Electric conductivity in finite-density SU(2) lattice gauge theory with dynamical fermions

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Electric conductivity of QCD matter

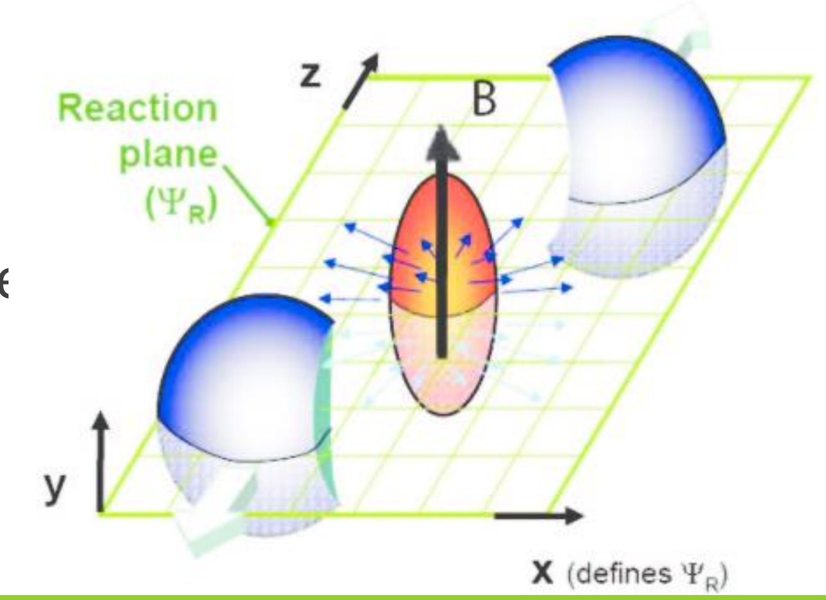
- Soft photon/Dilepton emission rate in heavy-ion collisions [McLerran, Toimela, PRD31(1985)545]

$$\frac{R}{V} = -4e^4 \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} L^{\mu\nu}(p_1, p_2) \frac{\sigma_{\mu\nu}(q)}{q^4}$$

- Essential part of magnetohydrodynamics of quark-gluon plasma

- Important for detecting anomalous transport phenome

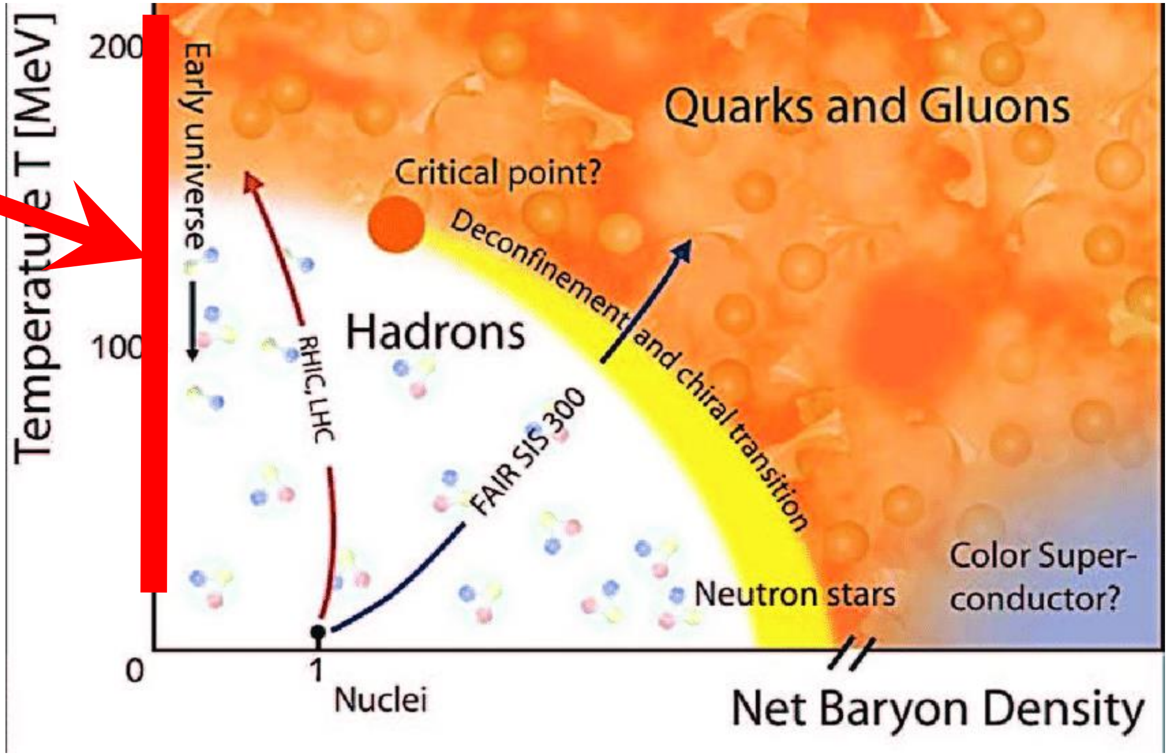
- Determines the lifetime of magnetic field created in off-central heavy-ion collision [McLerran,Skokov, 1305.0774]



What we know about QCD electric conductivity?

What we know for sure

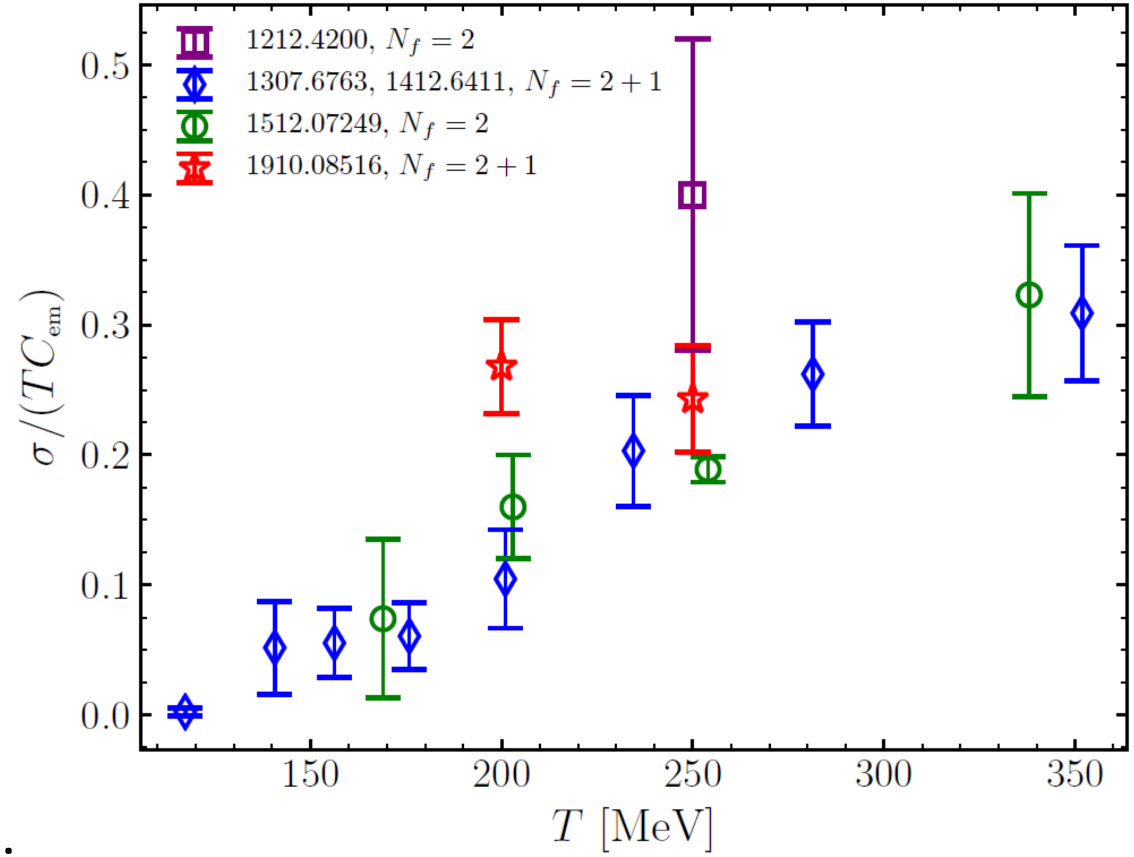
Nonzero density remains
to a large extent
unexplored …
SIGN PROBLEM!!!



Density dependence of electric conductivity unexplored from first principles!

What we know about QCD electric conductivity

- Quark gluon plasma is a good conductor
- Hadronic matter is not such a good conductor
- Pion gas conductivity few times smaller than quark gas conductivity (same T)
- Conductivity drops with temperature
- Minimal conductivity around crossover
- Crossover between two conductance mechanisms



[Summary plot from 2008.12326]

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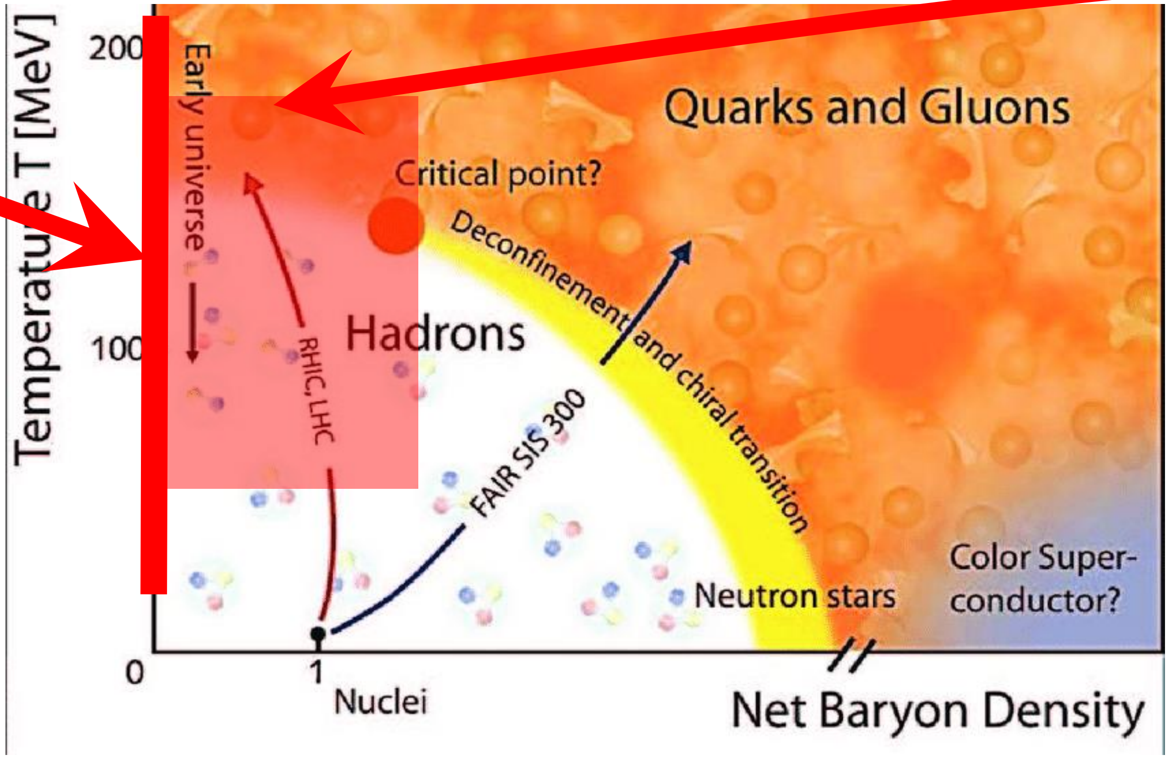


Can be reached
by Taylor expansion
or reweighting,
also using QCD-like theories

Density dependence of electric conductivity unexplored from first principles!

QCD conductivity at moderate densities

Conductivity is an even function of μ and can be expanded as:

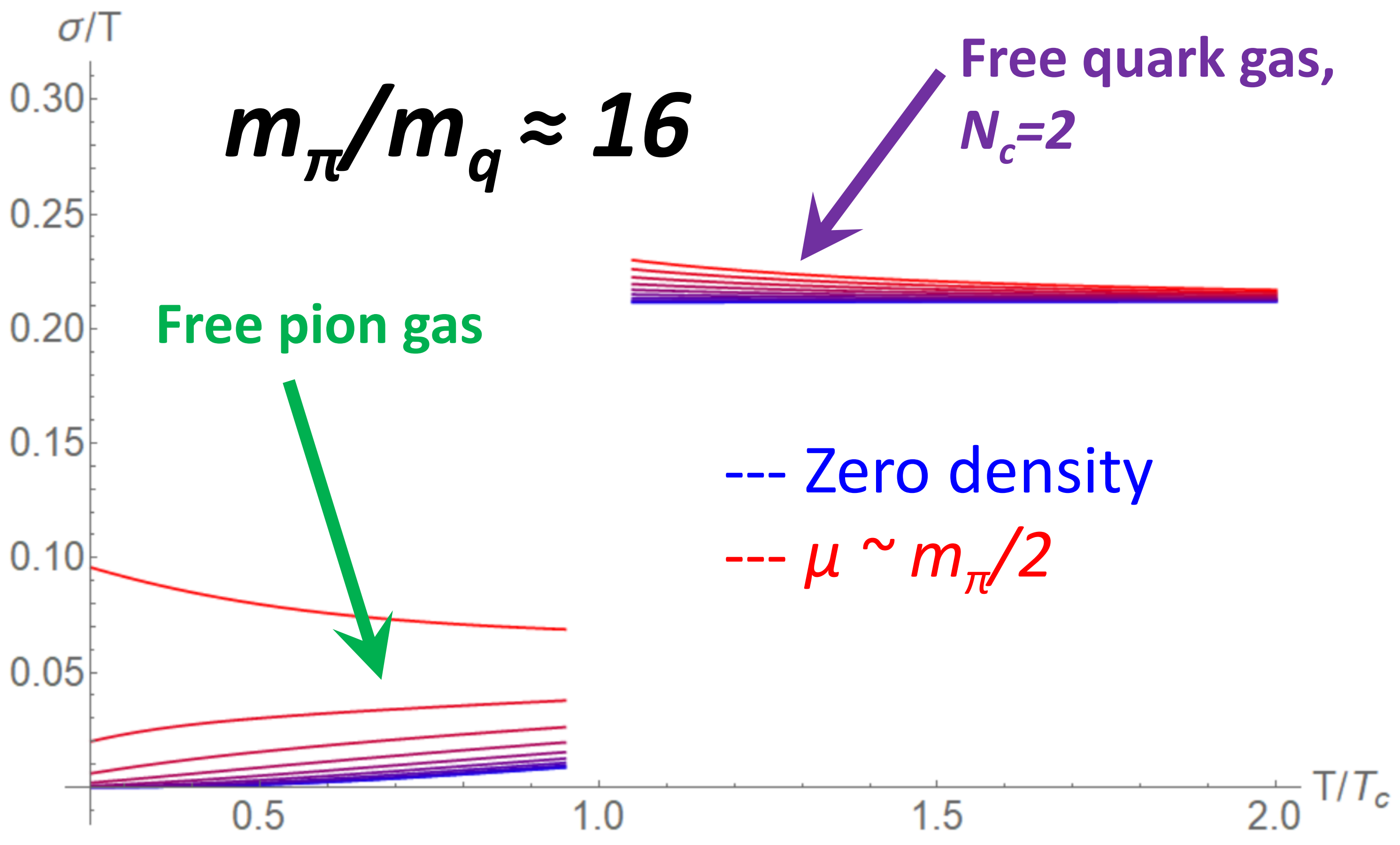
$$\frac{\sigma(T,\mu)}{T} = \frac{\sigma(T,0)}{T} \left(1 + c(T) \left(\frac{\mu}{T}\right)^2 + O\left(\mu^4\right)\right)$$

Some model estimates:

- *c(T) ≈ 0.5* at $T \sim T_c$ from Parton-Hadron String Dynamics [Cassing, Steinert, 1312.3189] and Boltzmann equation [Srivastava,Thakur,Patra,1501.03576]
- Potentially strong dependence on μ at *μ/T ~ 1*
- Dynamical quasiparticle model [Soloveva,Moreau,Bratkovskaya,1911.08547] and Functional Renormalization Group [Tripolt,Jung,Tanji,von Smekal,Wambach,1807.04952] imply much weaker μ dependence
- *c(T) ≈ 0.057* for free massless quarks – rather weak dependence!

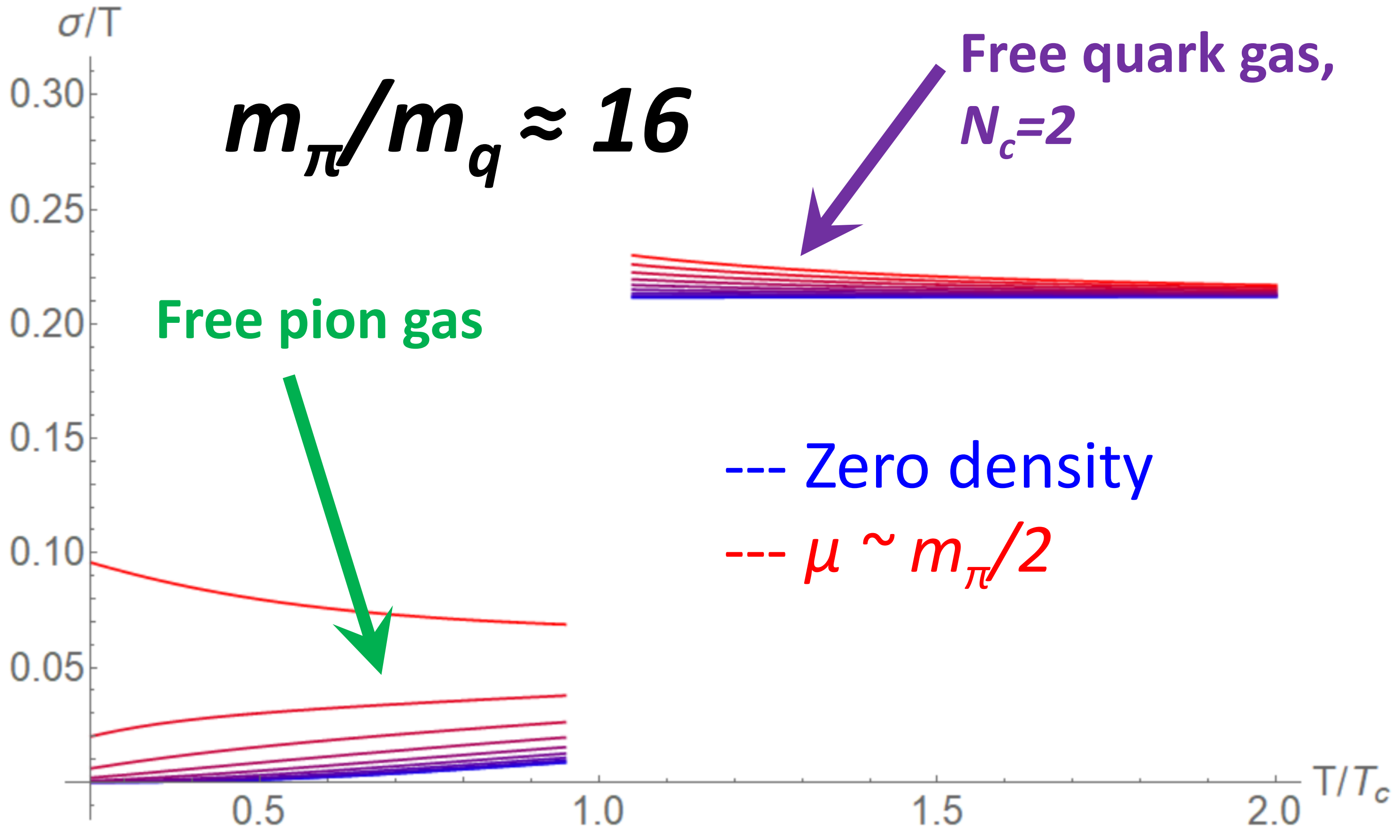
Low- and high-temperature limits: free quarks and pions

- We use the "lattice-practical" definition of conductivity $\sigma(w)$ smeared over $w\sim T$
- Pion gas conductivity much smaller
- Pion gas conductivity much more sensitive to density!



Low- and high-temperature limits: free quarks and pions

- For fermions, the effect of finite density grows at low temperatures
- For free pions, finite density has larger effect at larger temperatures
- Fermi surface vs. Bose condensation – two different conductance mechanisms!
- **c(T): peak around T_c !!!**



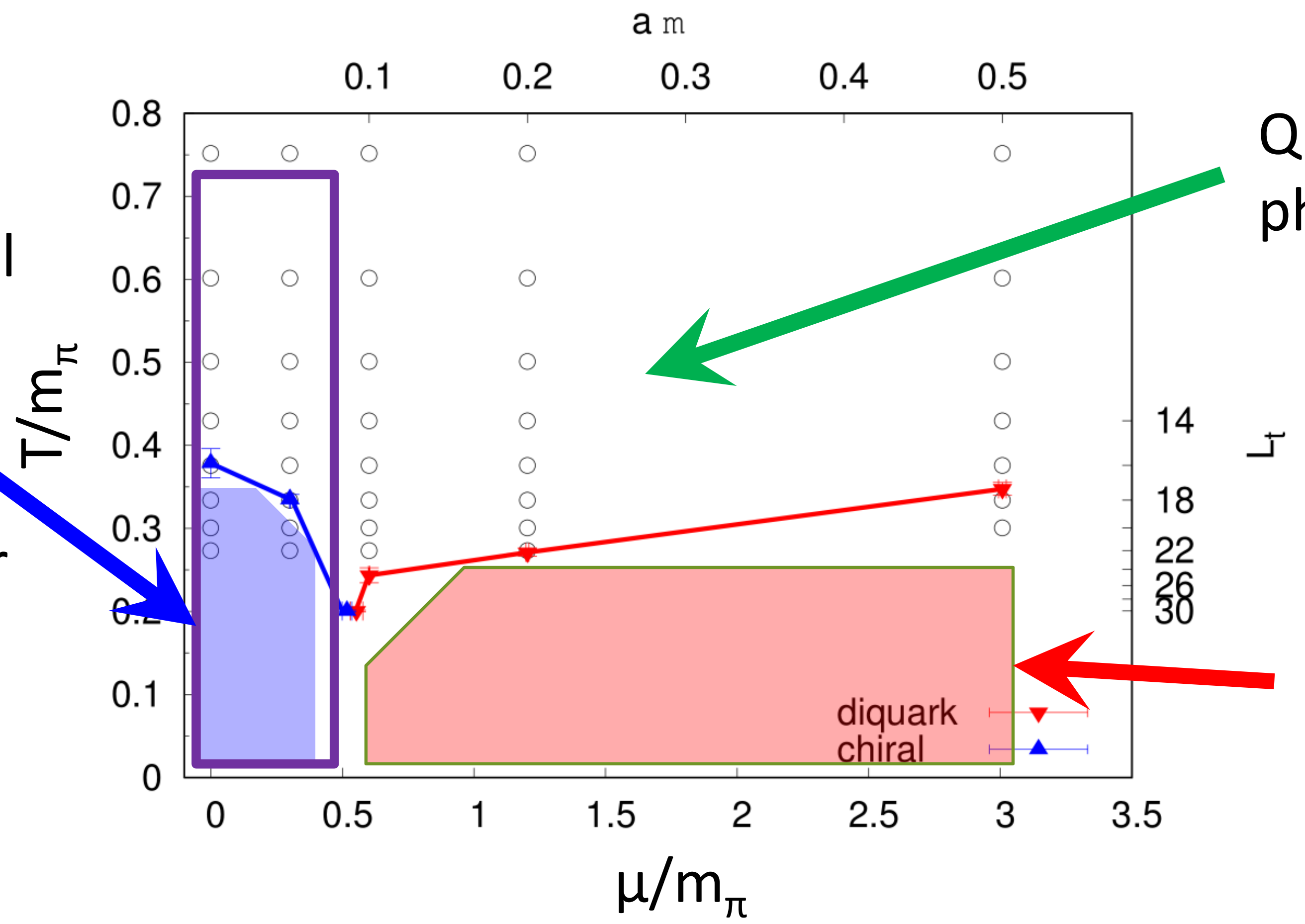
QCD Electric conductivity at finite density: ways to explore

- Direct Taylor expansion would require correlators of four currents for c(T) – computationally very challenging task! (Disconnected contributions, multiple fermion diagrams, noise issues, difficulties of implementing conserved currents…)
- Reweighting would most likely be noisy
- **In this work: Use QCD-like theory which is similar to QCD at small μ**
- We get qualitative insight into what might happen in QCD
- We use **finite-density SU(2) gauge theory**, free of sign problem [Kogut,Sinclair,Hands,Morrison,hep-lat/0105026]

Phase diagram of SU(2) gauge theory

QCD-like low-temperature phase, broken chiral symmetry, pion excitations

Qualitatively similar to QCD !!!

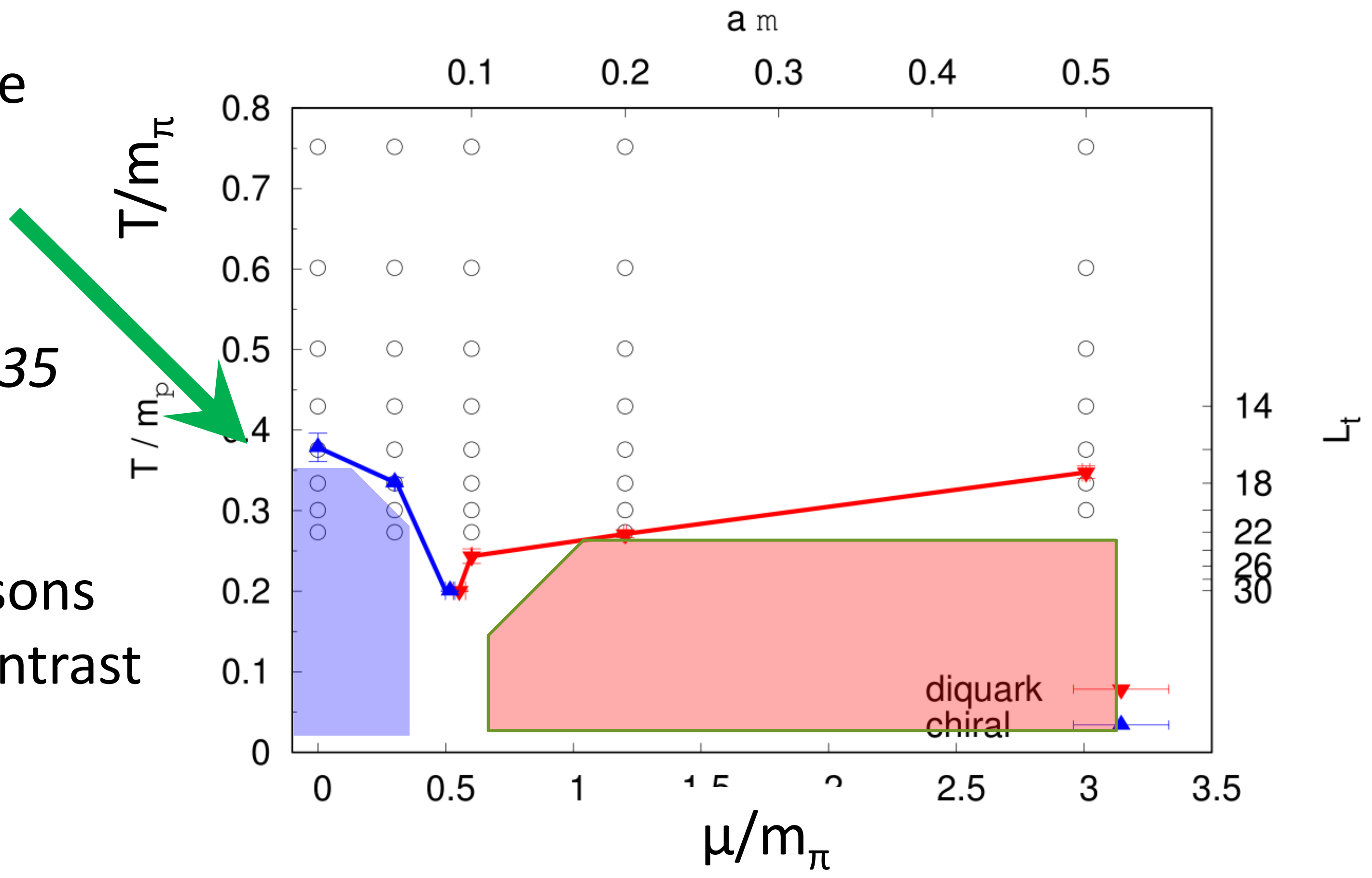


Quark-gluon plasma phase

Diquark condensation phase, absent in QCD

Phase diagram of SU(2) gauge theory

- Interesting feature of SU(2) gauge theory:
- Small value of $T_c/m_\pi \approx 0.4$
- In real QCD, $T_c \approx 155$ *MeV*, $m_\pi \approx 135$ *MeV*, $T_c/m_\pi \approx 1.15$
- Possible reason: 5 Goldstone bosons in $N_f=2$ *SU(2)* gauge theory, in contrast to 3 pions in $N_f=2$ QCD [Kogut *et al.*,hep-ph/0001171]



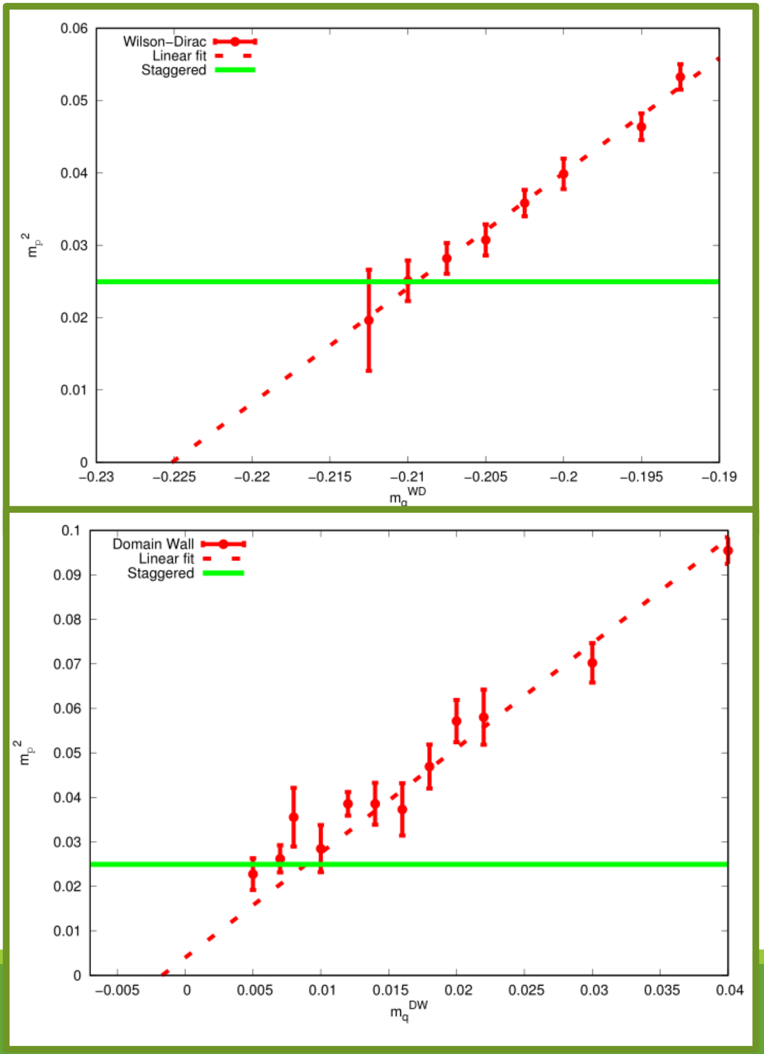
Lattice setup: sea quarks & gauge action

- $N_f=2$ light flavours with $m_u=m_d = 0.005$, pion mass $m_\pi = 0.158$
- Rooted staggered sea quarks
- Tadpole-improved gauge action
- Spatial lattice sizes $L_s=24$ and $L_s=30$
- Single gauge coupling = single lattice spacing
- Temporal lattice sizes $L_t=4 \ldots 26$
- Standard Hybrid Monte Carlo
- Acceleration using **GPUs**

- Small diquark source term added for low temperatures to facilitate diquark condensation

Lattice setup: valence quarks

- Wilson-Dirac and Domain-Wall valence quarks
- HYP-smeared gauge links in the Dirac operator: reduces additive mass renormalization and lattice artifacts
- Better quality of signal than for staggered quarks
- Bare mass for Wilson-Dirac/Domain-Wall quarks tuned to match the pion mass calculated with sea quarks
- GMOR relation works with good precision



Numerical measurement of electric conductivity

Green-Kubo relations:

$$\frac{1}{V}\sum_{\vec{x}}\langle\, j_i\left(\tau,\vec{x}\right) j_i\left(0,\vec{0}\right)\rangle \equiv G\left(\tau\right) = \int_0^\infty d\omega\, K\left(\tau,\omega\right)\, \sigma\left(\omega\right)$$

$$K\left(\tau,\omega\right) = \frac{\omega}{\pi}\,\frac{\cosh\left(\omega\left(\tau-\frac{1}{2T}\right)\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

- On the lattice, τ takes O(10) values, while ω is continuous
- $K(\tau, \omega)$ is an ill-defined kernel
- An ill-defined numerical analytic continuation problem

Simplest option: midpoint estimator

$$G(\tau/2) = \int_0^{+\infty} \frac{d\omega}{\pi} \frac{\omega}{\sinh\left(\frac{\omega}{2T}\right)} \sigma(\omega)$$

- Estimates the low-frequency conductivity smeared over frequency range $w \leq 4.4\ T$
- Completely model-independent

Backus-Gilbert method

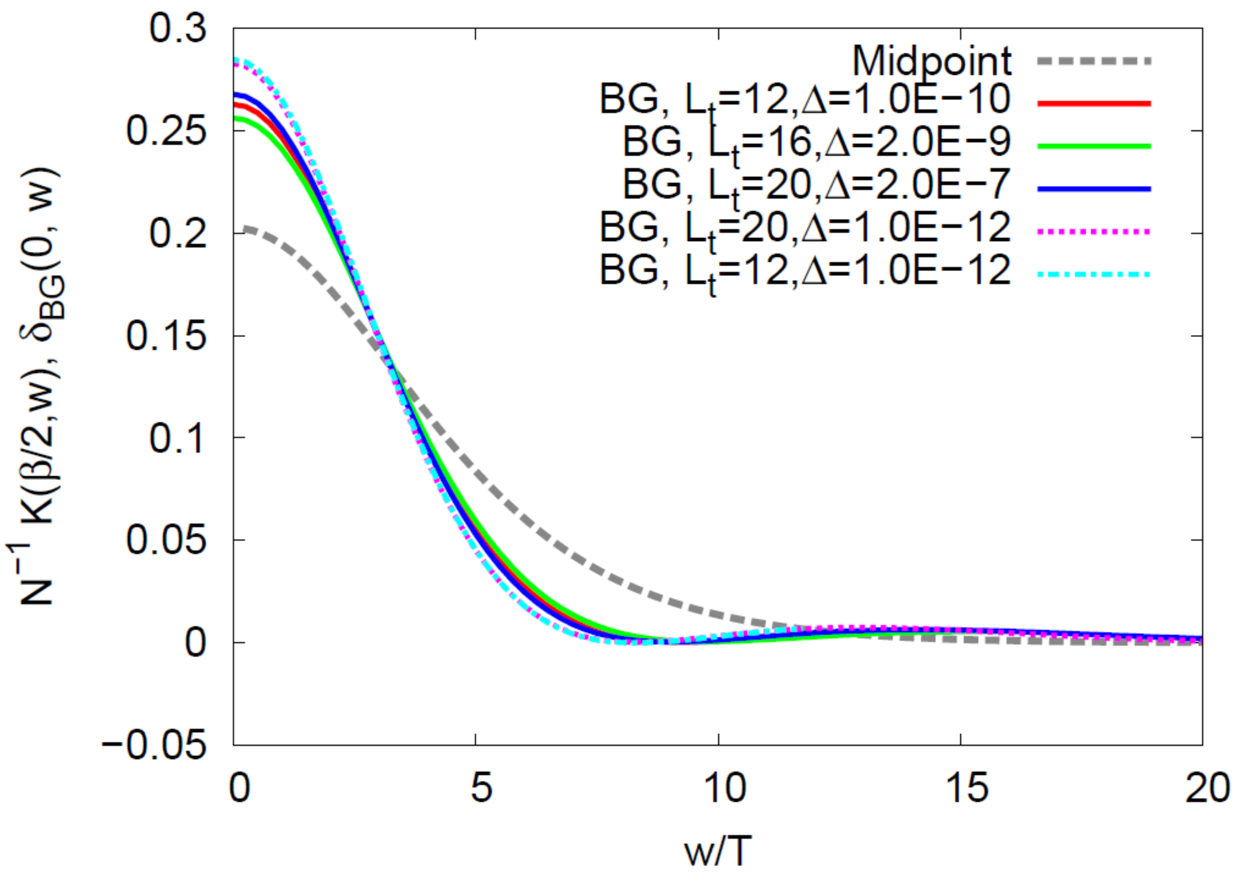
- Instead of exact spectral function, an estimate smeared using the "regularized delta-function":

$$\sigma_{BG}(\omega) = \sum_{\tau} q_{\tau}(\omega) G(\tau) = \int_{0}^{+\infty} \delta_{BG}(\omega, \omega')\sigma(\omega')$$

$$\delta_{BG}(\omega, \omega') = \sum_{\tau} q_{\tau}(\omega) K(\tau, \omega')$$

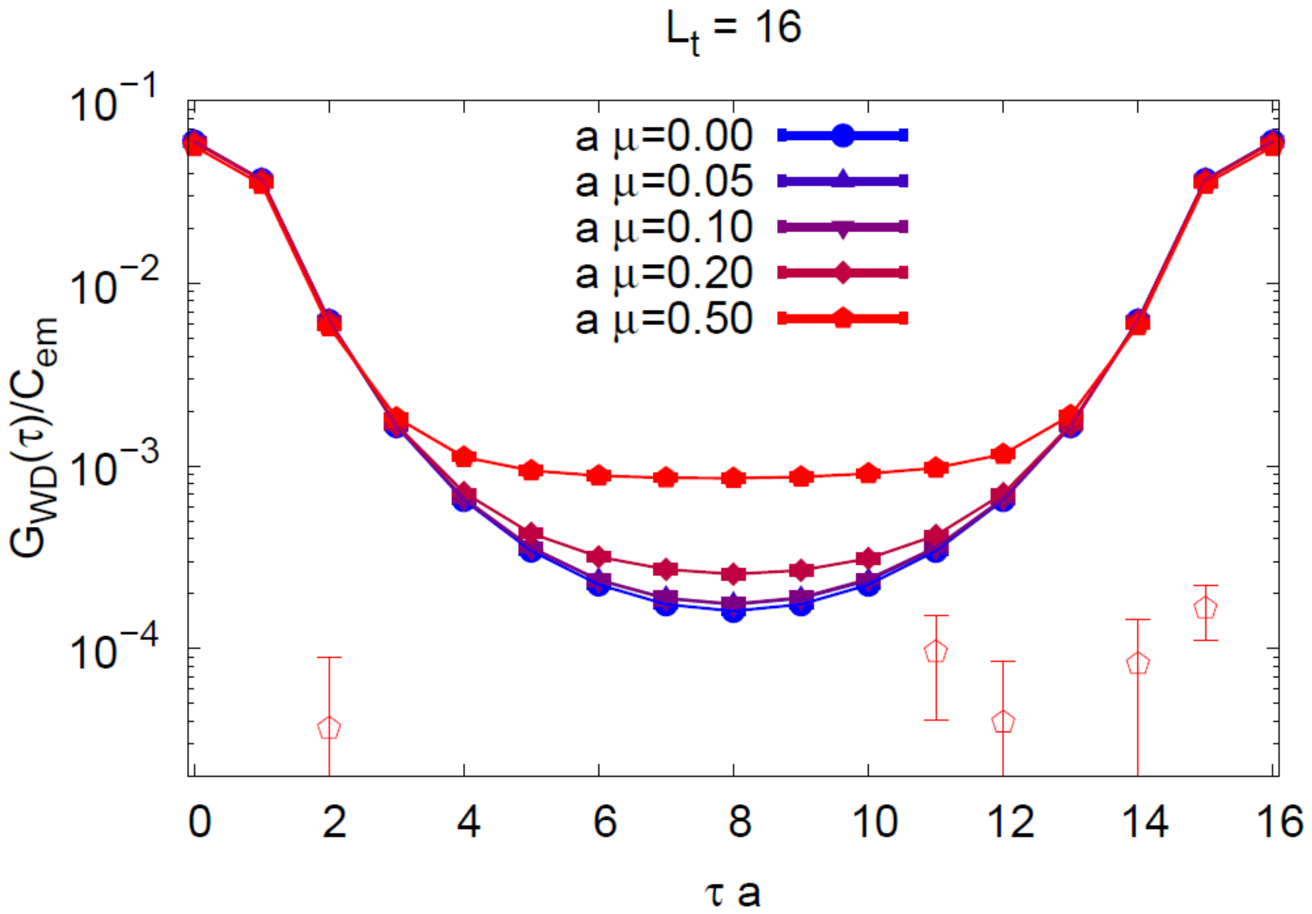
- $q_{\tau}(w)$ chosen such that the width of $\delta_{BG}(w,w')$ is minimized
- Tikhonov regularization for minimization problem [Ulybyshev, Winterowd, Zafeiropoulos, 1707.04212]: $\frac{1}{\lambda_i} \to \frac{\lambda_i}{\lambda_i^2 + \delta^2}$

Backus-Gilbert vs. midpoint resolution functions



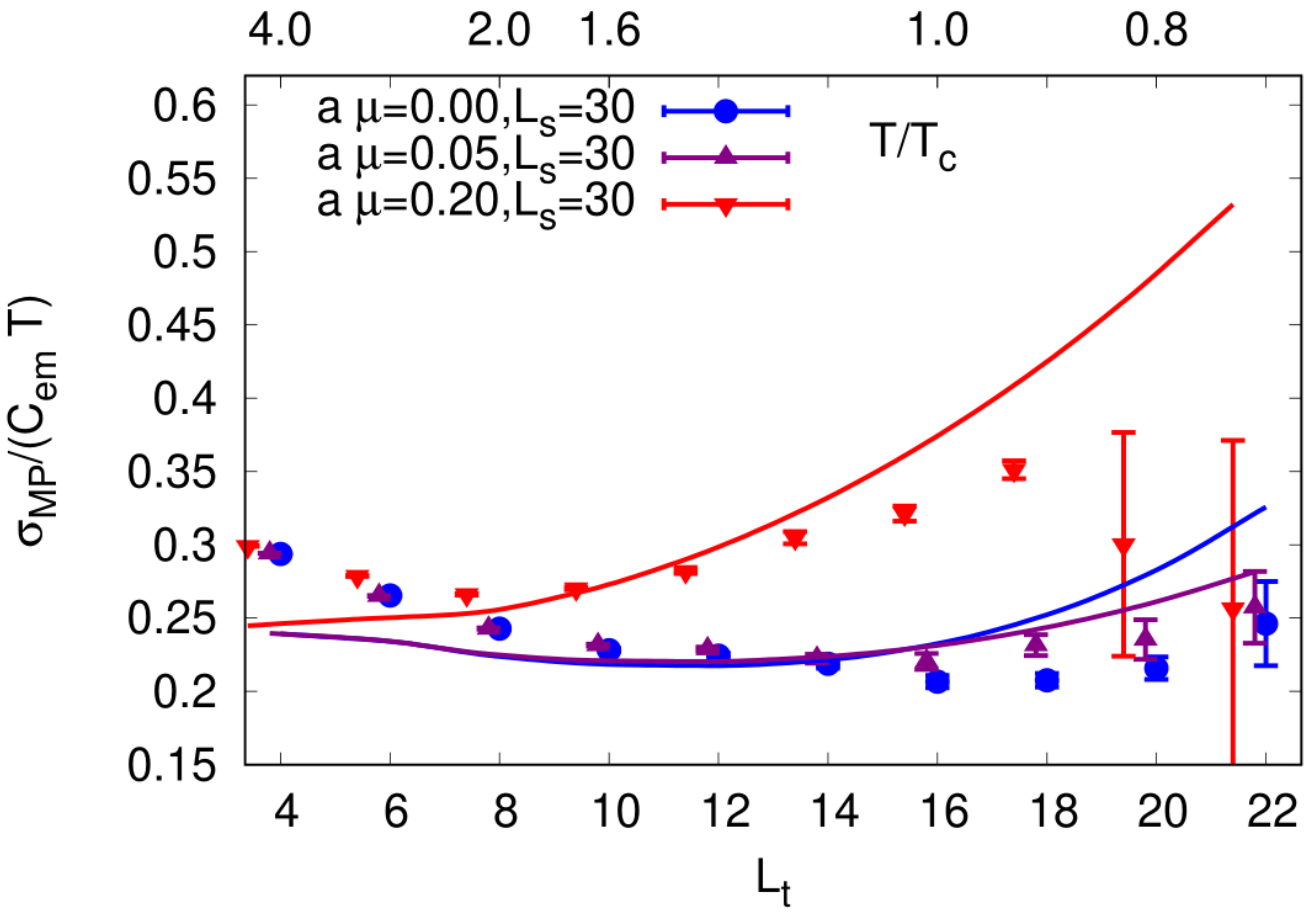
- Backus-Gilbert method yields ~50% narrower resolution function, at the expense of regularization dependence

Current-current correlators vs. density



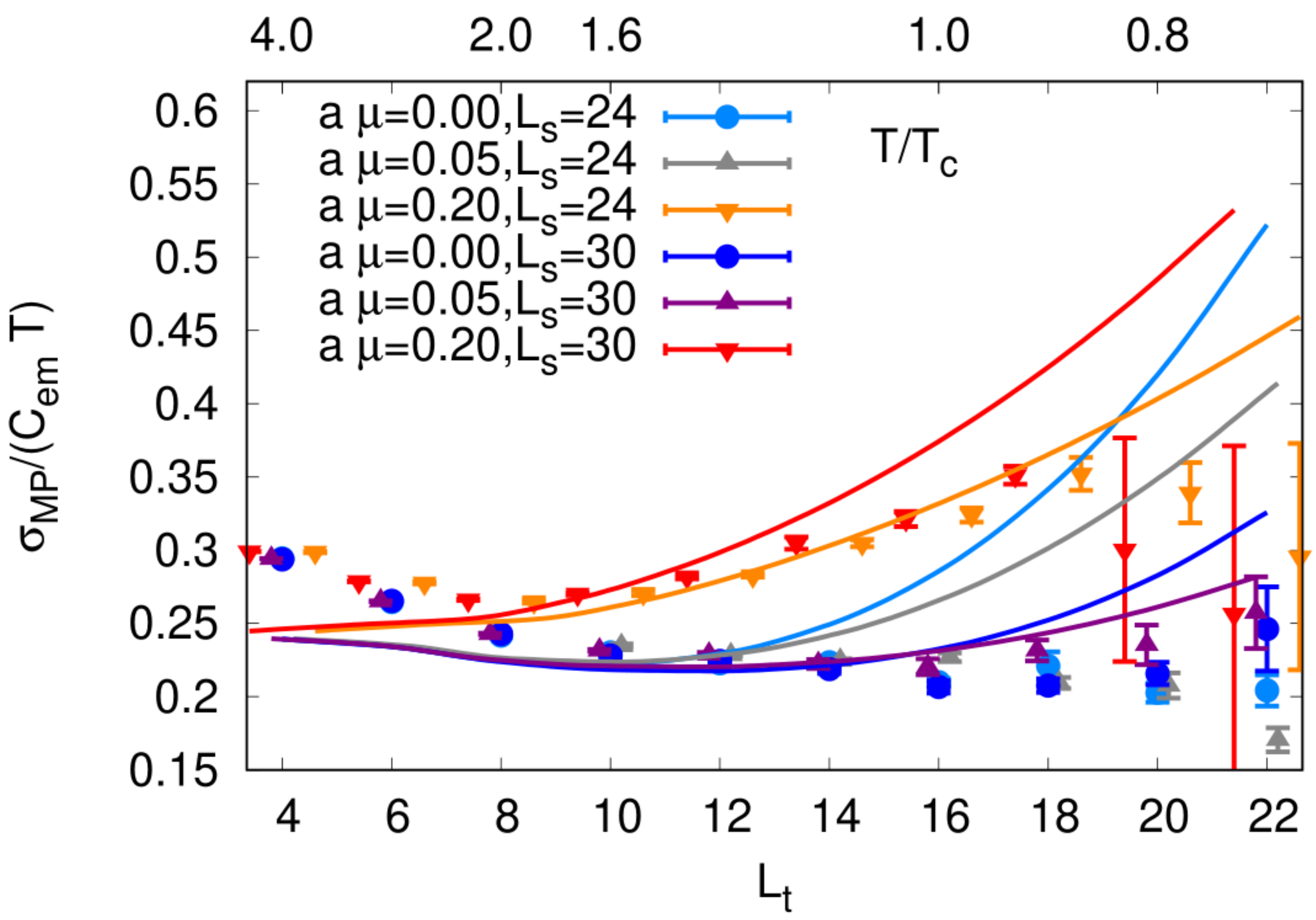
- Large-τ (infrared) correlators grow with density
- Implies the growth of low-frequency conductivity
- Deviations from free-fermion correlators not very large
- Disconnected contributions much smaller than the connected ones
- The importance of disconnected contribution grows with density
- $C_{em} = \sum_f q_f^2 = 5/9$

Midpoint estimator vs. density



- At zero and small densities, conductivity has a minimum around crossover
- At large densities, conductivity quickly grows
- > 50% smaller than the free-fermion result at low temperatures

Finite-volume effects: 24^3 vs 30^3 lattices



- Results on larger lattices are closer to the free fermion result
- Quite significant volume dependence, as conductivity determined by number of near the Fermi surface

$$\frac{2\pi n}{L_s} \approx \mu$$

Anatomy of free quark spectral function

$$\sigma_q(\omega) = \frac{\alpha_q N_c}{24\pi T}\delta(\omega) +$$
$$+ \frac{N_c}{24\pi}\mathrm{Re}\left(\omega^2 - 4m^2\right)^{\frac{1}{2}}\left(1 + \frac{2m^2}{\omega^2}\right) \times$$
$$\times \frac{\sinh\left(\frac{\omega}{2T}\right)}{\cosh\left(\frac{\omega-2\mu}{4T}\right)\cosh\left(\frac{\omega+2\mu}{4T}\right)},$$

Transport peak

AC conductivity

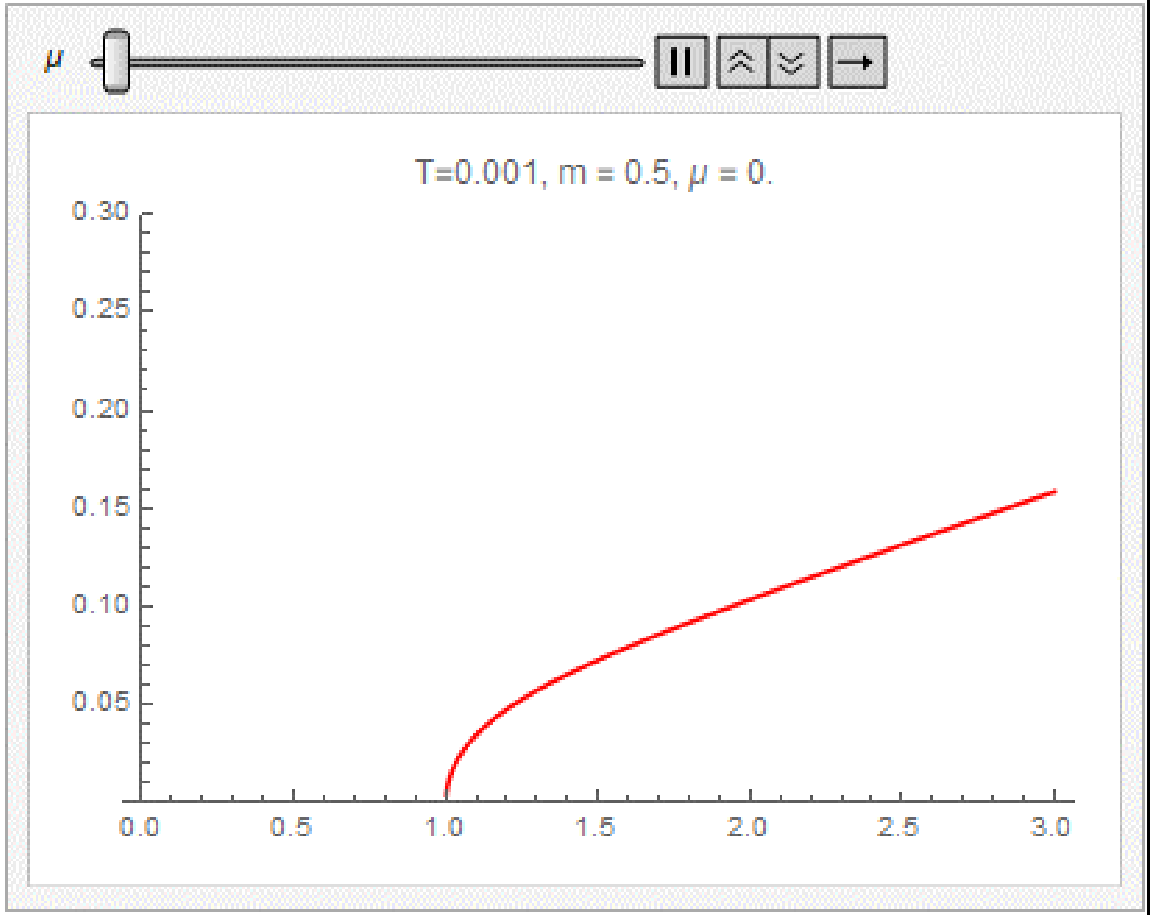
Mass gap threshold (density of states)

Fermi surface threshold (Fermi distribution)

$$\alpha_q = \int_m^\infty d\epsilon \frac{\left(\epsilon^2 - m^2\right)^{\frac{3}{2}}}{\epsilon}\left(\frac{1}{\cosh^2\left(\frac{\epsilon-\mu}{2T}\right)} + \frac{1}{\cosh^2\left(\frac{\epsilon+\mu}{2T}\right)}\right)$$

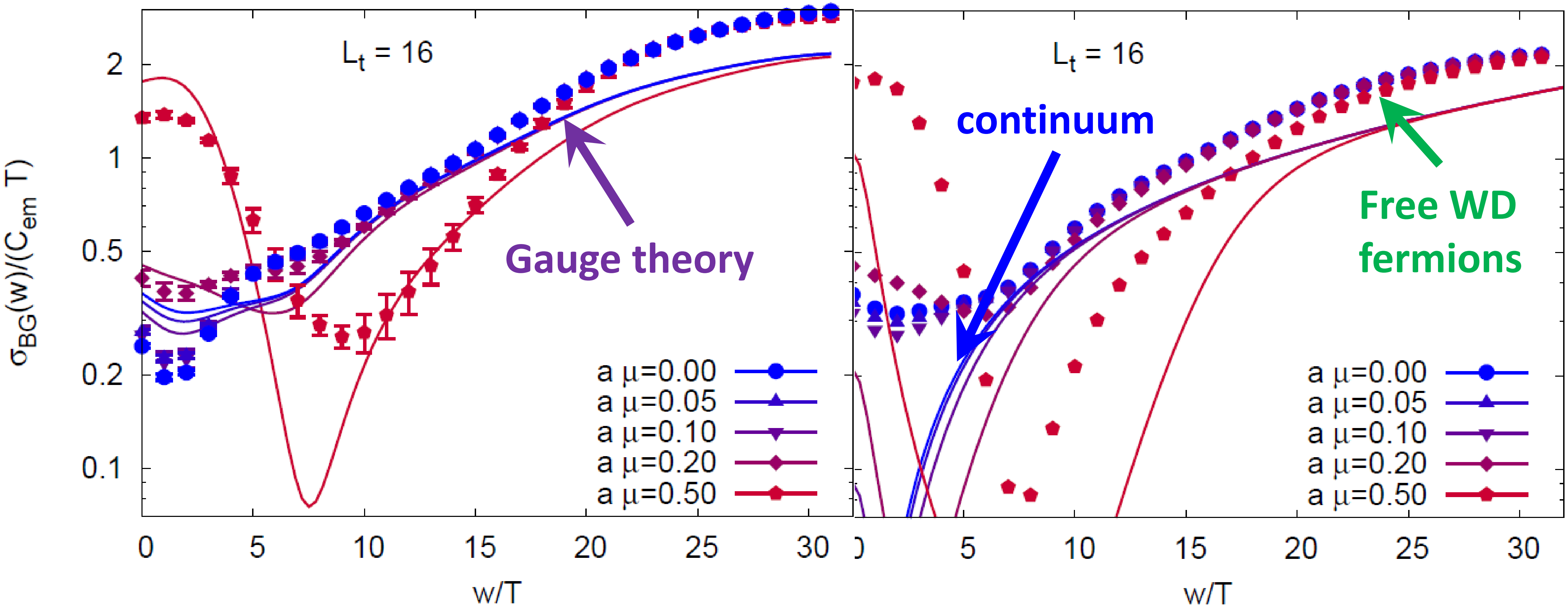
Grows as μ^2 at large μ

Anatomy of free quark spectral function

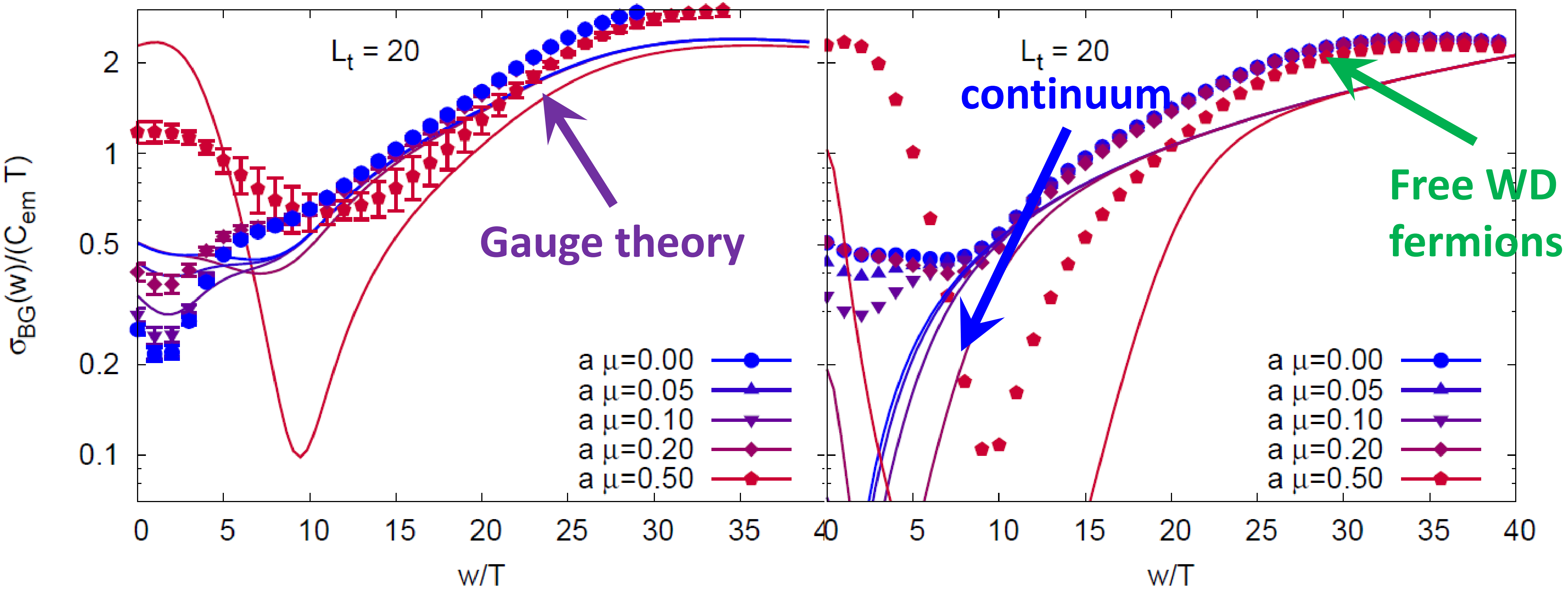


- Bare mass is *m = 0.5*
- Temperature is *T = 0.001*
- The δ-function in the transport peak was replaced with the Breit-Wigner profile of width 0.01 (for illustrative purposes)

Spectral functions at finite density



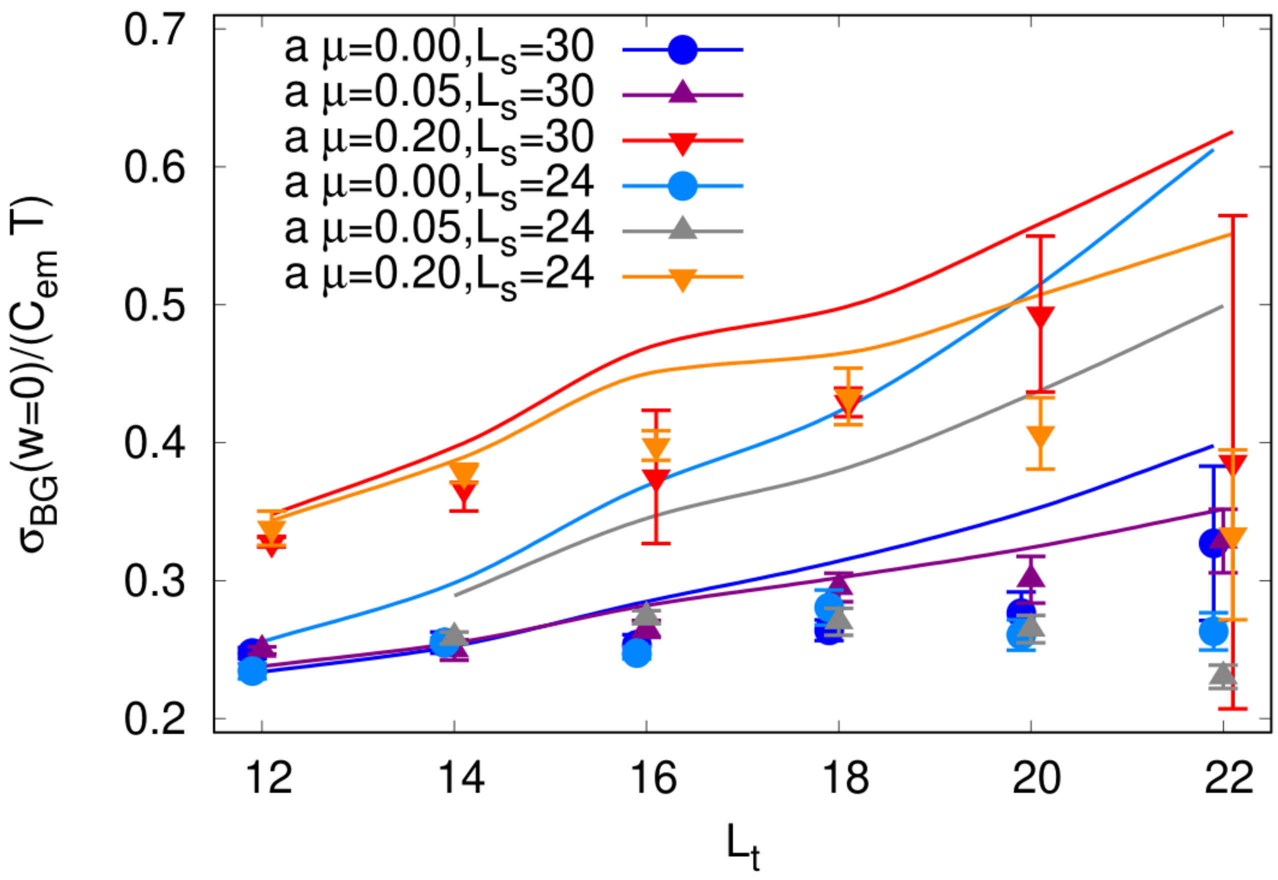
Spectral functions at finite density



Spectral functions at finite density

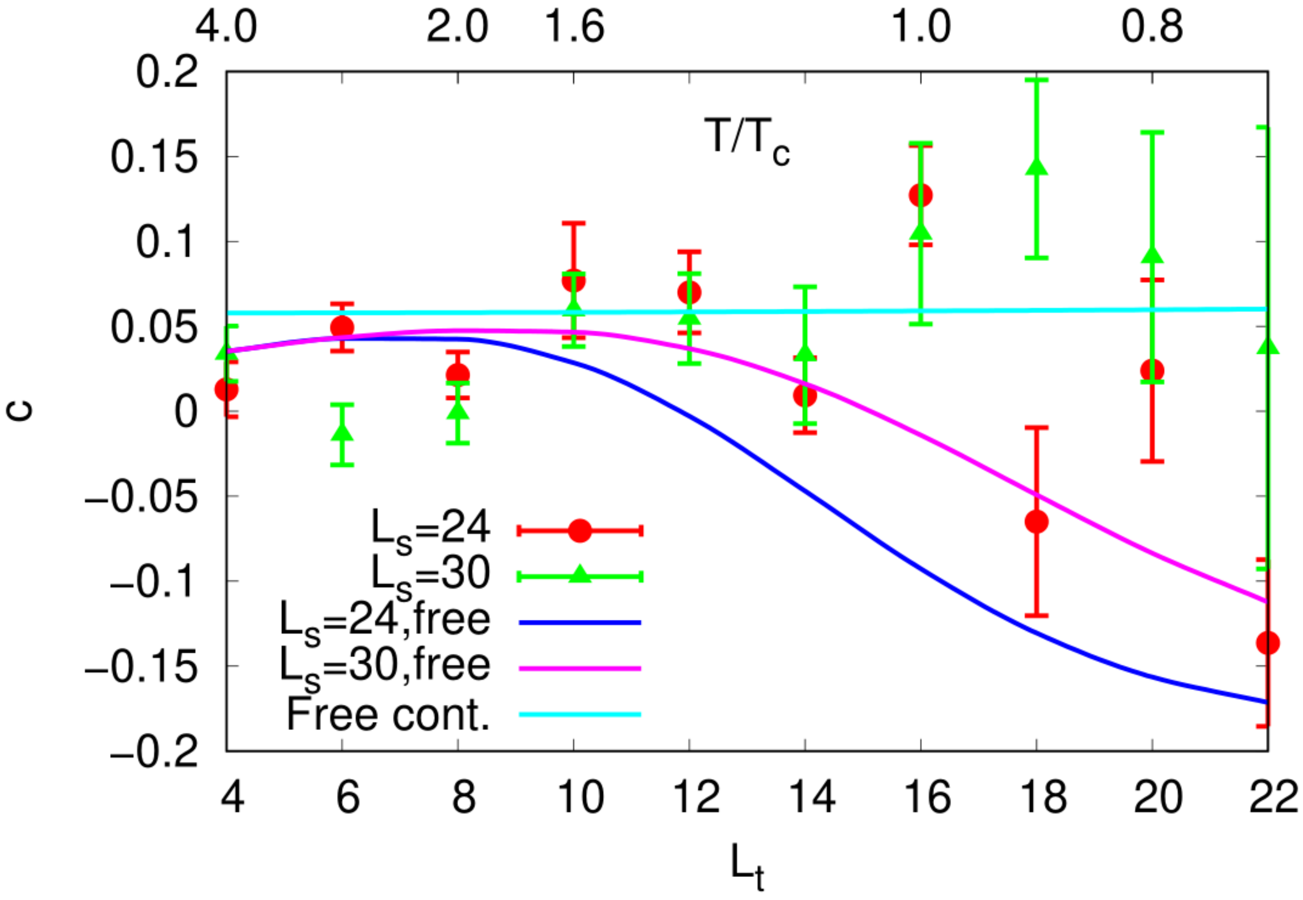
- Nontrivial interplay of threshold effects and finite-volume effects
- Significantly larger spectral function at $w/T \sim 0.4$ – ρ-meson peak
- At low temperatures, low-frequency conductivity becomes very different from the free fermion result
- Density dependence also very different at low temperatures
- At large densities, ρ-meson peak and transport peak seem to merge

Conductivity from the Backus-Gilbert method



Expansion coefficient c(T) in

$$\frac{\sigma(T,\mu)}{T} = \frac{\sigma(T,0)}{T} \left(1 + c(T) \left(\frac{\mu}{T}\right)^2 + O\left(\mu^4\right)\right)$$



- *c(T)* has its maximal value *c(T) ≈ 0.15 +/- 0.05* around crossover temperature
- Finite-volume effects large for free fermions at low temperatures, but not in gauge theory
- The effect of finite density on electric conductivity should not be very large even at *μ/T ~ 1*

Conclusions

- For small densities, dependence of conductivity on finite density is not very strong
- Even $\mu/T \sim 1$ changes the conductivity by 20-30%
- Conductivity is most strongly sensitive to density around crossover temperature
- These conclusions obtained in QCD-like low-density phase and should be at least qualitatively relevant for real QCD
- Strong effect of finite density at large μ in the diquark condensation phase