Self-testing of quantum states using symmetric local hidden state model

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What is LHS model?

- To describe the traditional LHS model, we need a game---known as steering game

- Consider a situation where Alice creates a bipartite state and sends one of its parties to Bob.

- Unlike Bell scenario, where an Unknown source creates the state and Both does not trust each-other and the source, in this scenario, Alice is the trustless party i.e. she does not need to trust anyone. And Bob does not trust Alice

- This introduces an asymmetry in the steering game.

LHS model

- This asymmetry in the steering game introduces a scenario, where for some states, Alice
- can steer Bob but Bob cannot!

- Although, it perfectly makes sense given the asymmetric nature of the game, this characteristic does not fit to a measure/an witness of correlation

- In this circumstances, we ask the following question:

Can we come up with a game which is symmetric in nature, which gives rise to a symmetric form of the hidden state model and a new form of correlation which is symmetric with respect to the parties as well?

Symmetric hidden state model

- Answer to this question is easy!

- We already know that Bell locality does not imply complete quantum locality!

- Steerability is the prime example of that. There are Bell local states for which one party can still influence the other nonlocally. (Ex. Werner state, I’ll show this with a diagram later.)

- Therefore, there must be an witness or measure of such correlations, which is symmetric. We must be able to quantify such correlations in a scenario where both the parties are trustless just like Bell scenario.

SLHS model and Bell locality model

- Let us then ask a question: How will it then be different from the Bell model?

- Bell model does not consider quantum states. Instead, only considers probability statistics generated by the parties.

- In the SLHS scenario, unknown source is sending quantum states.

SLHS model

- In the symmetric hidden state model, we must be able to express the bipartite state in terms of the local hidden states, i.e.,

$$\rho_{AB} = \sum_{\lambda} p(\lambda) \rho_{\lambda}^{A} \otimes \rho_{\lambda}^{B}, \quad \text{Eq 1}$$

- However, this is just a seperable state and any state other than the above form is just an entangled state.

SLHS and seperable states

- Question: does that mean SLHS model just produces seperable states?
- Ans: no.
- why?
- Entanglement witness or measures need Alice and Bob to trust each-other.

SLHS model

- In the LHV model, we consider the joint probability distribution.
- We use the principle of locality and determinism to write the joint distribution in terms of local probability distributions and hidden variables.
- The locality in Bell's formalism does not imply EPR locality. Non-locality in a different form may still exist. It is just that the non-locality is not reflected in its local probability distributions.

SLHS model

- Here, we assume that both Alice and Bob are receiving quantum states from an unknown source. However, quantum states provide more information than just probability distributions.
- One can also extract information about certain quantities, which have no classical counterparts unlike probability distributions. We consider a global or joint property with no classical counterpart and using the principle of locality and hidden states, express in terms of the local property of the hidden states.
- With these assumptions, all the elements of a bipartite density matrix must be expressible by some unknown local hidden states ρ_A^λ and ρ_B^λ generated by the unknown source with a distribution $p(\lambda)$.

SLHS model

$$\langle i^a j^b | \rho_{AB} | i^{a'} j^{b'} \rangle = \sum_\lambda p(\lambda) \langle i^a | \rho_A^\lambda | i^{a'} \rangle \langle j^b | \rho_B^\lambda | j^{b'} \rangle$$

-----(Eq2)

- We only consider terms for which $a \neq a'$ and $b \neq b'$
- needless to mention, i and j are indices to denote bases and a, b are vectors in the particular basis.

SLHS model

- Transition probability can also be used to express a relatively weaker version of our hidden state model in the following form,

$$p\left(|i^{a-a'}\rangle^A, |j^{b-b'}\rangle^B\right) = \sum_\lambda p(\lambda) p\left(|i^{a-a'}\rangle^A \middle| \lambda\right) p\left(|j^{b-b'}\rangle^B \middle| \lambda\right).$$

-----Eq3

- where $p(|i^{a-a'}\breve{\rangle}) = |\langle i^a|\rho|i^{a'}\rangle|.$

SLHS Inequality

$$\sum_{\lambda} p(\lambda) p\left(|i^{a-a'}\rangle^{A} | \lambda \right) p\left(|j^{a-a'}\rangle^{B} | \lambda \right)$$

$$\leq \sqrt{\sum_{\lambda'} p(\lambda') p^{2}\left(|i^{a-a'}\rangle^{A} \middle| \lambda' \right) \sum_{\lambda} p(\lambda) p^{2}\left(|j^{a-a'}\rangle^{B} \middle| \lambda \right)}$$

$$= \sqrt{\overline{p^{2}\left(|i^{a-a'}\rangle^{A} \right)} . \overline{p^{2}\left(|j^{a-a'}\rangle^{B} \right)}} \leq \frac{1}{4}$$

$$\implies \sum_{\substack{a, a', \\ a \neq a'}} p(|i^{a-a'}\rangle^{A}, |j^{a-a'}\rangle^{B}) \leq \frac{1}{2}, \qquad \text{Eq4}$$

Example

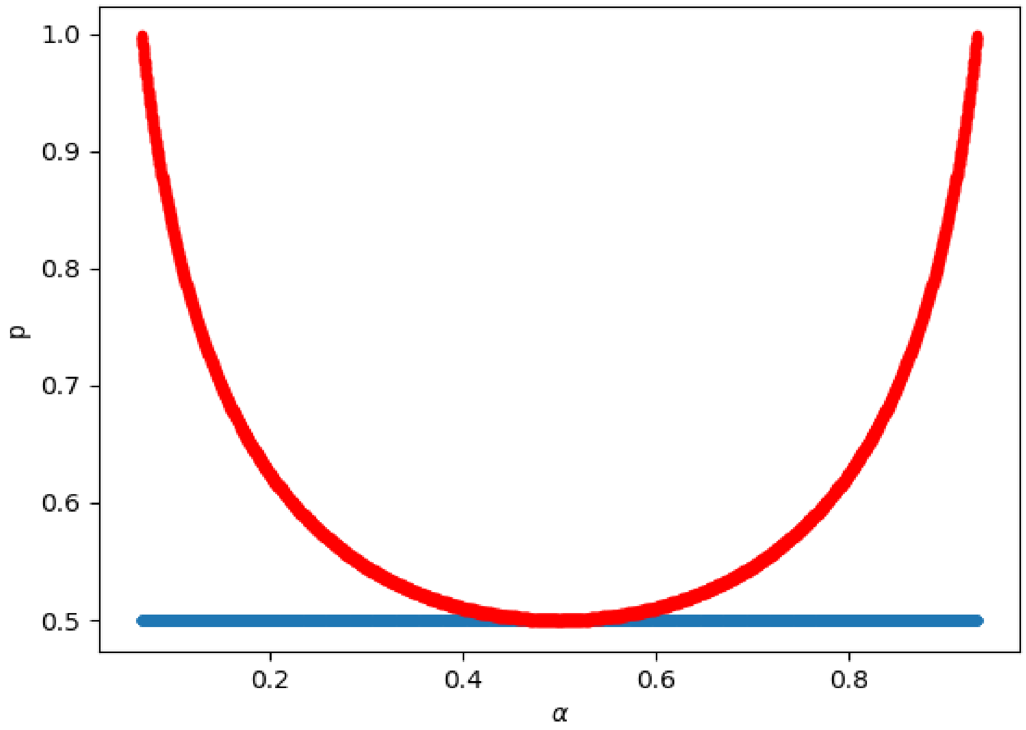


FIG. 1: We plot (in red) the left hand side of the Eq. with respect to the parameters for the Werner state $\rho_w^{\phi+} = p|\phi_+^\alpha\rangle\langle\phi_+^\alpha| + \frac{1-p}{4} I_4$, where $|\phi_+^\alpha\rangle = \sqrt{\alpha}|00\rangle + \sqrt{1-\alpha}|11\rangle$, when both Alice and Bob measures the transition probabilities in the σ_3 basis. The straight line parallel to the α-axis represents the bound $p > \frac{1}{2}$, for which the state is nonlocal.

Example

- Consider the Werner state for which the state can be represented as

$$\begin{pmatrix} \frac{\mathrm{pw}+1}{4} & 0 & 0 & \frac{\mathrm{pw}}{2} \\ 0 & \frac{1-\mathrm{pw}}{4} & 0 & 0 \\ 0 & 0 & \frac{1-\mathrm{pw}}{4} & 0 \\ \frac{\mathrm{pw}}{2} & 0 & 0 & \frac{\mathrm{pw}+1}{4} \end{pmatrix}$$

- Therefore, the left hand side of the inequality turns out to be p_w. So, for p_w>1/2 the state has EPR correlation.

Generalized Inequality

$$\sum_{\substack{a,a'=0 \\ a\neq a'}}^{d-1} p(|i^{a'-a}\rangle^A, |j^{a-a'}\rangle^B) \leq \frac{d-1}{d}.$$

-------------Eq5

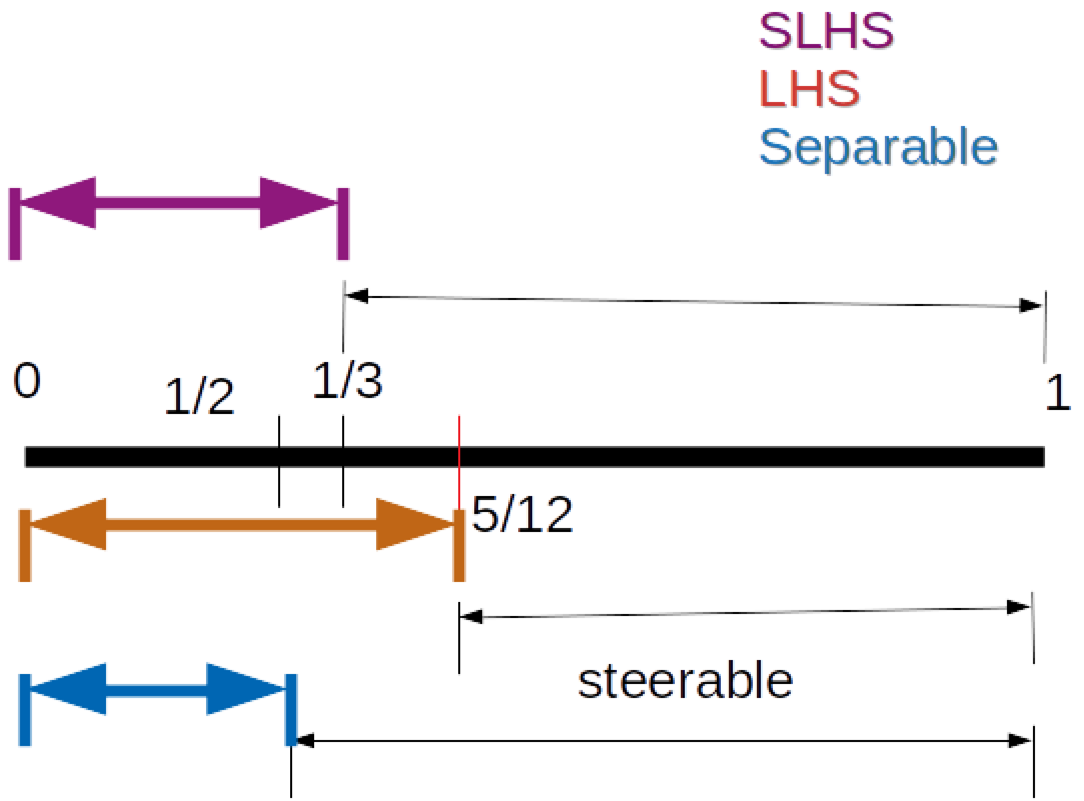
Nonlocality in the generalized scenario

- For a dxd dimensional Isotropic state of the form of

$$\rho_{AB} = (1-p)\frac{I_{d^2}}{d^2} + \frac{p}{d}\sum_{i,j} |ii\rangle\langle jj|$$

the state is steerable for $p > \frac{\sum_{r=2}^{d} \frac{1}{r}}{d-1}$, entangled for $p > \frac{1}{d+1}$ and violates SLHS model and thus, nonlocally correlated for $p > \frac{1}{d}$.

Nonlocality in the two-qutrit scenario



Nonlocality in the generalized scenario

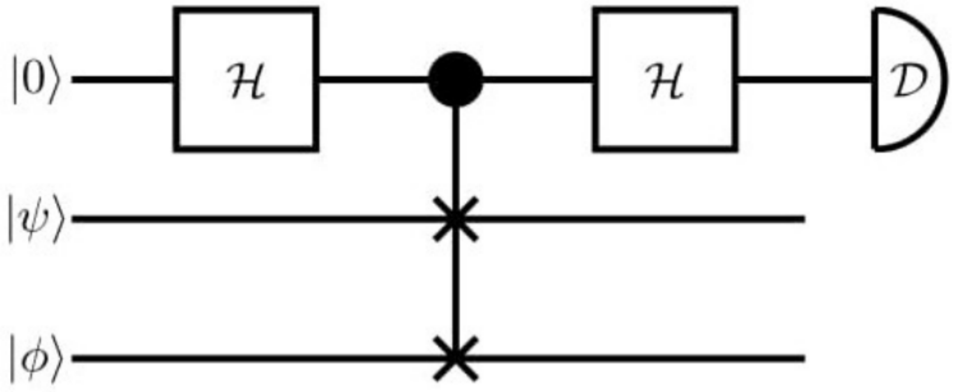
- SLHS cannot single out the whole set of LHS (in the higher dimensional state space. d>2x2)
- EPR correlation predicted by the SLHS model is stronger than the existing bounds predicted by the LHS model.

Experimental Realizations

- We know

$$\langle ++|\rho|++\rangle + \langle +-|\rho|+-\rangle = \frac{1}{2}\left(\langle 00|\rho|00\rangle + \langle 10|\rho|10\rangle + \langle 01|\rho|01\rangle + \langle 11|\rho|11\rangle + \langle 00|\rho|11\rangle + \langle 11|\rho|00\rangle\right) \qquad \text{Eq6}$$

- One can measure all the diagonal terms by measuring the overlaps between the two states.
- The last two terms can be calculated from the measurment results.



$$P = \frac{1}{2}\left(1 + |\langle\psi|\phi\rangle|\right)$$

Experimental realization

- Now using the triangle inequality for two complex numbers, we can write

$$|\langle 00|\rho|11\rangle + \langle 11|\rho|00\rangle)| \leq |\langle 00|\rho|11\rangle| + |\langle 11|\rho|00\rangle)| \qquad \text{Eq7}$$

- Now, we already knew, the right hand side is bounded from above by 1/2 from the inequality Eq. 4.

- Thus, we can just measure the left hand side of the inequality 7 using the Eq6.

Self-testing

- In this context, a state $|\psi_{AB}\rangle$ and the measurement basis $|\phi^a\rangle$ are self-testable by the correlation measured via joint transition probability $p(|\phi^{a-a'}\rangle^A, |\phi^{b-b'}\rangle^B)$, if all states ρ_{AB} and measurements $|\widetilde{\phi}^a\rangle^A$ compatible with $p(|\phi^{a-a'}\rangle^A, |\phi^{b-b'}\rangle^B)$ turns out to be the same state $|\psi_{AB}\rangle$ and measurement $|\phi^a\rangle$.

Here a= 0, 1, 2, 3, ... up to the dimension of the local party.

self-testing

- To self-test one of the Bell states and measurement bases, we start with the assumptions that the transition amplitudes of an arbitrary state ρ_{AB} in the measurement bases $|\phi^a\rangle$ and $|\xi^a\rangle^A$ take the maximal possible value of 1/2, i.e.,

$$\langle\phi_A^0\phi_B^0|\rho_{AB}|\phi_A^1\phi_B^1\rangle = \frac{1}{2}, \ \langle\xi_A^0\xi_B^0|\rho_{AB}|\xi_A^1\xi_B^1\rangle = \frac{1}{2}$$

- and

$$\langle\phi_A^0\phi_B^1|\rho_{AB}|\phi_A^1\phi_B^0\rangle = 0,$$

where we define $|\xi^0_{A(B)}\rangle = \frac{1}{\sqrt{2}}(|\phi^0_{A(B)}\rangle + |\phi^1_{A(B)}\rangle)$ and $|\xi^1_{A(B)}\rangle = \frac{1}{\sqrt{2}}(|\phi^0_{A(B)}\rangle - |\phi^1_{A(B)}\rangle)$.

self-testing

- We can write the state as

$$\rho_{AB} = \sum_{\substack{j,l \in S_{d_B} \\ i,k, \in S_{d_A}}} \alpha_{ij}^{kl} |\phi_A^i \phi_B^j\rangle\langle\phi_A^k \phi_B^l|$$

without loss of generality.

$$\sum_{ij} \alpha_{ij}^{ij} = 1.$$ Normalization condition.

$$\alpha_{ij}^{kl} = \alpha_{kl}^{ij*}$$ Hermiticity condition

self-testing

- First two equations and the hermiticity condition imply

$$\alpha_{11}^{00} = \alpha_{00}^{11} = \frac{1}{2}$$

$$\alpha_{01}^{10} = \alpha_{10}^{01} = 0.$$

The third equation and the hermiticity condition imply

$$\sum_{\substack{i,j=0\\k,l}}^{1} (-1)^{i+j} \alpha_{ij}^{kl} = 2. \quad \text{and} \quad \sum_{\substack{i,j=0\\k,l}}^{1} (-1)^{k+l} \alpha_{ij}^{kl} = 2$$

self-testing

- Adding the last two equations and putting the values of

  $\alpha_{11}^{00} = \alpha_{00}^{11}$ and $\alpha_{01}^{10} = \alpha_{10}^{01}$, we get

$$\alpha_{00}^{00} - \alpha_{01}^{01} - \alpha_{10}^{10} + \alpha_{11}^{11} = 1.$$

- Subtracting the equation from the normalization condition, we get

$$2\alpha_{01}^{01} + 2\alpha_{10}^{10} + \sum_{i,j\neq 0,1} \alpha_{ij}^{ij} = 0.$$

This implies all the diagonal terms are zero(since diagonal terms are non-negative) except α_{00}^{00} and α_{11}^{11}

self-testing

- Thus, we get

$$\alpha_{00}^{00} + \alpha_{11}^{11} = 1.$$

- Using the Cauchy-Schwartz inequality, one can show that

$$|\alpha_{ij}^{kl}|^2 \leq |\alpha_{ij}^{ij}||\alpha_{kl}^{kl}|$$

We use the inequality to show that for all vanishing diagonal elements, i.e., $\alpha_{ij}^{ij} = 0$, all the off-diagonal elements are zero, i.e., $\alpha_{ij}^{kl} = \alpha_{kl}^{ij} = 0 \forall i, j, k, l$.

self-testing

Using the same inequality, one can also show that $\alpha_{00}^{00}\alpha_{11}^{11} \geq \frac{1}{4}$. From the bound together with the Eq. one can show that

$$(\alpha_{00}^{00} - \alpha_{11}^{11})^2 = {\alpha_{00}^{00}}^2 + {\alpha_{11}^{11}}^2 - 2\alpha_{00}^{00}\alpha_{11}^{11} = 1 - 4\alpha_{00}^{00}\alpha_{11}^{11} \leq 0.$$

This implies $0 \leq (\alpha_{00}^{00} - \alpha_{11}^{11})^2 \leq 0$

which in turn implies $\alpha_{00}^{00} = \alpha_{11}^{11} = \frac{1}{2}$

Self-testing

- Thus, we are left with

$$\alpha_{11}^{00} = \alpha_{00}^{11} = \alpha_{00}^{00} = \alpha_{11}^{11} = \frac{1}{2}$$

which is nothing but the Bell state and the corresponding measurement bases are x and z bases upto local isometry.

Conclusions

- There was a formidable belief that the only way to describe the lhs game is by giving up the freedom whether to trust or not to trust a party and retaining it for the other, thereby introducing an asymmetry in the game.
- In the existing lhs game, one of the parties is considered to be an un-trusted party.
- We provide alternative description of the lhs model, which is symmetric, trust-less and can be used in self-testing the states and measurements.
- Therefore, it provides a better and operationally more efficient alternative to the Bell inequality for the self-testing and other device-independent protocols
- The symmetric form of lhs model reveals more nonlocality than the existing lhs or lhv models (particularly in higher dimensions $d > 2\otimes 2$) opening a new frontier for the
- experimentalists to test the new boundary of quantum

correlation.

Thank You