Demystifying locality problem in Aharonov-Bohm effect

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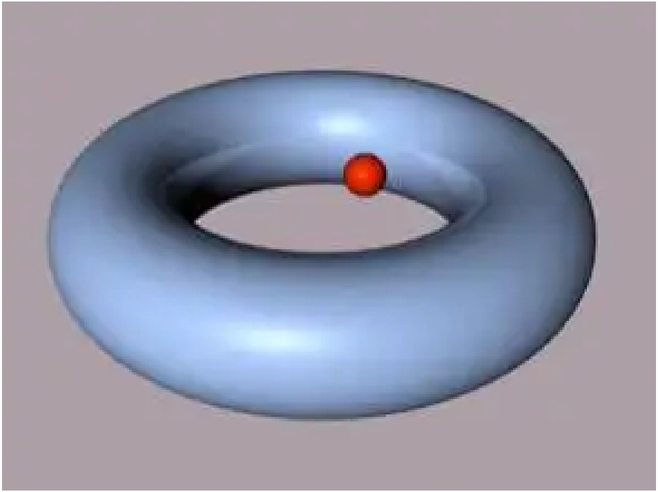
Overview

Motivation

Resolution

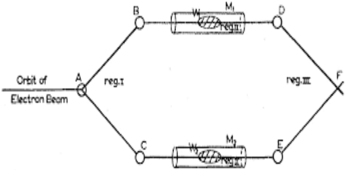
One of 'seven wonders of the quantum world'

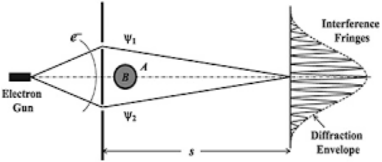


It starts with a doughnut-shaped magnet...

Aharonov & Bohm, 1959

Physical Review. 115 (3): 485–491





Electric charges can be affected by EM field where the field is absent!

Significance of EM potentials

Classical physics

$\Phi' = \Phi - \frac{\partial \chi}{\partial t}$
$\mathbf{A}' = \mathbf{A} + \nabla \chi$

Potentials: good, but they are not essential

Quantum physics

- A-B effect resolved using potentials in field free region
- Changes in potential contain information of fields
- Equations and physical observables: gauge invariant

- $\psi \rightarrow \psi' = e^{i\phi}\psi$, $\phi(x, t)$ related to gauge potential
- Potentials fundamental physical entities, rather than fields

Arguments in favour of potential

Locality, simplicity - naturally incorporated in theory

Arguments against potential

Gauge dependent (locally arbitrary) & unobservable

B vs A

Is the vector potential merely a device which is useful in making calculations ... or is the vector potential a real field? ... What we mean here by a "real" field is this; a real field is a mathematical function we use for avoiding the idea of action at a distance... A "real" field is then a set of numbers we specify in such a way that what happens at a point depends only on the numbers at that point. We do not need to know any more about what's going on at other places... for a long time it was believed that [the potential] was not a real field. It turns out, however, that there are phenomena involving quantum mechanics which show that the potential is in fact a "real" field in the sense we have defined it.

Feynman Lectures, *Vol II-15-4*

A variational principle for curl-free fields

Electrostatics[1]

$\delta \int_A^B \mathcal{E}\ ds = 0$

Euler-Lagrange equation consistent with $\nabla \times \vec{\mathcal{E}} = \mathbf{0}$

Magnetostatics

$\delta \int_A^B \mathcal{H}\ ds = 0$

-valid for simply connected region devoid of free current

- Unmistakable similarity to Fermat's principle: $\delta \int_A^B n\ ds = 0$
- Hamiltonian formulation of ray optics
- Quantum theory of light rays[2]:

$$\lambda\!\!\!^{-2}\nabla^2 u + n^2 u = 0$$

$$\implies -\frac{\lambda\!\!\!^{-2}}{2 \times 1}\nabla^2 u - \frac{n^2}{2} u = 0 \cdot u$$

[1] K Bhattacharya. Journal of Electrostatics (Elsevier), 71(5):926–930, 2013.

[2] D Gloge and D Marcuse. Journal of Optical Society of America, 59(12):1629–1631, 1969.

Schrödinger's equations: static EM fields

$$-\frac{\bar{\gamma}^2}{2\times 1}\nabla^2\psi_E - \frac{\mathcal{E}^2}{2}\psi_E = 0\cdot\psi_E$$

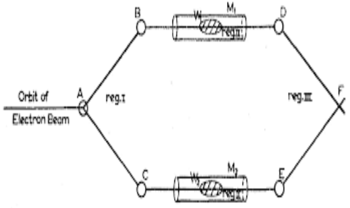
Dimension of $\bar{\gamma}$: potential
ψ_E wave function of $\vec{\mathcal{E}}$ field

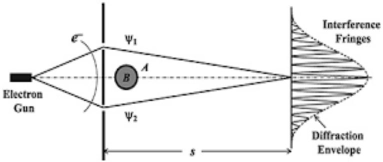
$$-\frac{\bar{\eta}^2}{2\times 1}\nabla^2\psi_M - \frac{\mathcal{H}^2}{2}\psi_M = 0\cdot\psi_M$$

Dimension of $\bar{\eta}$: electric current
ψ_M wave function of $\vec{\mathcal{H}}$ field

- Bosonic nature of ψ_E and ψ_M
- Consistent with facts: refraction of field lines
- ψ_E, ψ_M may be non-zero, even if $\mathcal{E}, \mathcal{H} = 0$

Implication on A-B effect





Φ specified on metal tubes:

$$\nabla^2\Phi = 0$$

$$\implies \psi_E \propto \Phi \implies \psi_E = \frac{\Phi}{\bar{\gamma}}$$

$$\vec{\mathcal{H}} = \mathbf{0} \implies \vec{\mathcal{A}} = -\nabla\mathcal{V}$$

$$\nabla^2\mathcal{V} = 0$$

$$\implies \psi_M \propto \mathcal{V} \implies \psi_M = \frac{\mathcal{V}}{\kappa}$$

Resolution of locality

Field quantum $\psi(x,t)$ operates on a material wave function ψ_e through unitary operation: $U\psi_e = e^{i\psi(x,t)}\psi_e$

Electrostatic A-B effect

Electrons in two different paths will have two different phases $e^{i\frac{\Phi_1}{\bar{\gamma}}}$, $e^{i\frac{\Phi_2}{\bar{\gamma}}}$.

- Phase difference: $\frac{\Delta\Phi}{\bar{\gamma}}$

Magnetic A-B effect

Electrons in two different paths will have two different phases $e^{i\frac{\mathcal{V}_1}{\kappa}}$, $e^{i\frac{\mathcal{V}_2}{\kappa}}$.

- Phase difference: $\frac{\Delta\mathcal{V}}{\kappa}$

$$\Delta\mathcal{V} \propto \int_{\mathcal{C}_1} \vec{A}\cdot d\vec{l} - \int_{\mathcal{C}_2} \vec{A}\cdot d\vec{l}$$
$$= \oint \vec{A}\cdot d\vec{l} = \int_{\mathcal{S}} \nabla\times\vec{A}\cdot d\vec{S}$$
$$= \text{magnetic flux}$$

$$S = -q \int A_\mu dx^\mu = -q \int \phi dt + q \int \vec{\mathcal{A}} \cdot d\vec{l}$$

Electrostatics

$$\vec{\mathcal{A}} = \mathbf{0}$$

$$\text{Minimum } \phi \equiv \bar{\gamma} \Leftrightarrow \hbar$$

$$\implies \bar{\gamma} = -\frac{\hbar}{qt}$$

$$e^{-i\frac{q\Phi t}{\hbar}}$$

Magnetostatics

$$\Phi = 0$$

$$\text{Minimum } \int \vec{\mathcal{A}} \cdot d\vec{l} \equiv \kappa \Leftrightarrow \hbar$$

$$\implies \kappa = \frac{\hbar}{q}$$

$$e^{i\frac{q\int \vec{\mathcal{A}}\cdot d\vec{l}}{\hbar}}$$

Takeaway messages

- A quantum theory of static fields can explain locality issue of the Aharonov-Bohm effect

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- In presence of classical field, material wave function ψ_q of charge q transforms as: $e^{i\psi}\psi_q$ where ψ is not proportional to Φ or $\int \vec{A} \cdot d\vec{l}$.

Thank You

Questions, comments welcome...

Back up

$$S_E = \int (T - V)\, dt \approx -q \int \Phi\, dt \implies \hbar = -q\bar{\gamma}\; t$$

$$\begin{aligned}
\kappa &:= \frac{\mu_0}{4\pi}\bar{\eta} D \\
&= \frac{\mu_0 \epsilon_0}{4\pi\epsilon_0} e v_d \ldots\ldots\ldots (|J| = nev) \\
&= \frac{e\beta c}{4\pi\epsilon_0 c^2} \\
&= \alpha\beta\frac{\hbar}{e}
\end{aligned}$$