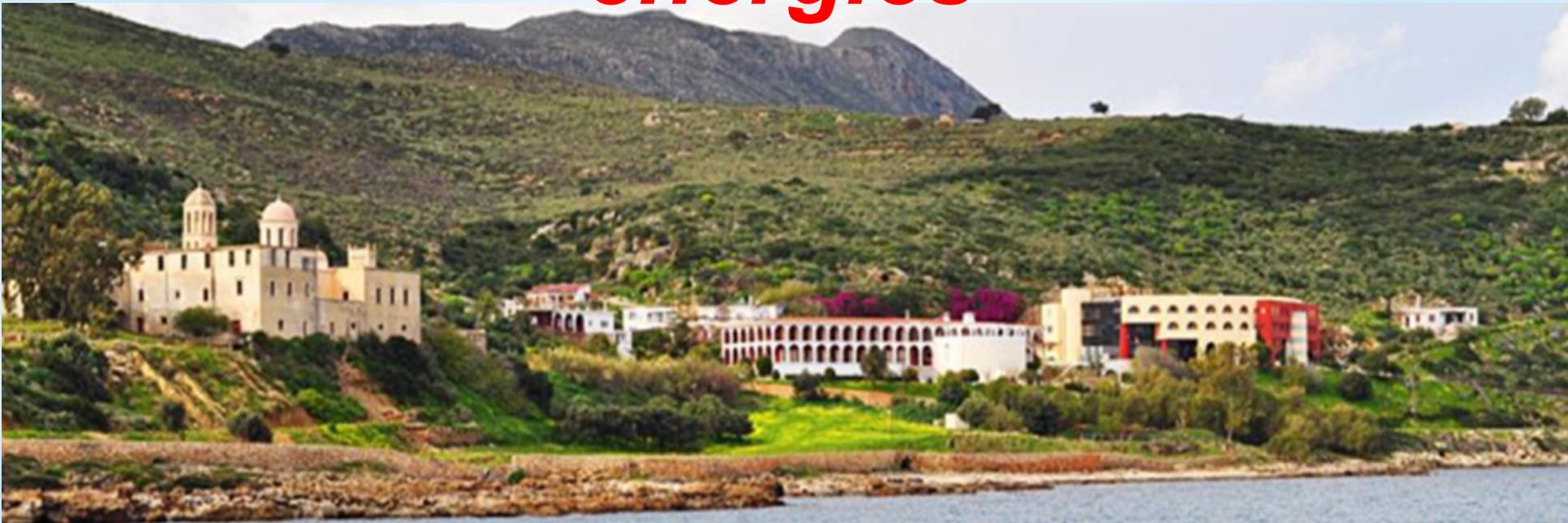


Shear viscosity of nucleons and pions in A+A collisions at NICA energies



E. Zabrodin,
in collaboration with
M. Teslyk, O. Vitiuk and L. Bravina



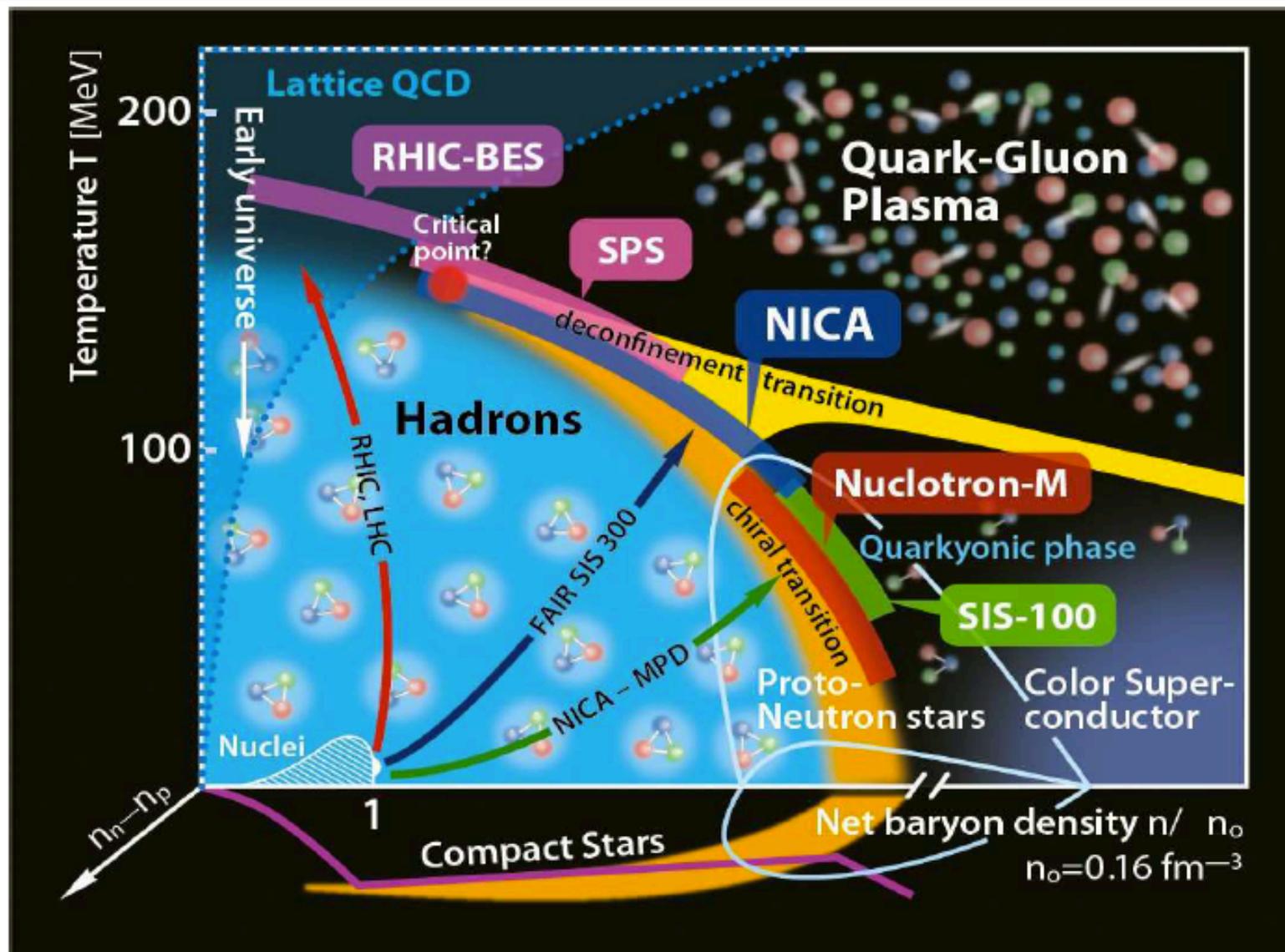
9-th International Conference on New Frontiers in Physics ICNFP-2020
Kolymbari, Crete, Greece, 4-11.09.2020



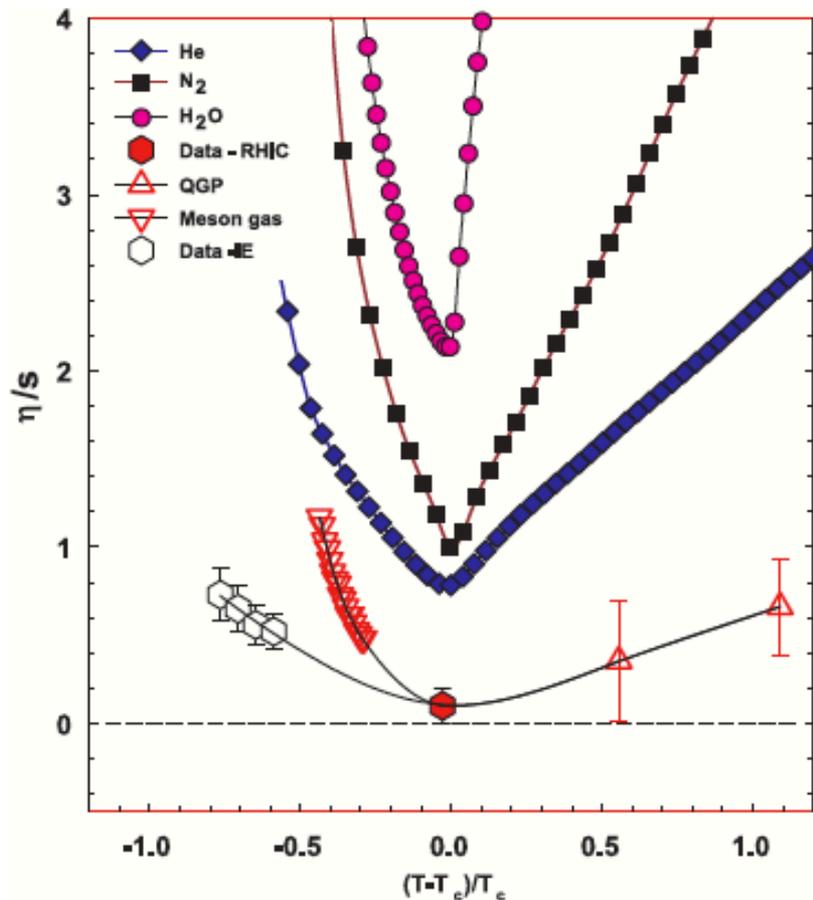
Based on publications:

- 1. M.Teslyk, L.Bravina, O.Panova, O.Vitiuk, E.Z.,
PRC 101 (2020) 014904**
- 2. E.Z., M.Teslyk, O.Vitiuk, L.Bravina,
Phys. Scripta 95 (2020) 7, 074009**

Motivation



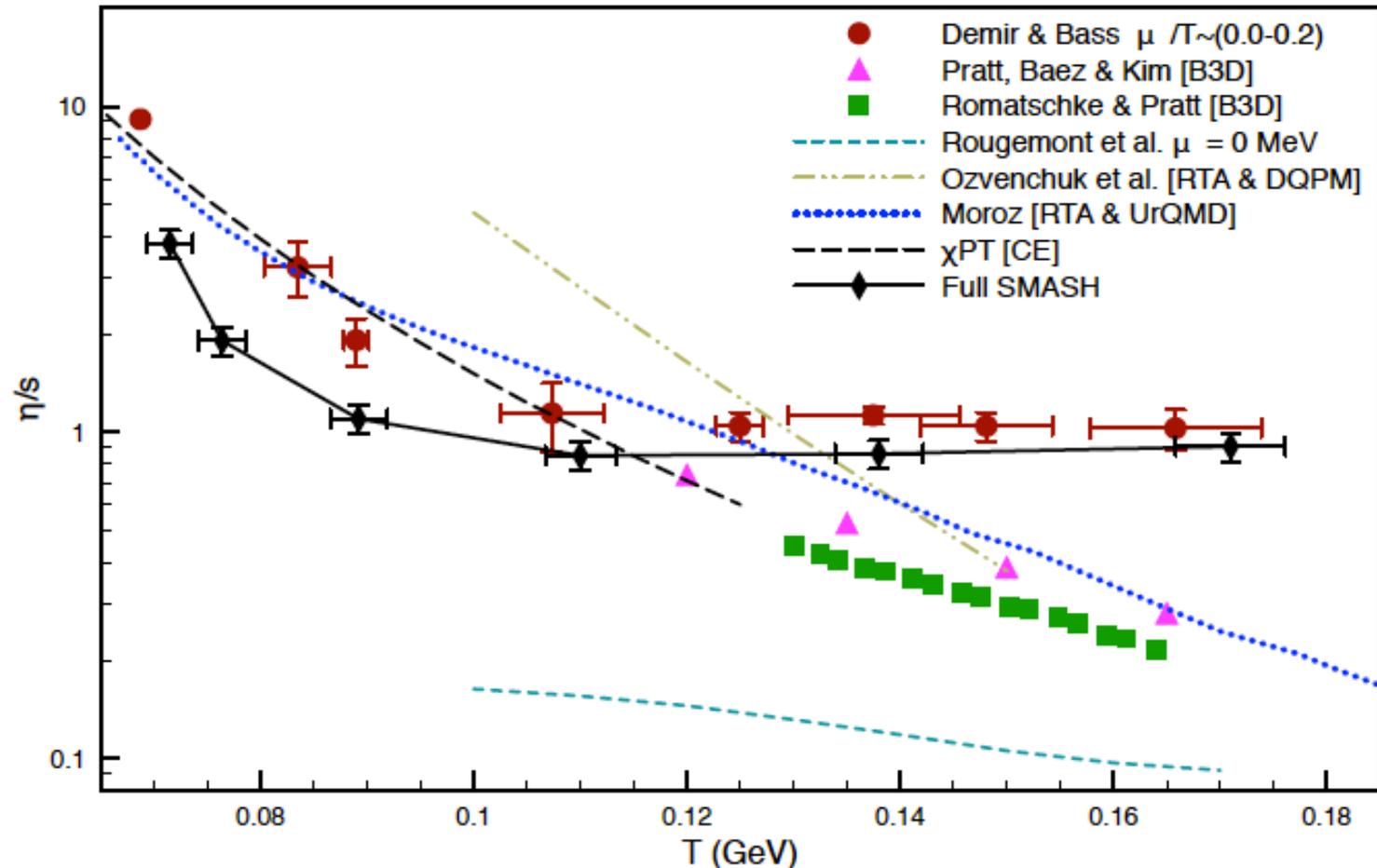
Motivation



courtesy of R. Lacey

- P.Kovtun, D.T.Son, O.Starinets. PRL 94, 111601 (2005)
- A.Muronga. PRC 69, 044901 (2004)
- L.Csernai, J.Kapusta, L.McLerran. PRL 97, 152303 (2006)
- P.Romatschke, U.Romatschke. PRL 99, 172301 (2007)
- S.Plumari et al. PRC 86, 054902 (2012)
- ALICE collaboration, CERN COURIER (14.10.2016)
- ...

Motivation



Comparison of several calculations for the hadron gas η/s at $\mu_B = 0$
[from *J. Rose et al. PRC97 (2018) 055204*]

Green-Kubo: shear viscosity η may be defined as:

$$\eta(t_0) = \frac{1}{\hbar} \frac{V}{T} \int_{t_0}^{\infty} dt \langle \pi(t) \pi(t_0) \rangle_t = \frac{\tau}{\hbar} \frac{V}{T} \langle \pi(t_0) \pi(t_0) \rangle,$$

where

$$\begin{aligned} \langle \pi(t) \pi(t_0) \rangle_t &= \frac{1}{3} \sum_{\substack{i,j=1 \\ i \neq j}}^3 \lim_{t_{\max} \rightarrow \infty} \frac{1}{t_{\max} - t_0} \int_{t_0}^{t_{\max}} dt' \pi^{ij}(t+t') \pi^{ij}(t') \\ &= \langle \pi(t_0) \pi(t_0) \rangle \exp\left(-\frac{t-t_0}{\tau}\right) \end{aligned}$$

with

$$\pi^{ij}(t) = \frac{1}{V} \sum_{\text{particles}} \frac{p^i(t) p^j(t)}{E(t)}$$

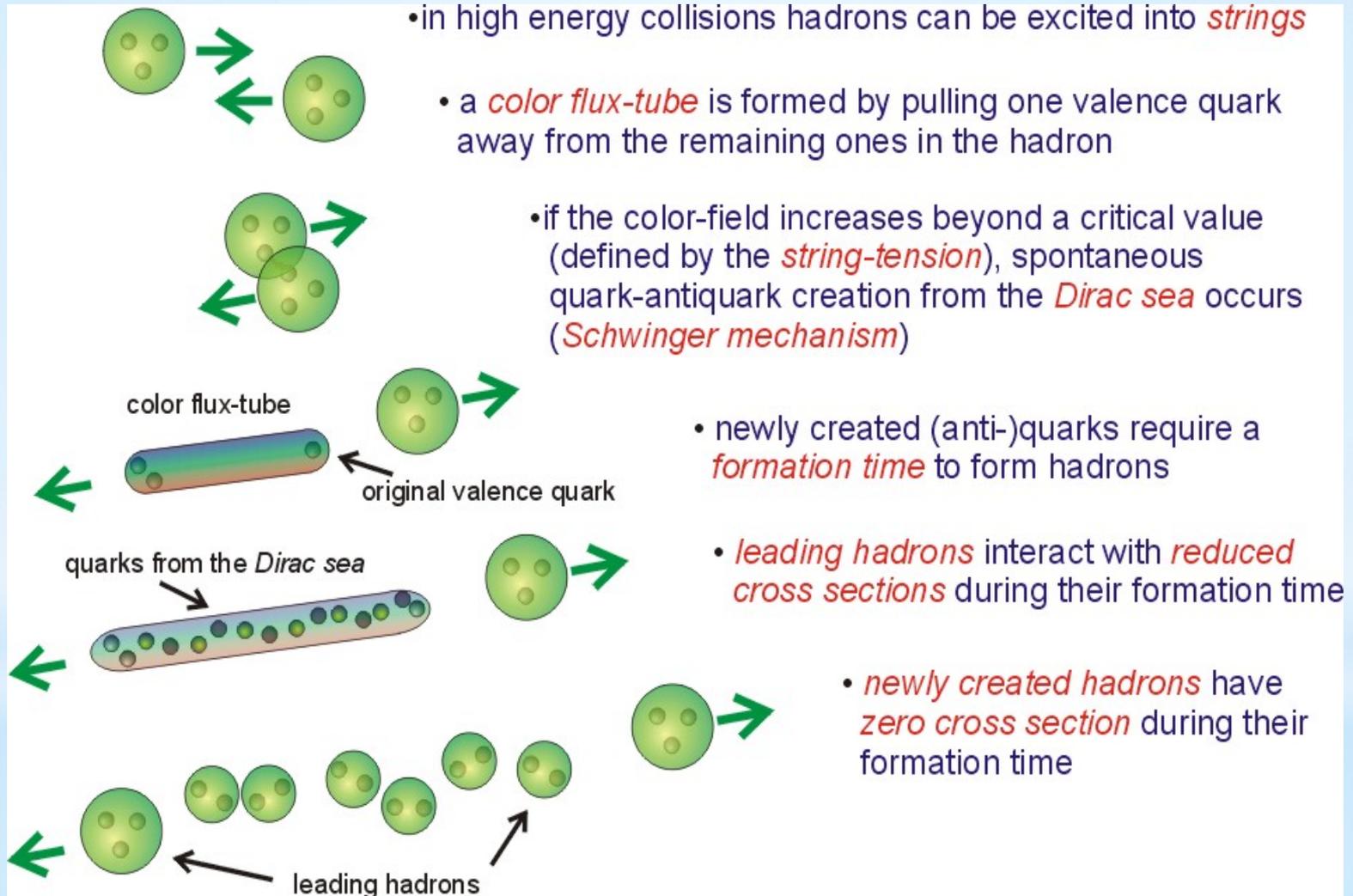
t_0 : initial cut-off time to start with

The UrQMD Model

- * elementary degrees of freedom: hadrons, const. (di)quarks
- * classical trajectories in phase-space (relativistic kinematics)
- * initial high energy phase of the reaction is modeled via the excitation and fragmentation of strings
- * 55 baryon- and 32 meson species, among those 25 N^* , Δ^* resonances and 29 hyperon/hyperon resonance species
- * full baryon-antibaryon and isospin symmetry
- ideal for the description of excited hadronic matter
- * main physics input and parameters:
 - * cross sections: total and partial cross sections, angular distributions
 - * resonance parameters: total and partial decay widths
 - * string fragmentation scheme: fragmentation functions, formation time
- * An interaction takes place if at the time of closes approach d_{min} of two hadrons the following condition is fulfilled:

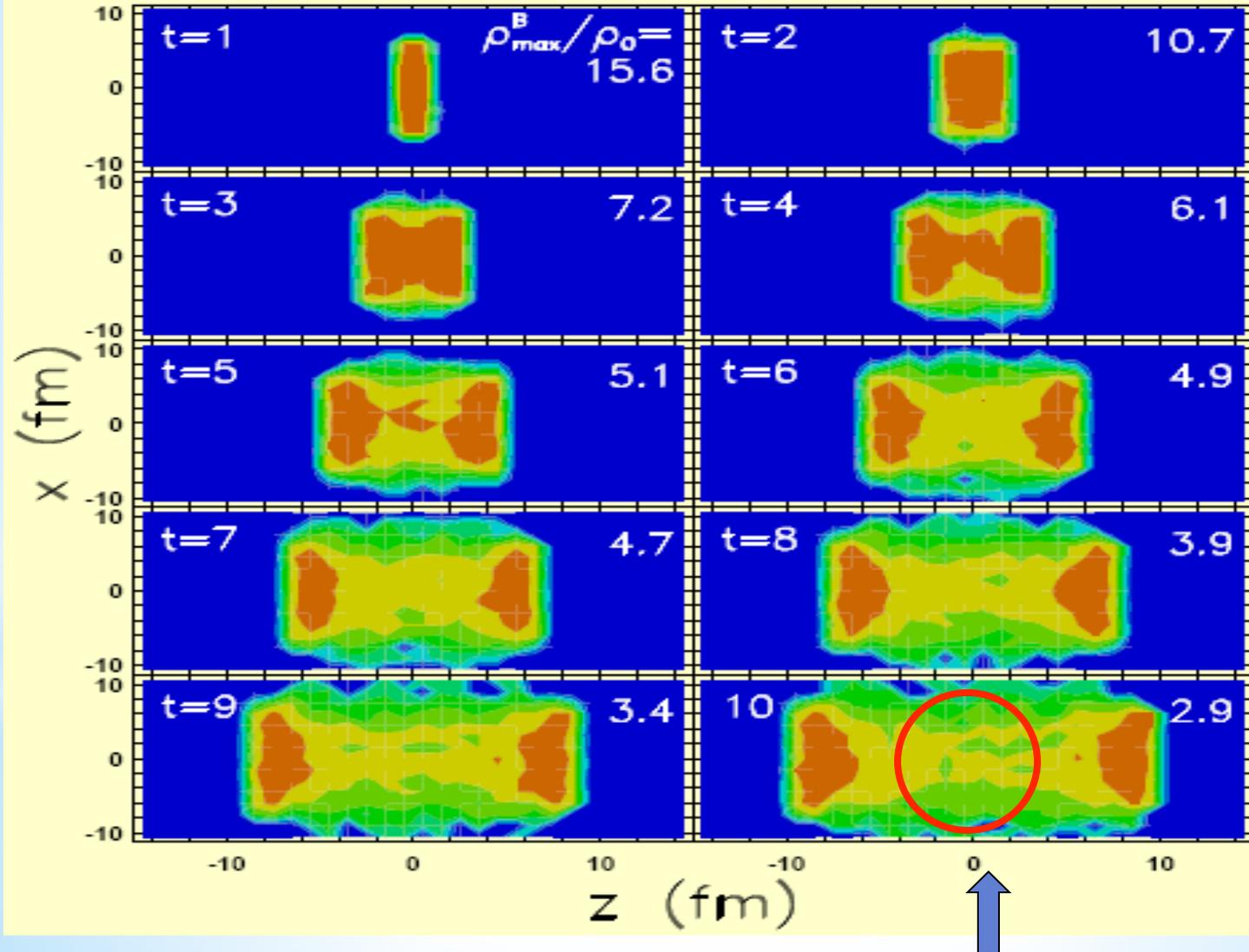
$$d_{min} = \sqrt{\frac{\sigma_{tot}}{\pi}} \quad \text{with} \quad \sigma_{tot} = \sigma_{tot}(\sqrt{s}, |h_1\rangle, |h_2\rangle)$$

Initial Particle Production in UrQMD



Pre-equilibrium: Homogeneity of baryon matter

L.Bravina et al., PRC 60 (1999) 024904



The local equilibrium in the central zone is quite possible

Model setup: cell calculations

- UrQMD calculations, central Au+Au collisions at energies $E \in [5, 10, 20, 30, 40]$ AGeV of the projectile, 51200 events per each
- central cell $5 \times 5 \times 5 \text{ fm}^3 \Rightarrow \{\varepsilon, \rho_B, \rho_S\}$ at times $t_{\text{cell}} = 1 \div 20 \text{ fm}/c$
- statistical model (SM): $\{\varepsilon, \rho_B, \rho_S\} \Rightarrow \{T, s, \mu_B, \mu_S\}$

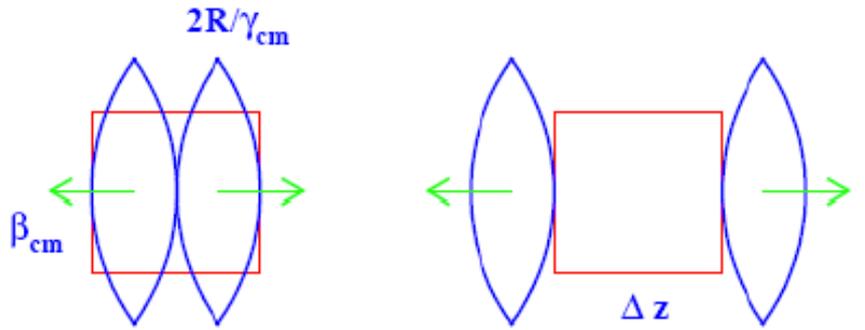
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Equilibration in the Central Cell



$$t^{\text{cross}} = 2R/(\gamma_{\text{cm}} \beta_{\text{cm}})$$

$$t^{\text{eq}} \geq t^{\text{cross}} + \Delta z/(2\beta_{\text{cm}})$$

Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

L.Bravina et al., PLB 434 (1998) 379;
JPG 25 (1999) 351

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Chemical equilibrium:

Particle yields are reproduced by SM with the same values of (T, μ_B, μ_S) :

$$N_i = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values

output values

$$\epsilon^{\text{mic}} = \frac{1}{V} \sum_i E_i^{\text{SM}}(T, \mu_B, \mu_S),$$

$$\rho_B^{\text{mic}} = \frac{1}{V} \sum_i B_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S),$$

$$\rho_S^{\text{mic}} = \frac{1}{V} \sum_i S_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S).$$

Multiplicity

Energy

Pressure

Entropy density

$$N_i^{\text{SM}} = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 f(p, m_i) dp,$$

$$E_i^{\text{SM}} = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \sqrt{p^2 + m_i^2} f(p, m_i) dp$$

$$P^{\text{SM}} = \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 \frac{p^2}{3(p^2 + m_i^2)^{1/2}} f(p, m_i) dp$$

$$s^{\text{SM}} = - \sum_i \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty f(p, m_i) [\ln f(p, m_i) - 1] p^2 dp$$

Model setup: box calculations

- UrQMD box calculations at $\{\varepsilon, \rho_B, \rho_S\}$ for every energy and cell time t_{cell} from cell calculations, 100 points in total, 12800 events per each

ρ_B is included as $N_p : N_n = 1 : 1$

ρ_S is included via kaons K^-

box size: $10 \times 10 \times 10 \text{ fm}^3$

box boundaries: transparent

- $\pi^{ij}(t)$ data extraction: $t = 1 \div 1000 \text{ fm}/c$ in box time, all types of hadrons are taken into account

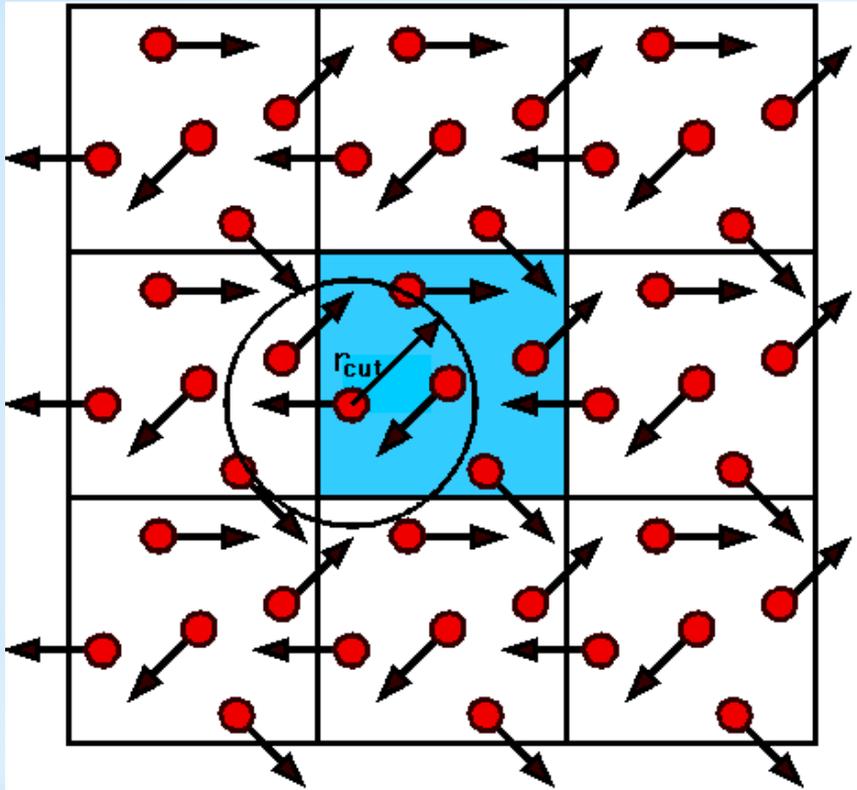
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Box with periodic boundary conditions



M.Belkacem et al., PRC 58, 1727 (1998)

Model employed: UrQMD
55 different baryon species
(N, Δ , hyperons and their resonances with $m \leq 2.25 \text{ GeV}/c^2$)

32 different meson species
(including resonances with $m \leq 2 \text{ GeV}/c^2$) and their respective antistates.

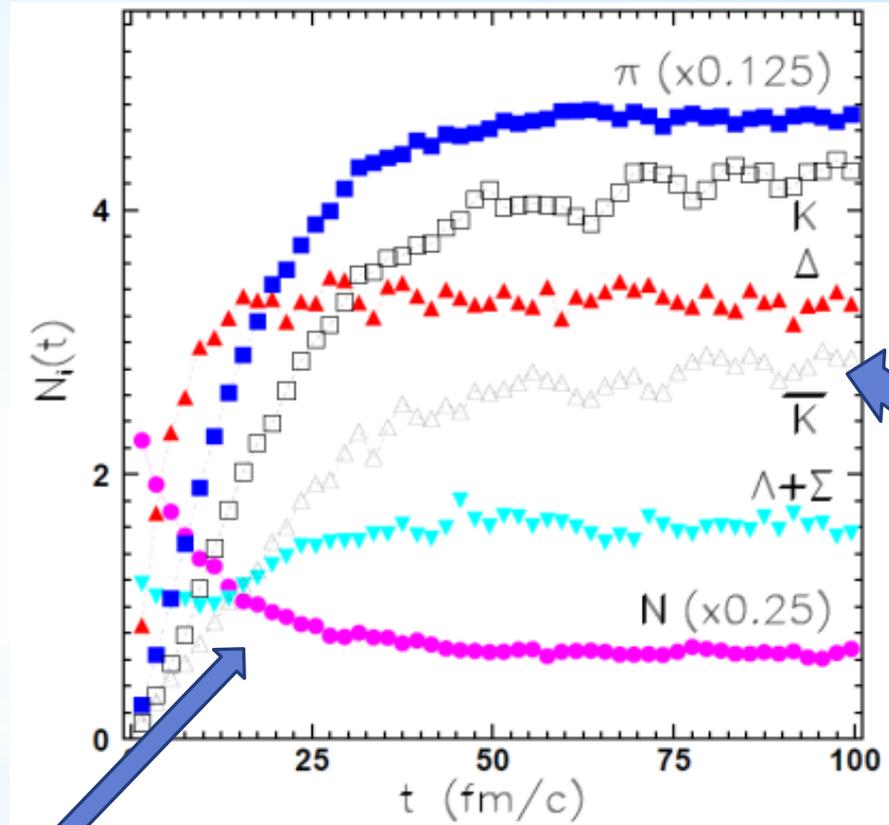
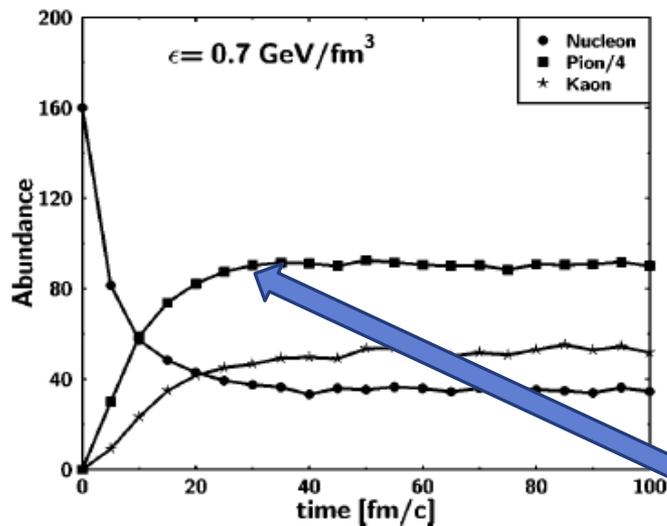
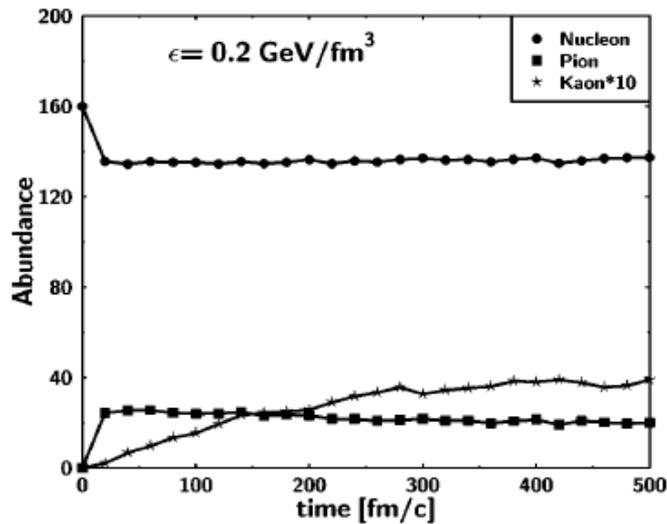
For higher mass excitations a string mechanism is invoked.

Initialization: (i) nucleons are uniformly distributed in a configuration space;
(ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra

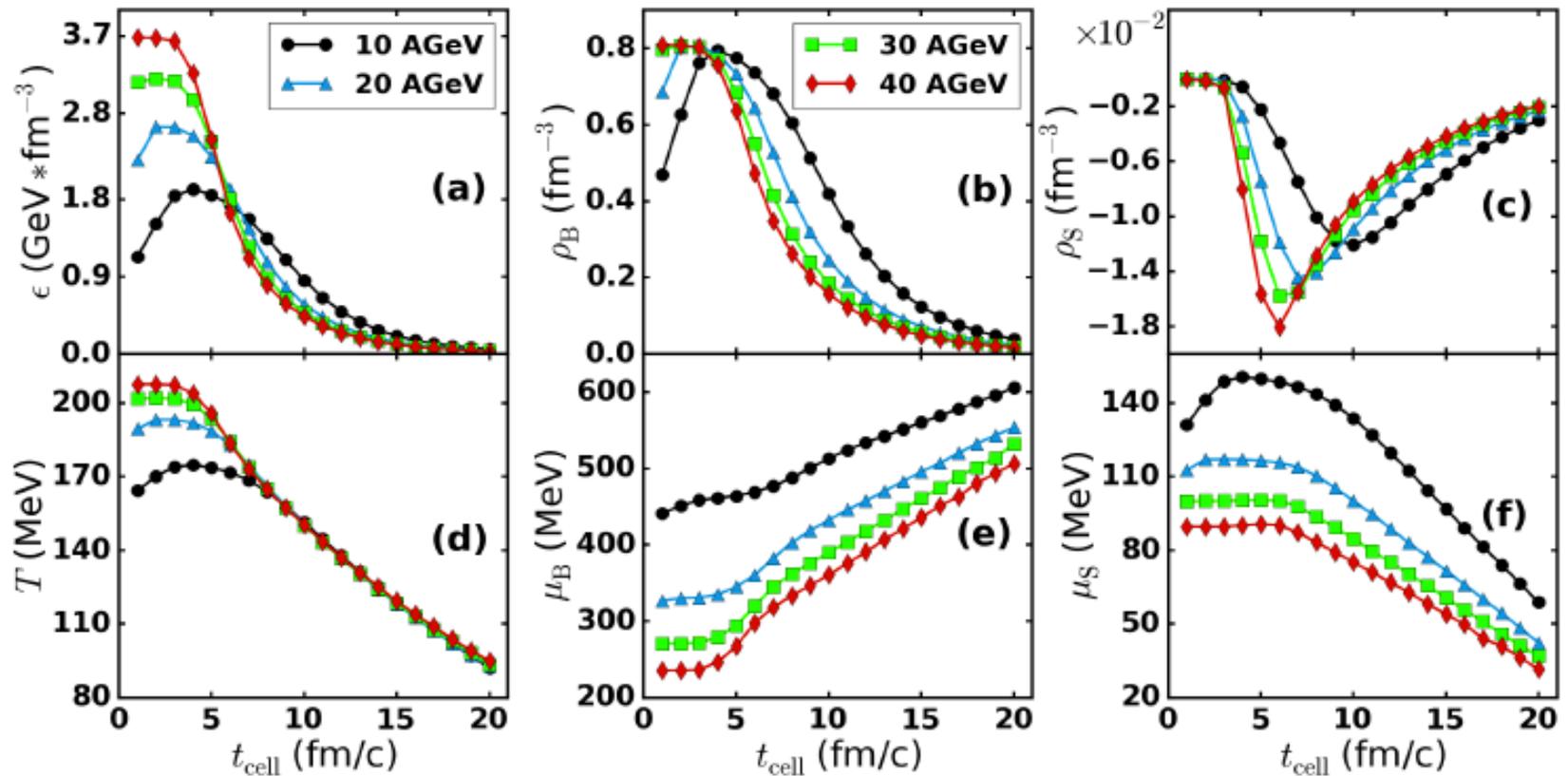
Box: particle abundances

M. Belkacem et al., PRC 58, 1727 (1998)



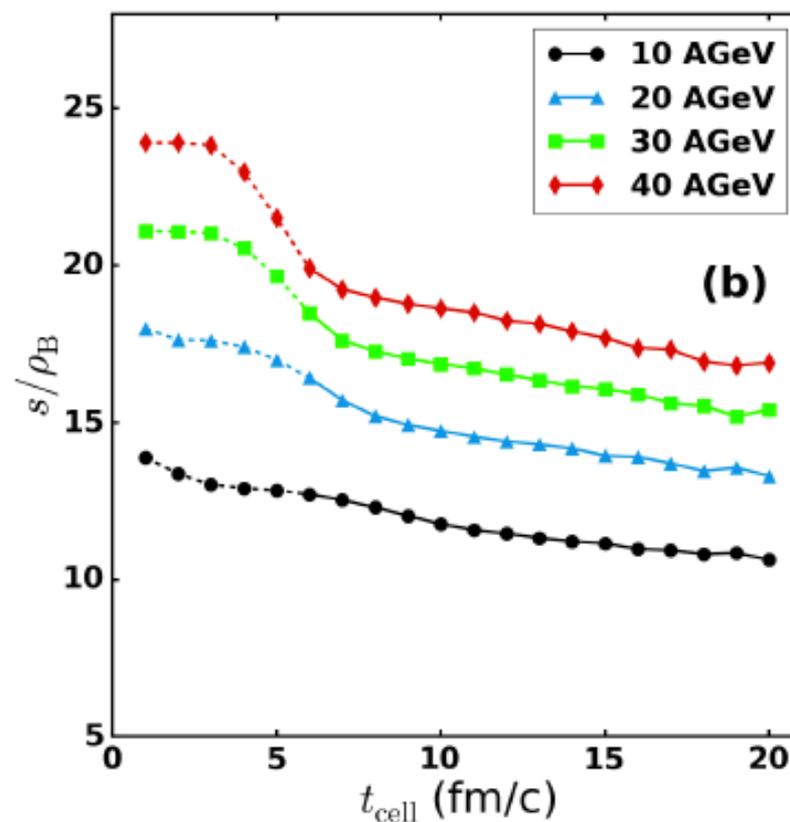
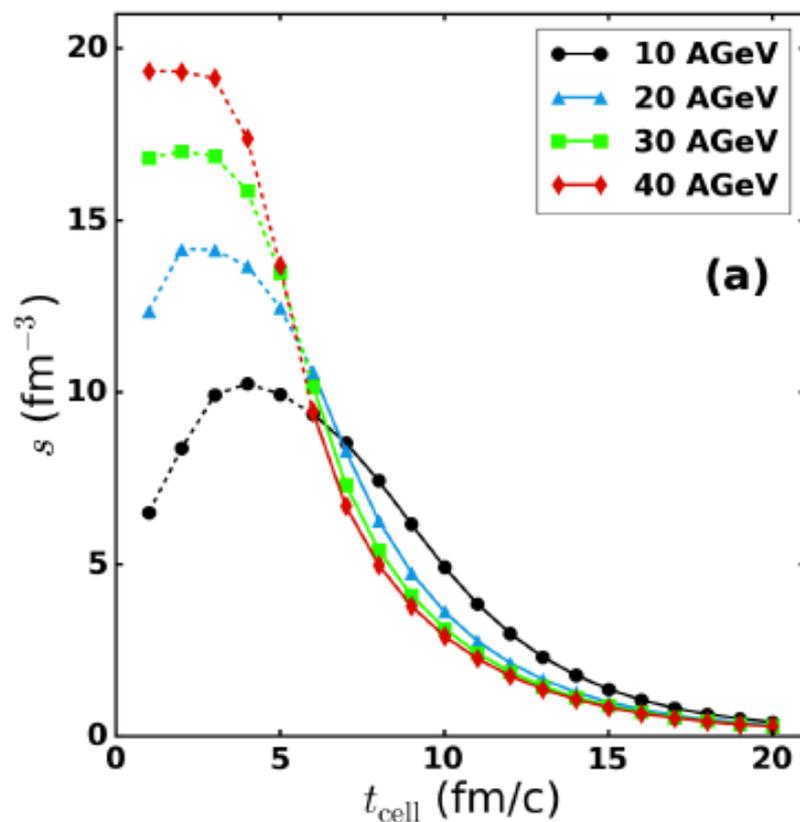
L. Bravina et al., PRC 62, 064906 (2000)

Saturation of yields after a certain time. Strange hadrons are saturated longer than others (at not very high energy densities)



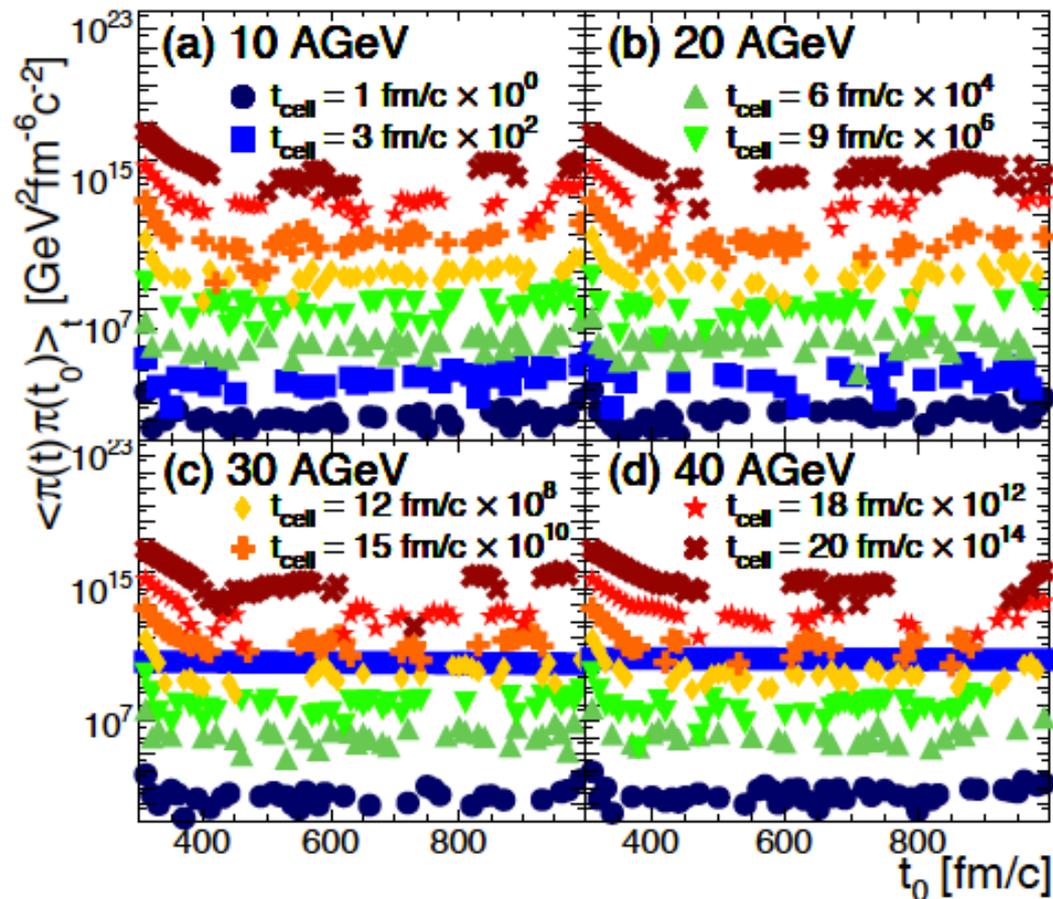
Dependence of ϵ, ρ_B, ρ_S (from cell) and of T, μ_B, μ_S (from SM) on t_{cell}

SM, Boltzmann entropy s



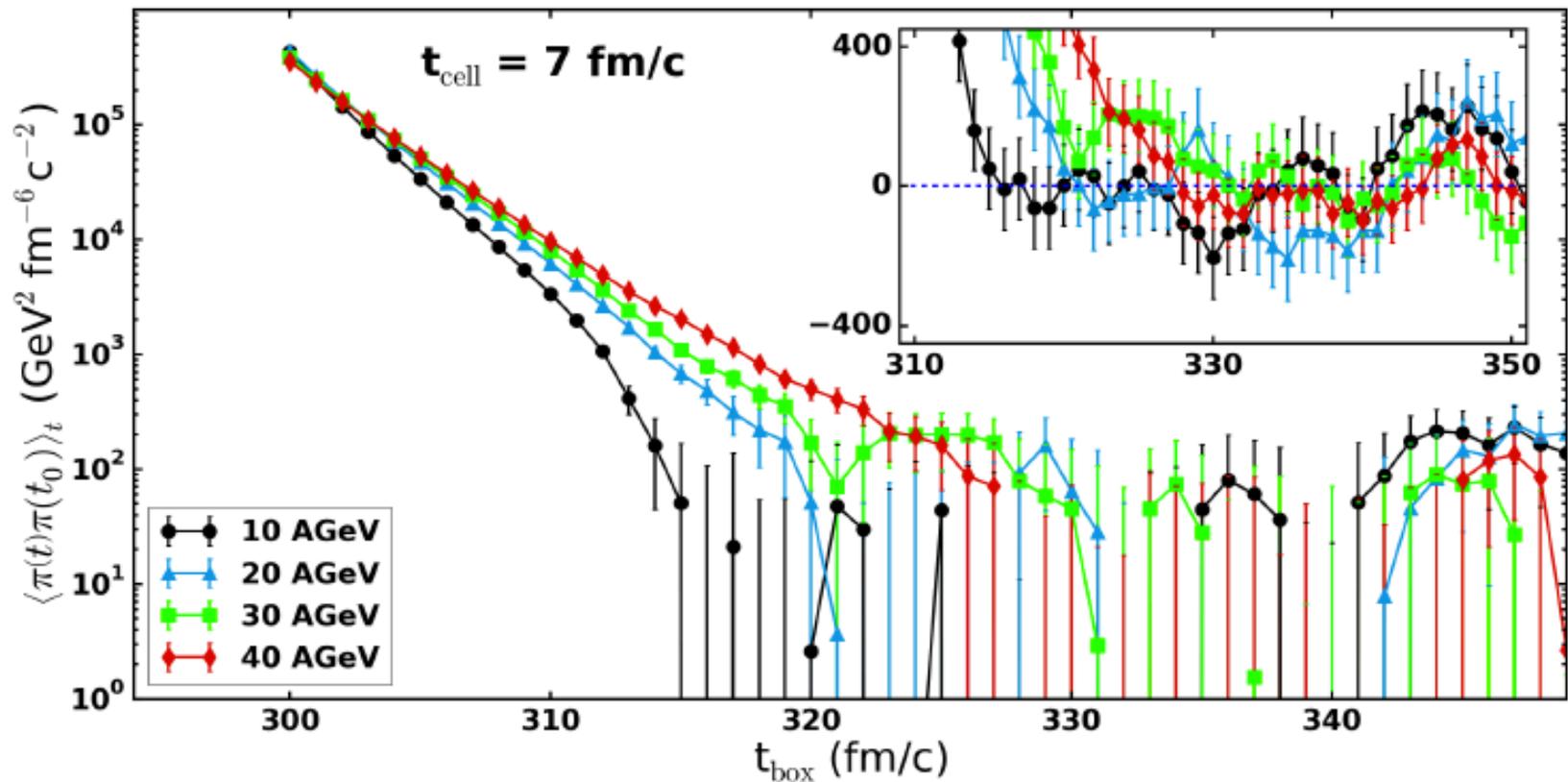
Dynamics of Boltzmann entropy density s and of s/ρ_B in cell

Results: $\langle \pi(t) \pi(t_0) \rangle_t$ at $E \in [10, 20, 30, 40]$ AGeV



Time dependence of correlators $\langle \pi(t) \pi(t_0) \rangle_t$
 $t_0 = 300 \text{ fm/c}$; $t_{\text{cell}} \in \{1 \div 20\} \text{ fm/c}$

Results: $\langle \pi(t) \pi(t_0) \rangle_t$ at fixed t_{cell}



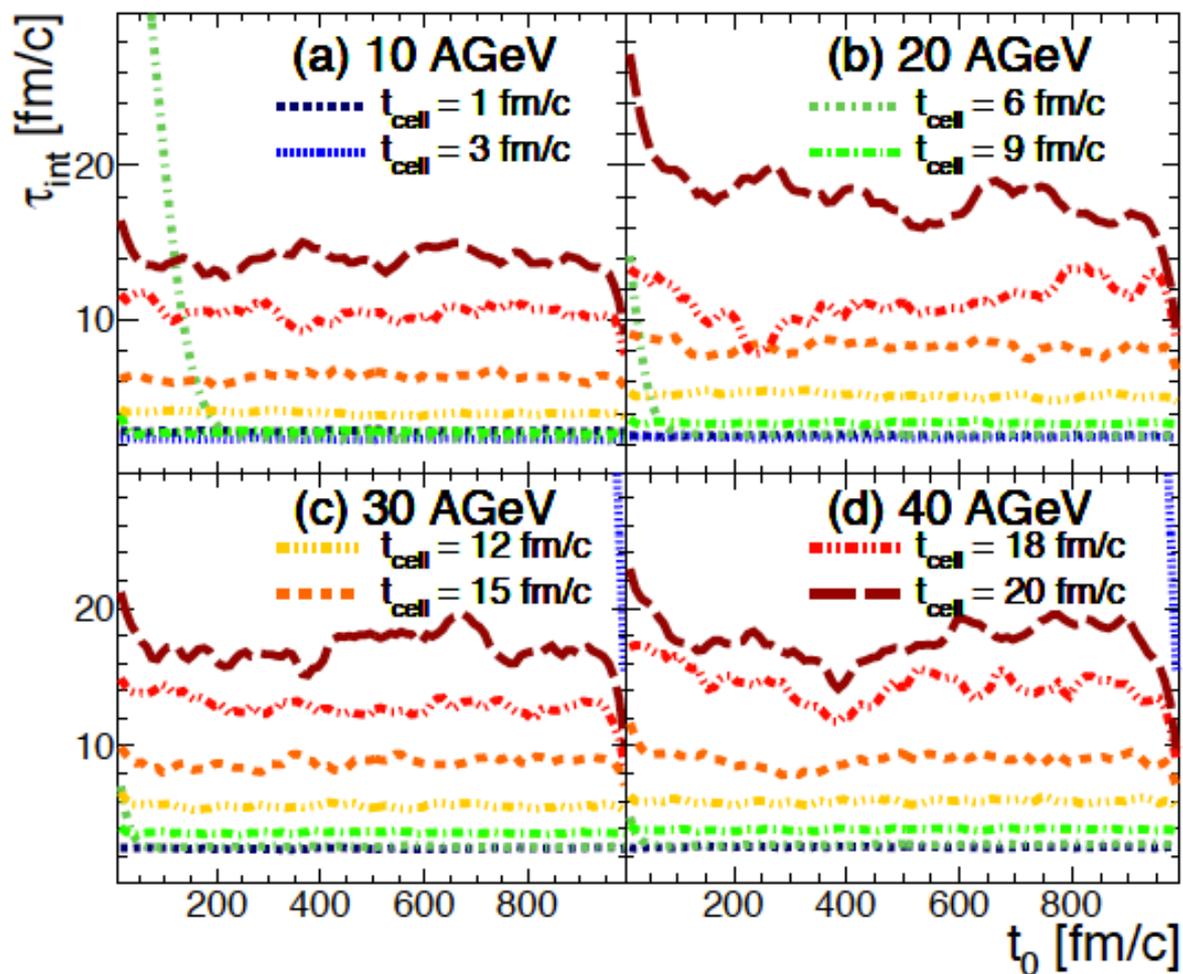
Time dependence of correlators $\langle \pi(t) \pi(t_0) \rangle_t$

Subplot: the same but at linear scale

$t_0 = 300 \text{ fm/c}$

$t_{\text{cell}} = 7 \text{ fm/c}$

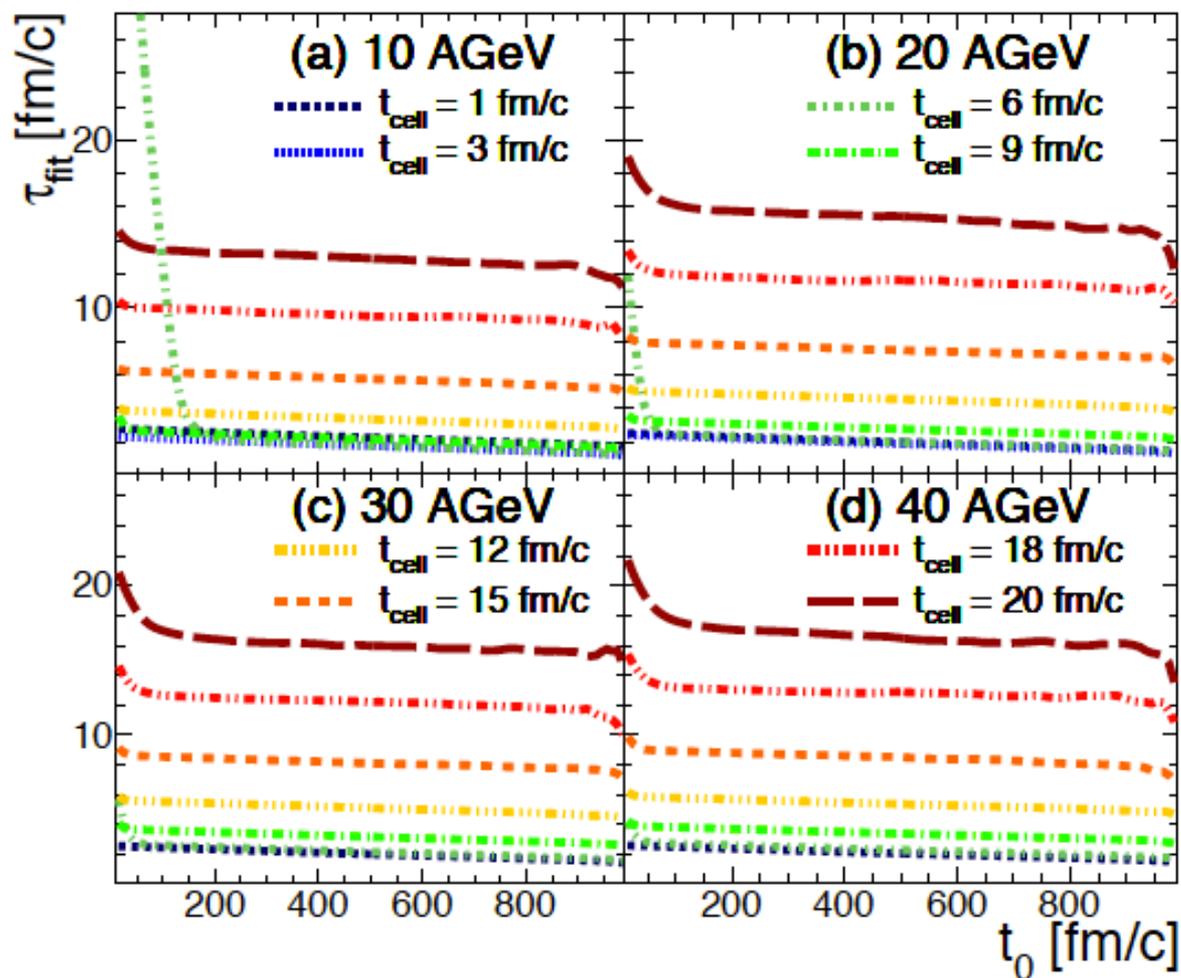
Results: $\tau(t_0)$



Dependence of τ_{int} on t_0

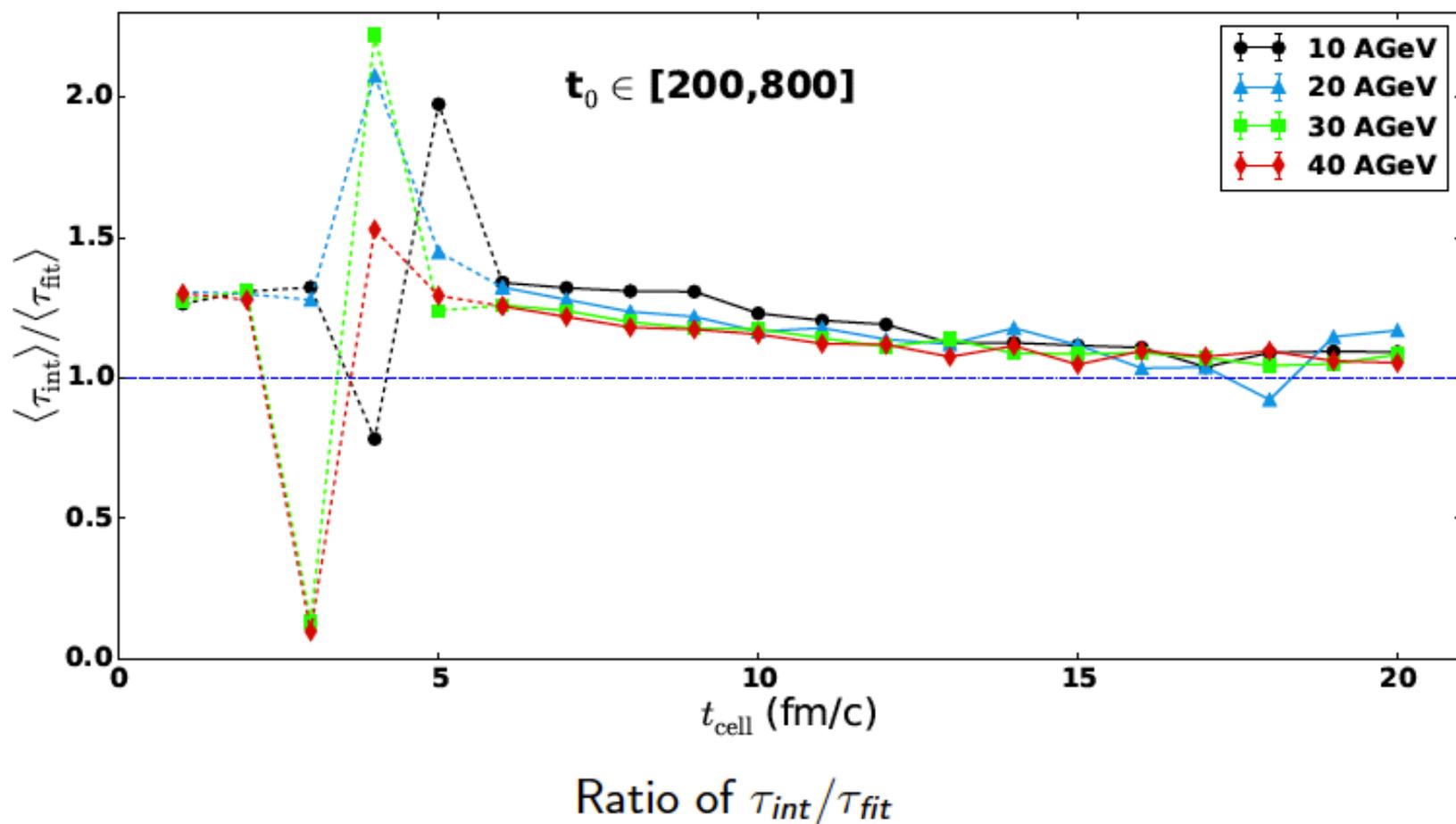


Results: τ from the fit

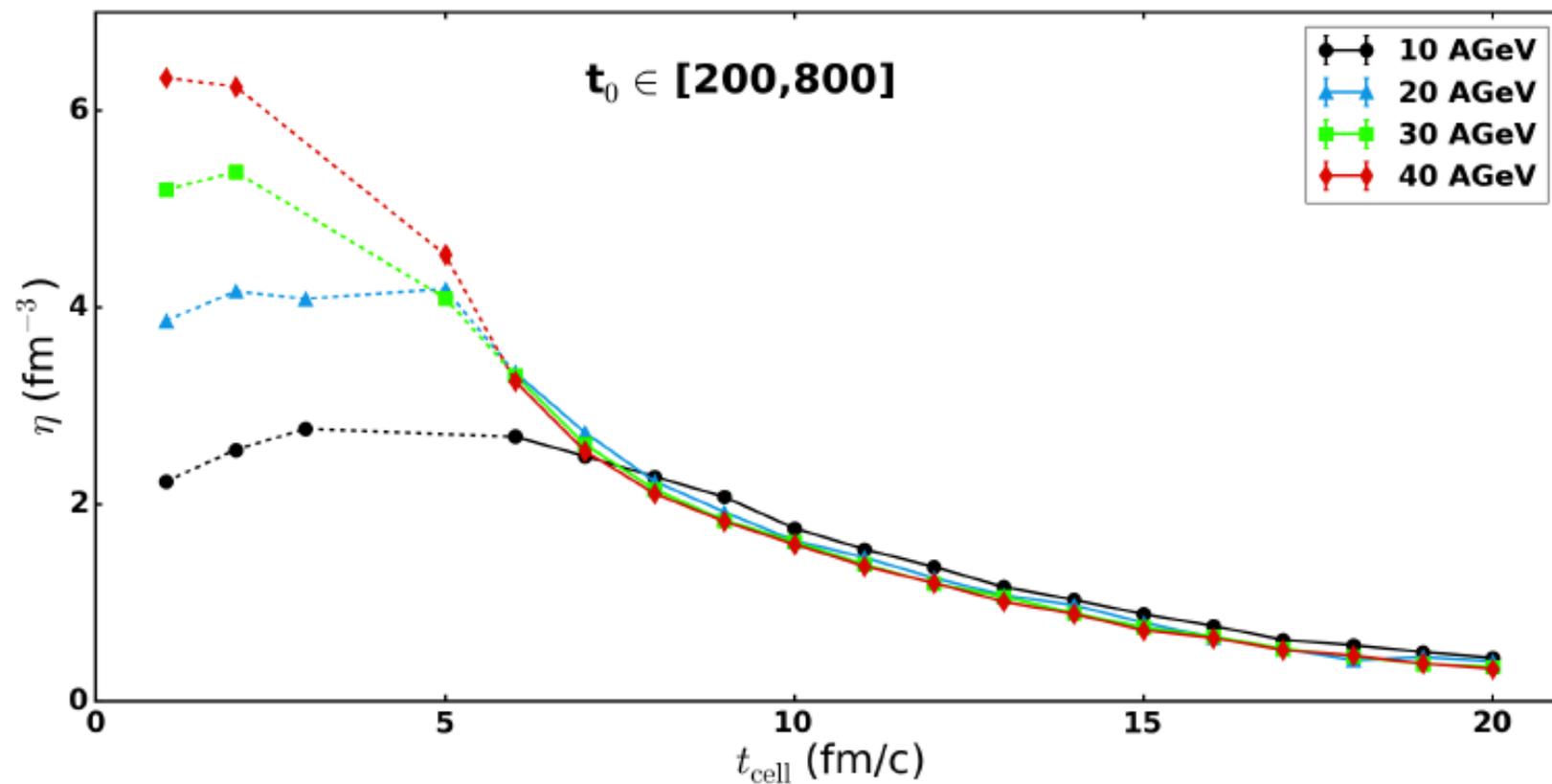


Dependence of τ_{fit} on t_0

Results: Comparison of τ_{int} and τ_{fit}



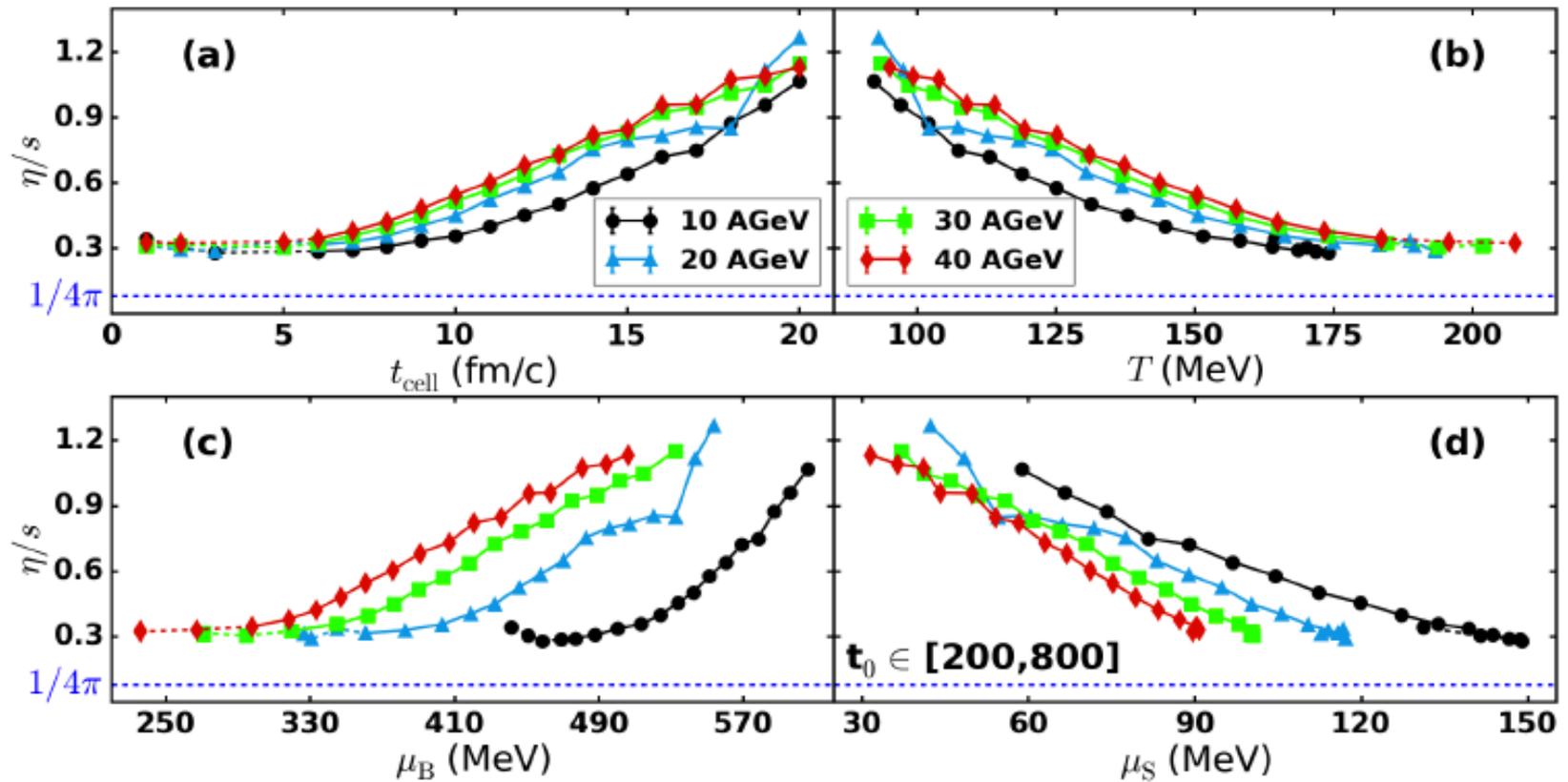
Results: viscosity $\eta(t_{\text{cell}})$



Dynamics of η in cell

All curves sit on the top of each other for $t_{\text{cell}} \geq 7$ fm/c

Results: η/s



Dynamics of η/s in cell
as function of time, T , μ_B , μ_S

Entropy density of nonequilibrium state

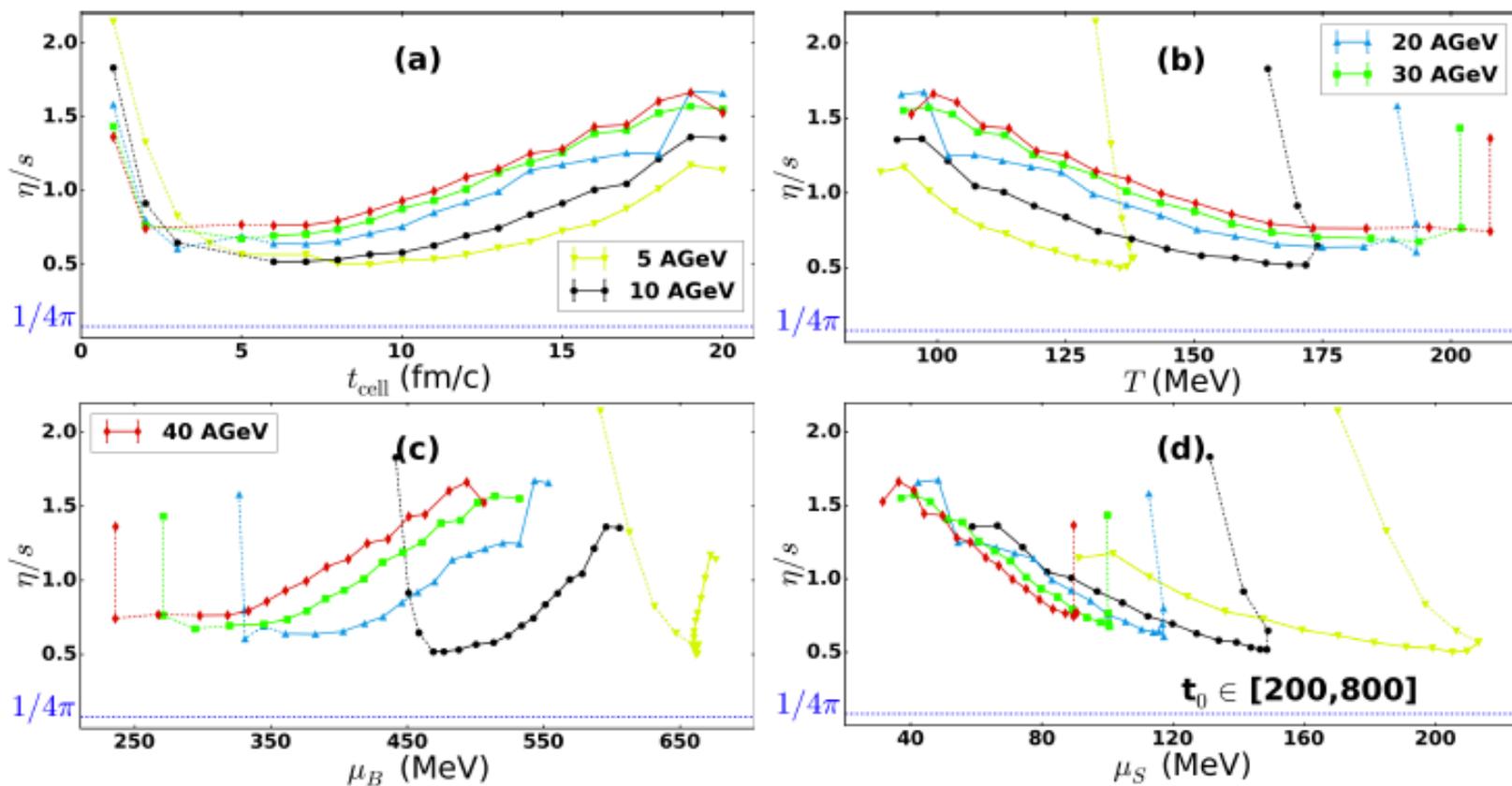
Entropy density s is defined as:

$$s = \sum_i \frac{g_i}{(2\pi\hbar)^3} \int_0^\infty f_i(p, m_i) [\ln f_i(p, m_i) - 1] d^3p$$

where microscopic distribution function reads

$$f_i^{mic}(p) = \frac{(2\pi\hbar)^3}{Vg_i} \frac{dN_i}{d^3p}$$

Results: $\eta/s_{noneq.}$

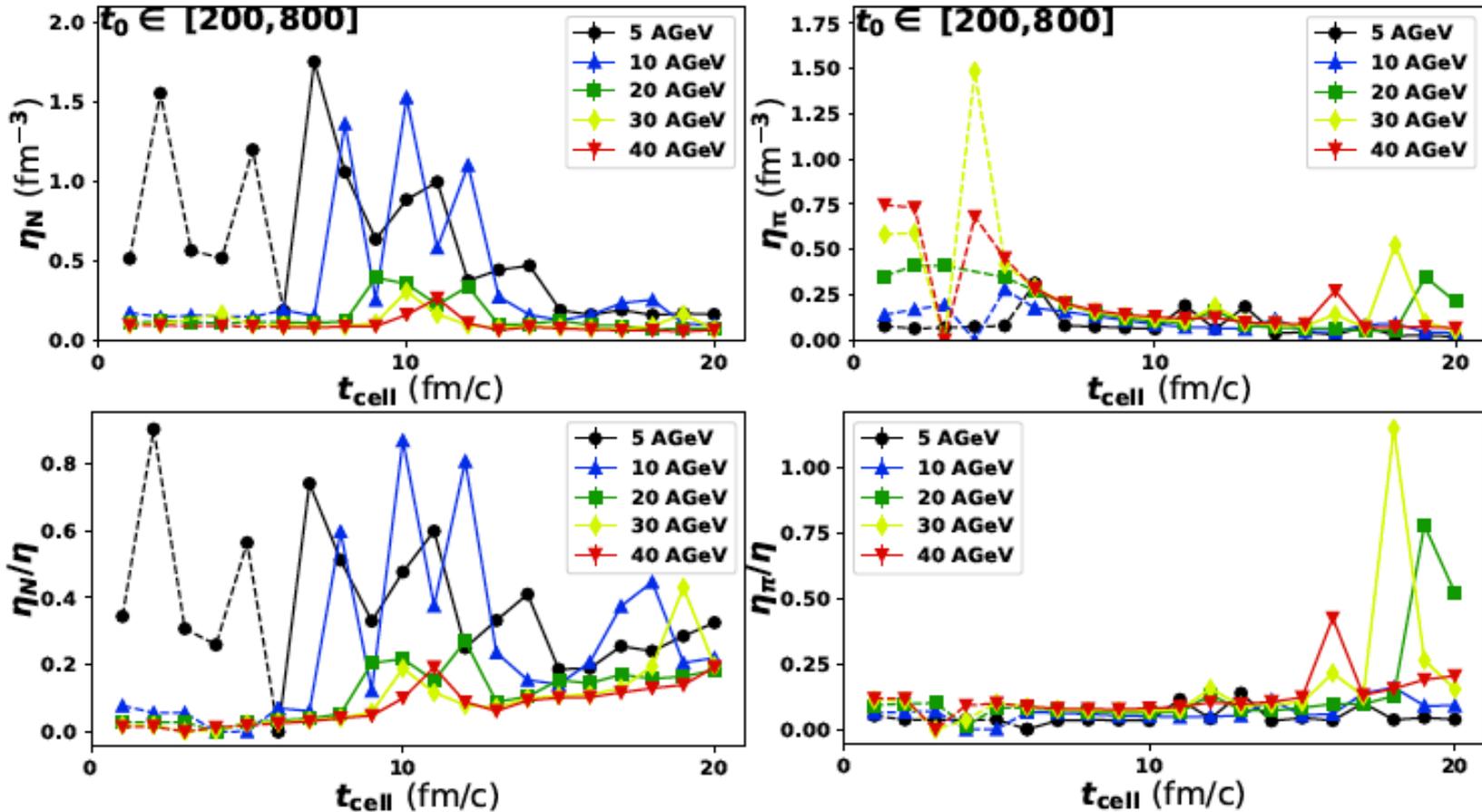


Dynamics of $\eta/s_{noneq.}$ in cell

η/s drops with time for $t_{cell} \leq 6$ fm/c. Then it increases for all five energies

Pronounced minima for all reactions

Results: η of nucleons and pions



Partial η from $N - N$ and $\pi - \pi$ correlators

What makes the main contribution to shear viscosity?

Conclusions

- data from central cell of UrQMD calculations are used as input for SM to calculate temperature T and entropy density s , and for UrQMD box calculations to estimate shear viscosity η
- box data are taken within the range $200 \leq t_0 \leq 800$ fm/c because:
 - values at $t_0 < 200$ fm/c are distorted by the initial fluctuation in the box
 - values at $t_0 > 800$ fm/c may be disturbed by the analog of Brownian motion
- it is shown that for all tested energies η and s in the cell drop with time
- ratios η/s reach minima about 0.3(0.5) at $t \approx 5$ fm/c for all energies. Then, the ratios rise to $1.0 \div 1.2$ ($1.3 \div 1.6$) at $t = 20$ fm/c
- partial contributions of nucleons and pions to shear viscosity seem to be low. πN ? Resonances?