Gluons, Heavy and Light Quarks in the Instanton Vacuum

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Outline

QCD vacuum

- Instanton $=$ tunneling path between the C-S states $\Rightarrow$
- Instanton Liquid Model (ILM) $\sim$ collective coordinates ξ, $DA \rightarrow n(\rho)D\xi$ (Shuryak1981, Diakonov-Petrov1983).
- KvBLL instantons ($T \neq 0$) $\sim$ dyons can describe large instantons $\Rightarrow$ Liquid Dyon Model (LDM)(Diakonov2009, Shuryak et al 2015) $\Rightarrow$ confinement–deconfinement.
- Small size instantons still in terms of collective coordinates.

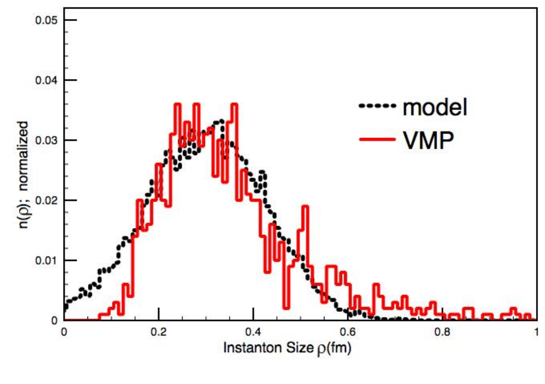
At the typical values of the parameters ILM vacuum energy density $\epsilon \approx -500\,\mathrm{MeV/fm^3}$.

ILM and its main parameters

- Instanton sum ansatz $A = \sum_{\pm} A_{\pm}$, $N_+ = N_- = N/2$, instanton collective coordinates ξ:
  $4\ (position) + 1\ (size) + (4N_c - 5)\ (orientations)\ =\ 4N_c$
- Two main parameters: instanton size $\rho \approx$ average size $\bar{\rho}$ and inter-instanton distance $R = (V/N)^{1/4}$;
- Estimates:
  - $R \approx 0.89\ fm$, $\rho \approx 0.36\ fm$ – lattice;
  - $R \approx 1.00\ fm$, $\rho \approx 0.33\ fm$ – phenomenological;
  - $R \approx 0.76\ fm$, $\rho \approx 0.32\ fm$ – our estimate with account of $1/N_c$ corrections correspond ChPT (Goeke et al 2007).

  Thus within $10 - 15\%$ uncertainty different approaches give similar estimates.
- *Packing parameter* $\lambda = (\rho/R)^4 \sim 0.01 - 0.03$
  $\Rightarrow$ Independent averaging over instanton positions and orientations.

Instanton vs hadron sizes.ILM & DLM.



Instanton size $n(\rho)$ – lattice vs ILM (Millo,Faccioli2011).
ILM $\bar{\rho} \sim 0.3\,\mathrm{fm}$(Shuryak1981, Diakonov-Petrov1983).
DLM $\bar{\rho} \sim 0.45 - 0.5\,\mathrm{fm}$ (Diakonov2009, Shuryak et al 2015).

Instanton vs hadron sizes.

Quarkonium states and its sizes in non-relativistic potential model (see Satz2012).

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ'_b	Υ''
mass [Gev]	3.07	3.53	3.68	9.46	9.99	10.02	10.26	10.36
size r [fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

- $r_{J/\psi} = 0.25\ fm, \ \ r_\Upsilon = 0.14\ fm.$
- A model estimates of nucleon quark core size $r_N \sim 0.3 - 0.5$ fm (see Weise1985).
- Small quark core size hadrons are insensitive to the confinement, we may safely apply ILM.

Light quarks in ILM.

Zero modes $(\hat{p} + g\hat{A}_\pm)\Phi_{\pm,0}(x,\xi_\pm) = 0$ dominance $\Rightarrow$ light quarks partition function $Z[\eta^+,\eta] =$

$= \int D\xi \, \mathrm{Det}_{low}(\hat{p} + g\hat{A} + im) \exp\left(-\eta^+(\hat{p} + g\hat{A} + im)^{-1}\eta\right) =$

$$= \int D\xi \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger(\hat{p} + im_f)\psi_f + \psi_f^\dagger\eta_f + \eta_f^+\psi_f\right)$$

$$\times \prod_f \left\{\prod_+^{N_+} V_{+,f}[\psi^\dagger,\psi] \prod_-^{N_-} V_{-,f}[\psi^\dagger,\psi]\right\}, \;\; V_{\pm,f}[\psi^\dagger,\psi] =$$

$$= i \int dx \left(\psi_f^\dagger(x)\,\hat{p}\Phi_{\pm,0}(x;\xi_\pm)\right) \int dy \left(\Phi_{\pm,0}^\dagger(y;\xi_\pm)(\hat{p}\,\psi_f(y)\right).$$

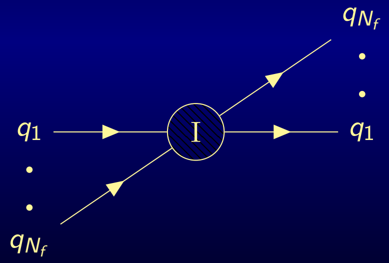
$\psi^\dagger, \psi$ are constituent quarks.

Instanton generated light quarks interactions in ILM.

Small packing parameter $\lambda \approx 0.01 \Rightarrow$ independent averaging:

$$< V_\pm[\psi^\dagger, \psi] >= \int d\xi_\pm \prod_f V_{\pm,f}[\psi^\dagger, \psi]$$

$\Rightarrow$ non-local ($\sim \rho$) t'Hooft-like vertex with $2N_f$-legs:

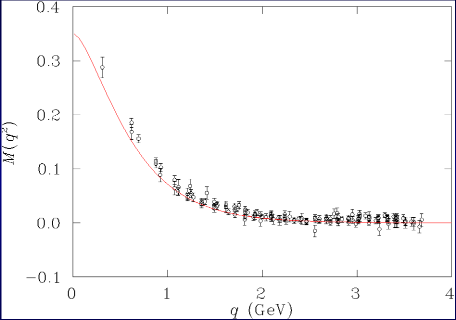


The coupling κ is defined by saddle-point Eqs.

$$N = \mathrm{Tr}\frac{iM(p)}{\hat{p} + i(m + M(p))}, \quad M(p) = \frac{\kappa}{N_c}(2\pi\rho F(p))^2$$

Spontaneous Breaking of the Chiral Symmetry.

Z in saddle-point approximation (leading order on $1/N_c$) $\rightarrow$ SBCS $\rightarrow$ Dynamical quark mass $M(q)$: ILM at $\rho = 0.33\,fm$, $R = 1\,fm$ vs lattice (Bowman2005).



$M(0) \approx 360$ *MeV*$\sim \lambda^{1/2}\rho^{-1} \sim$ *strength of light quark-instanton interaction, which is large!!* $\Rightarrow$ at $1/N_c$ leading order successful reproducing of quark condensate, pion and nucleon properties etc!! (see e.g. Schafer,Shuryak1996, Diakonov2002).

Next to leading order $(m, 1/N_c)$ corrections. Successful reproducing of ChPT (Goeke et al 2007).

Scalar "gluons" in ILM.

No zero modes $\Rightarrow$ existence of the propagators
$\Delta = (p^2 + \sum_i (\{p, A_i\} + A_i^2) + \sum_{i \neq j} A_i A_j)^{-1}$,
$\Delta_i = (p^2 + \{p, A_i\} + A_i^2)^{-1}$, $\Delta_0 = p^{-2}$.
Define: $\tilde{\Delta} = (p^2 + \sum_i (\{p, A_i\} + A_i^2))^{-1}$.
The propagator in ILM is $< \Delta >= \int D\xi \, \Delta$, $< \tilde{\Delta} >= \int D\xi \, \tilde{\Delta}$.
Multi-scattering expansion

$$\tilde{\Delta} - p^{-2} = \sum_I (\Delta_I - p^{-2}) + \sum_{I \neq J} (\Delta_I - p^{-2}) p^2 (\Delta_J - p^{-2})$$

$$+ \sum_{I \neq J, J \neq K} (\Delta_I - p^{-2}) p^2 (\Delta_J - p^{-2}) p^2 (\Delta_K - p^{-2}) + \ldots .$$

Averaging of this one – essential point here: instantons I, J, K are different or whether some of them coincide.

Main tool – the extension of Pobylitsa equation (Pobylitsa89).

The extension of Pobylitsa Eq. for scalar "gluons"

Follow Pobylitsa89, Diakonov,Petrov,Pobylitsa89.
N_c counting – density of instantons $N/V \sim N_c$, averaging over instanton color orientation $\sim 1/N_c$.

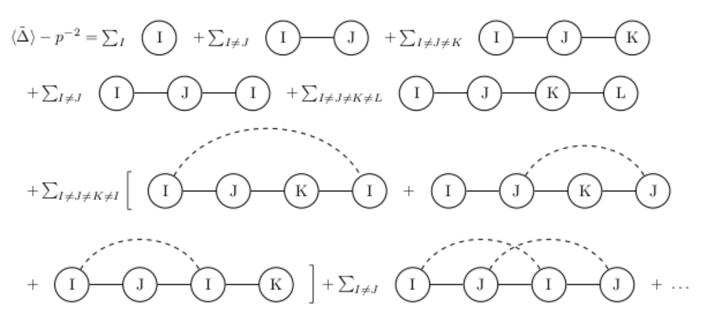


Fig.1. A circle with I inside is $(\Delta_{\rm I} - p^{-2})$, solid line is p^2. Dashed lines connect the circles with the same instanton. At a large N_c the planar graphs dominate: neglect by crossed dashed lines. Here last one is a non-planar graph, which is negligible.

The extension of Pobylitsa Eq. for scalar "gluons"

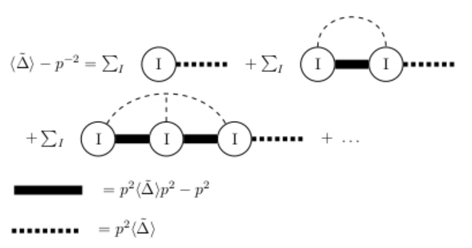


Fig.2 represent summed-up planar graphs given as a skeleton expansion for the operator $\tilde{\Delta} - p^{-2} \Rightarrow$ the extension of Pobylitsa Eq. for scalar "gluons" in ILM as

$$< \tilde{\Delta} >^{-1} - p^2 = \sum_i < \left[< \tilde{\Delta} > + (\Delta_i^{-1} - p^2)^{-1} \right]^{-1} > .$$

The solution at $O(\lambda)$ $< \Delta > = < \tilde{\Delta} >$ and

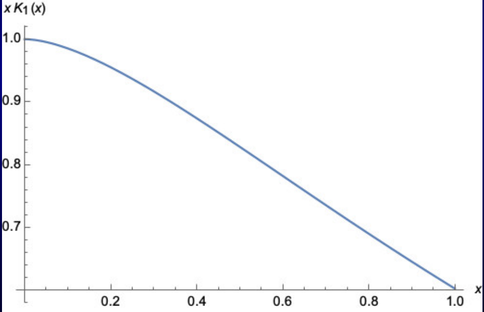
$$< \Delta >^{-1} - p^2 = N\, p^2 (< \Delta_{\mathrm{I}} > - p^{-2}) p^2 ,$$

Scalar "gluon" dynamical mass in ILM.

Well-known result for the $\Delta_{\rm I}$ (Brown et al 1978) $\Rightarrow$

$$M_s(q) = \left[\frac{12\pi^2\lambda}{(N_c^2-1)}\right]^{1/2} \rho^{-1}(q\rho K_1(q\rho))$$

where the form-factor $q\rho K_1(q\rho)$



At $\rho = 0.33\,\mathrm{fm}$, $R = 1\,\mathrm{fm}$

$M_s(0) = 256\ MeV \sim \lambda^{1/2}\rho^{-1}$ *is a strength of scalar "gluon"-instanton interaction.*

Gluons in ILM. Zero-modes problem.

Gluons effective action in single instanton field is
$(a_\mu M^{\rm I}_{\mu\nu} a_\nu) + O(a^3, a^4)$,
$4N_c$ zero-modes $M^{\rm I}_{\mu\nu}\phi^i_\nu = 0$
The single instanton gluon propagator $S^{\rm I}_{\mu\nu}$ is
$M^{\rm I}_{\mu\nu} S^{\rm I}_{\nu\rho} = \delta_{\mu\nu} - P^{\rm I}_{\mu\nu}$
$P^{\rm I}_{\mu\nu}$ – zero-modes projection operator.
The explicit solution $S^{\rm I}_{\mu\nu}$ was given by (Brown et al 1978).
Zero-modes problem was solved by (Brown1978) introducing artificial gluon mass m_g.
Repeating the way to Pobylitsa Eq. for ILM "scalar" gluon propagator $< \Delta >$ we have at $m_g \to 0$ limit

$$M_g^2 \delta_{\rho\nu} = N {S^0}^{-1}_{\rho\sigma} (< S^{\rm I}_{\sigma\mu} > - {S^0}_{\sigma\mu}) {S^0}^{-1}_{\mu\nu}$$

Dynamical gluon mass in ILM.

From well-known $S^{\mathrm{I}}_{\sigma\mu}$ we conclude

$$M_g^2(q) = 2M_s^2(q).$$

At $\rho = 0.33\,\mathrm{fm}, R = 1\,\mathrm{fm}$

$M_g(0) = 362\,MeV \approx M(0) \sim \lambda^{1/2}\rho^{-1}$ *strength of gluon-instanton interaction.*

It is essentially modify $Q\bar{Q}$ one-gluon exchange potential at the distances $r \sim M_g^{-1} \sim 0.5\,fm$.

Perturbative corrections in ILM

ILM partition function $Z[j]$ ($Z[0]=1$) with account of perturbative gluons a_μ and their sources j_μ is approximated by

$$Z[j]=\int D\xi Da e^{-[S_{eff}[a,A(\xi)]+(ja)]}\approx\int D\xi e^{-\frac{1}{2}(j_\mu S_{\mu\nu}(\xi)j_\nu)},$$

Here $\mathcal{O}(a^3,a^4)$ are neglected, $(ja)=\int d^4x j^a_\mu(x)a^a_\mu(x)$,
$(j_\mu S_{\mu\nu}(\xi)j_\nu)=\int d^4xd^4y j^a_\mu(x)S^{ab}_{\mu\nu}(x,y,\xi)j^b_\nu(y)$,
$S^{ab}_{\mu\nu}(x,y,\xi)$ – a gluon propagator in the ILM background $A(\xi)$.
To account perturbative and nonperturbative effects both $\Rightarrow$

double expansion series in terms of α_s and λ.

We assume $\alpha_s\sim\lambda^{1/2}$ and keep $\mathcal{O}(\lambda),\mathcal{O}(\alpha_s\lambda^{1/2})$ terms during our calculations.

Heavy quarks in ILM

Heavy quark Q and antiquark $\bar{Q}$ Lagrangians are

$$L_Q = Q^+(\theta^{-1} - ga - gA + ...)Q,\; L_{\bar{Q}} = \bar{Q}^+(\theta^{-1} - g\bar{a} - g\bar{A} + ...)\bar{Q},$$

Here $< t_1|\theta|t_2 >= \theta(t_1 - t_2)$, $a_4 \equiv a = a_a\lambda_a/2$, $\bar{a}_4 \equiv \bar{a} = a_a\bar{\lambda}_a/2$, $\bar{\lambda}_a = -\lambda_a^{\mathrm{T}}$. The same for A fields.
Now ILM heavy quark propagator

$$w = \int D\xi \left[\int \left(\theta^{-1} - g\frac{\delta}{\delta j} - g\sum_i A_i \right)^{-1} \exp\left\{ \frac{1}{2}(jS(\xi)j) \right\} \right]_{j=0}.$$

Here $j \equiv j_4$, $S \equiv S_{44}$. It is easy to prove that

$$\left[\frac{1}{\theta^{-1} - g\frac{\delta}{\delta j} - gA(\xi)} \exp\left(\frac{1}{2} jS(\xi)j \right) \right]_{j=0}$$

$$= \left[\exp\left(\frac{1}{2}\frac{\delta}{\delta a_a} S_{ab}(\xi) \frac{\delta}{\delta a_b} \right) \frac{1}{\theta^{-1} - ga - gA(\xi)} \right]_{a=0}$$

This equation can be extended to any heavy quark correlator.

Heavy quark Q propagator in ILM with perturbative corrections

is:

$$w = \int D\xi \left[\theta^{-1} - \sum_i \left(gA_i(\xi_i) - g^2\left(\Delta S^i(\xi_i)\theta\right)\right)\right]^{-1},$$

where $\Delta S^i(\xi_i) = S^i(\xi_i) - S^0$ is single instanton contribution to Q propagator and last term is $\mathcal{O}(\alpha_s \lambda^{1/2})$ ILM perturbative mass operator.

w and its $g \to 0$ limit are similar on their ξ_i dependencies and we may easily extend Pobylitsa equations (DPP1989). The solution in $\mathcal{O}(\lambda, \alpha_s \lambda^{1/2})$ approximation is

$$w^{-1} = \theta^{-1} - \sum_i \int d\xi_i \theta^{-1}\left[\left(\theta^{-1} - gA_i(\xi_i)\right)^{-1} - \theta\right]\theta^{-1}$$

$$-g^2\left[(\bar{S} - S^0)\theta\right].$$

Second term $\to \mathcal{O}(\lambda)\ \Delta M_Q^{\rm dir}$, third one $\to \mathcal{O}(\alpha_s\lambda^{1/2})\ \Delta M_Q^{\rm pert}$.

ILM contribution to the heavy quark mass.

Direct instanton media contribution to the heavy quark mass

$$\Delta M_Q^{\rm dir} = 16\pi i_0(0)\lambda\rho^{-1}/N_c, \quad i_0(0) = 0.55.$$

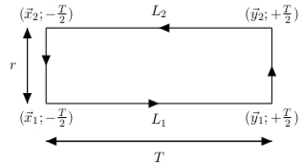
At $\rho = 0.33\,\mathrm{fm}$, $R = 1\,\mathrm{fm}$

$\Delta M_Q^{\rm dir} \approx 70\,\mathrm{MeV} \sim \lambda\rho^{-1} \sim$ *strength of a heavy quark-instanton interaction!!*

$$\Delta M_Q^{\rm pert} \approx \frac{2}{N_c}\,\alpha_s M_g(0) = 70\,\mathrm{MeV} \sim \alpha_s\lambda^{1/2}\rho^{-1}$$

at $N_c = 3$, $\alpha_s = 0.3$, $\rho = 1/3\,\mathrm{fm}$, $R = 1\,\mathrm{fm}$.
Perturbative gluon-instanton interaction $\rightarrow$ sizable changes of the perturbative gluon corrections.

$Q\bar{Q}$ correlator in ILM with perturbative gluons



$$W = \int D\xi \left[\exp \frac{1}{2} \sum_{i,j=1}^{2} \left(\frac{\delta}{\delta a_a^{(i)}} S_{ab}^{(ij)}(\xi) \frac{\delta}{\delta a_b^{(j)}} \right) \frac{1}{D_i^{(1)} - g a^{(1)}} \frac{1}{\bar{D}_i^{(2)} - g \bar{a}^{(2)}} \right] \Big|_{a=0},$$

where $D_i^{(1)} = \theta^{-1} - g\bar{A}_i^{(2)}(\xi_i)$ $(\bar{D}_i^{(2)} = \theta^{-1} - g\bar{A}_i^{(2)}(\xi_i))$, $a^{(1)}, A^{(1)}$ $(\bar{a}^{(2)}, \bar{A}^{(2)})$ are fields projections to the line L_1 (L_2).

$Q\bar{Q}$ potential $V_{\rm ILM}(r)$

Solution of Pobylitsa's equation extension in the $\mathcal{O}(\lambda, \alpha_s \lambda^{1/2})$ approximation is

$$W^{-1} = w^{(1)-1}\bar{w}^{(2)-1} - \sum_i \int d\xi_i$$

$$\times\theta^{(1)-1}\left(\frac{1}{D_i^{(1)}} - \theta^{(1)}\right)\theta^{(1)-1}\theta^{(2)-1}\left(\frac{1}{\bar{D}_i^{(2)}} - \theta^{(2)}\right)\theta^{(2)-1}$$

$$-g^2\frac{\lambda_a}{2}\frac{\bar{\lambda}_b}{2}\int D\xi\, S_{ab}^{(12)},$$

right side first and second terms lead to direct instanton potential $V_{\rm dir}(r)$ derived at DPP1989, while third term gives ILM modified one-gluon exchange potential $V_{\rm pert}(r)$.

$$V_{\rm ILM}(r) = V_{\rm dir}(r) + V_{\rm pert}(r)$$

Direct instanton contribution to the heavy quark-antiquark ($Q\bar{Q}$) potential in ILM

at colorless state is a smooth positive function

$$V_{\rm dir}(r) = \frac{N}{2VN_c}\sum_{\pm}\int d^3 z_{\pm}{\rm tr}_c\,[1-$$

$$P\exp\left(i\int_{L_1} dtA_{\pm,4}\right)P\exp\left(-i\int_{L_2} dtA_{\pm,4}\right)\Bigg],$$

$V_{\rm dir}(0) = 0$ and $V_{\rm dir}(r\to\infty)\to 2\Delta M_Q^{\rm dir}$. At $r<\rho$

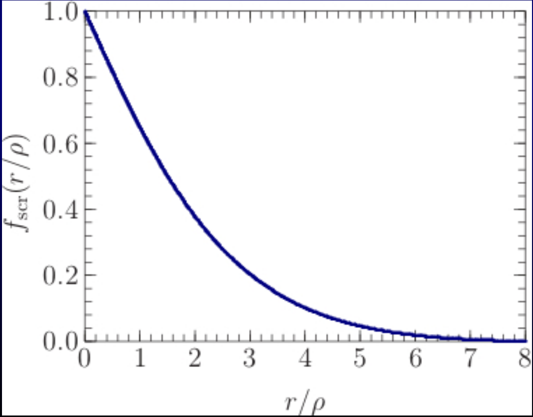
$$V_{\rm dir}(r)\approx\frac{4\pi\rho^3}{R^4N_c}1.345\left(\frac{r^2}{\rho^2}-0.372\frac{r^4}{\rho^4}\right).$$

Average size of charmonium $r_c\sim\rho$ while for botomonium $r_b<\rho$. So, r^2- electric dipole approximation work for botomonium much better then for charmonium case.

Perturbative one-gluon exchange singlet potential in ILM

is

$$V_{\rm pert}(r) = -\frac{4}{3}\, g^2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2 + M_g^2(q)} = -\frac{4\alpha_s}{3r} f_{\rm scr}(r/\rho)\,.$$



Instanton contribution to the charmonium states

Phenomenological Cornell potential = one-gluon exchange + confinement

$$V_{\rm Cornell}(r) = -(4/3)\frac{\alpha_s}{r} + \sigma r,$$

vs

$$V = V_{\rm ILM} + \sigma r = V_{\rm Cornell} + V_{\rm scr} + V_{\rm dir}$$

charmonium S-wave states energies $\Delta E_n = E_n - 2m_Q$

n	$V_{\rm Cornell}$	$\rho = 1/3\,\text{fm},\ R = 1\,\text{fm}$: $V_{\rm Cornell} + V_{\rm scr}$	$V_{\rm Cornell} + V_{\rm dir}$	V
1	519	579	561	622
2	1061	1114	1132	1186
3	1485	1529	1569	1613
4	1855	1893	1946	1984

$m_Q = 1275\,\text{MeV}$, $\sigma = 0.17\,\text{GeV}^2$ and $\alpha_s = 0.2$

$n = 1$, $V_{\rm scr}$-*corrections* $\sim$ $V_{\rm dir}$-*corrections* $\sim 10\%$.

Heavy quark-light quarks interactions in ILM.

With light quarks: $D\xi \Rightarrow D\xi\,\mathrm{Det}_{low}(\hat{p} + g\hat{A} + im)$.
Heavy quark propagator is

$$w = \int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger(\hat{p} + im_f)\psi_f\right)$$
$$\times \prod_\pm \left(< V_\pm[\psi^\dagger, \psi] >\right)^{N_\pm} w[\psi, \psi^\dagger],$$

where

$$w[\psi, \psi^\dagger] = \int \prod_\pm^{N_\pm} \frac{V_\pm[\psi^\dagger, \psi]}{< V_\pm[\psi^\dagger, \psi] >} d\xi_\pm \,(\theta^{-1} - g \sum_\pm A_\pm(\xi_\pm))^{-1}$$

Solution of extended Pobilitca Eq. at $O(\lambda)$ is

$$w^{-1}[\psi, \psi^\dagger] = \theta^{-1} - \frac{N}{2} \sum_\pm \frac{1}{< V_\pm[\psi^\dagger, \psi] >} \Delta_{H,\pm}[\psi^\dagger, \psi],$$

$$\Delta_{H,\pm}[\psi^\dagger, \psi] = \int d\xi_\pm V_\pm[\psi^\dagger, \psi]\theta^{-1}(w_\pm - \theta)\theta^{-1}.$$

Heavy quark – N_f light quarks interaction term

is

$$S_{Q\psi} = -\kappa \sum_{\pm} \int d\xi_{\pm} V_{\pm}[\psi^{\dagger}, \psi] Q^{\dagger} \theta^{-1} (w_{\pm} - \theta) \theta^{-1} Q.$$

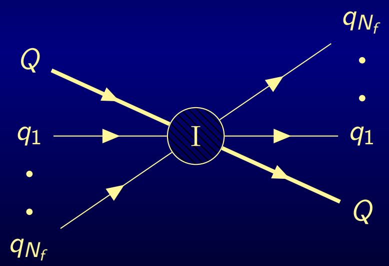


Figure: Instanton generated heavy Q N_f light q quarks interaction.

Heavy quark–light quarks interactions ($N_f = 1$).

is

$$S_{Q\psi} = i\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^3q}{(2\pi)^3}(2\pi)^4\delta^3(\vec{k}_2+\vec{k}_1-\vec{q})\delta(k_{2,4}-k_{1,4})$$

$$(M(k_1)M(k_2))^{1/2}\Delta M_Q^{\rm dir}R^4\frac{i_0(q\rho)}{i_0(0)}\left[\frac{2N_c^2-N_c}{2N_c^2-2}\psi^+(k_1)\psi(k_2)Q^+Q\right.$$

$$\left.+\frac{N_c^2-2N_c}{2N_c^2-2}(\psi^+(k_1)QQ^+\psi(k_2)+\psi^+(k_1)\gamma_5QQ^+\gamma_5\psi(k_2))\right]$$

First term is heavy quark–light meson interaction term, while second and third terms – Qq mesons degenerated on parity. It is similar to (Chernyshev etal94,95).

Heavy quark–pions interactions at $N_f = 2$

come from the co-product of heavy quark factor and the colorless light-quarks one. Bosonization ⇒ the effective action including heavy quark–pions interactions part:

$$S_{Q\pi} = -\frac{i}{2}\Delta M_Q^{\mathrm{dir}} R^4 F_{\pi Q}^2 \int d^4x \,\mathrm{tr}_f\, \partial_\mu U^\dagger(x)\, \partial_\mu U(x)$$

$$\times \int e^{-ipx} \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4} i_0(p\rho)/i_0(0)\, Q^\dagger(p_2) Q(p_1)$$

where pions $\vec{\phi}$ are given by $U = \exp(i\vec{\tau}\vec{\phi})$ and coupling

$$F_{\pi Q}^2 = 2N_c \int \frac{d^4p}{(2\pi)^4} \frac{M^2(p)p^2}{(p^2+M^2(p))^3} = 0.36 F_\pi^2,$$

$$F_\pi^2 = 4N_c \int \frac{d^4p}{(2\pi)^4} \frac{M^2(p)}{(p^2+M^2(p))^2} = (72\,\mathrm{MeV})^2.$$

Here were taken $\rho = 1/3\,\mathrm{fm}$, $R = 1\,\mathrm{fm}$.

$(Q\bar{Q})$-light quarks interactions.

Now averaged Wilson loop is given by

$$\int \prod_f D\psi_f D\psi_f^\dagger \exp \int \left(\psi_f^\dagger(\hat{p} + im_f)\psi_f\right) \prod_\pm \left(< V_\pm[\psi^\dagger, \psi] >\right)^{N_\pm}$$

$\mathrm{tr} < T|W[\psi, \psi^\dagger]|0 >$, where $< T|W[\psi, \psi^\dagger]|0 >=$

$$= \prod_\pm \left(< V_\pm[\psi^\dagger, \psi] >\right)^{-N_\pm} \int D\xi \prod_\pm^{N_\pm} V_\pm[\psi^\dagger, \psi]$$

$$P \exp(i \int_{L_1} dx_4 A_4) P \exp(-i \int_{L_2} dx_4 A_4)$$

Solution of extended Pobilitca Eq. at $O(\lambda)$ is

$$W^{-1}[\psi, \psi^\dagger] = w_1^{-1}[\psi, \psi^\dagger] \otimes w_2^{-1,T}[\psi, \psi^\dagger]$$

$$-\frac{N}{2} \sum_\pm \left(< V_\pm[\psi^\dagger, \psi] >\right)^{-1} \int d\xi_\pm V_\pm[\psi_f^\dagger, \psi_f]$$

$$\left(\theta^{-1}\left(w_\pm^{(1)} - \theta\right)\theta^{-1}\right) \otimes \left(\theta^{-1}\left(w_\pm^{(2)} - \theta\right)\theta^{-1}\right)^T$$

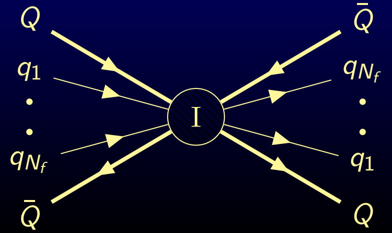
Instanton generated heavy $Q\bar{Q}$ in colorless state interaction with light quarks in ILM

$$S_{QQq} = -i\kappa \sum_{\pm} \int d^4 z_{\pm} dU_{\pm} \prod_{f=1}^{N_f} \frac{d^4 k_f}{(2\pi)^4} \frac{d^4 q_f}{(2\pi)^4} \exp(i(q_f - k_f) z_{\pm})$$

$$\times \frac{(2\pi\rho)^2 F(k_f) F(q_f)}{8} \psi^{+}_{f, a_f \alpha_f}(k_f) (\gamma_{\mu_f} \gamma_{\nu_f} \frac{1 \pm \gamma_5}{2})_{\alpha_f \beta_f}$$

$$\times (U^{a_f}_{\pm, i_f} (\tau^{\mp}_{\mu_f} \tau^{\pm}_{\nu_f})^{i_f}_{j_f} U^{\dagger j_f}_{\pm, b_f} \psi^{b_f}_{f, \beta_f}(q_f)$$

$$Q^{+} \bar{Q}^{+} \frac{1}{N_c} \mathrm{tr}_c \left[\left(\theta^{-1} w^{(1)}_{\pm} \theta^{-1} \right) \otimes \left(\theta^{-1} \bar{w}^{(2)}_{\pm} \theta^{-1} \right) - \theta^{-1} \otimes \theta^{-1} \right] Q \bar{Q}$$



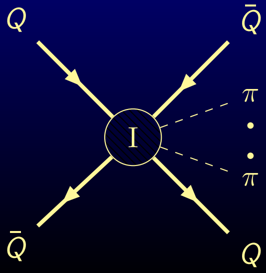
Instanton generated heavy quarkonium – pions interactions in ILM

Repeating the calculations leading to $S_{Q\pi}$ we find the action

$$S_{QQ\pi} = F_{\pi Q}^2 \int d^4z \, \mathrm{tr}_f \partial_\mu U(z) \partial_\mu U^+(z) \left(Q^+ \bar{Q}^+ \frac{1}{N_c} \mathrm{tr}_c \left[1 - \right.\right.$$

$$\left.\left. - P \exp\left(i \int_{L_1} A_{\pm,4}(\xi_\pm) dx_4 \right) P \exp\left(-i \int_{L_2} A_{\pm,4}(\xi_\pm) dx_4' \right) \right] Q \bar{Q} \right)$$

represented by the Fig.



$(Q\bar{Q})_{n'} \to (Q\bar{Q})_n\, \pi\pi$ amplitude in ILM is

$$A_{QQ\pi} = F_{\pi Q}^2 \int d^4z\, \mathrm{tr}_f \partial_\mu U(z)\partial_\mu U^+(z) \exp(i(\vec{p}\,' - \vec{p})\vec{z})$$

$$\times \int d^3y d^3r < n|\vec{r} > \exp(-i\vec{p}\vec{y}) \frac{1}{N_c} \mathrm{tr}_c\, [1-$$

$$-P \exp\left(i \int_{-\infty}^{\infty} d\tau_1 A_{\pm,4}(\vec{y} + \vec{r}/2, \tau_1) \right)$$

$$\times P \exp\left(-i \int_{-\infty}^{\infty} d\tau_2 A_{\pm,4}(\vec{y} - \vec{r}/2, \tau_2) \right) \Bigg] \exp(i\vec{p}\,'\vec{y}) < \vec{r}|n' >$$

where $\vec{y} = \vec{x} - \vec{z}$, the positions of Q and $\bar{Q}$ are taken as $\vec{x}_1 = \vec{x} + \vec{r}/2$ and $\vec{x}_2 = \vec{x} - \vec{r}/2$. Also, here $\exp(i\vec{p}\vec{x})|n>,\ \exp(i\vec{p}\,'\vec{x})|n'>$ are final and initial states of the $Q\bar{Q}$ with total initial momentum $\vec{p}\,' = (\vec{p}_1' + \vec{p}_2')$ and total final momentum $\vec{p} = (\vec{p}_1 + \vec{p}_2)$. They are the solutions of $H = \frac{\vec{p}_1^2}{2m_Q} + \frac{\vec{p}_2^2}{2m_Q} + V,\ V = V_{\mathrm{ILM}} + V_{\mathrm{conf}}$.

Phenomenology of $(Q\bar{Q})_{n'} \to (Q\bar{Q})_n \, \pi\pi$

From Mannel,Urech 1995: Chiral limit and $m_Q \to \infty$

$$\mathcal{L}_0 = g A_\mu^{(v)} B^{(v)\mu *} \mathrm{tr}[(\partial_\nu U)(\partial^\nu U)^\dagger] \, + O(v) + \, h.c.$$

Vector fields $A_\mu^{(v)}$ and $B^{(v)\mu} \sim (Q\bar{Q})_{n'}(2S)$ and $(Q\bar{Q})_n(1S)$

	$\psi(2S) \to J/\psi \pi^+\pi^-$	$\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$
g	0.30 ± 0.02	0.25 ± 0.02

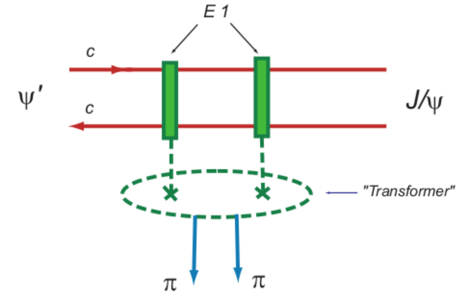
The couplings $\sim$ fit to PDG94. The errors are experimental. Our estimate is

$$g_{J/\psi} = \frac{F_{\pi Q}^2}{F_\pi^2} 1.345 \frac{r_{J/\psi}^2}{\rho^2} \left(1 - 0.372 \frac{r_{J/\psi}^2}{\rho^2}\right) = 0.28(1-0.2) = 0.22$$

it is obvious that $g_\Upsilon < g_{J/\psi}$.

Standard approach to quarkonium light hadron transitions

Hadronic transitions at the assumption $\lambda_g >> r_c, r_b \Rightarrow$ multipole expansion (see e.g. Voloshin12):



But $\lambda_g \approx \rho$. If $r_{c,b} << \rho$ then $(Q\bar{Q})$-light quarks interactions term is given by dipole approximation.
Compare $\rho = 0.33$ fm with charmonium size $r_{J/\psi} = 0.25$ fm.
Our estimation of the correction in charmonium case is -20%.

Discussion.

- Instantons with sizes $\rho \sim$ hadron quark core sizes r give most essential contribution to their properties. In this case ILM is applicable.
- The strength of a heavy quark-instanton interaction is defined by $\Delta M_Q^{\rm dir} \sim packing\ parameter\ \rho^{-1} \sim 70\, MeV$, at $\rho = 0.33\, fm$, $R = 1\, fm$.
- The strength of a gluon-instanton interaction is much more large and given by the dynamical gluon mass $M_g \sim (packing\ parameter)^{1/2}\ \rho^{-1} \sim 362\, MeV$.
- The analogous quantity for light quarks M is also large since has the same dependencies and $M \sim (packing\ parameter)^{1/2}\ \rho^{-1} \sim 360\, MeV$.
- Instantons naturally generate also heavy-light quarks interaction, which might be important for the heavy quarkonium and heavy-light quarks systems properties. It can be responsible for the SBCS effects in heavy quarks physics.

Future work.

- Apply ILM to the calculations of the spectrum of heavy quarkoniums.
- Take into account light quarks in the observables of heavy quark physics:
  - $Q\bar{Q}$ pions transitions;
  - heavy-light mesons etc.
- Consider ILM generated gluon-light quarks interactions and related problems of exotic hadrons.
- Extend the approach to non-zero temperature case and apply to the observables in heavy-heavy and heavy-light quark systems produced in hadron-hadron collisions.

All of these problems are on the consideration now.
Thank you for the attention.