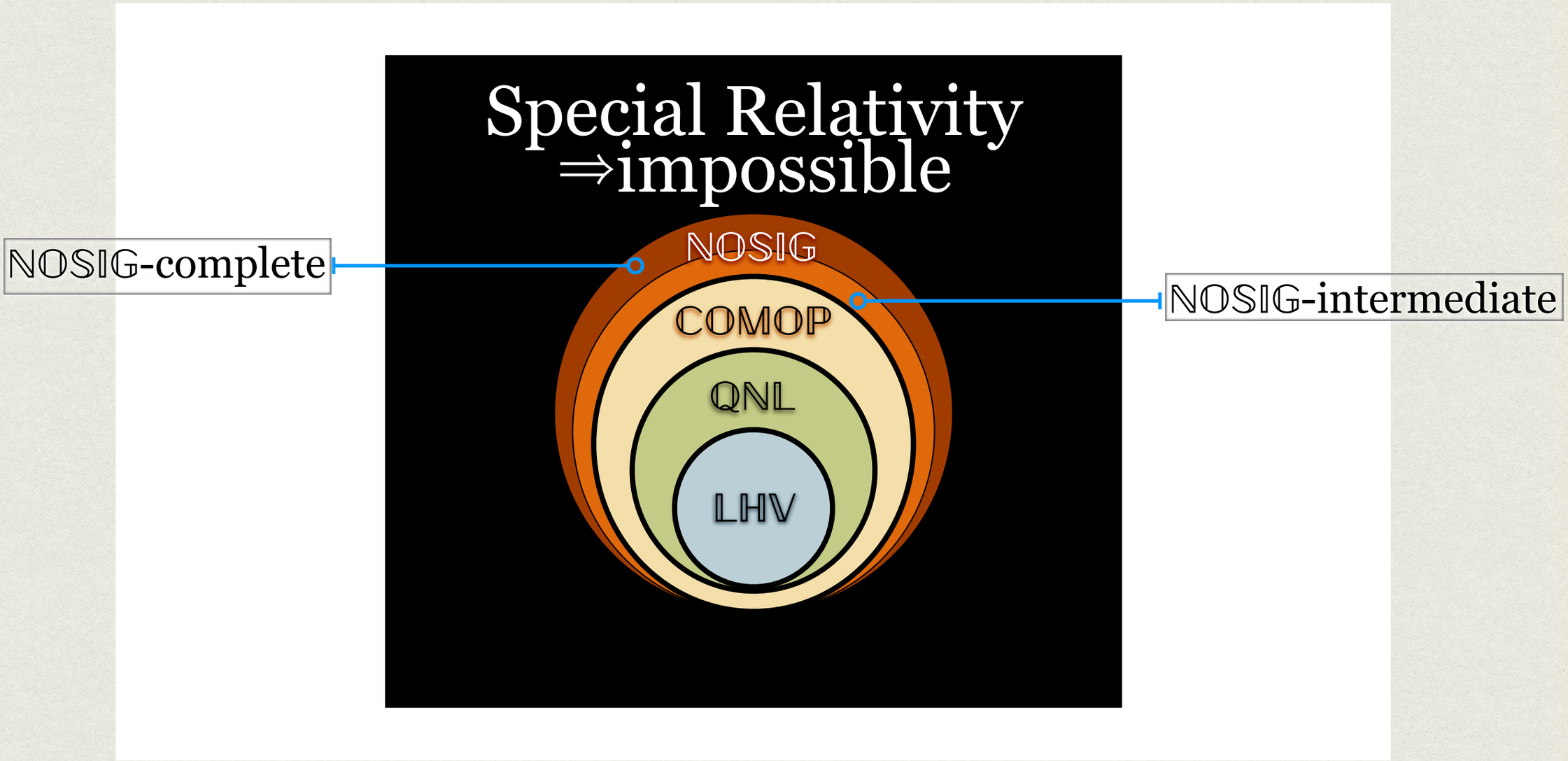
NON-LOCALITY HIERARCHY

At speeds greater than *c*



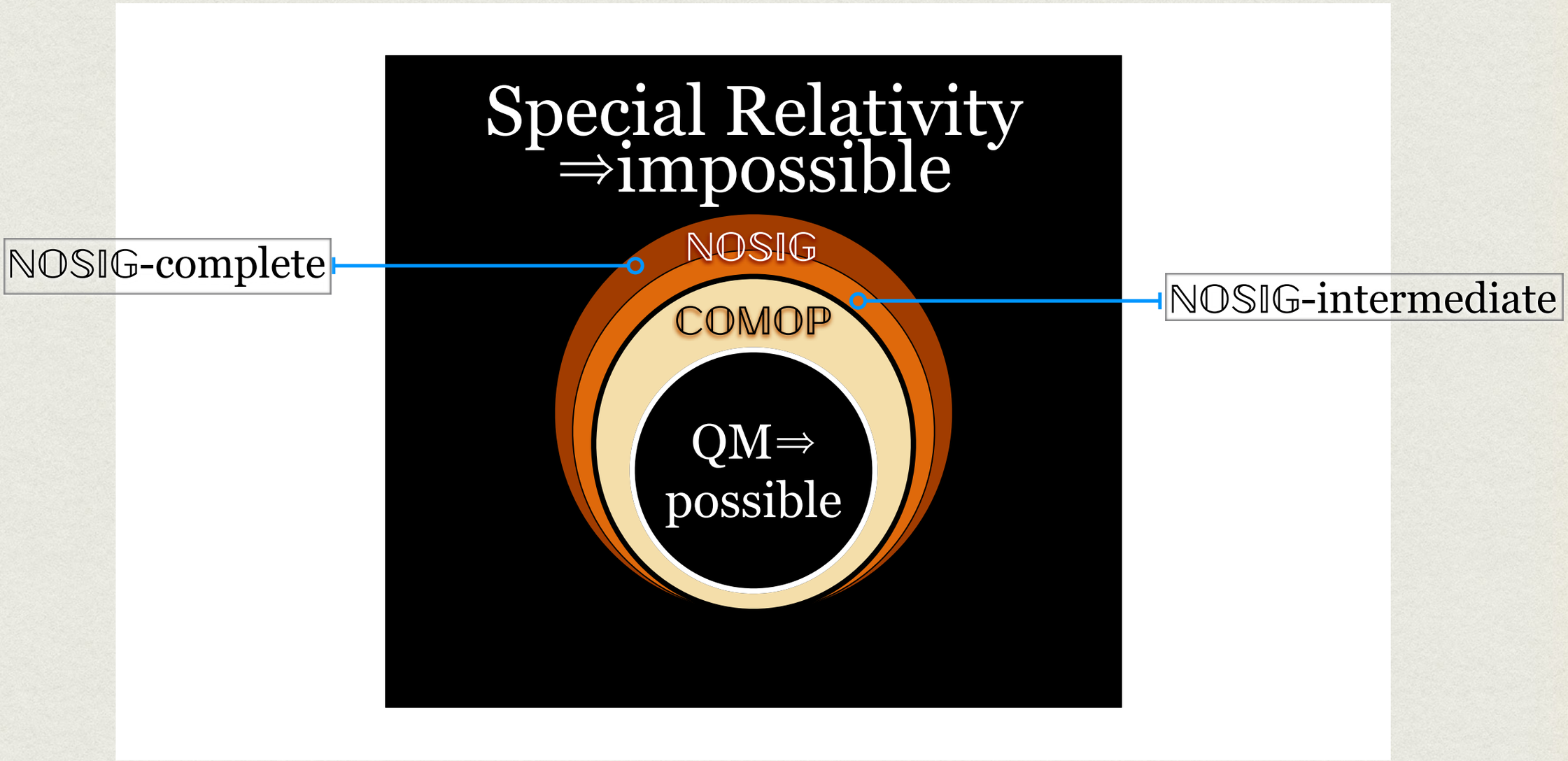
NON-LOCALITY HIERARCHY

At speeds greater than *c*



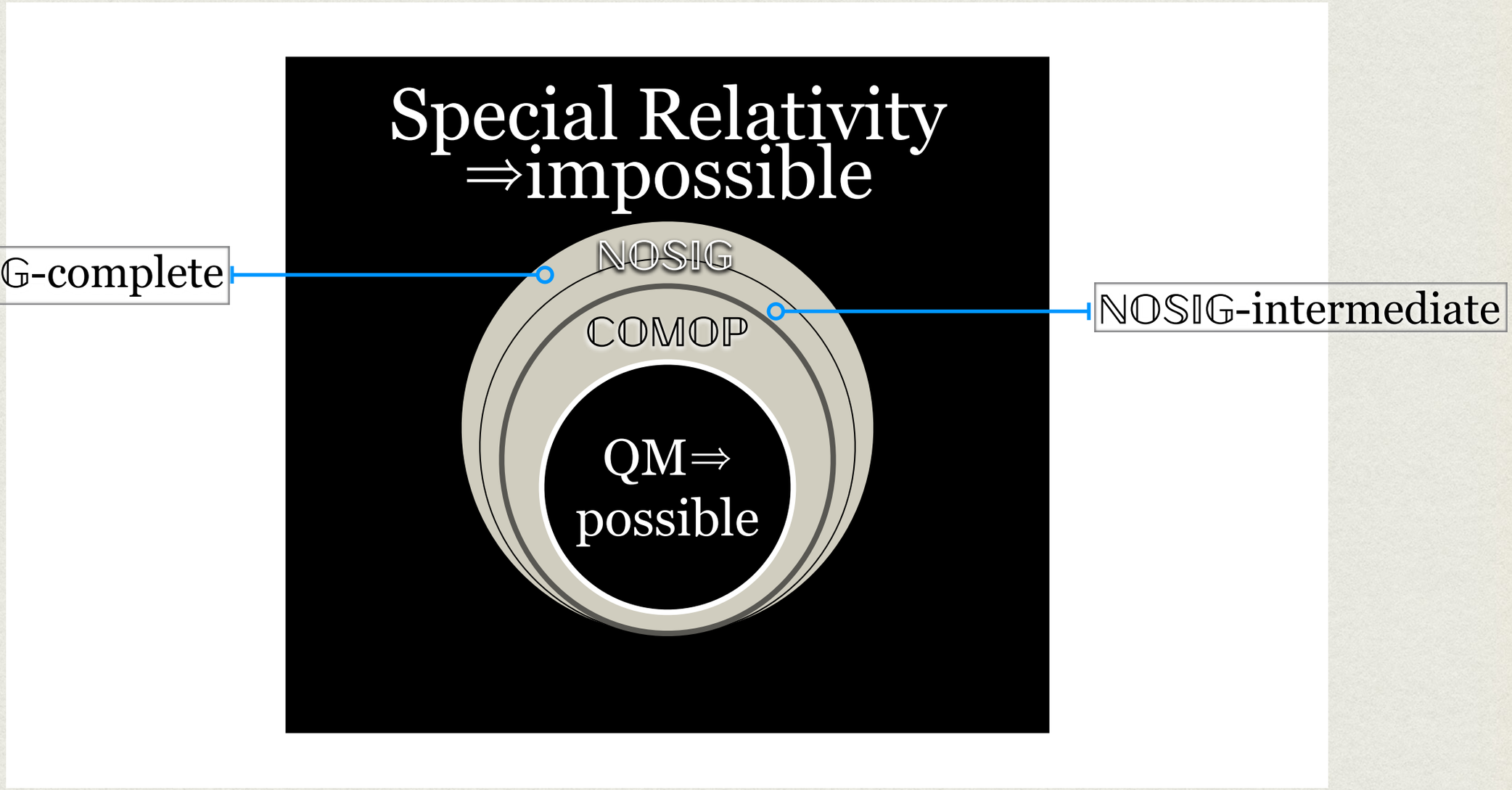
NON-LOCALITY HIERARCHY

At speeds greater than *c*



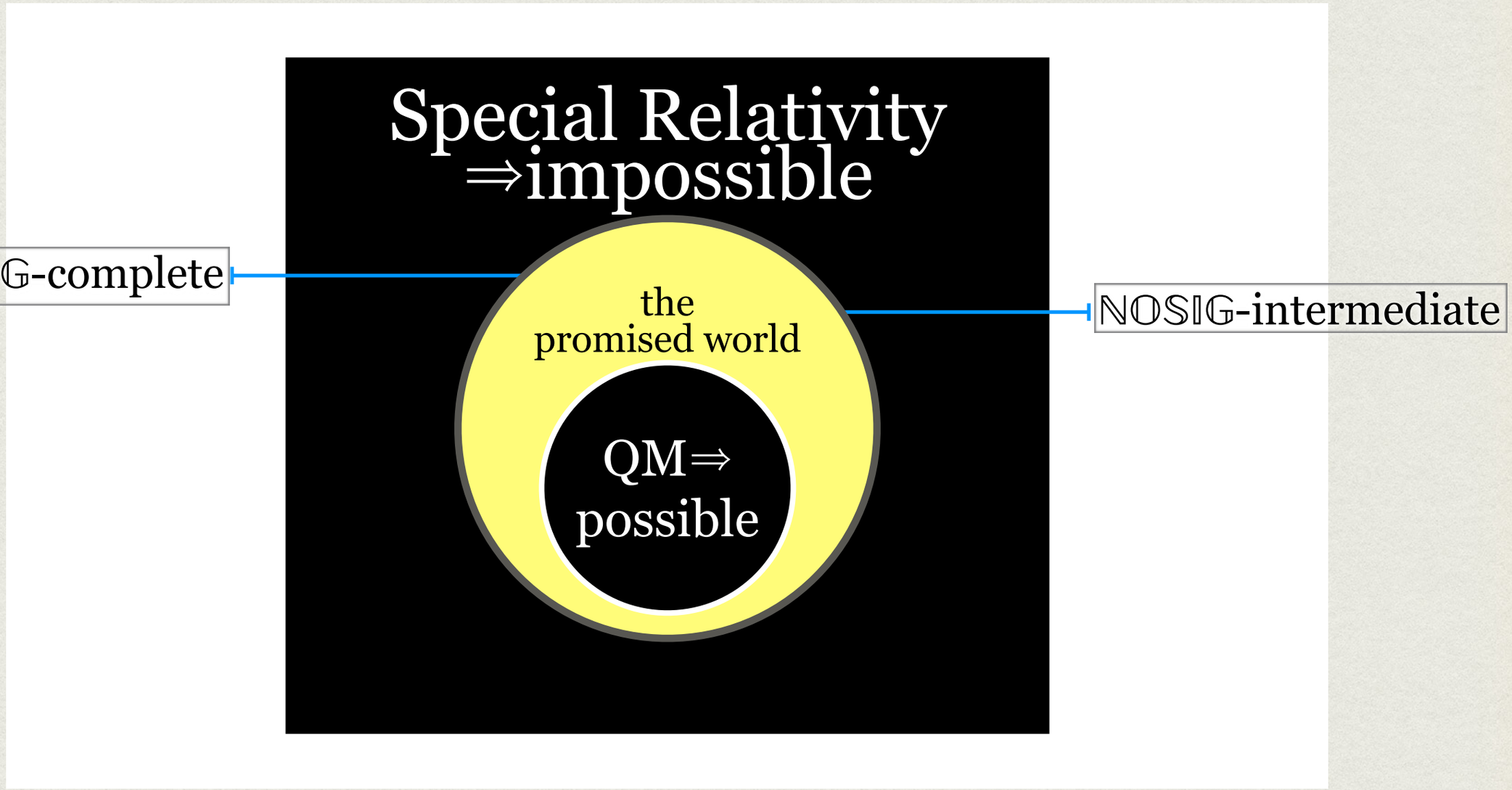
NON-LOCALITY HIERARCHY

At speeds greater than *c*



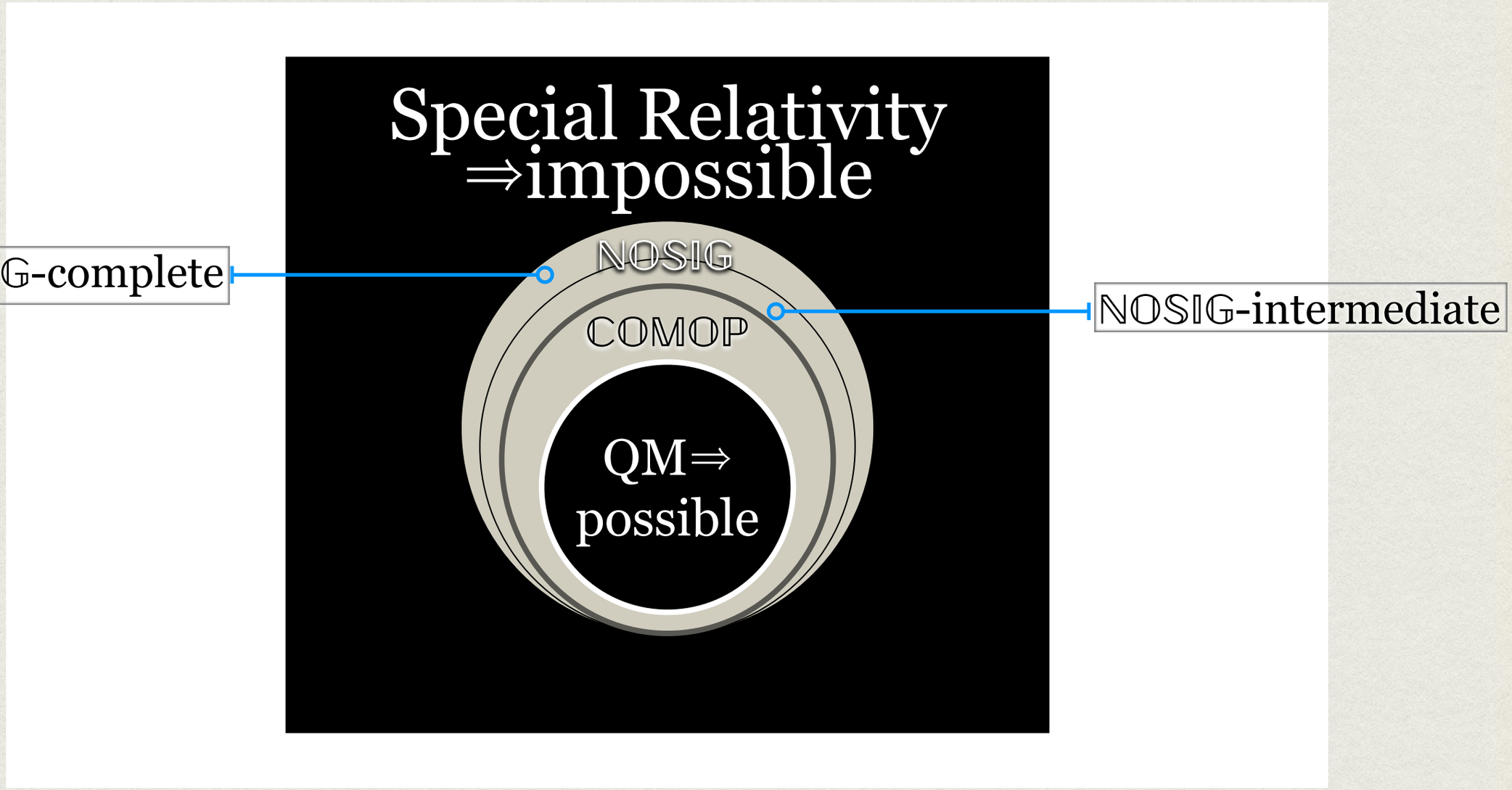
NON-LOCALITY HIERARCHY

At speeds greater than *c*



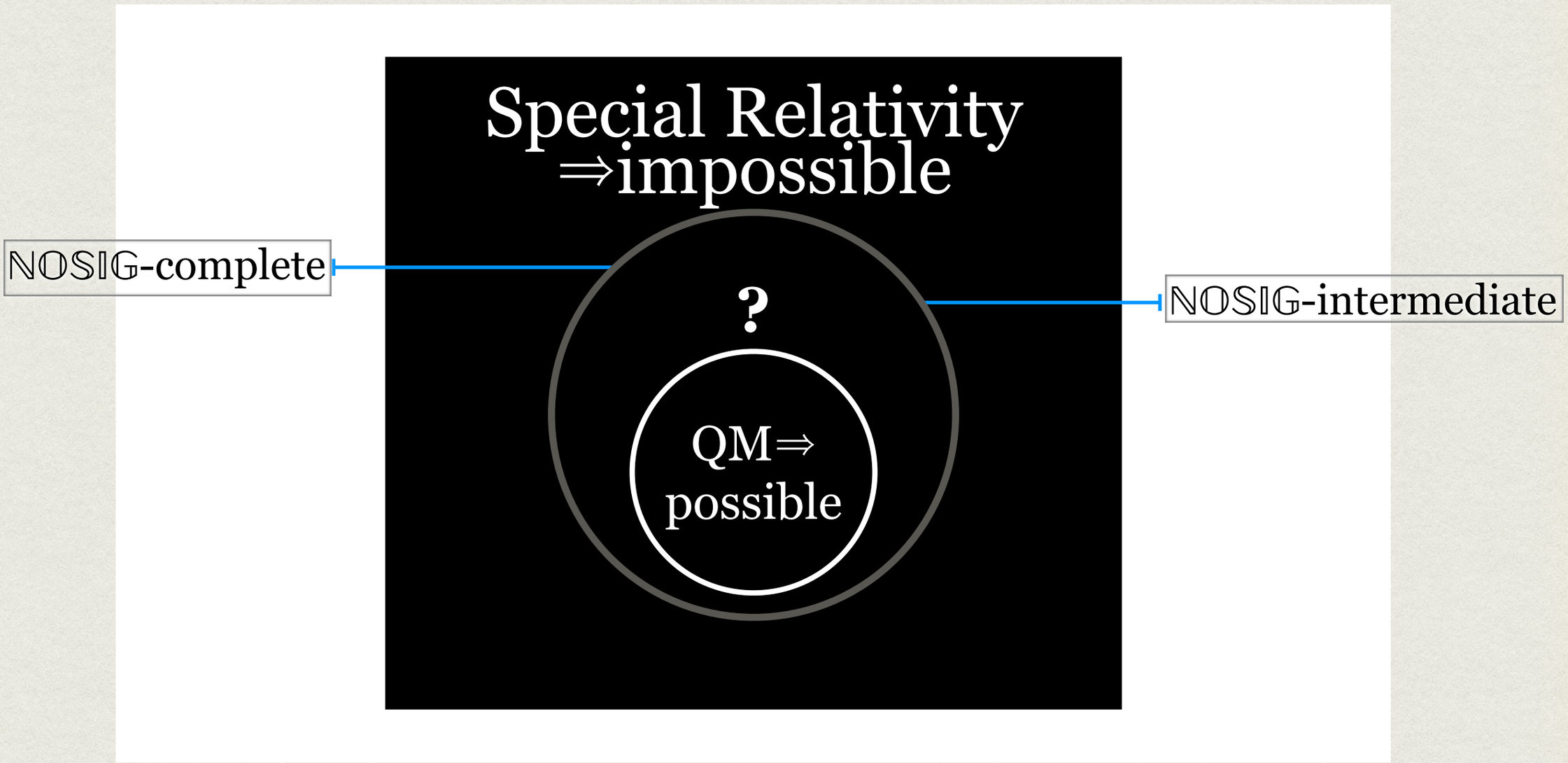
NON-LOCALITY HIERARCHY

At speeds greater than *c*



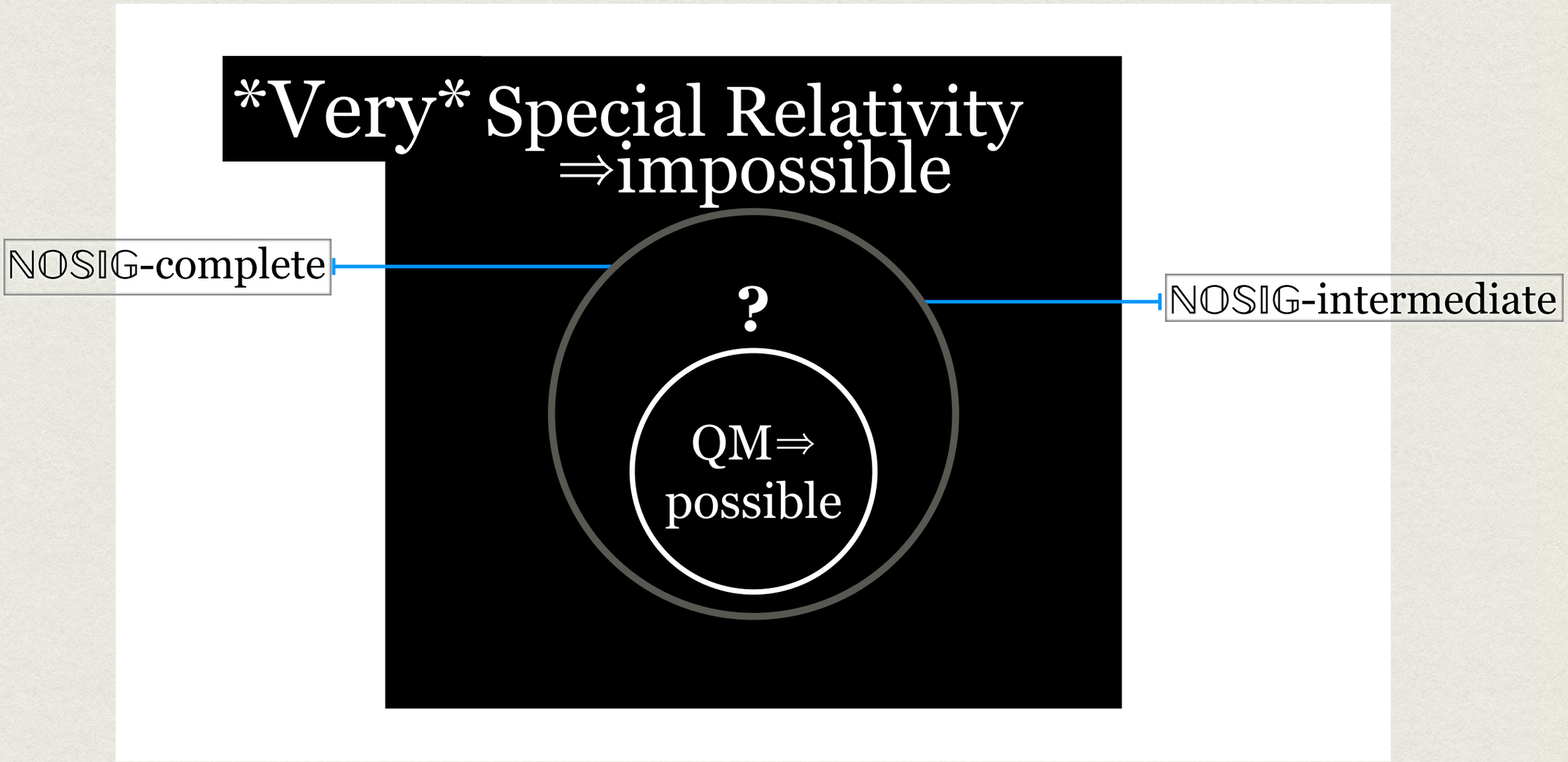
NON-LOCALITY HIERARCHY

At speeds greater than *c*



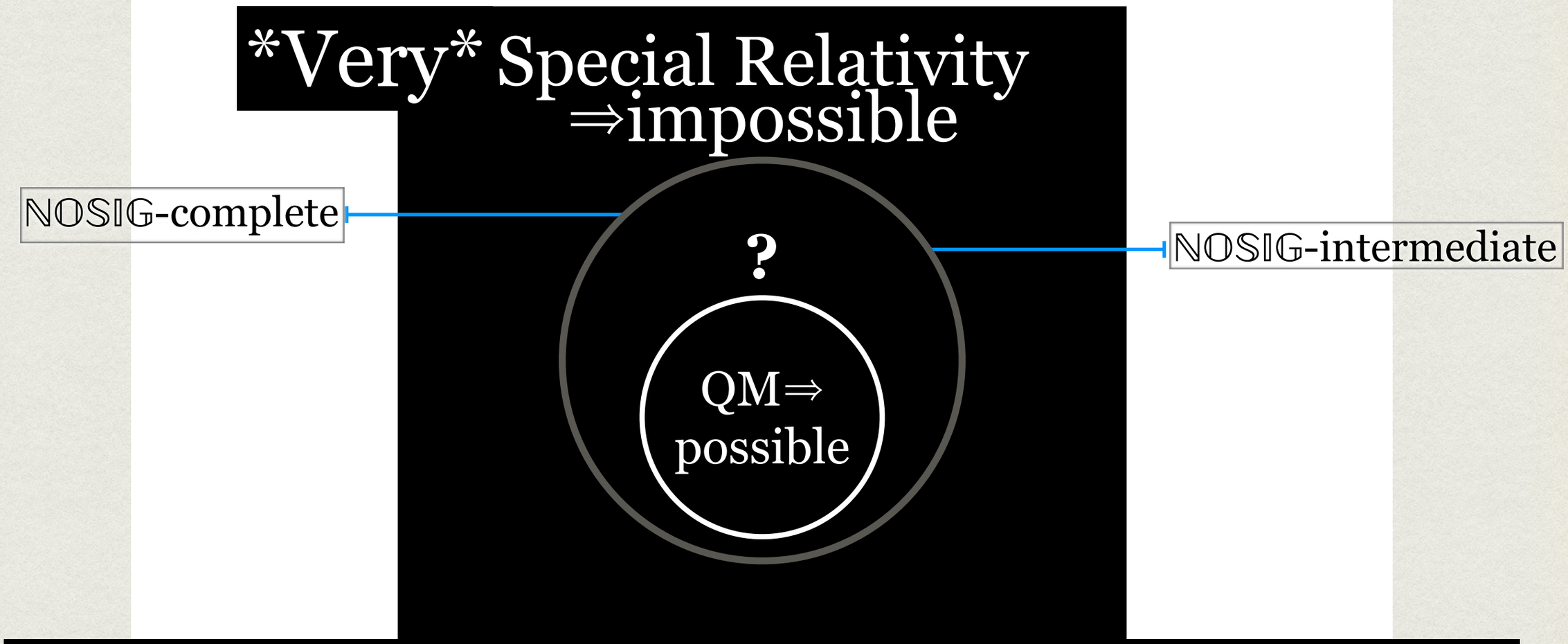
NON-LOCALITY HIERARCHY

At speeds greater than *c*



NON-LOCALITY HIERARCHY

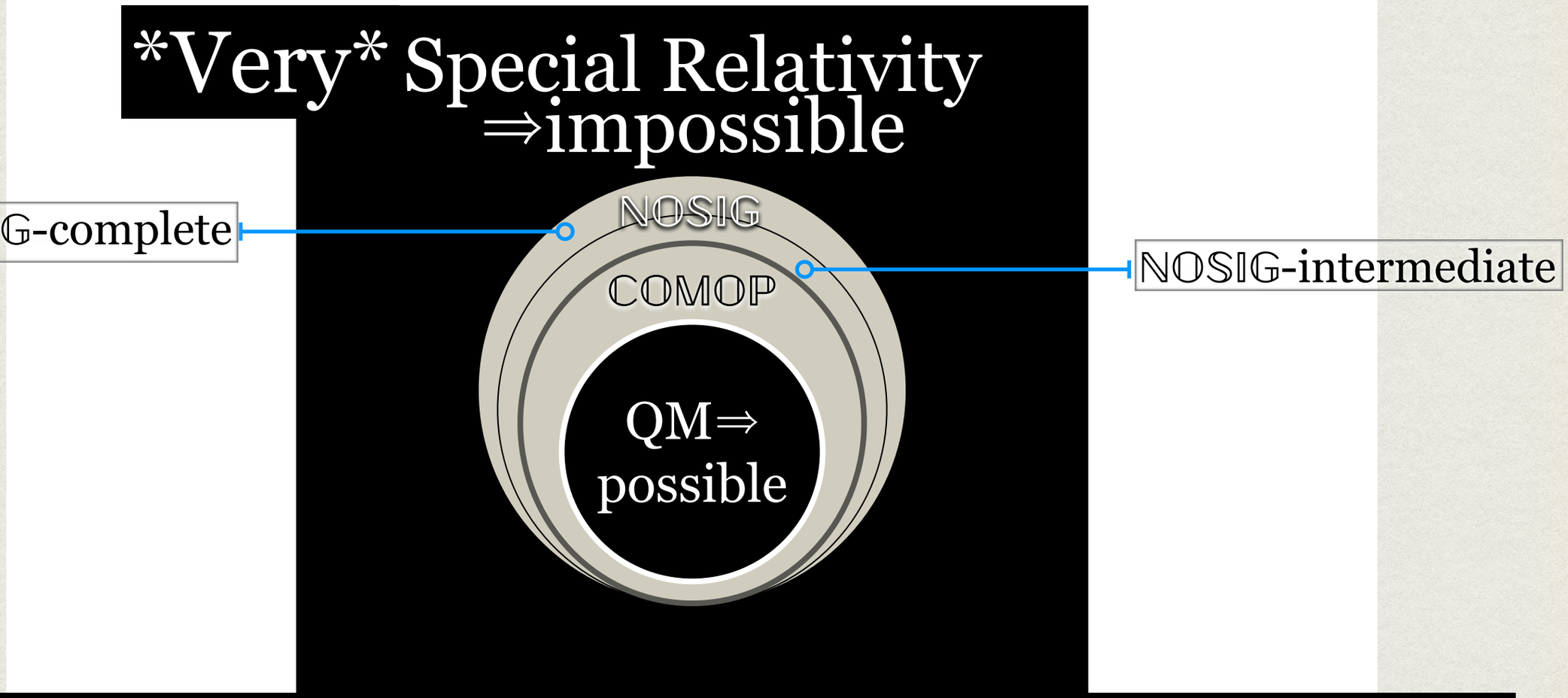
At speeds greater than *c*



VSR1 : no correlation beyond QNL can be achieved faster than speed *c*

NON-LOCALITY HIERARCHY

At speeds greater than *c*



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NON-LOCALITY HIERARCHY

At speeds greater than *c*

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NON-LOCALITY HIERARCHY

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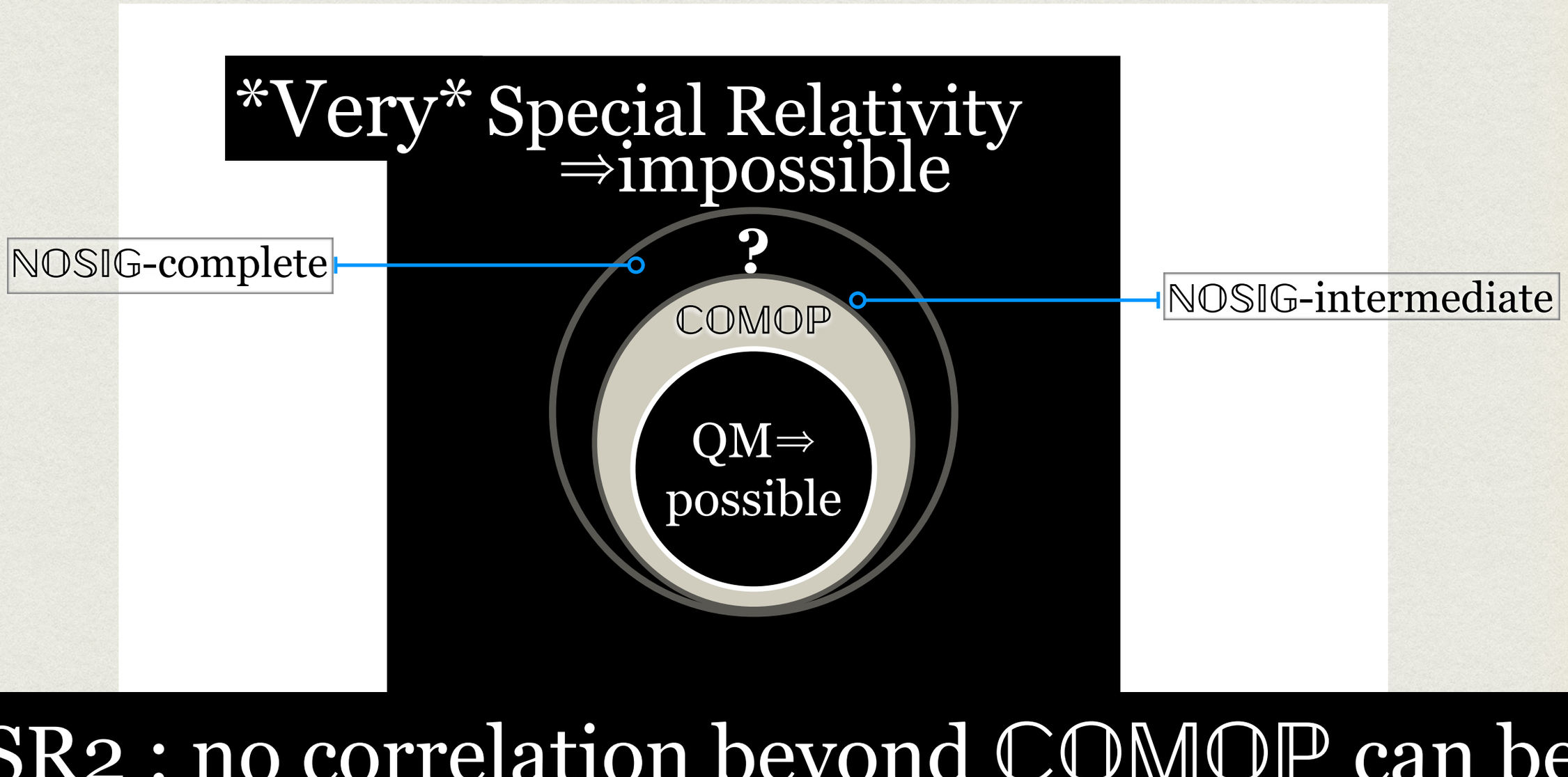
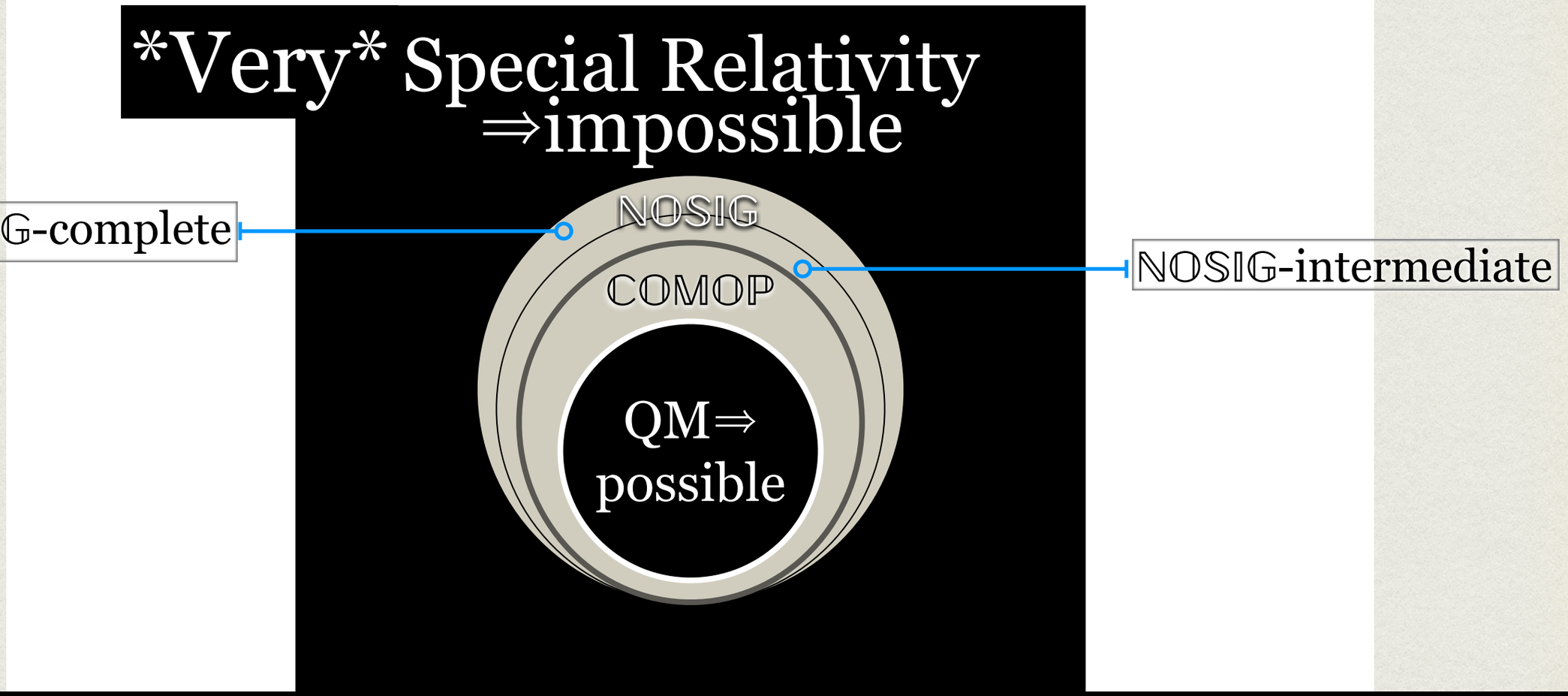


VSR2 : no correlation beyond COMOP can be achieved faster than speed *c*

NON-LOCALITY HIERARCHY

At speeds greater than *c*



VSR2 : no correlation beyond COMOP can be achieved faster than speed *c*

NON-LOCALITY HIERARCHY

At speeds greater than *c*

VSR2 : no correlation beyond COMOP can be achieved faster than speed *c*

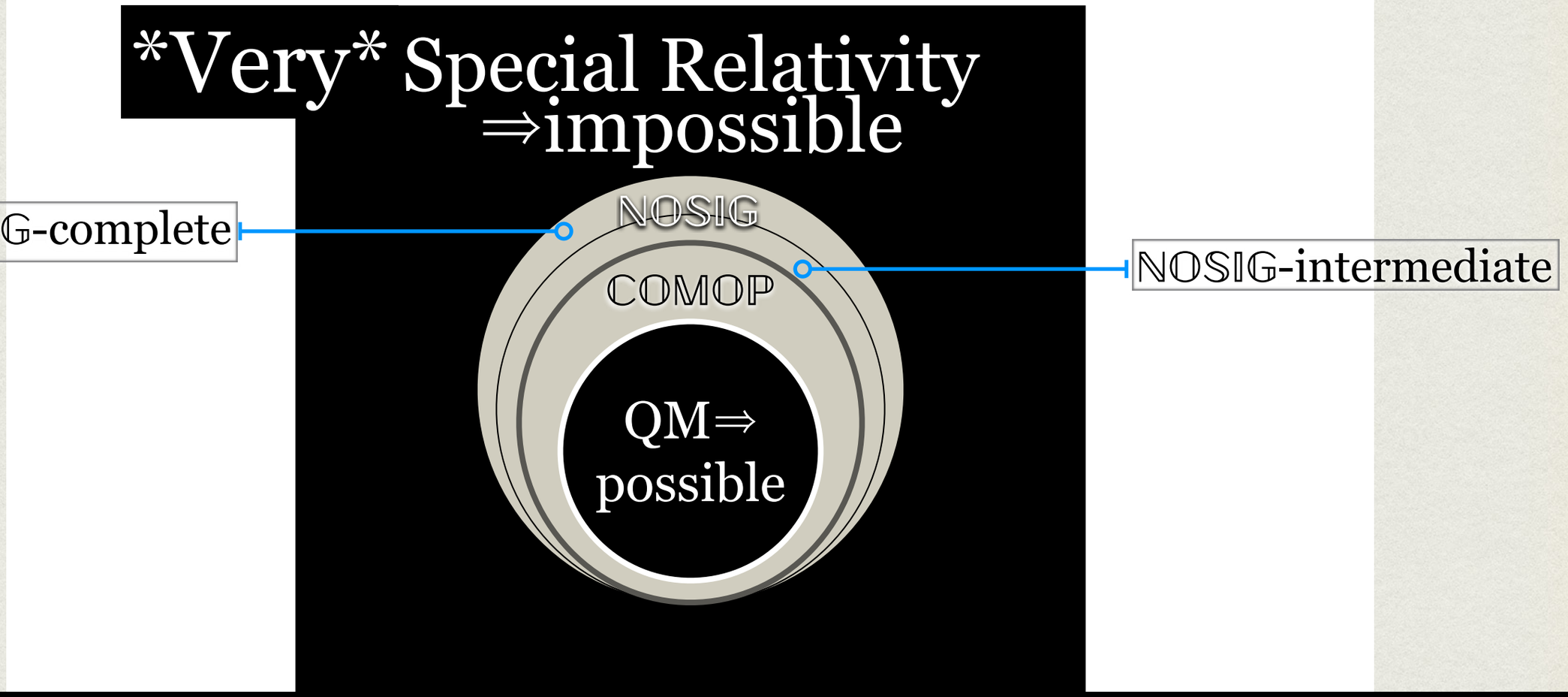
NON-LOCALITY HIERARCHY

At speeds greater than *c*

VSR3 : no correlation that is NOSIG-complete can be achieved faster than speed *c*

NON-LOCALITY HIERARCHY

At speeds greater than *c*

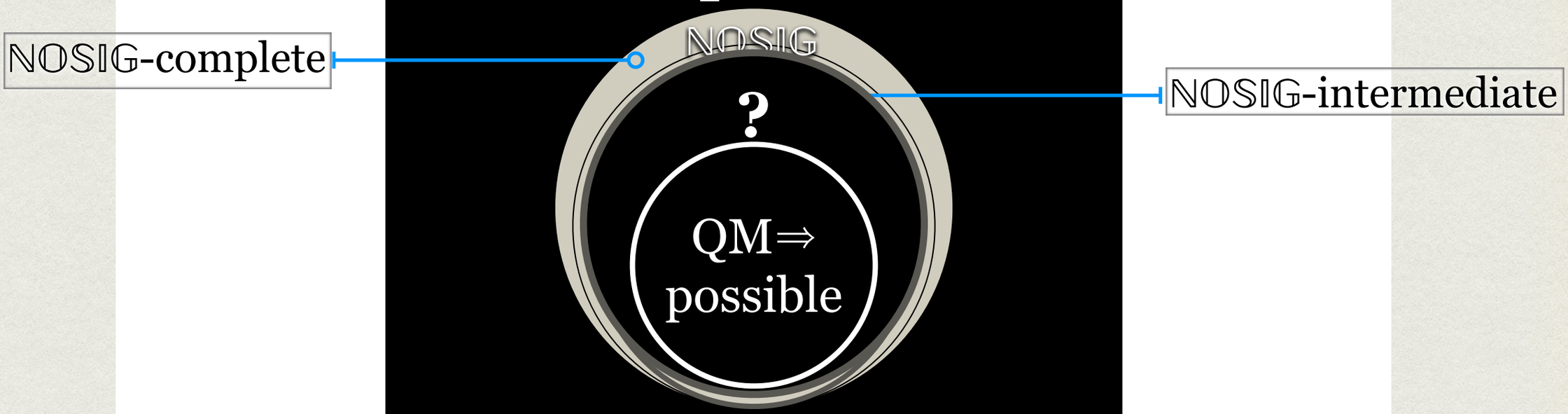


VSR3 : no correlation that is NOSIG-complete can be achieved faster than speed *c*

NON-LOCALITY HIERARCHY

At speeds greater than *c*

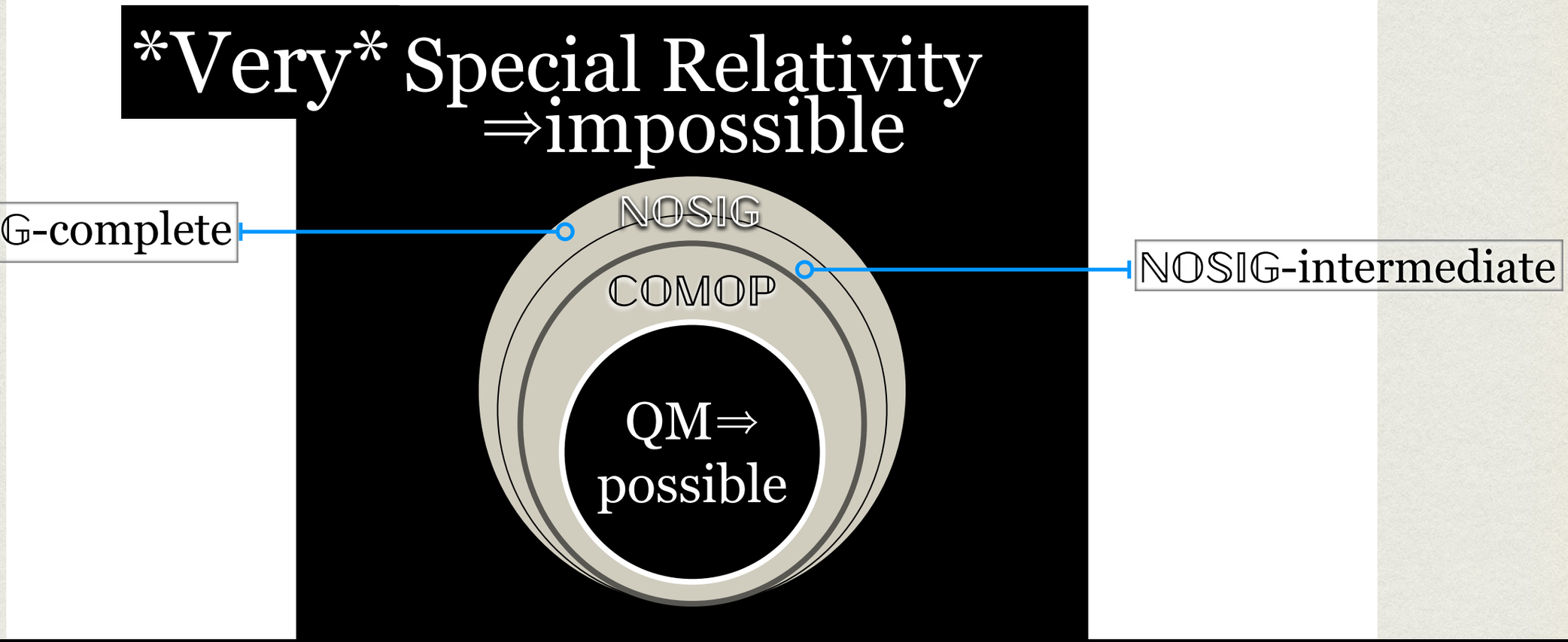
Very Special Relativity
⇒impossible



QM+1/2/3 : some correlation beyond QNL/COMOP/NOSIG-intermediate can be achieved faster than speed *c*

NON-LOCALITY HIERARCHY

At speeds greater than *c*



QM+1/2/3 : some correlation beyond QNL/COMOP/NOSIG-intermediate can be achieved faster than speed *c*

TO DO LIST

VSR1 : no correlation beyond QNL can be achieved faster than speed *c*

VSR2 : no correlation beyond COMOP can be achieved faster than speed *c*

VSR3 : no correlation that is NOSIG-complete can be achieved faster than speed *c*

QM+1: some correlation beyond QNL can be achieved faster than speed *c*

QM+2 : some correlation beyond COMOP can be achieved faster than speed *c*

QM+3 : all NOSIG correlations can be achieved faster than speed *c*

9th International Conference on
New Frontiers in Physics (ICNFP 2020)

A WHOLE WORLD LIES BETWEEN QUANTUM NON-LOCALITY AND COMMUNICATION

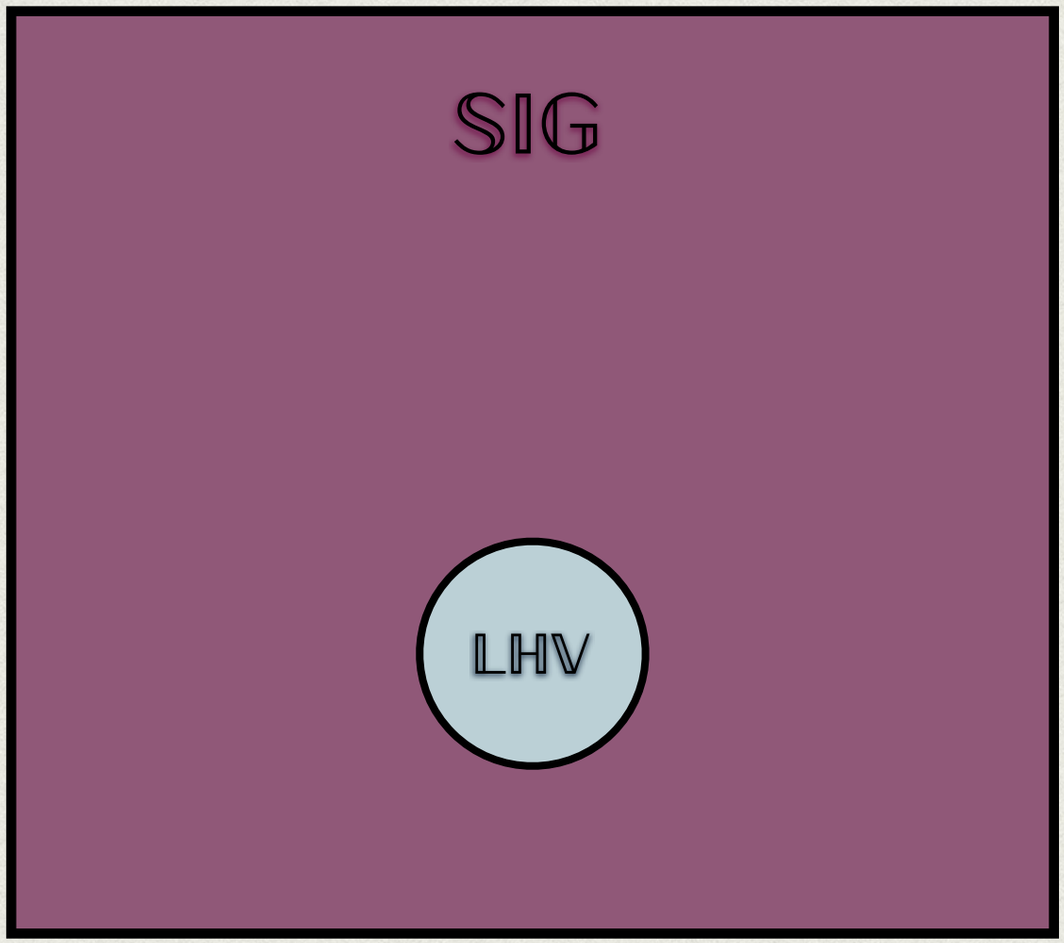
Claude Crépeau

McGill

NON-LOCALITY HIERARCHIES

NON-LOCALITY HIERARCHIES

NON-LOCALITY HIERARCHIES



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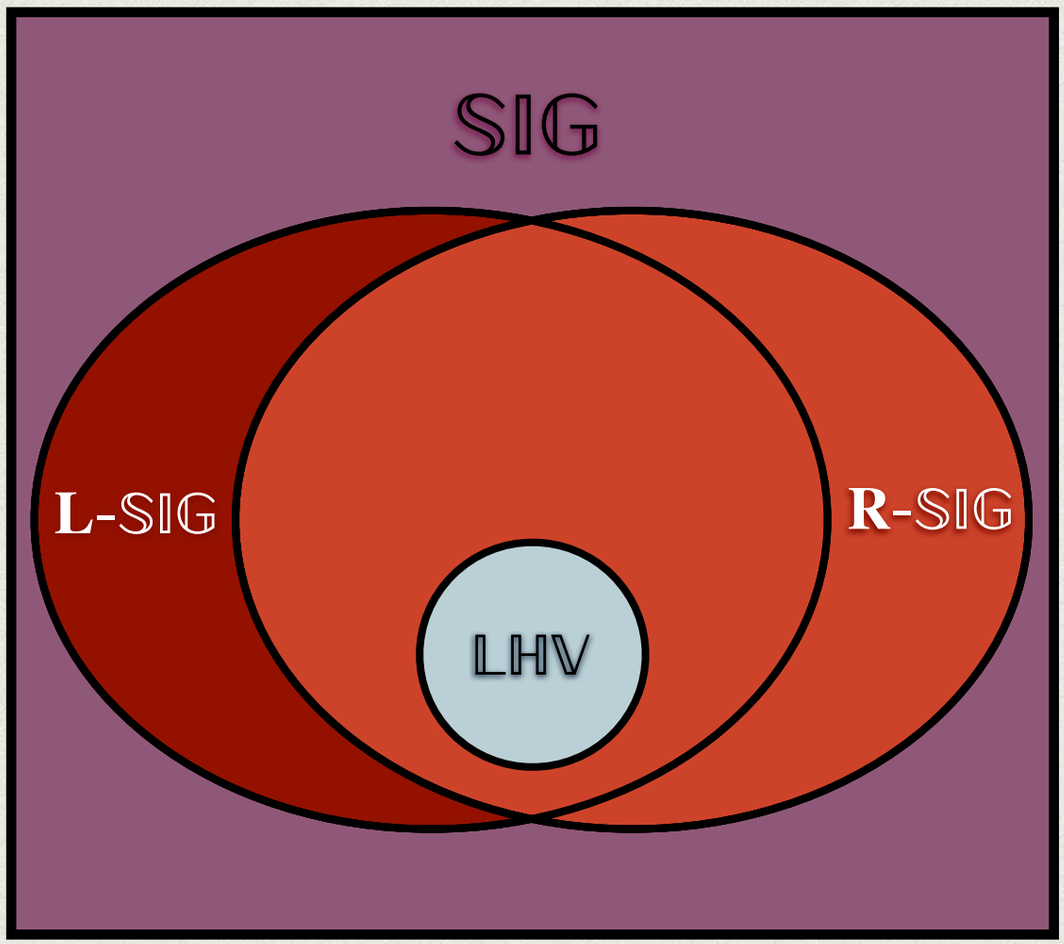
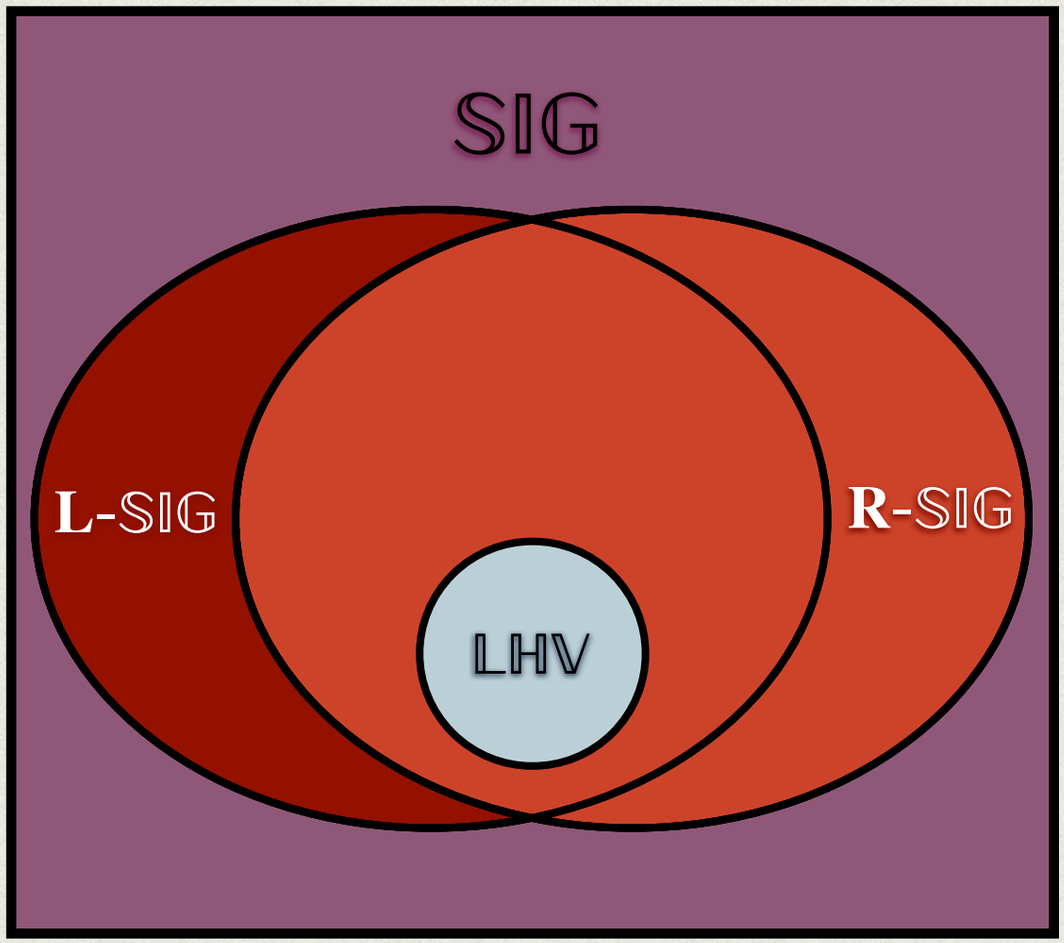


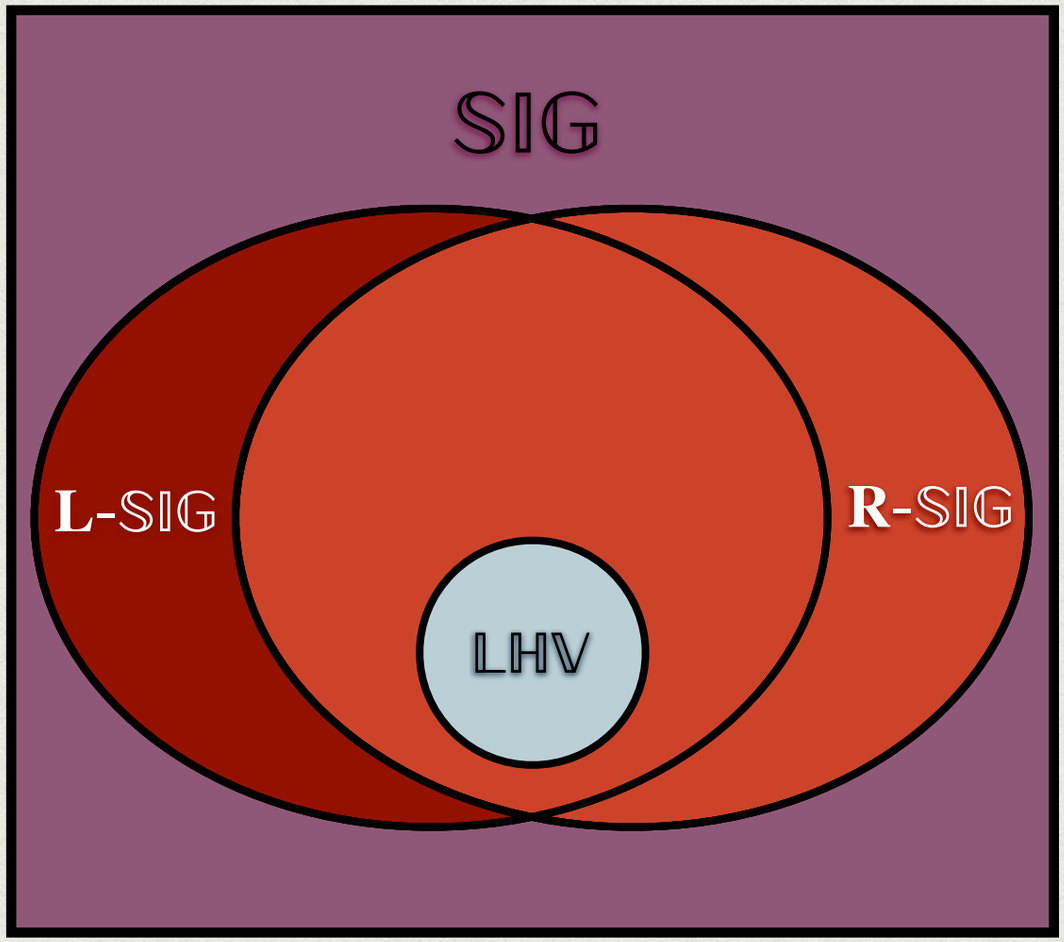
NON-LOCALITY HIERARCHIES

$|\mathbb{X}\rangle$ = Quantum analog of $\mathbb{X}$



NON-LOCALITY HIERARCHIES

$|X\rangle$ = Quantum analog of X



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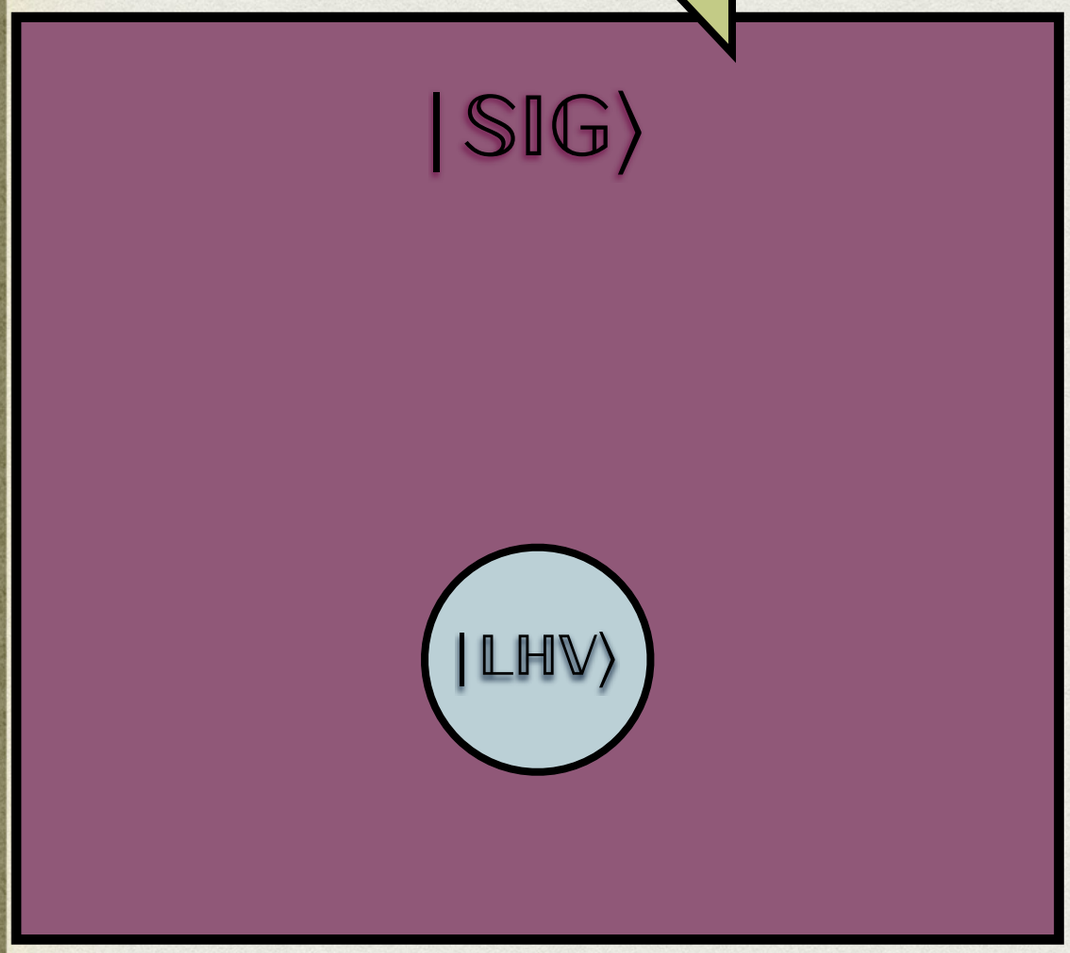
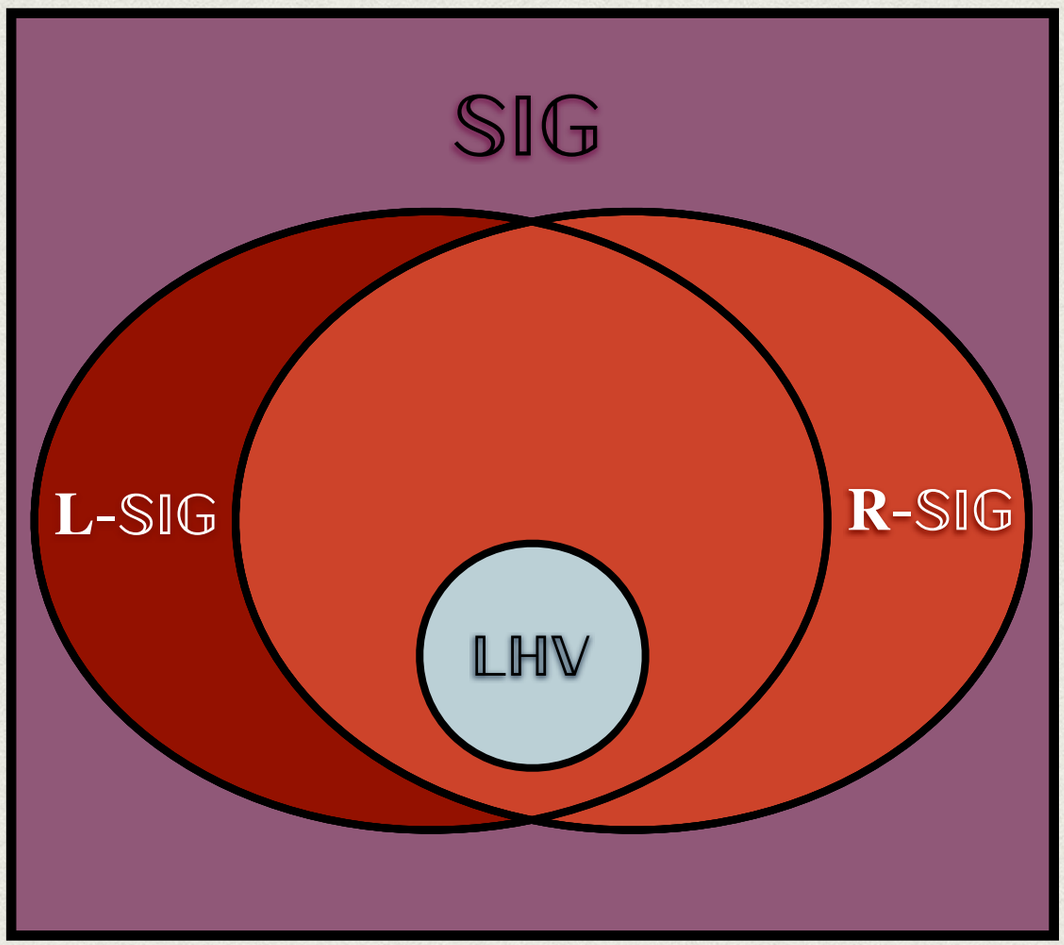
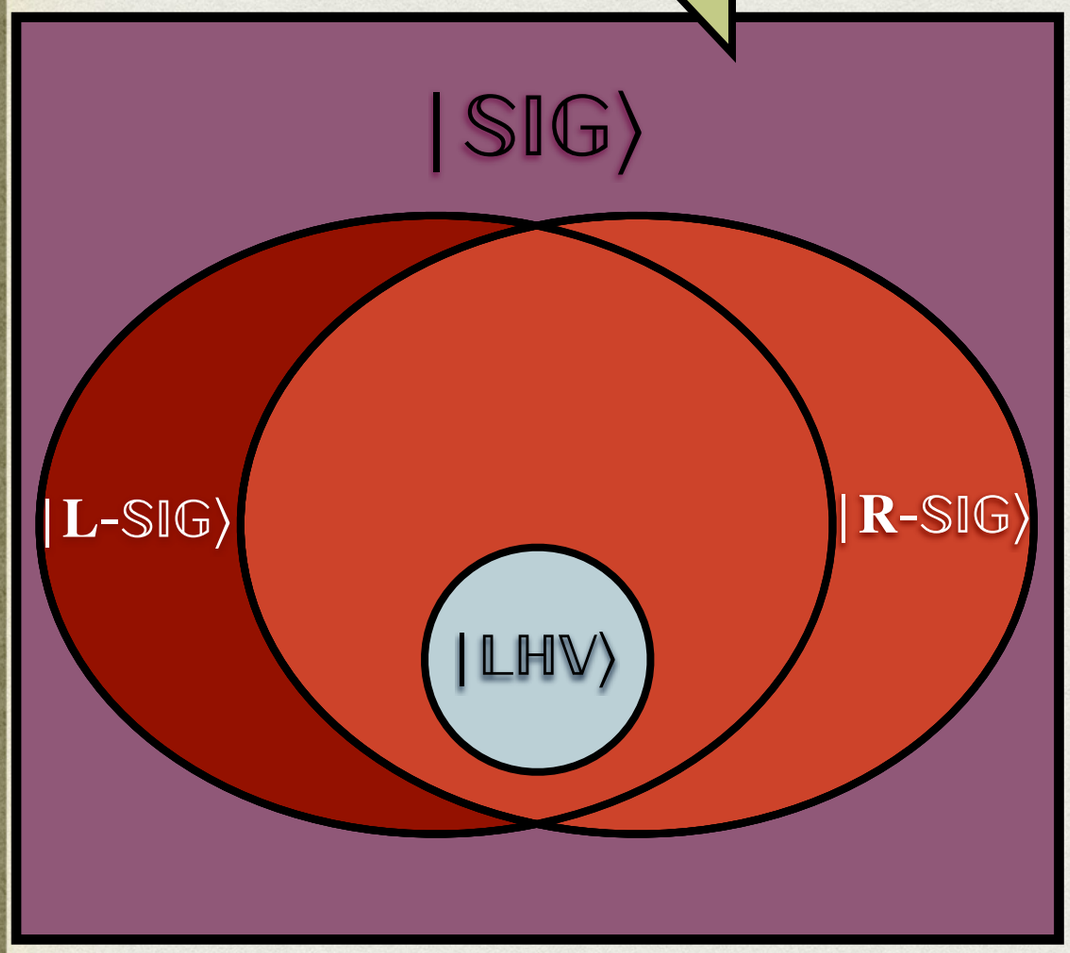
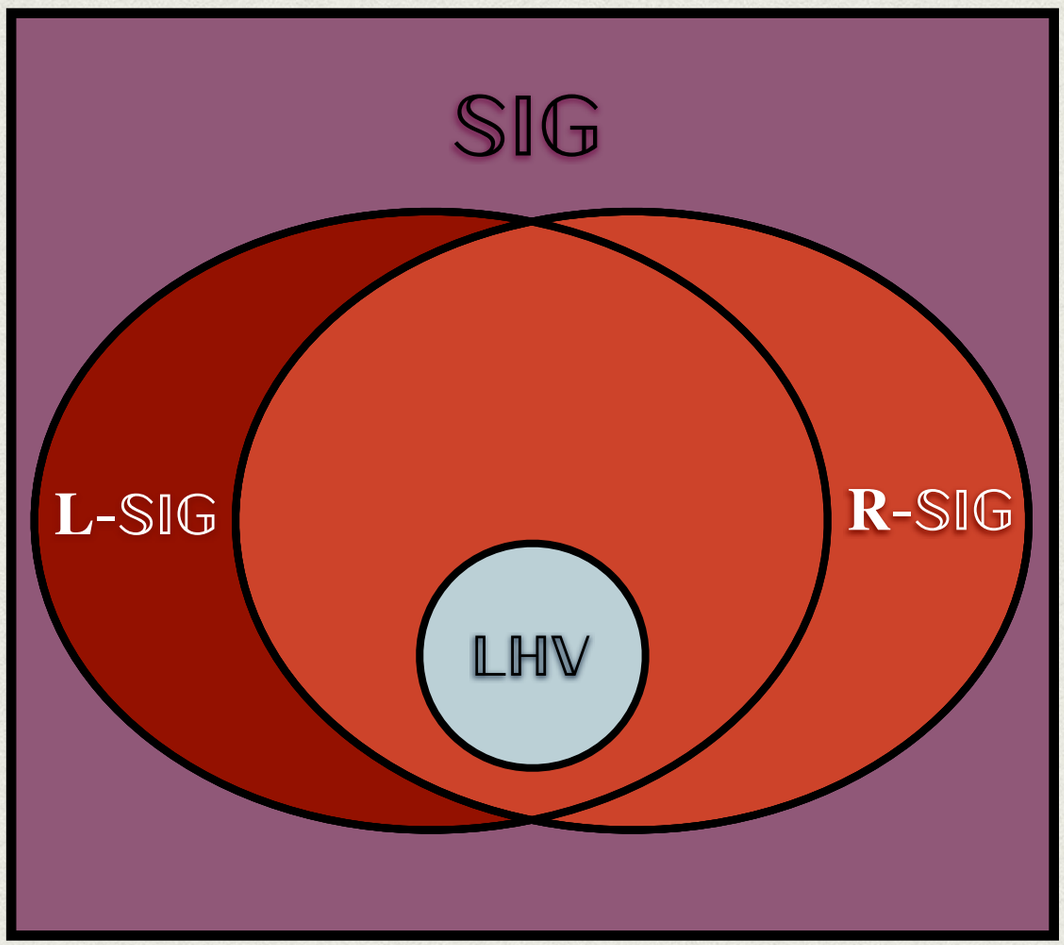




NON-LOCALITY HIERARCHIES

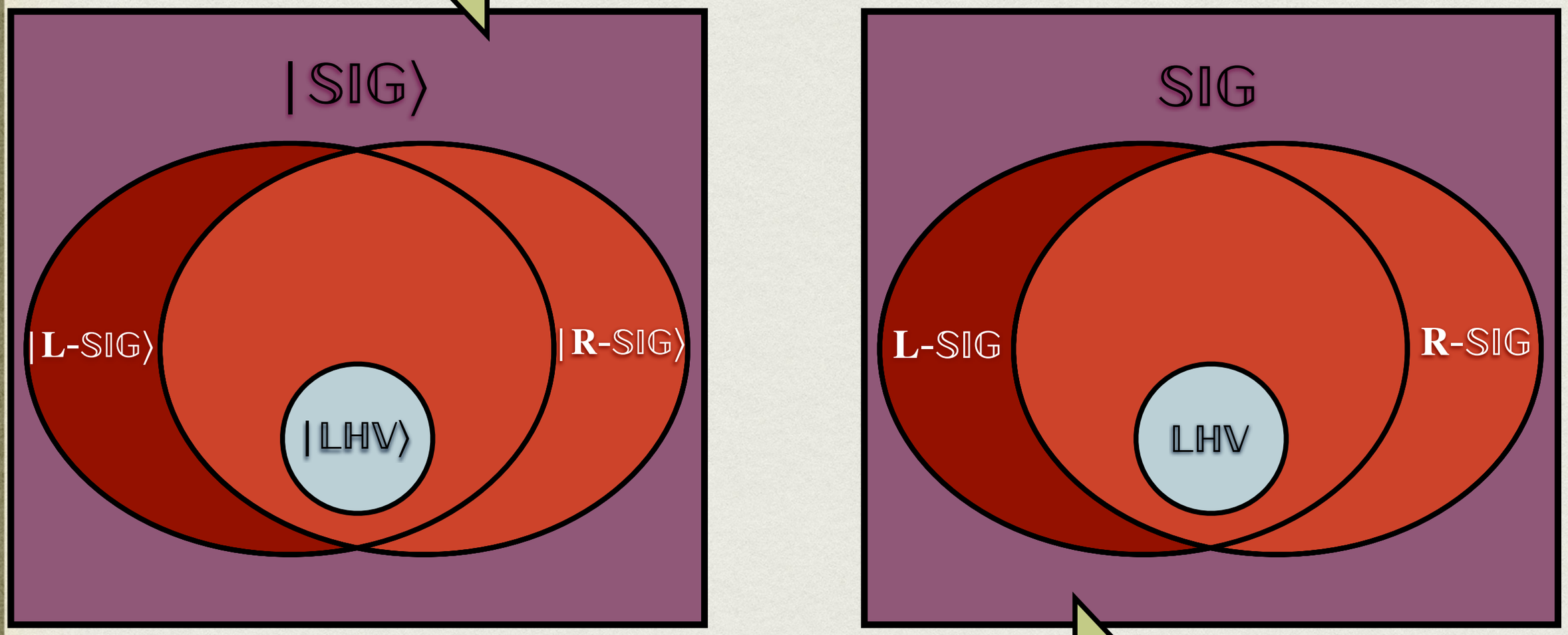
$|X\rangle$ = Quantum analog of X



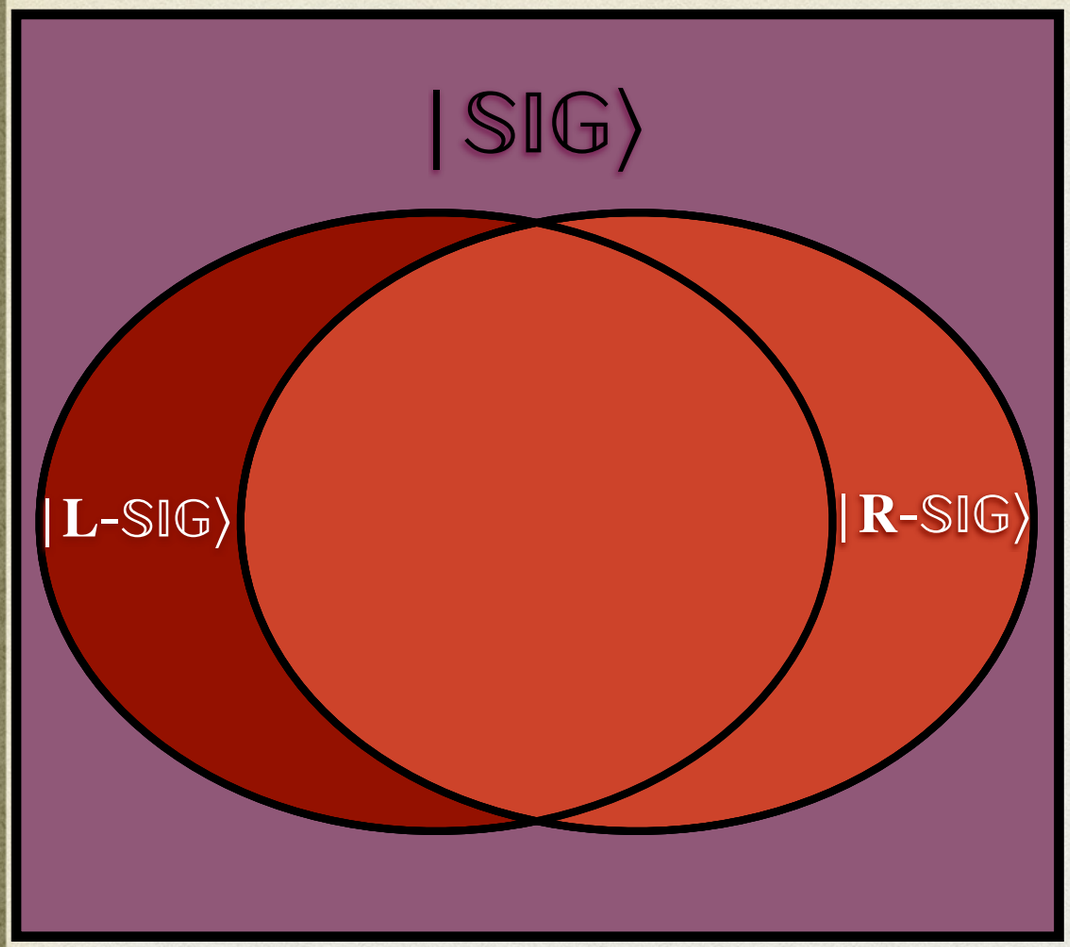


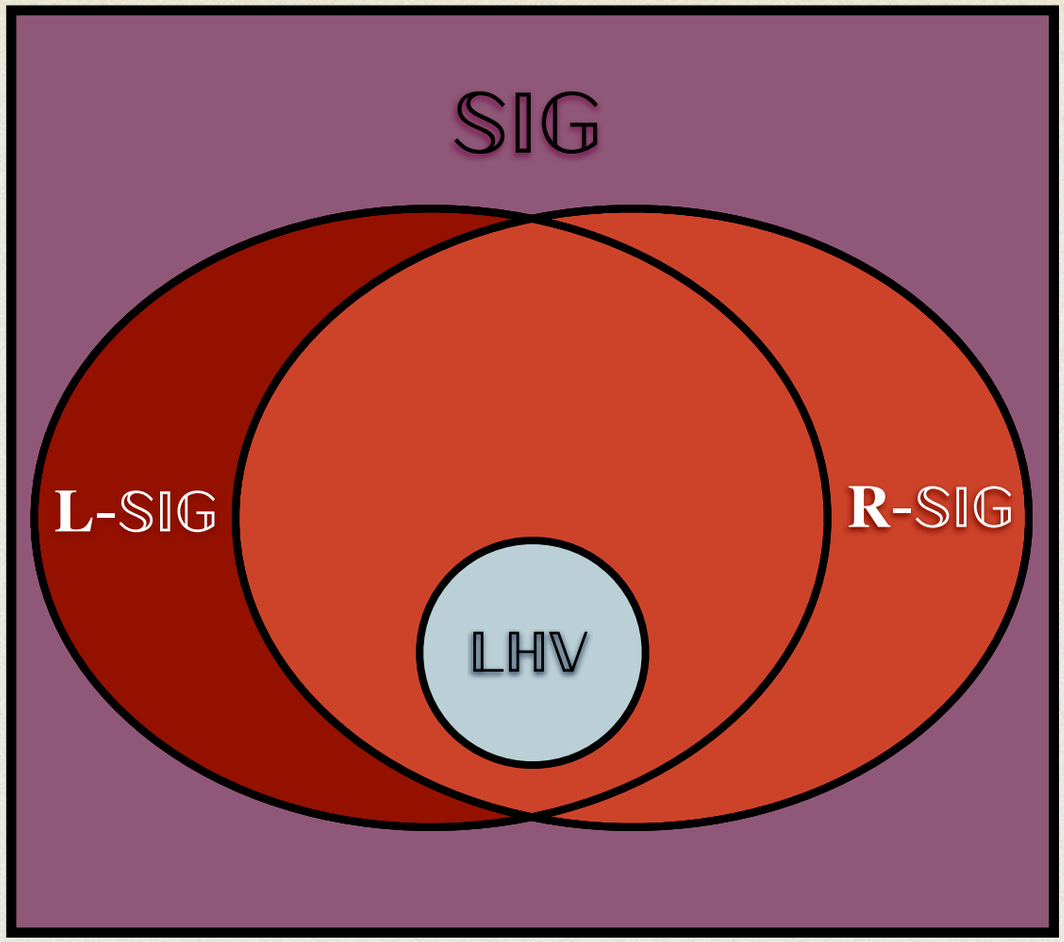
NON-LOCALITY HIERARCHIES

$|\mathbb{X}\rangle$ = Quantum analog of $\mathbb{X}$



$\downarrow \mathbb{X} \downarrow$ = Classical projection of $\mathbb{X}$



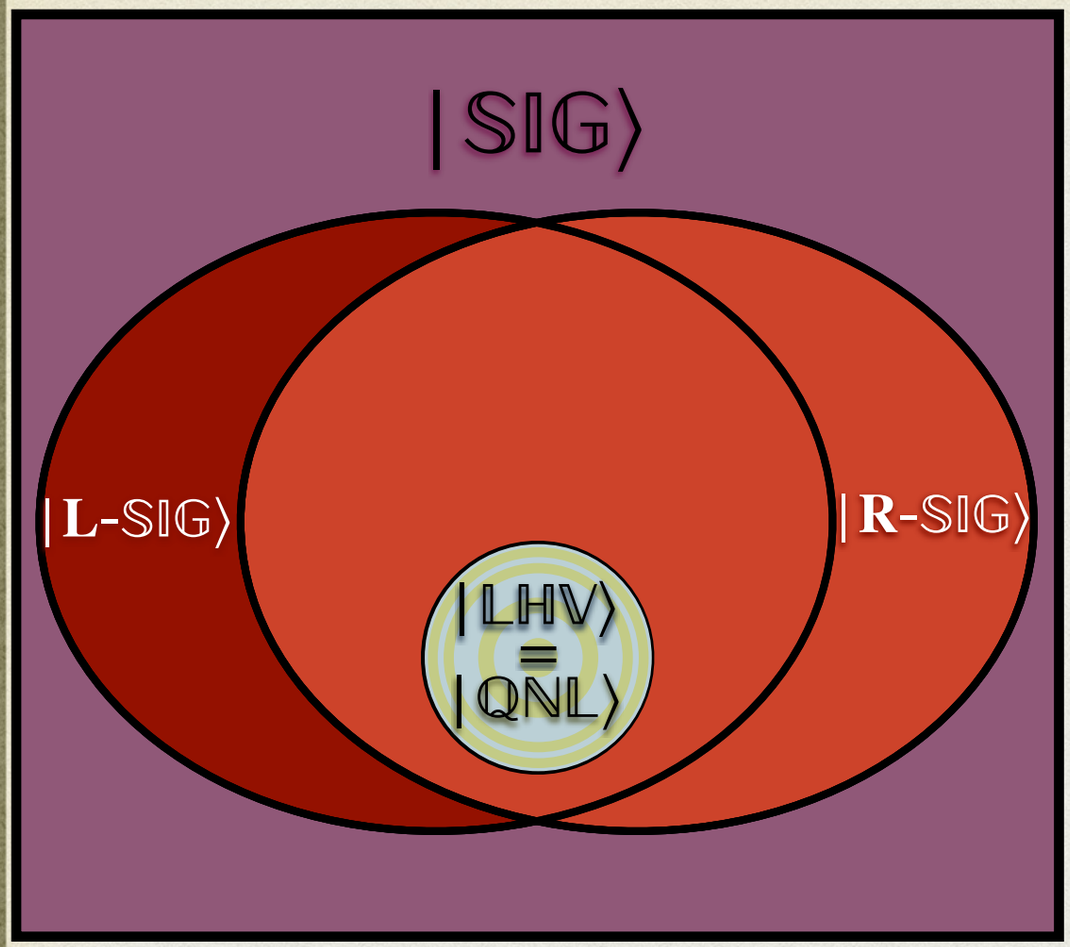


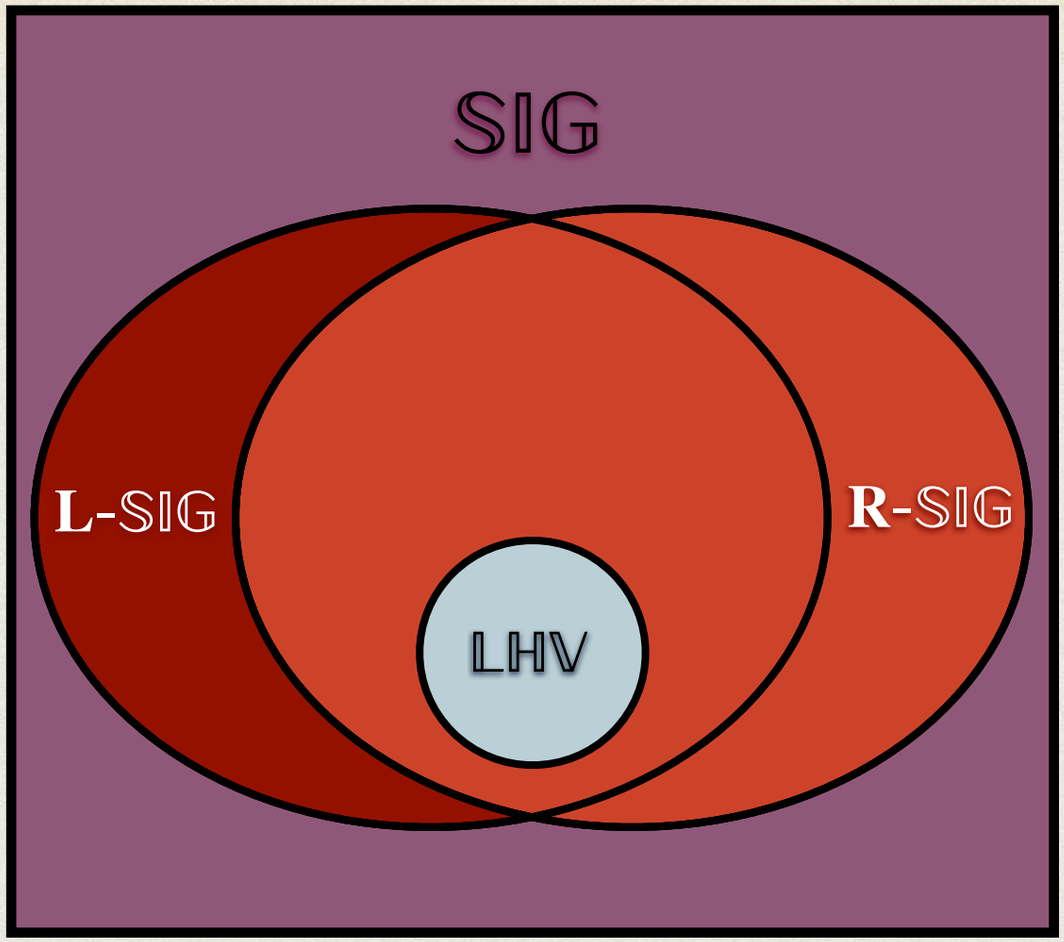
$$\left\downarrow |\mathrm{SIG}\rangle \right\downarrow = \mathrm{SIG}$$

$$\left\downarrow |\mathbf{L/R}\text{-SIG}\rangle \right\downarrow = \mathbf{L/R}\text{-SIG}$$

$$\mathrm{NOSIG} \stackrel{\mathbf{def}}{=} \mathbf{L}\text{-SIG} \cap \mathbf{R}\text{-SIG} = \left\downarrow |\mathrm{NOSIG}\rangle \right\downarrow$$

$$\mathrm{QNL} \stackrel{\mathbf{def}}{=} \left\downarrow |\mathrm{LHV}\rangle \right\downarrow, \quad |\mathrm{LHV}\rangle = |\mathrm{QNL}\rangle$$





$$\Big\downarrow |\mathrm{SIG}\rangle \Big\downarrow \;=\; \mathrm{SIG}$$

$$\Big\downarrow |\mathbf{L/R}\text{-}\mathrm{SIG}\rangle \Big\downarrow \;=\; \mathbf{L/R}\text{-}\mathrm{SIG}$$

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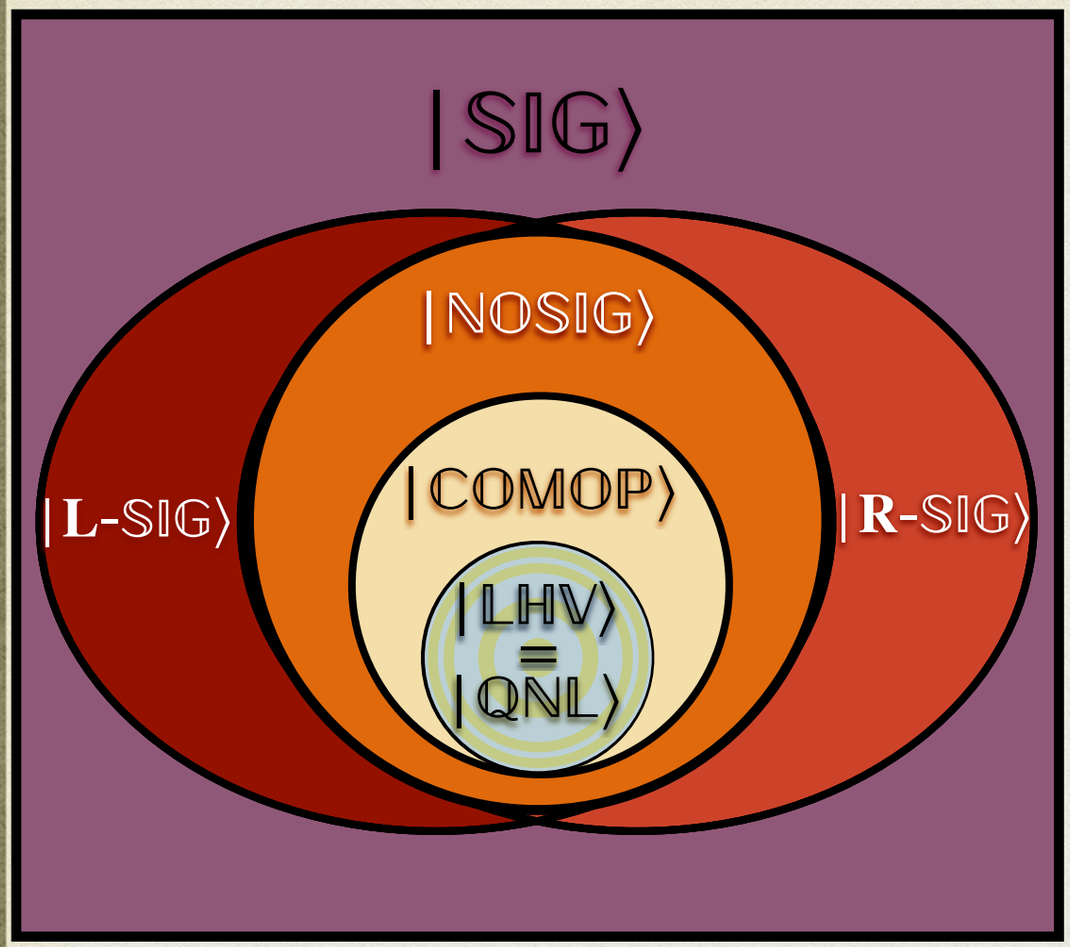

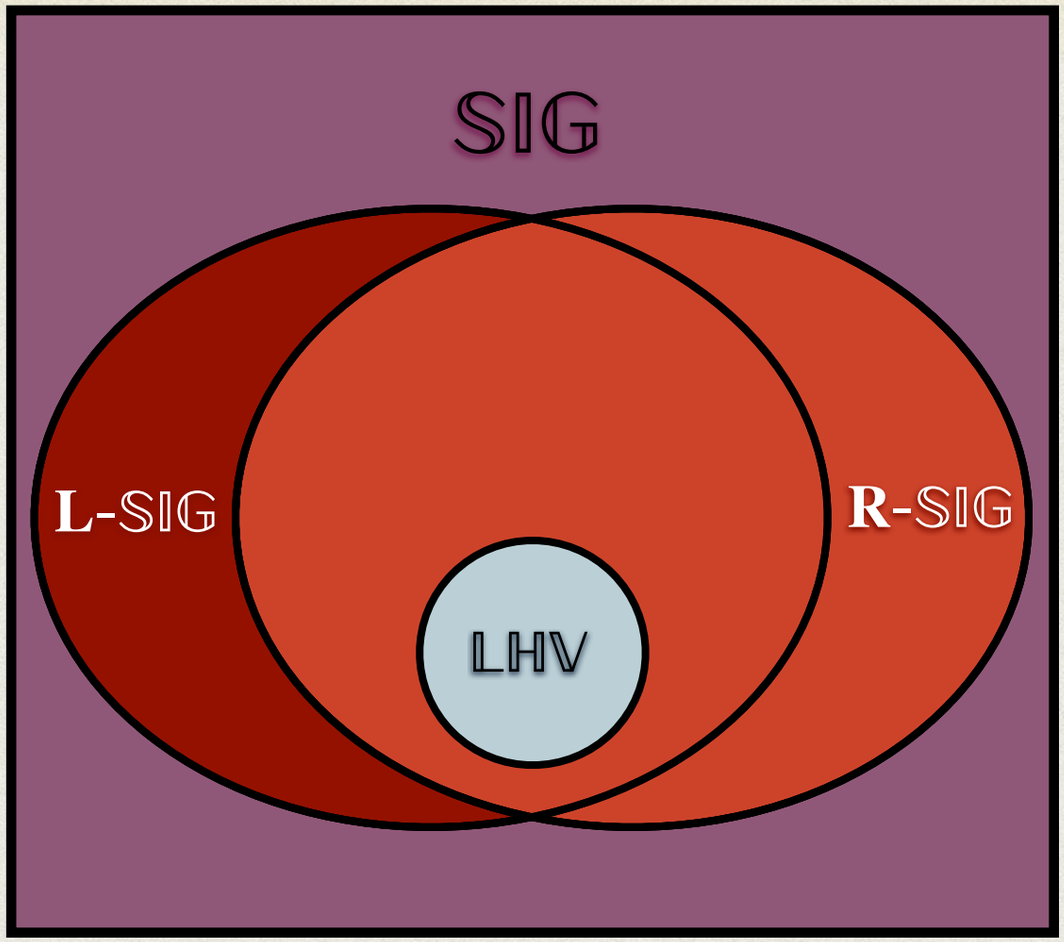

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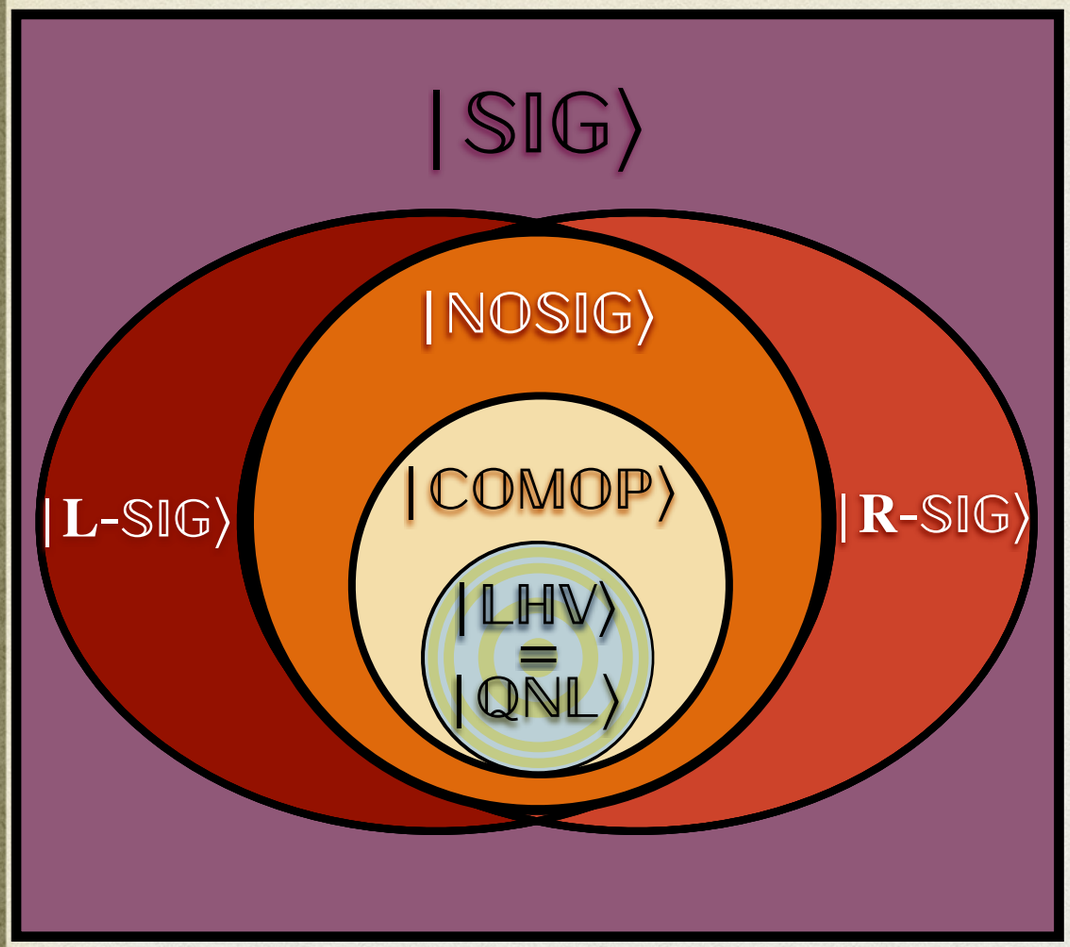

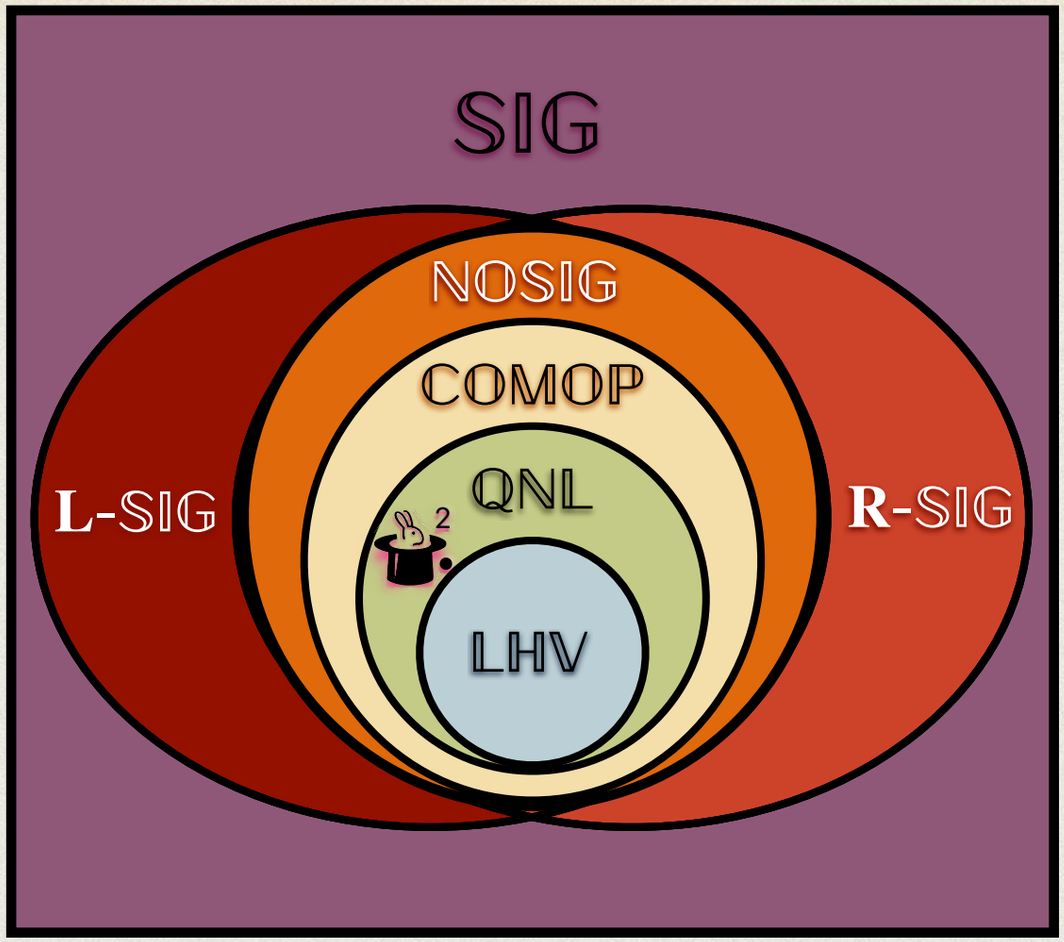

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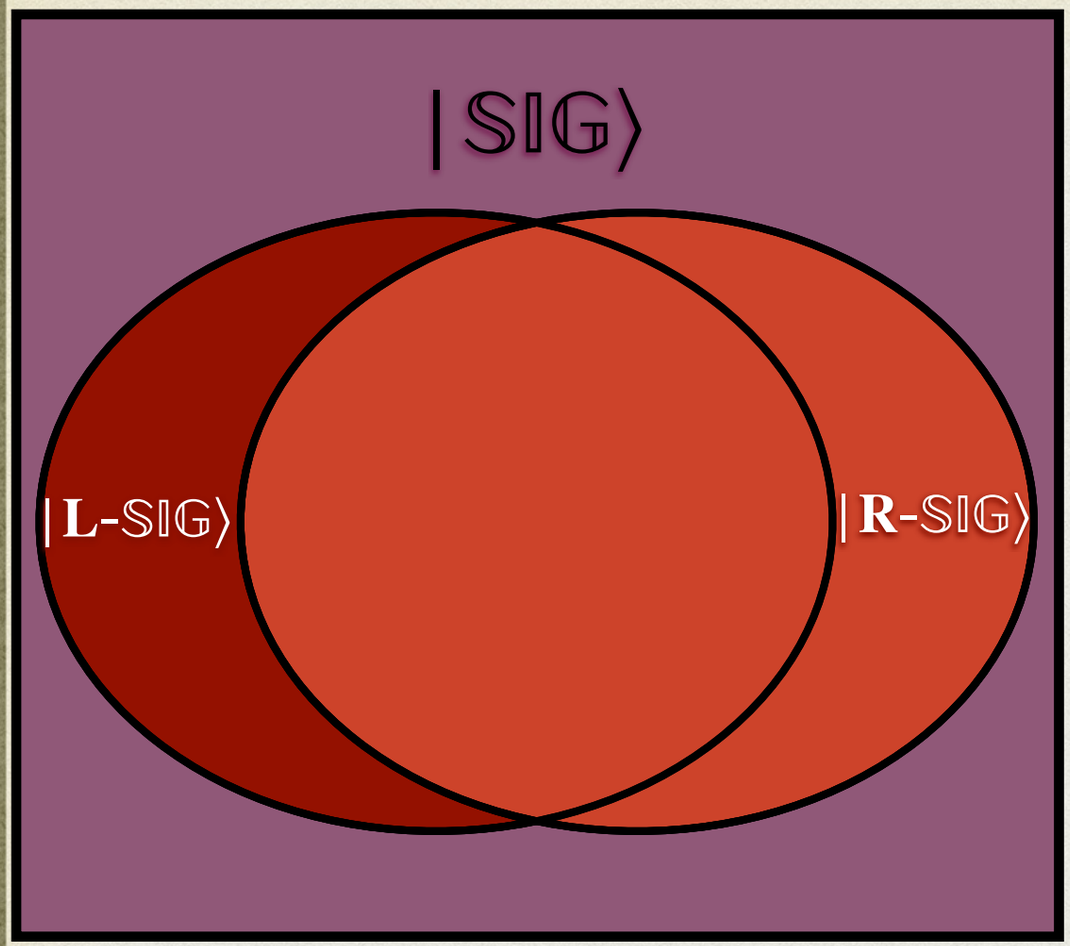
Quantum Teleportation:

$$\mathbf{R}\text{-}\mathbb{SIG} + |\mathbf{L}\text{-}\mathbb{SIG}\rangle = |\mathbb{SIG}\rangle = \mathbf{L}\text{-}\mathbb{SIG} + |\mathbf{R}\text{-}\mathbb{SIG}\rangle$$

$$|\mathbb{NOSIG}\rangle \stackrel{\text{def}}{=} |\mathbf{L}\text{-}\mathbb{SIG} \cap \mathbf{R}\text{-}\mathbb{SIG}\rangle \stackrel{?}{=} |\mathbf{L}\text{-}\mathbb{SIG}\rangle \cap |\mathbf{R}\text{-}\mathbb{SIG}\rangle$$

$$\mathbf{L}\text{-}\mathbb{SIG} + |\mathbb{LHV}\rangle = |\mathbf{L}\text{-}\mathbb{SIG}\rangle \qquad \mathbf{R}\text{-}\mathbb{SIG} + |\mathbb{LHV}\rangle = |\mathbf{R}\text{-}\mathbb{SIG}\rangle$$

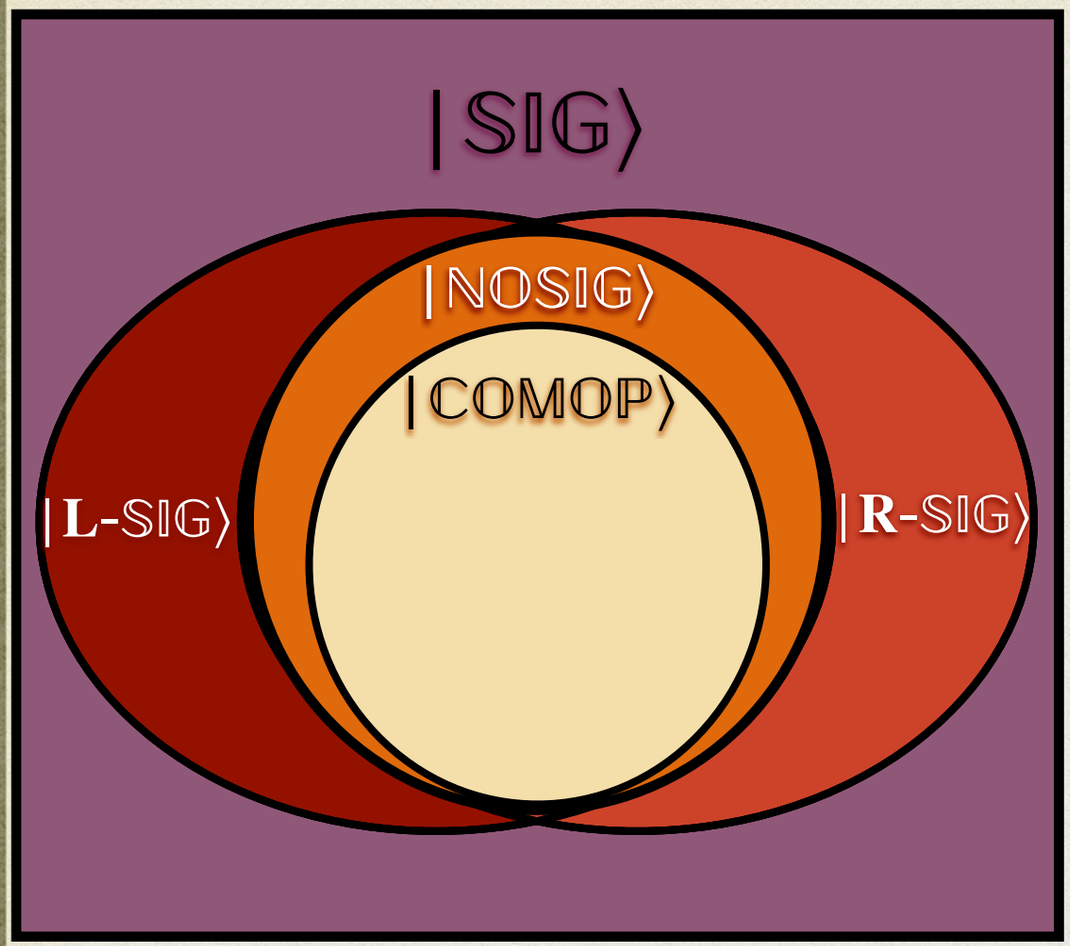


Quantum Teleportation:

$$\mathbf{R}\text{-SIG} + |\mathbf{L}\text{-SIG}\rangle = |\text{SIG}\rangle = \mathbf{L}\text{-SIG} + |\mathbf{R}\text{-SIG}\rangle$$

$$|\text{NOSIG}\rangle \stackrel{\text{def}}{=} |\mathbf{L}\text{-SIG} \cap \mathbf{R}\text{-SIG}\rangle \stackrel{?}{=} |\mathbf{L}\text{-SIG}\rangle \cap |\mathbf{R}\text{-SIG}\rangle$$

$$\mathbf{L}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{L}\text{-SIG}\rangle \qquad \mathbf{R}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{R}\text{-SIG}\rangle$$



Quantum Teleportation:

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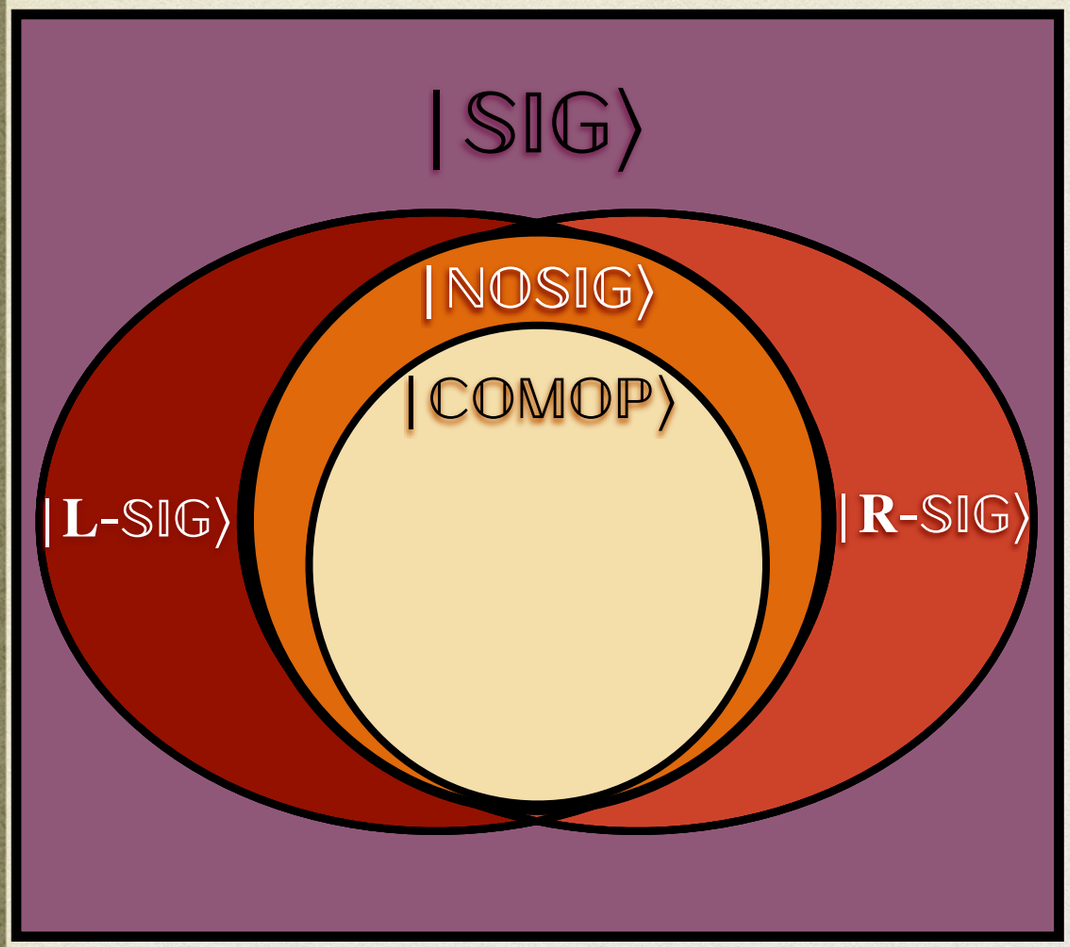
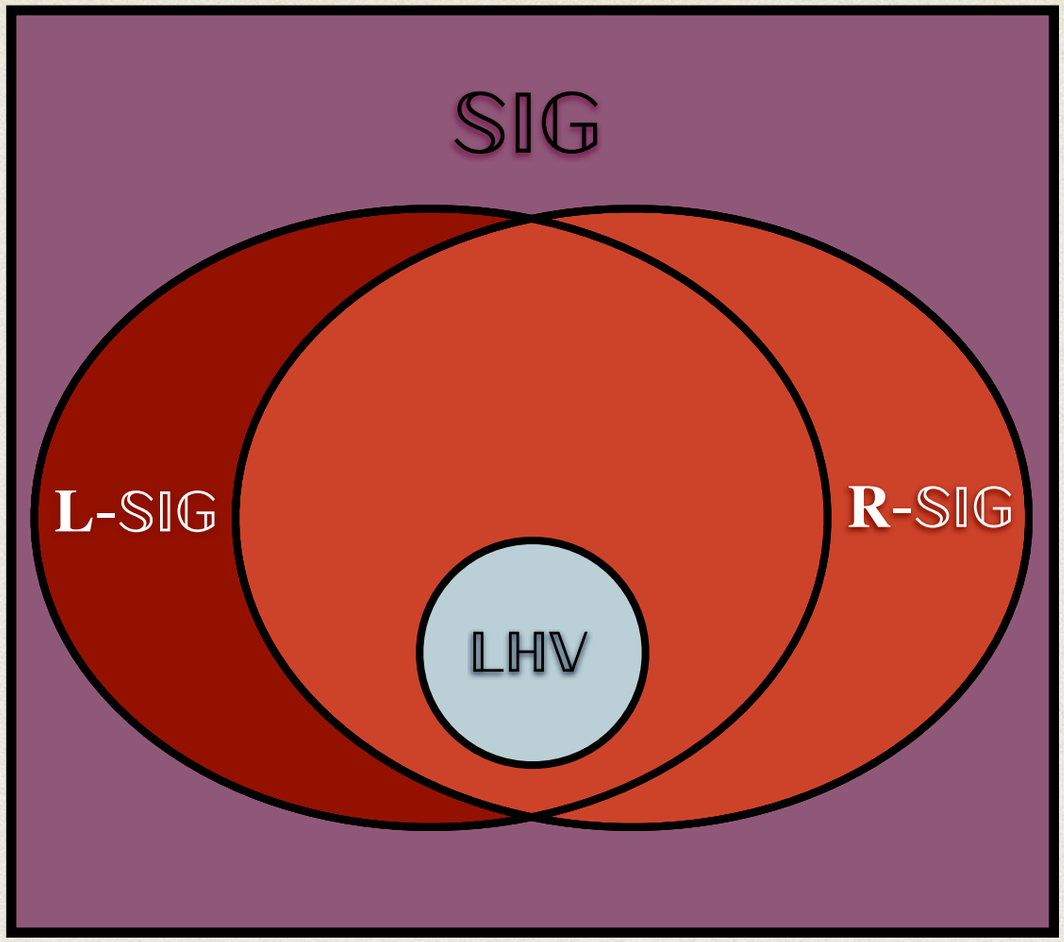
$$\mathbf{L}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{L}\text{-SIG}\rangle \qquad \mathbf{R}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{R}\text{-SIG}\rangle$$





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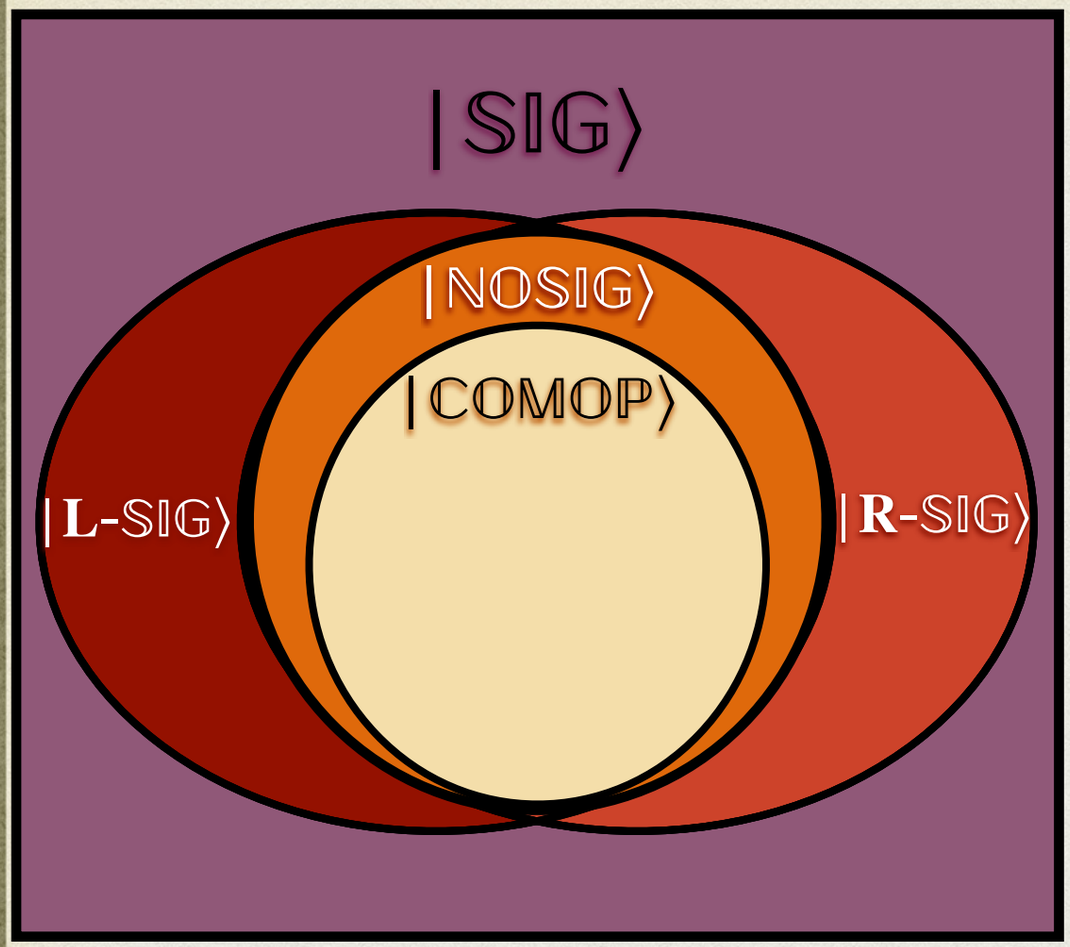
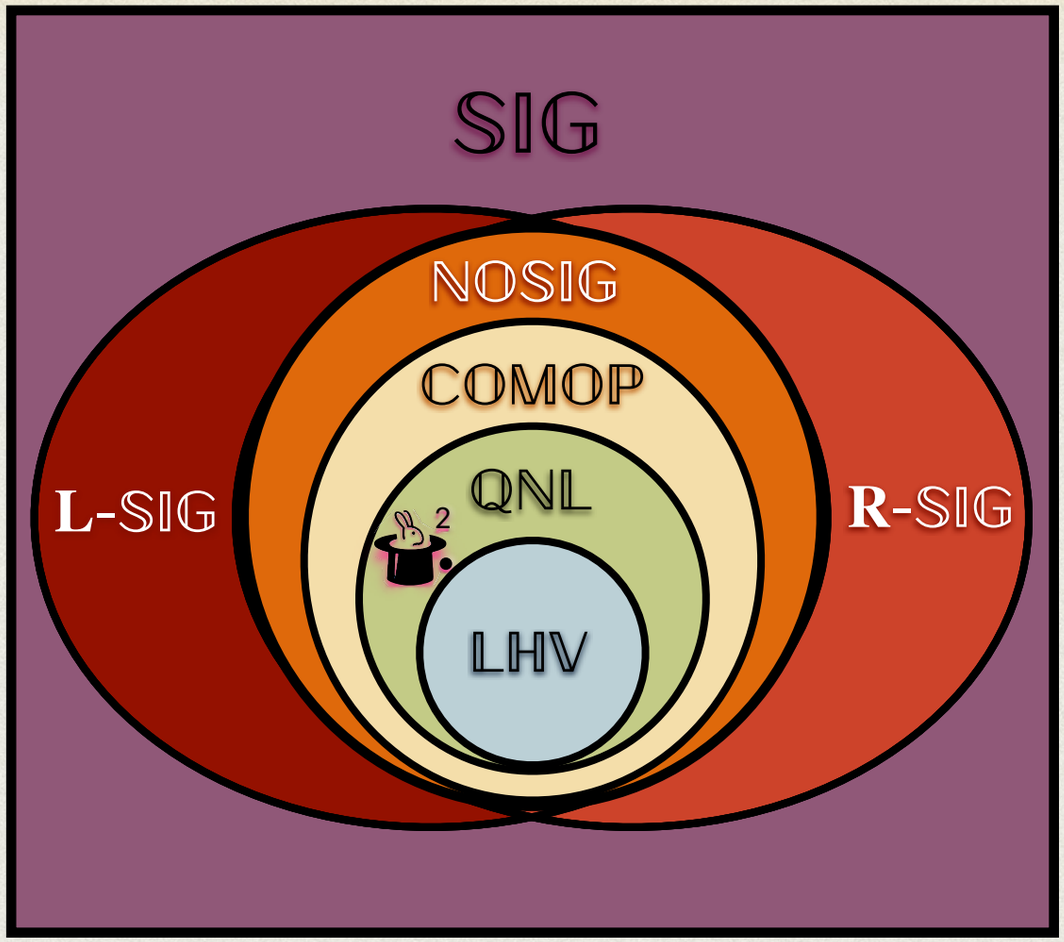
$$\mathbf{L}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{L}\text{-SIG}\rangle \qquad \mathbf{R}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{R}\text{-SIG}\rangle$$





Quantum Teleportation:

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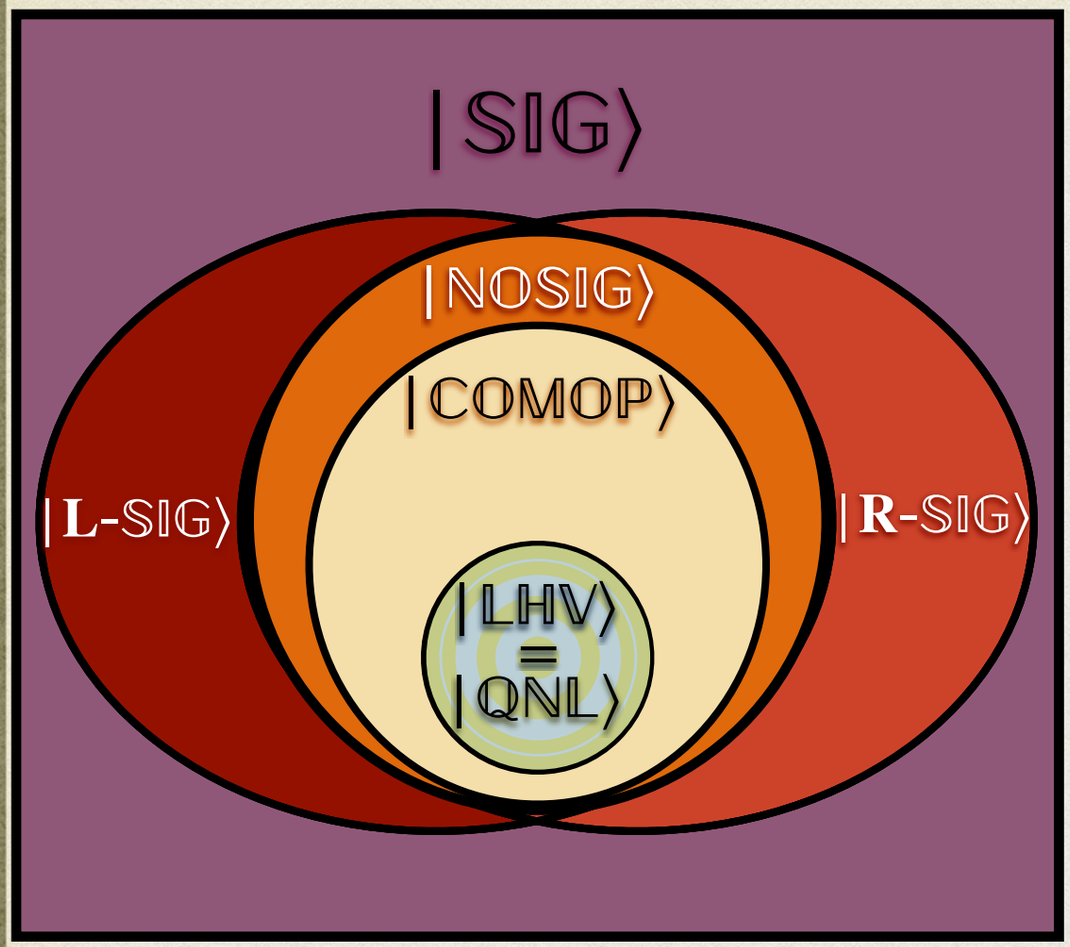
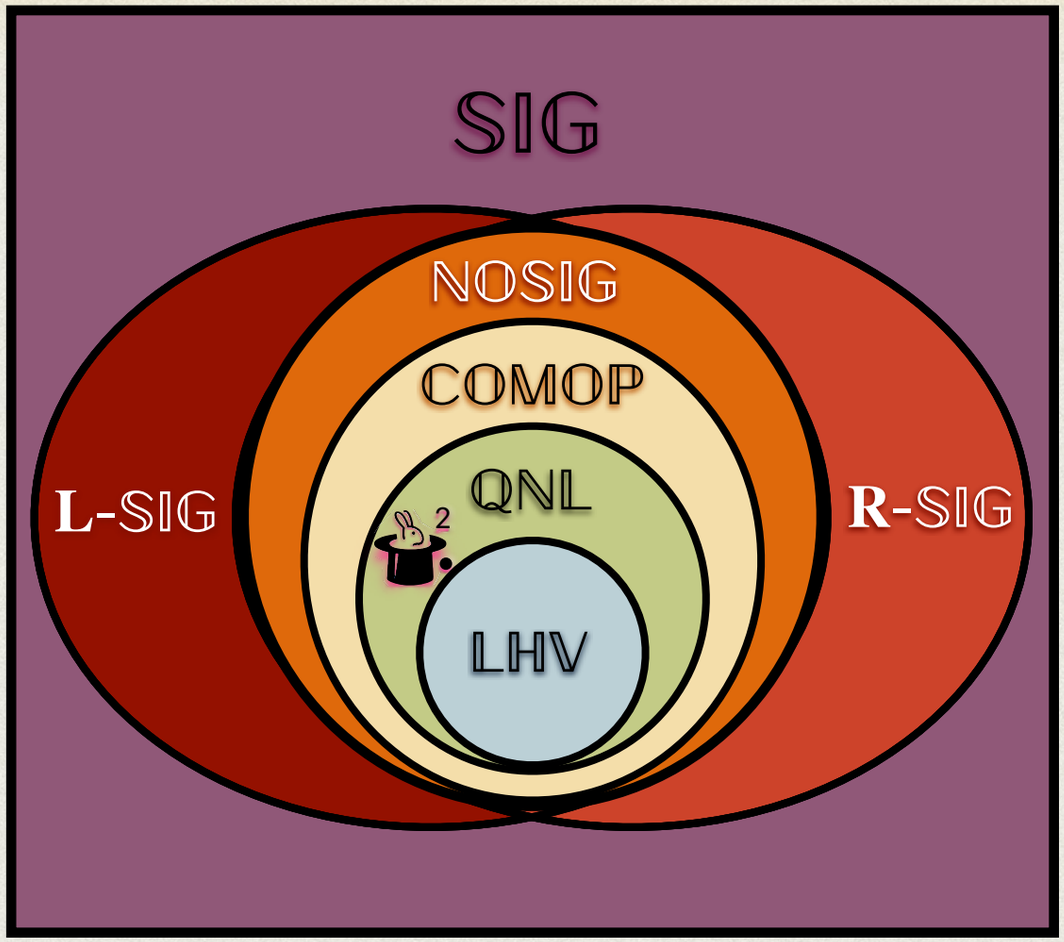
$$|\text{NOSIG}\rangle \stackrel{\text{def}}{=} |\mathbf{L}\text{-SIG} \cap \mathbf{R}\text{-SIG}\rangle \stackrel{?}{=} |\mathbf{L}\text{-SIG}\rangle \cap |\mathbf{R}\text{-SIG}\rangle$$

$$\mathbf{L}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{L}\text{-SIG}\rangle \qquad \mathbf{R}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{R}\text{-SIG}\rangle$$





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$$\mathbf{L}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{L}\text{-SIG}\rangle \qquad \mathbf{R}\text{-SIG} + |\text{LHV}\rangle = |\mathbf{R}\text{-SIG}\rangle$$

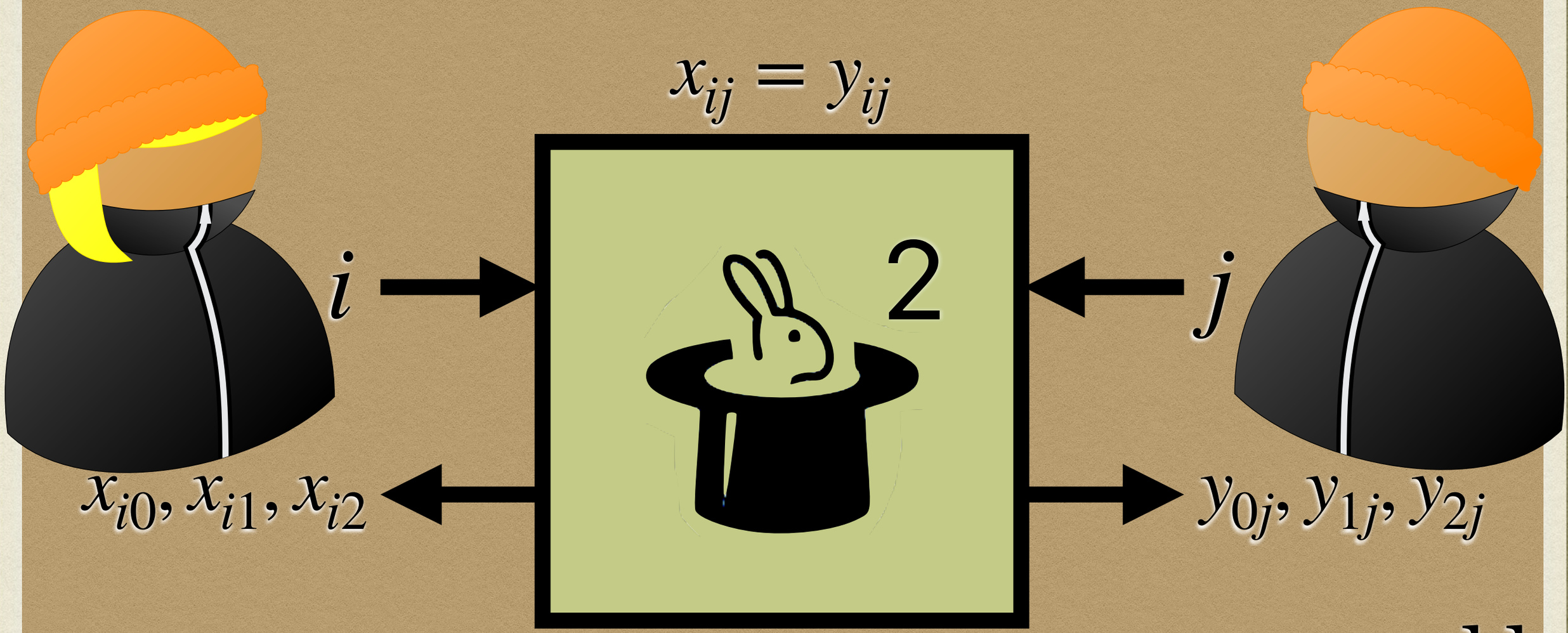
MAGIC

		y_{0j}
x_{i0}	x_{i1}	x_{i2} $=y_{2j}$
		y_{2j}

SQUARE

$$A=B=\{0,1,2\},\ X=Y=\{0,1\}^3$$

$$x_{ij} = y_{ij}$$



$$x_{i0} + x_{i1} + x_{i2} = \text{even} \qquad y_{0j} + y_{1j} + y_{2j} = \text{odd}$$