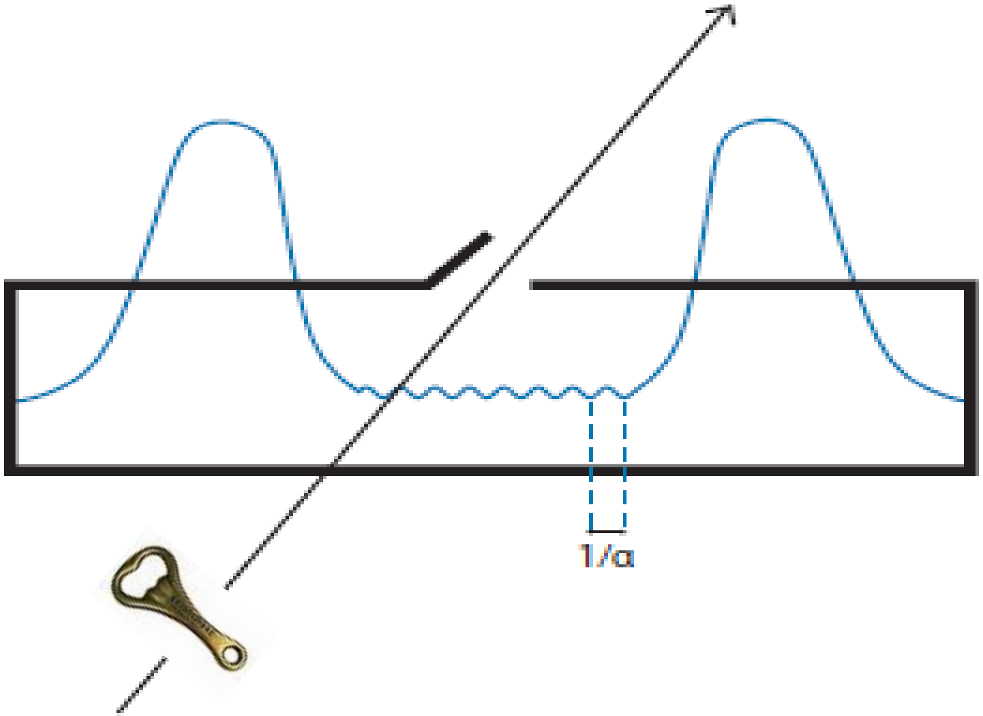
# Quantum conservation laws




Y. Aharonov, S. Popescu and D. Rohrlich,
*Proc. Nat. Acad. Sci.* (2020)

# Outline

1. Superoscillations
2. The experiment
3. The paradox
4. Energy conservation

## Outline

1. Superoscillations
2. The experiment
3. The paradox
4. Energy conservation

Common wisdom assumes that no function can oscillate faster than its fastest Fourier component. But consider $f(x)$:

$$f(x) = \left( \frac{1+\alpha}{2} e^{ix/N} + \frac{1-\alpha}{2} e^{-ix/N} \right)^N$$

The "fastest Fourier" components are $e^{ix}$ and $e^{-ix}$.

Common wisdom assumes that no function can oscillate faster than its fastest Fourier component. But consider $f(x)$ for $x \approx 0$:

$$f(x) = \left( \frac{1+\alpha}{2} e^{ix/N} + \frac{1-\alpha}{2} e^{-ix/N} \right)^N$$

The "fastest Fourier" components are $e^{ix}$ and $e^{-ix}$.

Common wisdom assumes that no function can oscillate faster than its fastest Fourier component. But consider $f(x)$ for $x \approx 0$:

$$f(x) = \left(\frac{1+\alpha}{2}e^{ix/N} + \frac{1-\alpha}{2}e^{-ix/N}\right)^N$$

$$\approx \left(\frac{1+\alpha}{2}(1 + i\frac{x}{N}) + \frac{1-\alpha}{2}(1 - i\frac{x}{N})\right)^N$$

Common wisdom assumes that no function can oscillate faster than its fastest Fourier component. But consider $f(x)$ for $x \approx 0$:

$$f(x) = \left(\frac{1+\alpha}{2}e^{ix/N} + \frac{1-\alpha}{2}e^{-ix/N}\right)^N$$

$$\approx \left(\frac{1+\alpha}{2}(1+i\frac{x}{N}) + \frac{1-\alpha}{2}(1-i\frac{x}{N})\right)^N$$

$$= \left(1+\frac{i\alpha x}{N}\right)^N \approx e^{i\alpha x}.$$

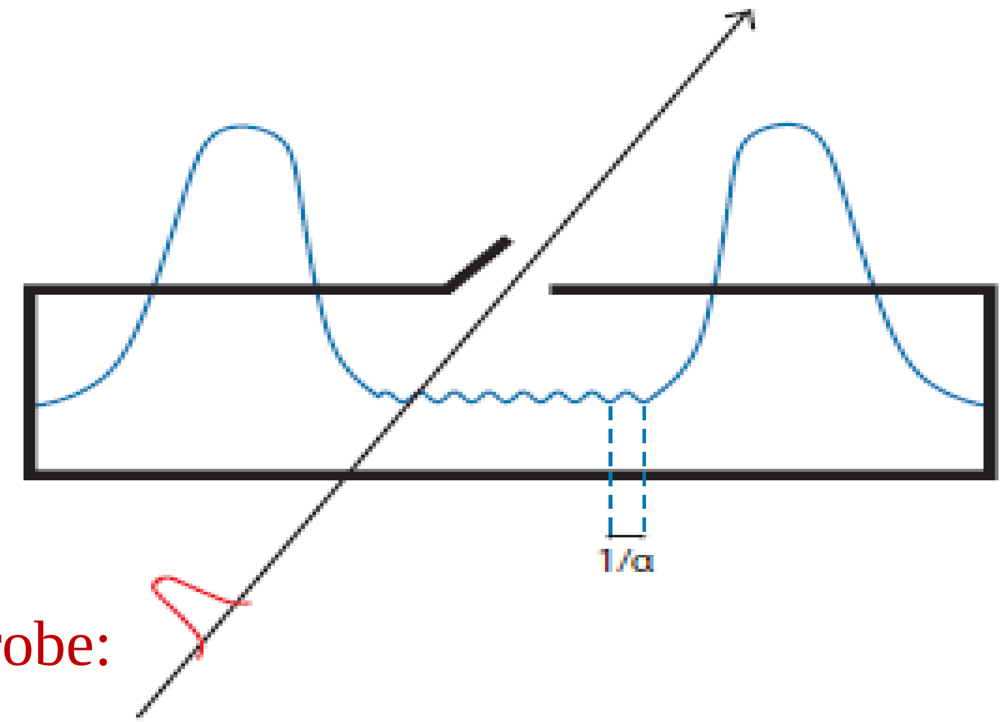
# Outline

1. Superoscillations

2. The experiment

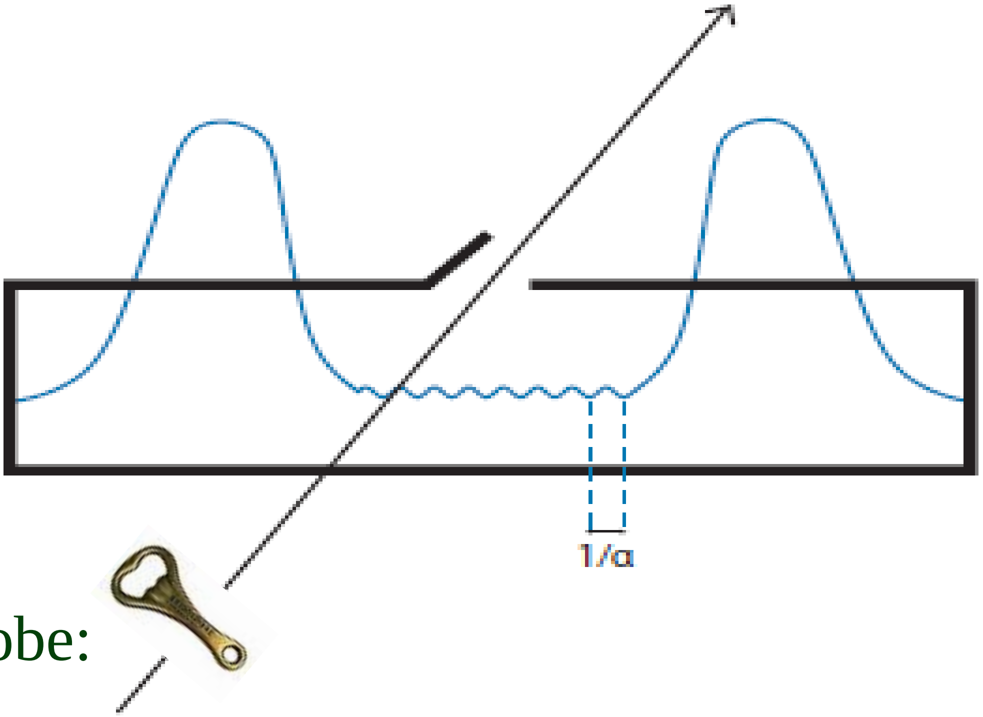
3. The paradox

4. Energy conservation

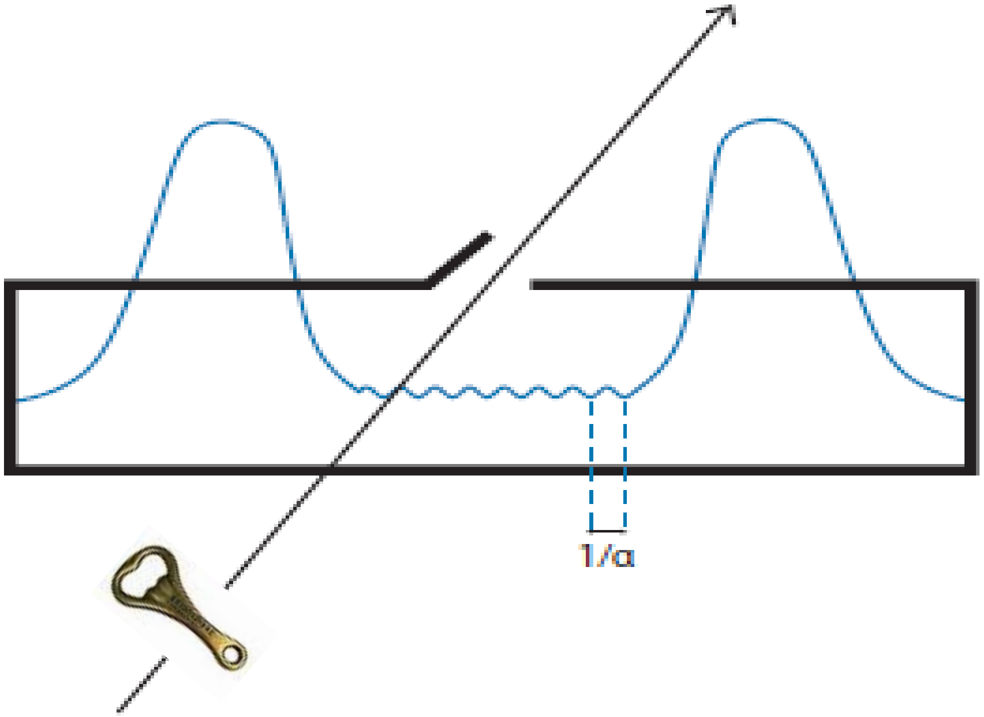



local probe:

local probe:



A local probe cannot distinguish between a true high-energy state and a mere superoscillation.

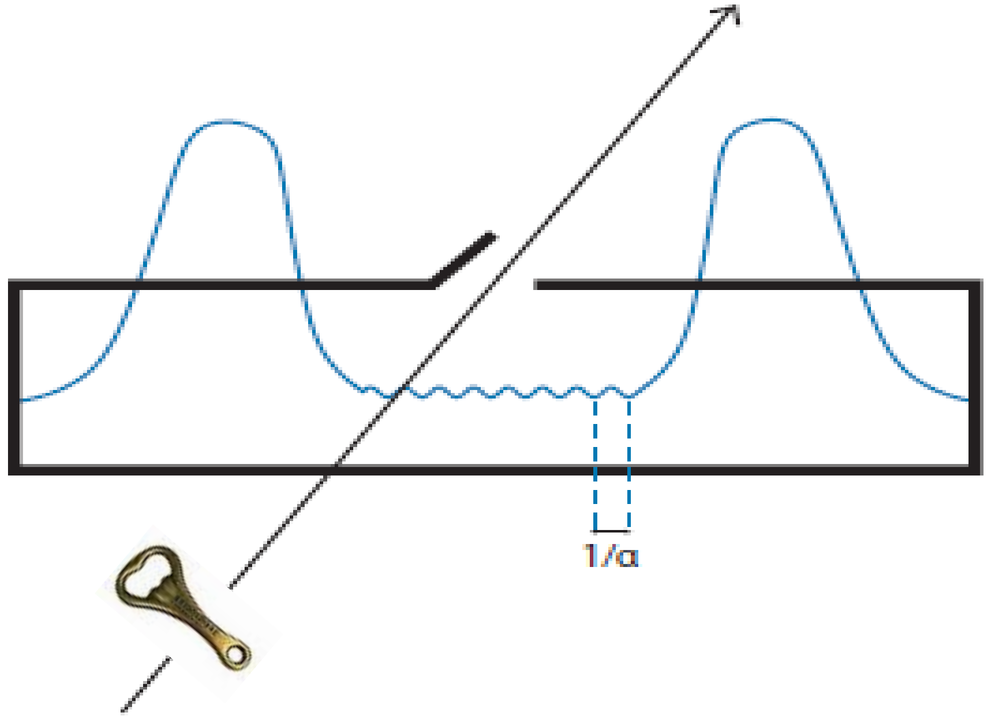



Hence the probe must find – *if it finds anything* – a true high-energy state!

# Outline

1. Superoscillations
2. The experiment
3. The paradox
4. Energy conservation

A local probe cannot distinguish between a true high-energy state and a mere superoscillation.




Where did the high energy come from?

Bohr might have guessed that the opener crossed the high-energy state in a time $T$, hence the uncertainty in the energy was at least $\hbar/T$.

Bohr might have guessed that the opener crossed the high-energy state in a time $T$, hence the uncertainty in the energy was at least $\hbar/T$.

Causality rules out this guess: the probe cannot distinguish between a true high-energy state and a superoscillation, hence nothing about the probe can distinguish between them.

# Outline

1. Superoscillations
2. The experiment
3. The paradox
4. Energy conservation

When we say that quantum mechanics conserves energy *in the average*, are we telling the complete story?