



9<sup>th</sup> International Conference on New Frontiers in Physics 2020

# Modeling (Anti-)deuteron Formation at RHIC with a Geometric Coalescence Model

Kittiratpattana, A. et al. (2020). *arXiv:2006.03052*.

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**Presenter:**

**Apiwit Kittiratpattana**

**September 4-11, 2020**



# Heavy-Ion Collisions

## Introduction

- Heavy ion Collision
- Coalescence Model

## Mrowczynski's Model

- Ansatz
- Parameterization
- Formation Rate
- Source Geometries

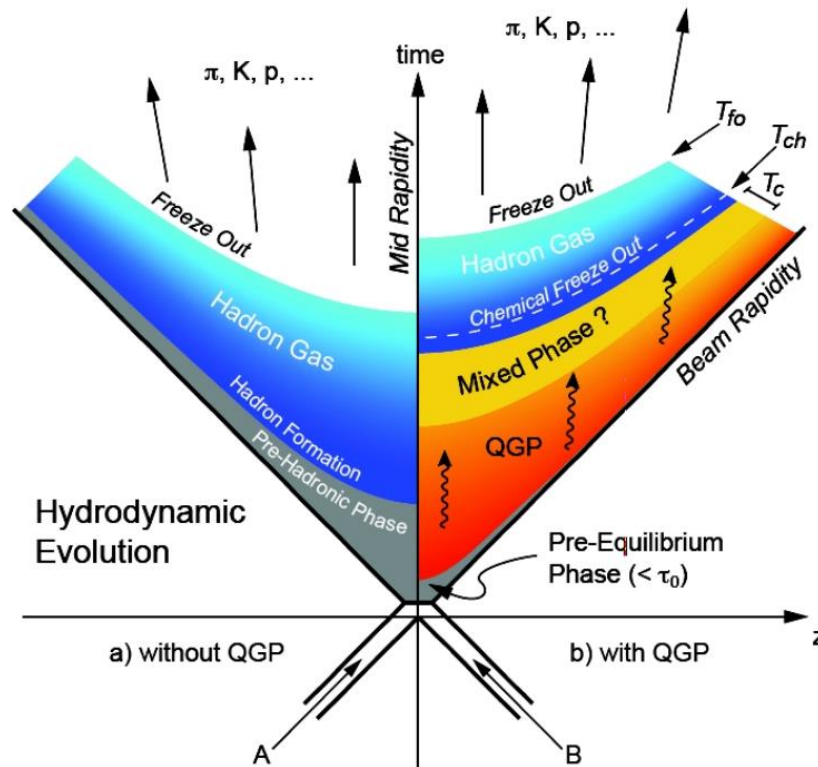
## Charged Volume

- Charged constrain
- Source Geometries

## UrQMD

- $r_T$  - Distribution
- Source Geometry

## Summary



Braun-Munzinger, P., & Dönigus, B. (2019). *Nuclear Physics A*, 987, 144-201.

<https://particlesandfriends.wordpress.com/2016/10/14/evolution-of-collisions-and-qgp/>

Light nuclei formation can give insightful information,

- Quark Gluon Plasma (QGP)
- Big Bang nucleosynthesis
- Emission source shape



# Coalescence Model

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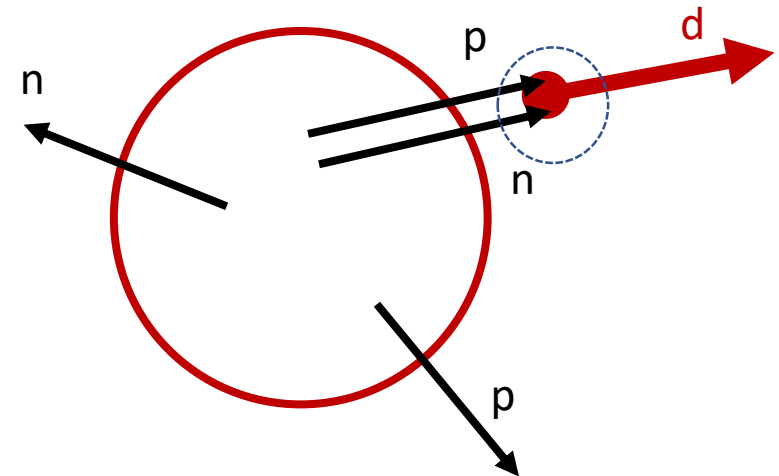
$$E \frac{d^3 \sigma_d}{d\mathbf{P}^3} = B_2 \left( \frac{E}{2} \frac{d^3 \sigma_p}{d\left(\frac{\mathbf{P}}{2}\right)^3} \right) \left( \frac{E}{2} \frac{d^3 \sigma_n}{d\left(\frac{\mathbf{P}}{2}\right)^3} \right) \quad (1)$$

$\sigma_i$  : Cross section

$E$  : Deuteron energy

$\mathbf{P}$  : Deuteron momentum

$B_2$ : Coalescence parameter.





# Mrowczynski's Coalescence Model

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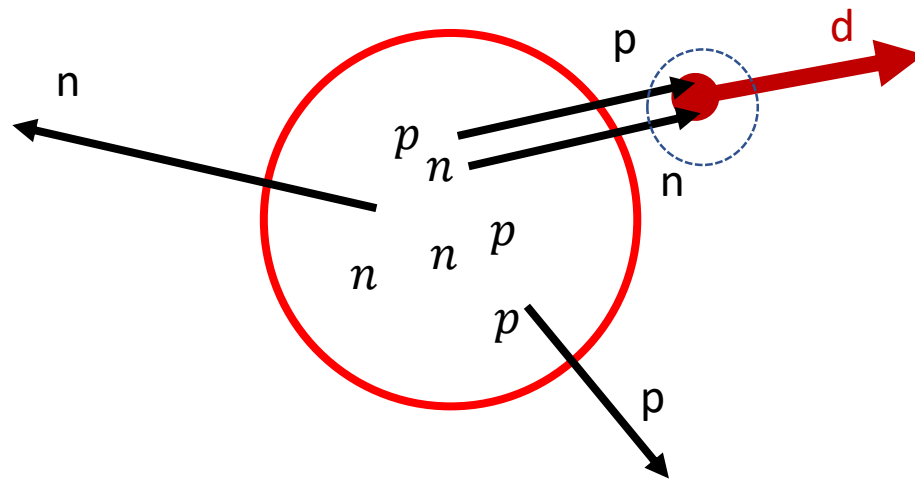
- Charged constrain
- Source Geometries

## UrQMD

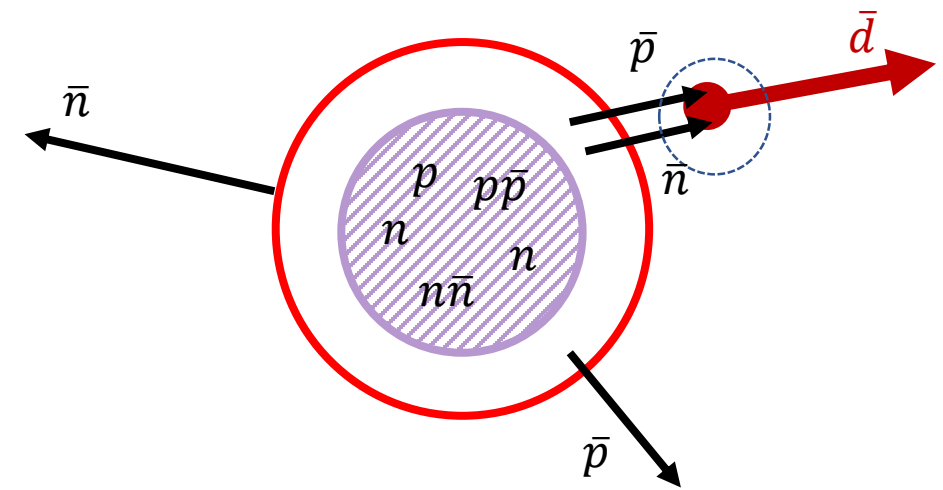
- $r_T$  - Distribution
- Source Geometry

## Summary

Nucleons are emitted from the **whole** source/fireball volume.



Antinucleons are emitted from the **outer shell** of the fireball volume.





# Source Geometry Parametrization

## Introduction

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- Source Geometries

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- $r_T$  - Distribution
- Source Geometry

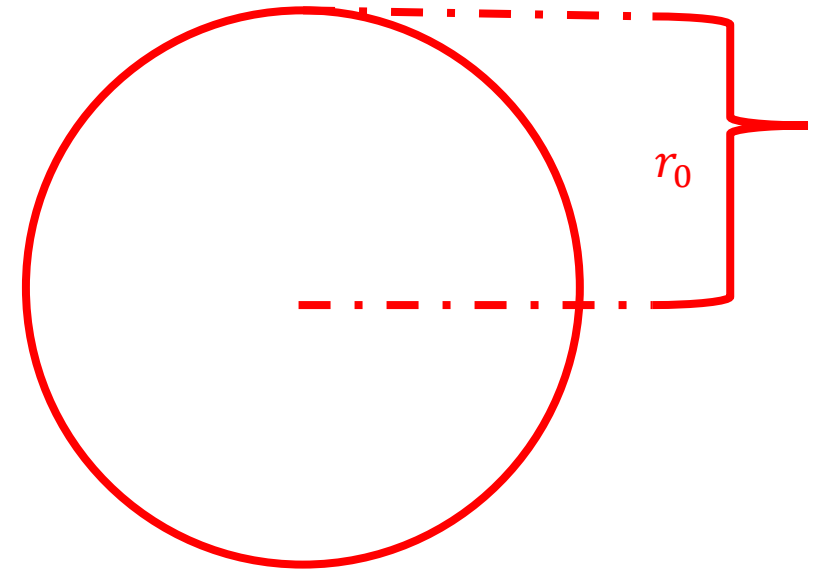
## Summary

### • **Nucleons :**

$$\bullet \mathcal{D}(r) = \frac{1}{(2\pi)^{\frac{3}{2}} r_0^3} \exp\left(-\frac{r^2}{2r_0^2}\right) \quad (2)$$

### • **Antinucleons :**

$$\bullet \bar{\mathcal{D}}(r) = \frac{1}{(2\pi)^{\frac{3}{2}} (r_0^3 - r_*^3)} \begin{bmatrix} \exp\left(-\frac{r^2}{2r_0^2}\right) \\ - \exp\left(-\frac{r^2}{2r_*^2}\right) \end{bmatrix}$$





# Source Geometry Parametrization

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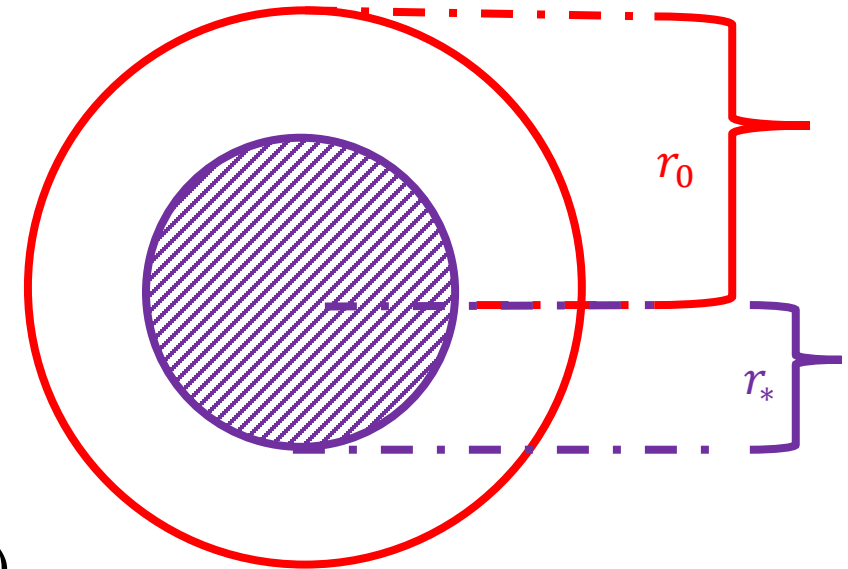
## Summary

## • Nucleons :

$$\bullet \mathcal{D}(r) = \frac{1}{(2\pi)^{\frac{3}{2}} r_0^3} \exp\left(-\frac{r^2}{2r_0^2}\right) \quad (2)$$

## • Antinucleons :

$$\bullet \bar{\mathcal{D}}(r) = \frac{1}{(2\pi)^{\frac{3}{2}} (r_0^3 - r_*^3)} \left[ \begin{array}{c} \exp\left(-\frac{r^2}{2r_0^2}\right) \\ - \exp\left(-\frac{r^2}{2r_*^2}\right) \end{array} \right] \quad (3)$$





# Formation Rate

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## Summary

The formation rate is  $\left(\frac{d^3N_d}{dP^3}\right)/\left(\frac{d^3N_p}{dP^3}\right)^2$  and is equal to the probability:

$$\mathcal{A}(r_0, r_*) = \int d^3r_p d^3r_n \mathcal{D}(\vec{r}_p) \mathcal{D}(\vec{r}_n) |\phi(\vec{r}_p - \vec{r}_n)|^2 \quad (4)$$

Then  $B_2$  is related to formation rate via,

$$\frac{m_N}{2} \mathcal{A}(r_0, r_*) = B_2 \quad (5)$$



- [2] R. Arsenescu et al. (NA52 (NEWMASS)), J. Phys. G25, 225 (1999)  
 [3] G. Van Buren (E864), Nucl. Phys. A661, 391 (1999)

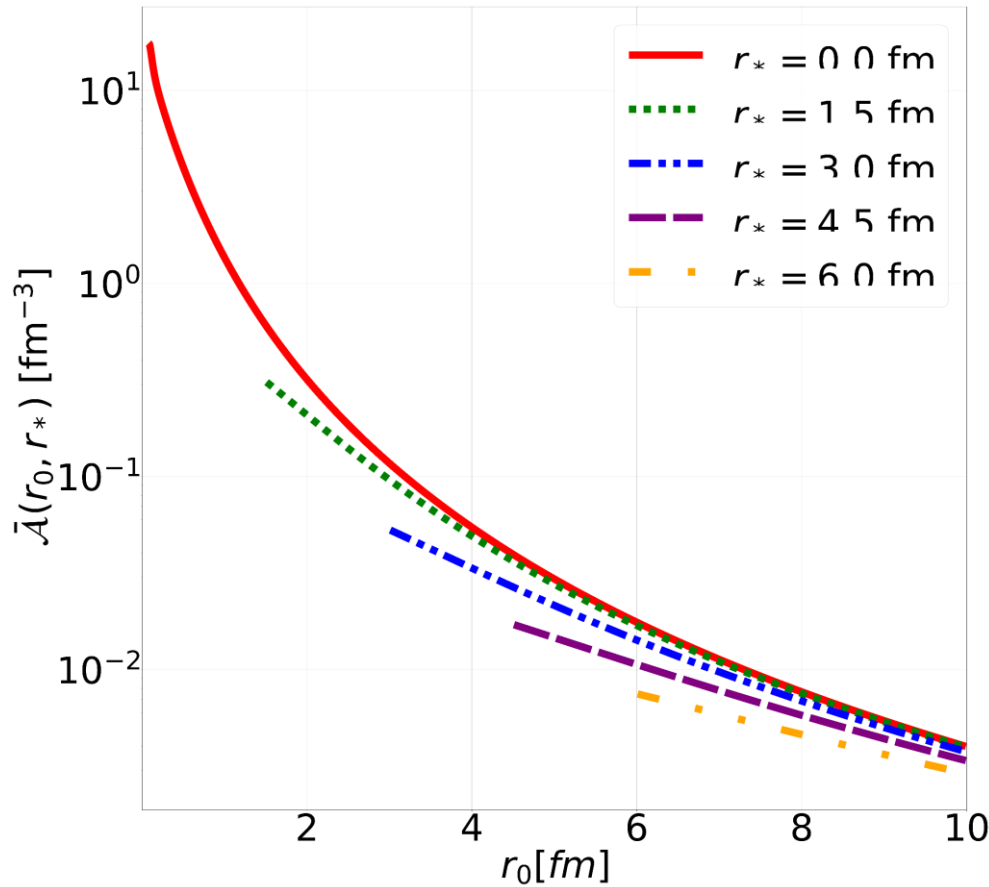


Figure 1: A few calculated formation rates (Eq. 4) varying  $r_*$  from 0-6 fm.

- [4] T. Armstrong et al. (E864), Phys. Rev. Lett.85, 2685(2000),nucl-ex/0005001  
 [5] J. Adam et al. (STAR), Phys. Rev. C99, 064905(2019),1903.11778

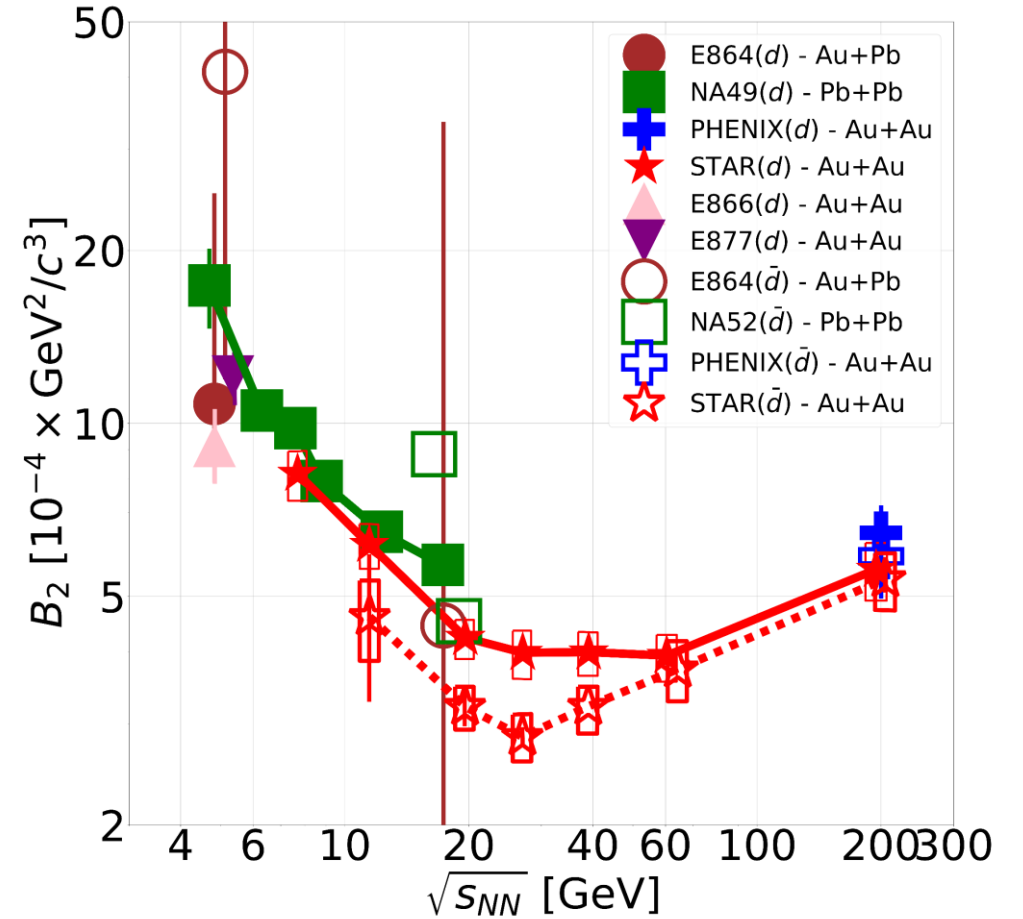


Figure 2: The  $B_2$  from experiment used for converting into formation rate as a function of  $\sqrt{s_{NN}}$  [2-5].

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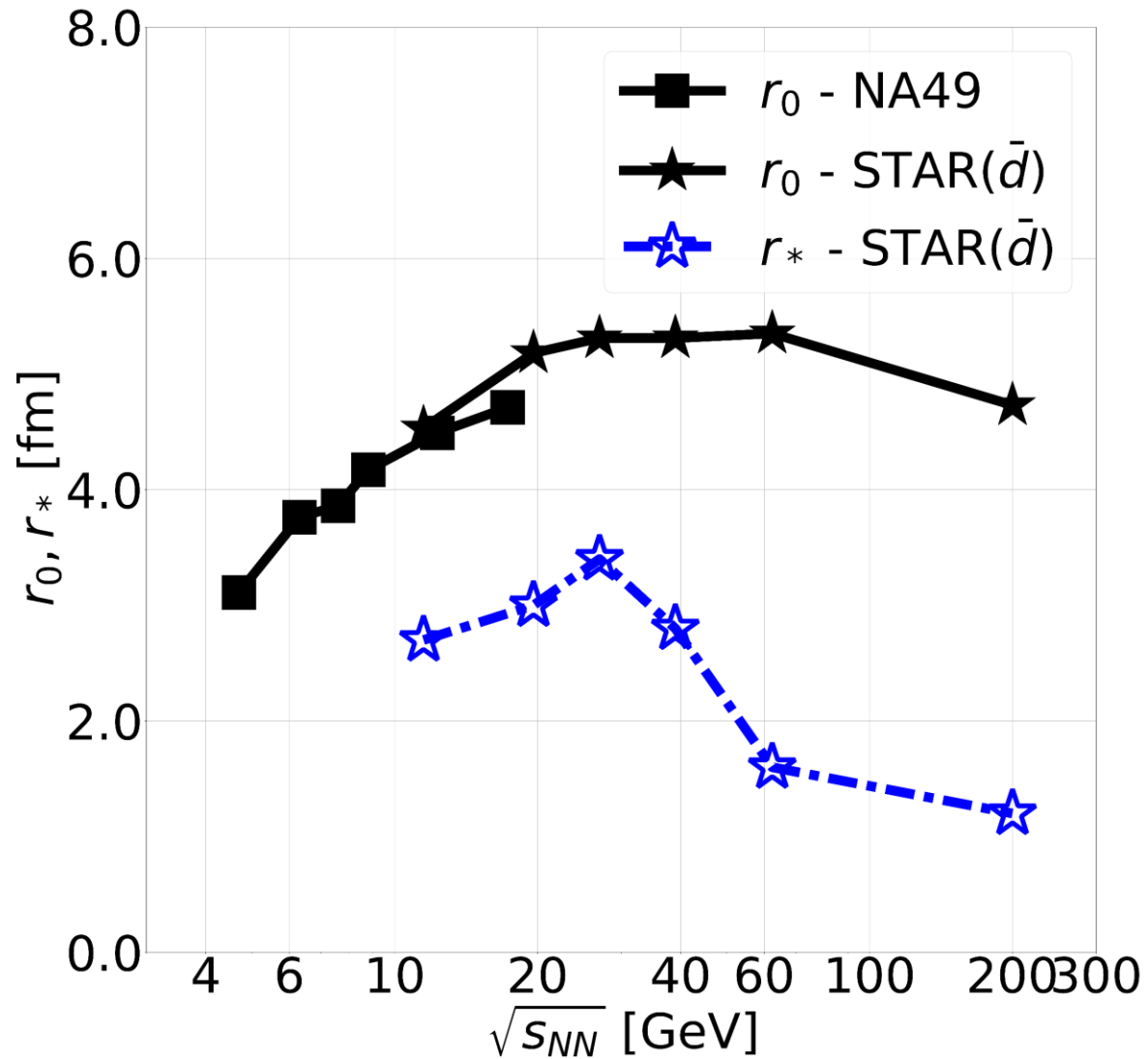


Figure 3: The (anti-)nucleon source radius and the suppression region as a function of energy.



# Charged Volume Constraint

## Introduction

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## Mrowczynski's Model

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## UrQMD

- $r_T$  - Distribution
- Source Geometry

## Summary

1. We constrain RMS radius with charged volume data,

$$RMS(r_0, r_*) = const \cdot N_{ch}^{\frac{1}{3}} \quad (6)$$

2. We use Eq. (5) & (6) to solve for the radii.

[6] - Adare, A. et al (2016). *Physical Review C*, 93(2), 024901.

[7] - Ghosh, S., (2017). *Physical Review C*, 96(2), 024912.

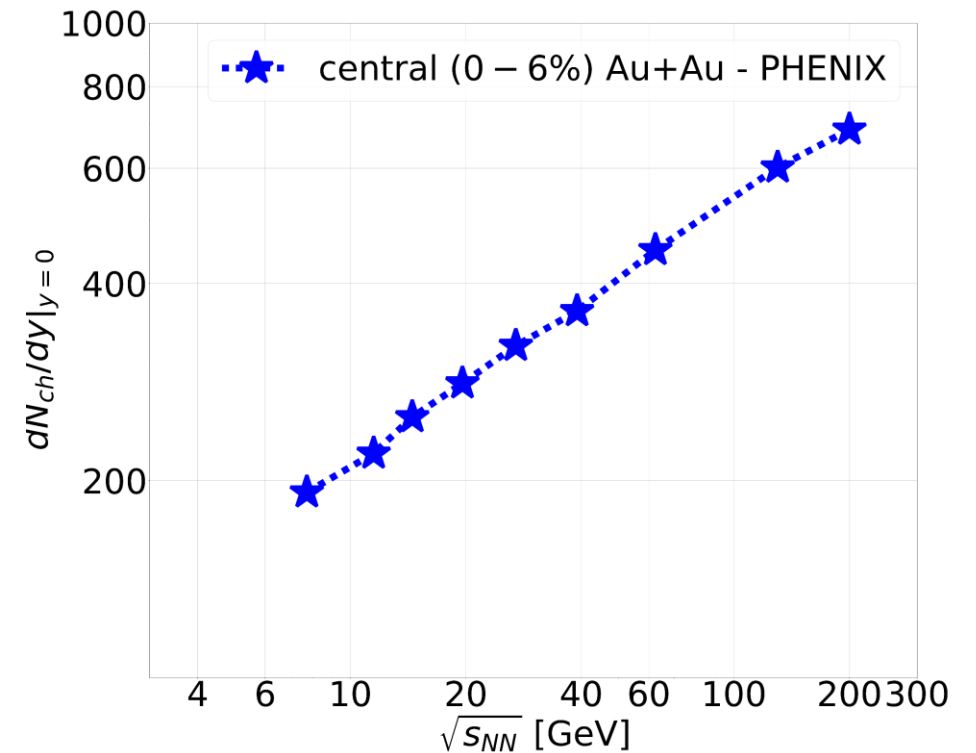


Figure 4: The energy dependence of total charged particle at mid-rapidity from PHENIX [6-7].



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## UrQMD

- $r_T$  - Distribution
- Source Geometry

## Summary

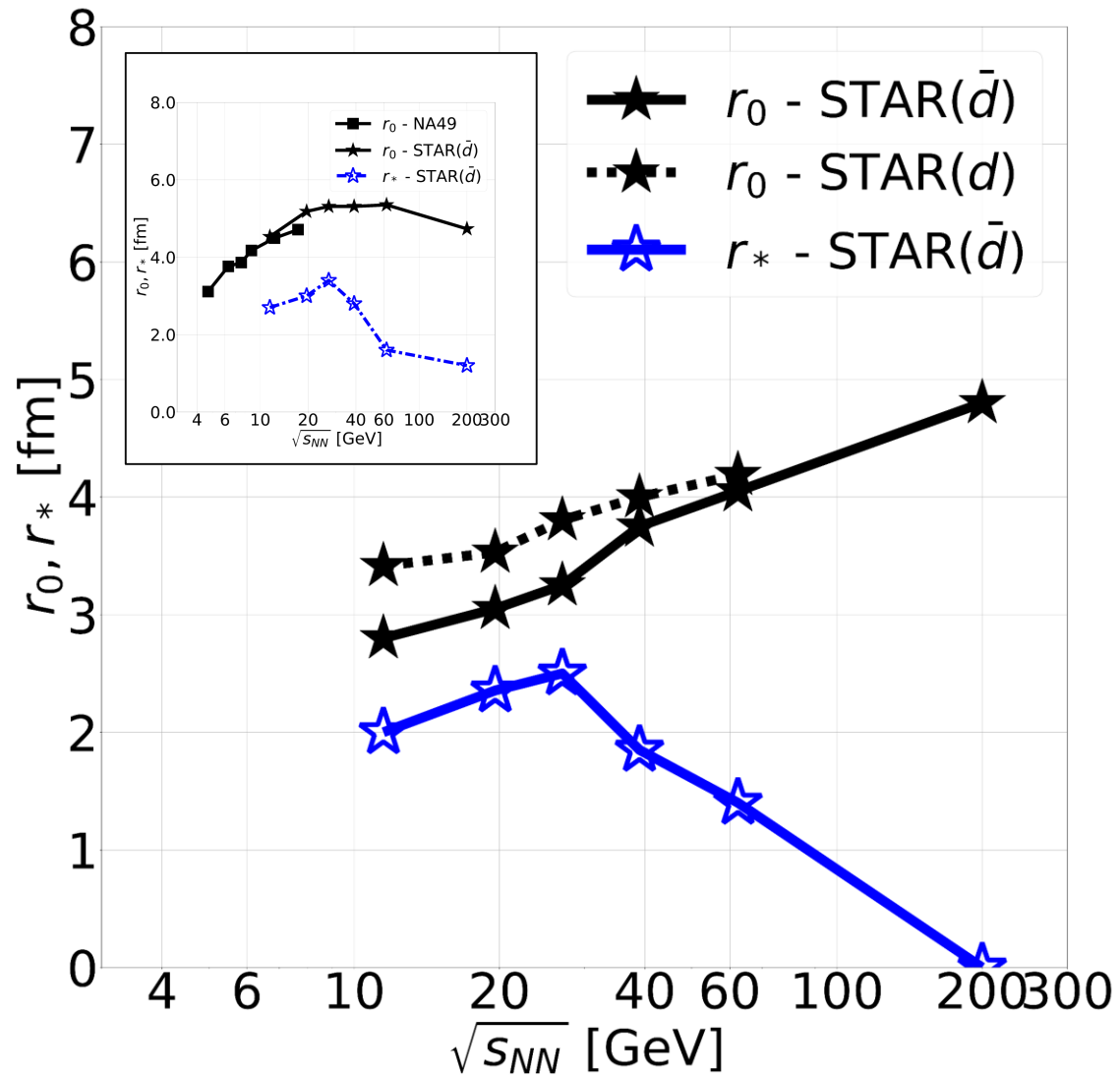


Figure 5: The (anti-)nucleon source radius from charged volume as a function of energy.



# UrQMD

## Introduction

- Heavy ion Collision
- Coalescence Model

## Mrowczynski's Model

- Ansatz
- Parameterization
- Formation Rate
- Source Geometries

## Charged Volume

- Charged constrain
- Source Geometries

## UrQMD

- $r_T$  - Distribution
- Source Geometry

## Summary

The Ultra-relativistic Quantum Molecular Dynamics (UrQMD) model is a microscopic transport model.

## Objective

To extract the freeze-out coordinate of the (anti)nucleons and fit directly with  $\mathcal{D}(r)$  and  $\bar{\mathcal{D}}(r)$  via transverse radius  $r_T$  to get the  $r_0$  and  $r_*$  at 7.7 – 200 GeV.



# $r_T$ - Distributions

## Introduction

- Heavy ion Collision
- Coalescence Model

## Mrowczynski's Model

- Ansatz
- Parameterization
- Formation Rate
- Source Geometries

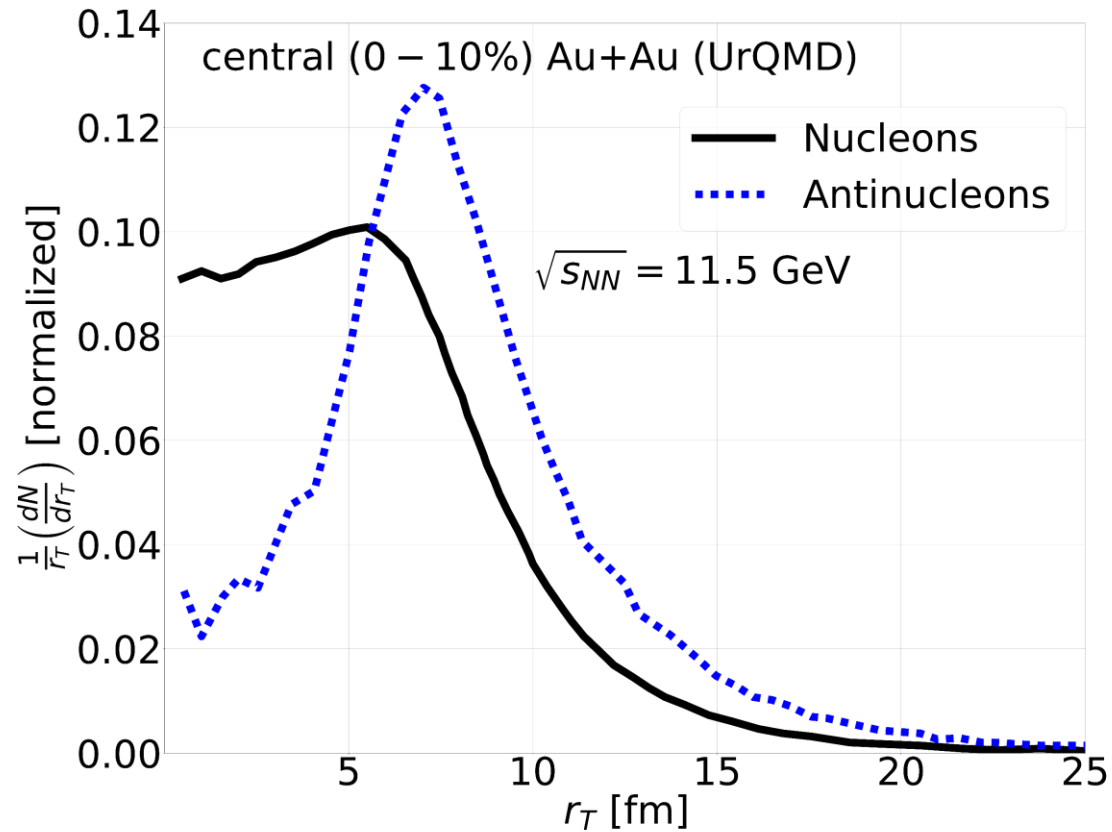
## Charged Volume

- Charged constrain
- Source Geometries

## UrQMD

- $r_T$  - Distribution
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## Summary



This agrees with the Gaussian source shape as well as the assumed source of an annihilation.

Figure 6: The (anti-)nucleon distributions along the transverse radius  $r_T$  at 11.5 GeV.



# $r_T$ - Distributions

## Introduction

- Heavy ion Collision
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## Mrowczynski's Model

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- Source Geometries

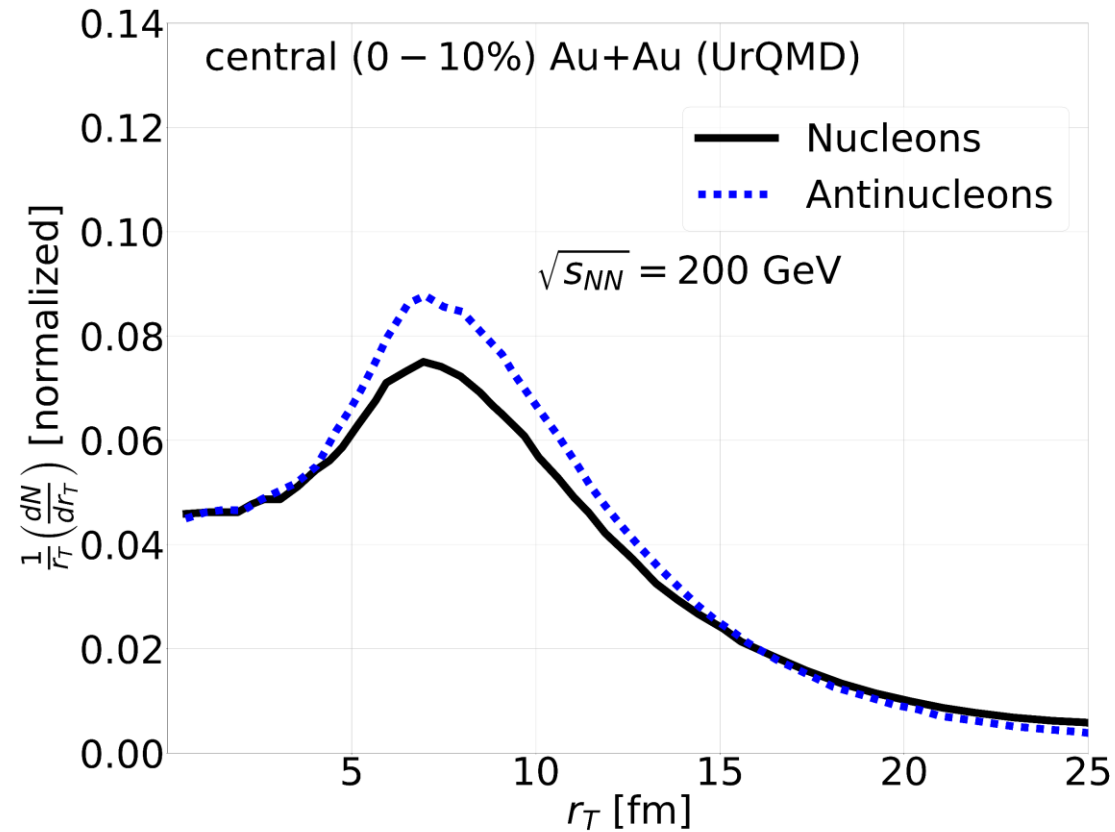
## Charged Volume

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## UrQMD

- $r_T$  - Distribution
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## Summary



This means that the nucleon source  $\mathcal{D}(r)$  is **ALSO** suppressed in the center.

Figure 7: The (anti-)nucleon distributions along the transverse radius  $r_T$  at 200 GeV.



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## UrQMD

- $r_T$  - Distribution
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## Summary

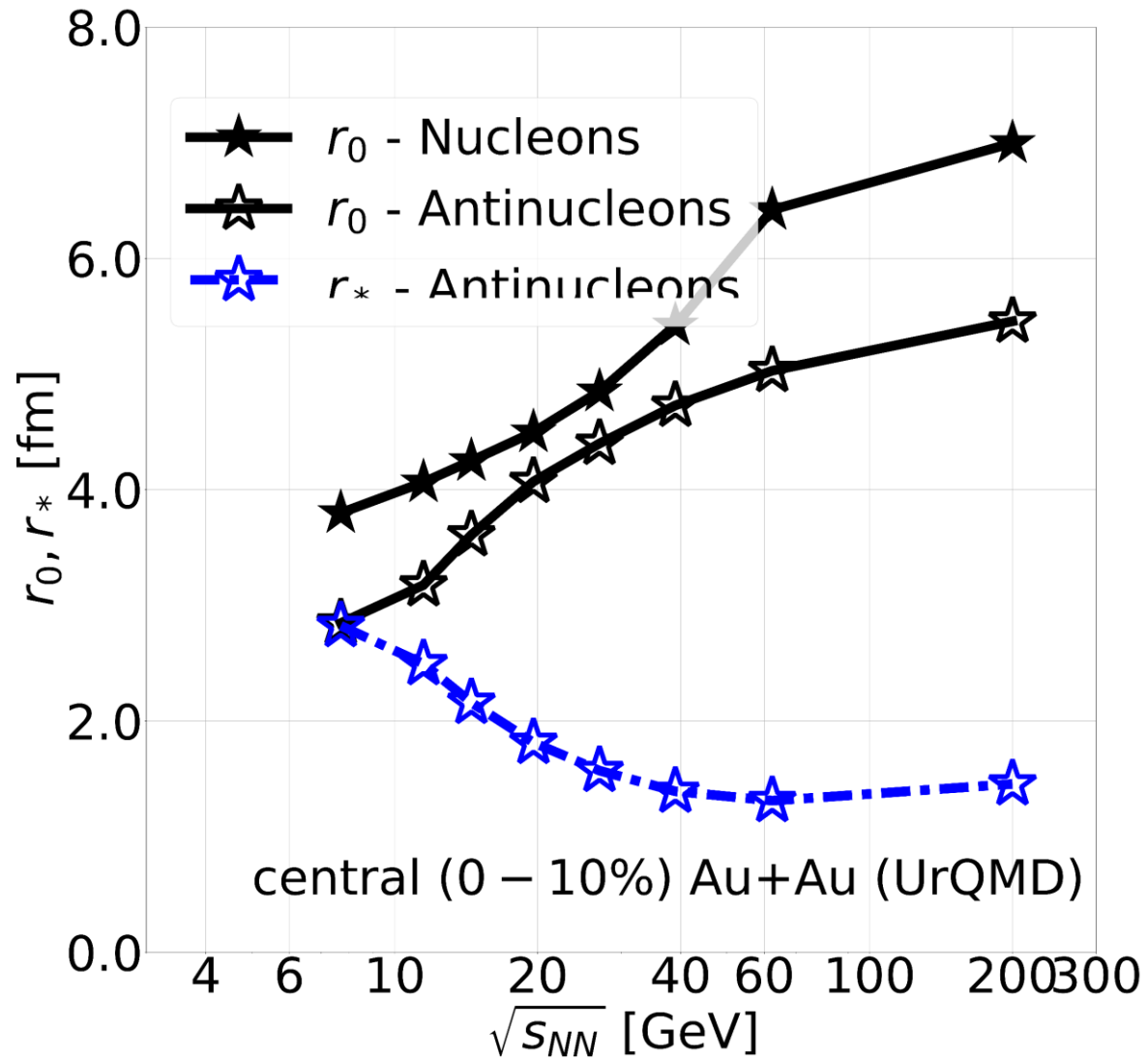


Figure 8: The fireball radius  $r_0$  and the antinucleon suppression region  $r_*$  UrQMD.



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## Summary

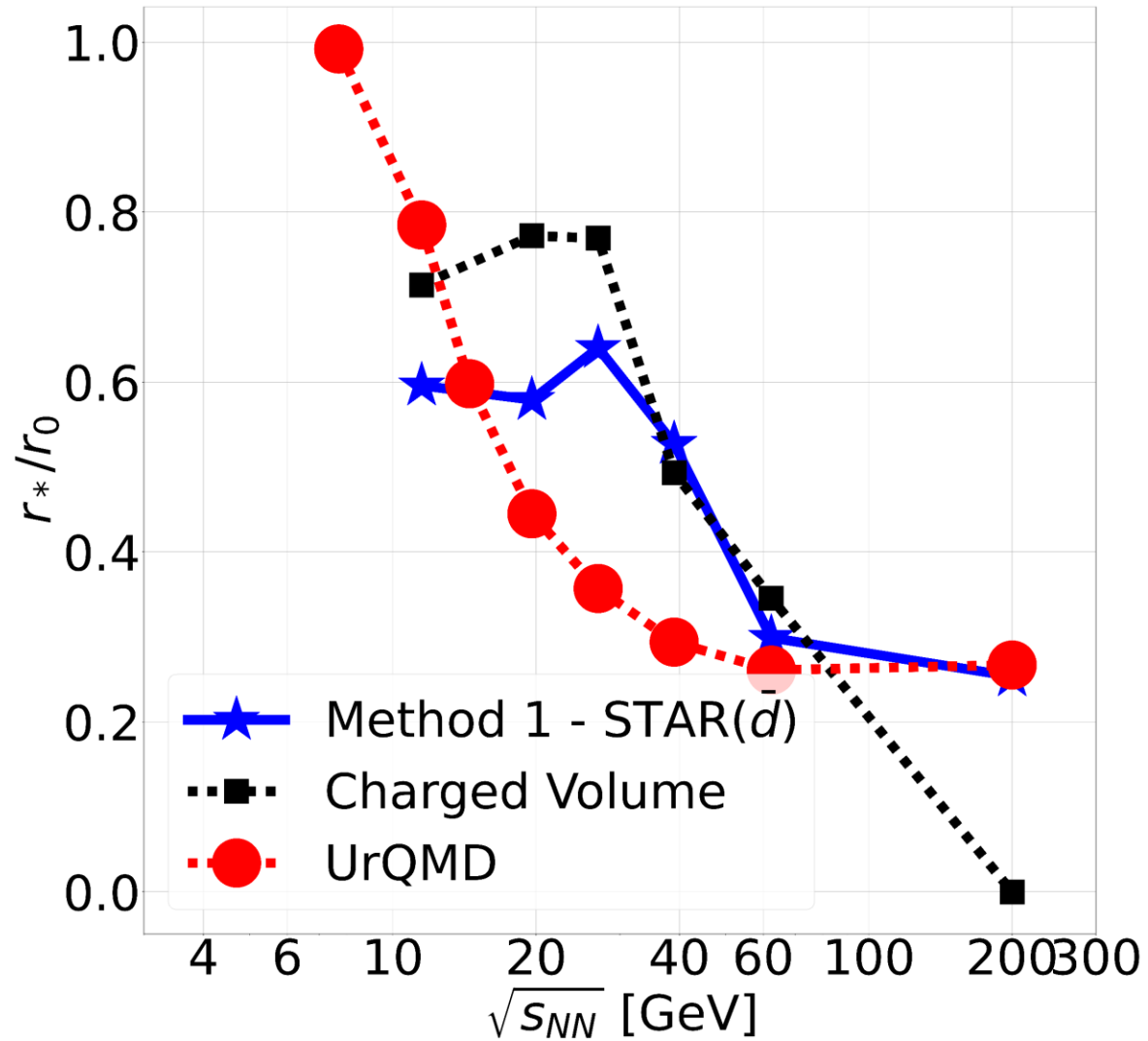


Figure 9: Comparison of  $r^*/r_0$  from different methods.





# Summary

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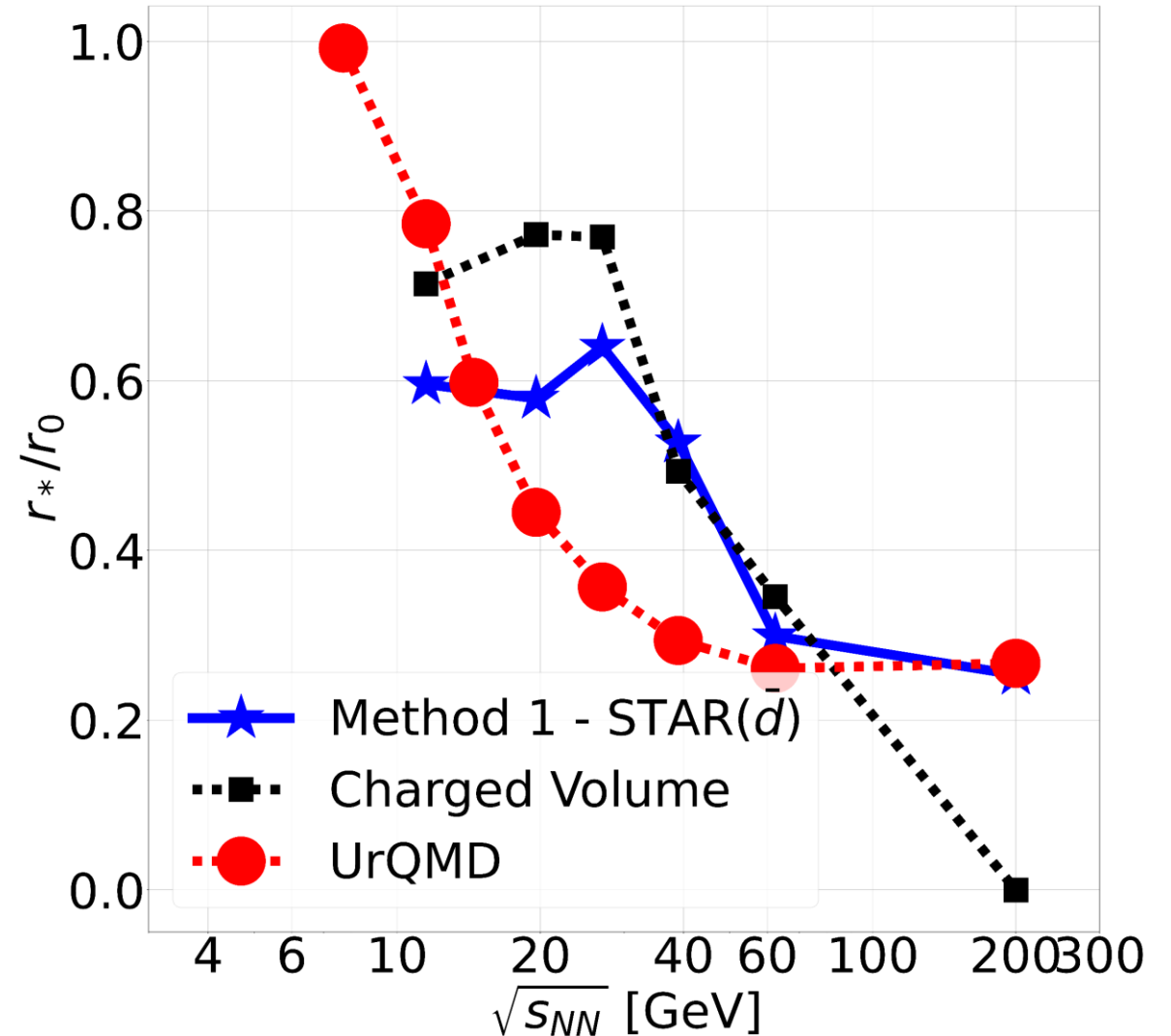
- $r_T$  - Distribution
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## Summary

- We use **Spatial Coalescence model** to extract the source geometry,  $(r_0, r^*)$  (Fig 3).
- We extract  $(r_0, r^*)$  from **the charged volume constraint** (Fig5).
- We extract the source geometry via **UrQMD** (Fig 8).

All results support our two explanations (Fig 9);

1. The antinucleons are emitted from the outer shell (supported by UrQMD).
2. The pionnic environment affects the source geometries.





# BACK UP SLIDES

## Introduction

- Heavy ion Collision
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## Summary

## 1. Density and Wave Function

## 2. Flow Effect

## 3. First Method Flow Chart

## 4. Second Method Flow Chart



# Density & Wave function

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## Summary

- **Hulthen Wave function:**  $\phi_d(r) = \left( \frac{\alpha\beta(\alpha-\beta)}{2\pi(\alpha+\beta)^2} \right)^{\frac{1}{2}} \left( \frac{e^{-\alpha r} - e^{-\beta r}}{r} \right)$  where  $\alpha = 0.23 \text{ fm}^{-1}$   $\beta = 1.61 \text{ fm}^{-1} - 1$

- **Formation rate in relative coordinate:**  $\mathcal{A}(r) \equiv (2\pi)^3 \int d^3r \mathcal{D}_r(r) |\phi_d(r)|^2$

$$\mathcal{D}_r(r) = \frac{1}{(4\pi)^{3/2} r_0^3} e^{-\frac{r^2}{4r_0^2}}$$

$$\bar{\mathcal{D}}_r(r) = \frac{r_0^3 e^{-\frac{r^2}{4r_0^2}} + r_*^3 e^{-\frac{r^2}{4r_*^2}} - \frac{2^{5/2} r_0^3 r_*^3}{3} e^{-\frac{r^2}{2(r_0^2 + r_*^2)}}}{(4\pi)^{3/2} (r_0^3 - r_*^3)^2}$$



# Flow Effect

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## Summary

In the Mrowczynski's model, the parameter  $r_0$  is to be replaced by

$$r'_0 = \sqrt{r_0^2 + v^2 \tau^2}$$

Here  $v$  is the velocity of the (anti-)deuterons and  $\tau$  is the freeze-out duration [7, 8].

[7] Maj, R., & Mrowczynski, S. (2009). Physical Review C, 80(3), 034907.

[8] Mrowczynski, S., & Slon, P. (2019). arXiv:1904.08320.

Flow effects become important only  $\sqrt{s_{NN}} \simeq 60$  GeV [9]

[9] Gaebel, V., Bonne, M., et al (2020). arXiv:2006.12951.



# Source Geometry

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## Summary

1. Convert  $B_2(E)$  data into formation rate  $\mathcal{A}(E)$

2. Calculate all possible formation  $\mathcal{A}(r_0, r_*)$  from any give  $r_0$  and  $r_*$

3. Fit  $\mathcal{A}(r_0)$  to extract  $r_0$  first. Then fit  $\bar{\mathcal{A}}(r_0, r_*)$  with one from the experiment  $\bar{\mathcal{A}}(E)$  to extract the radii.

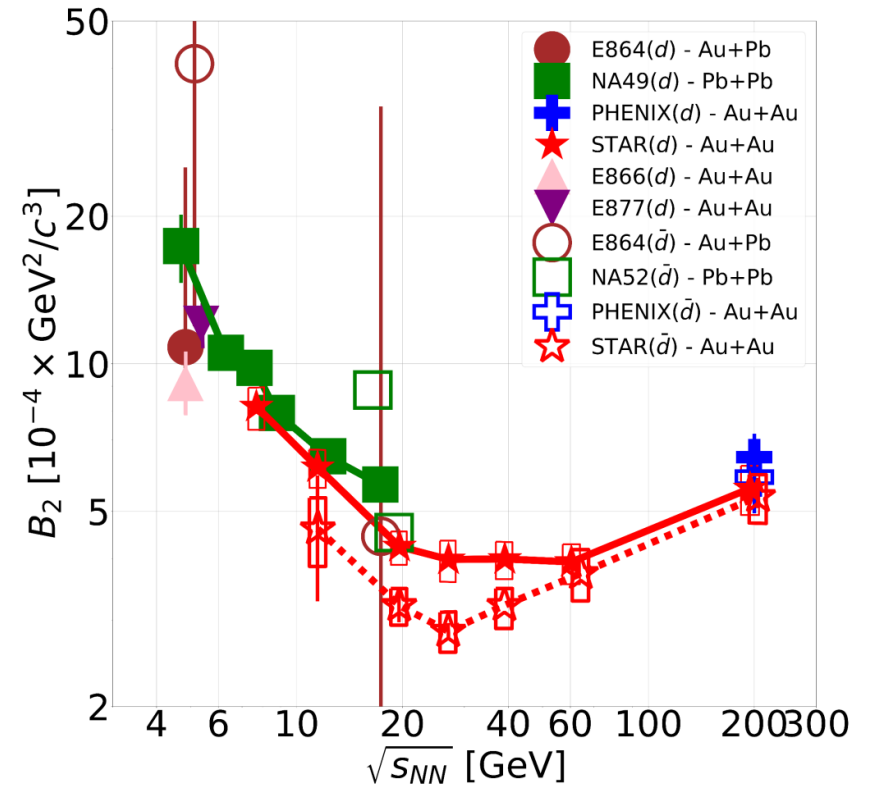


Figure 7: The  $B_2$  from experiment used for converting into formation rate as a function of  $\sqrt{s_{NN}}$ .



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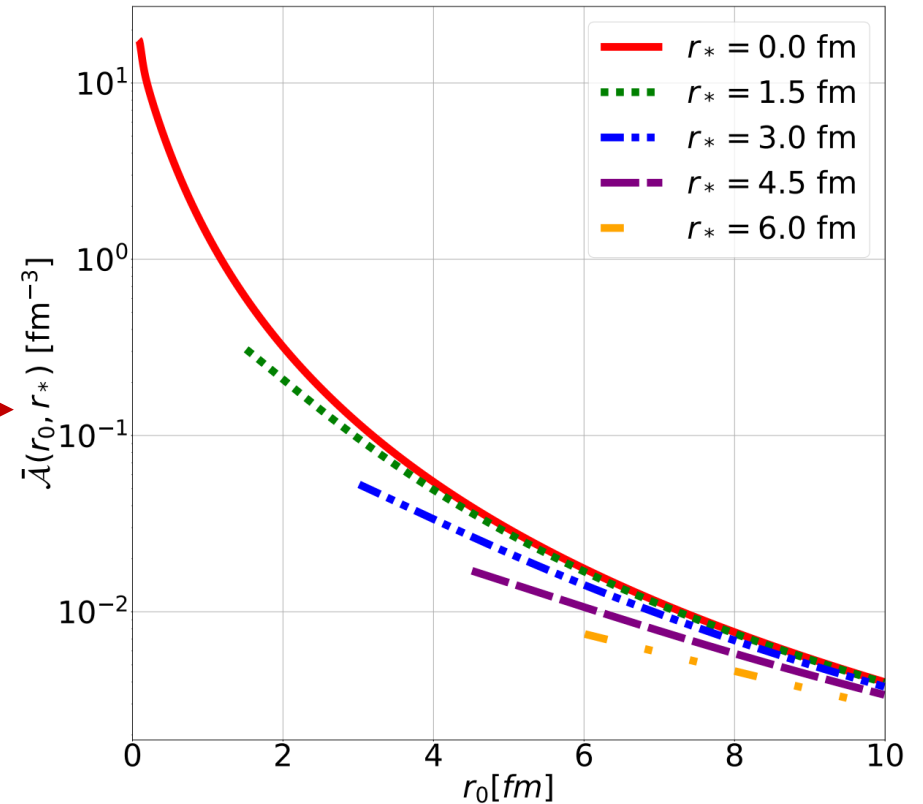


Figure 8: A few calculated formation rates varying  $r_*$  from 0-6 fm.



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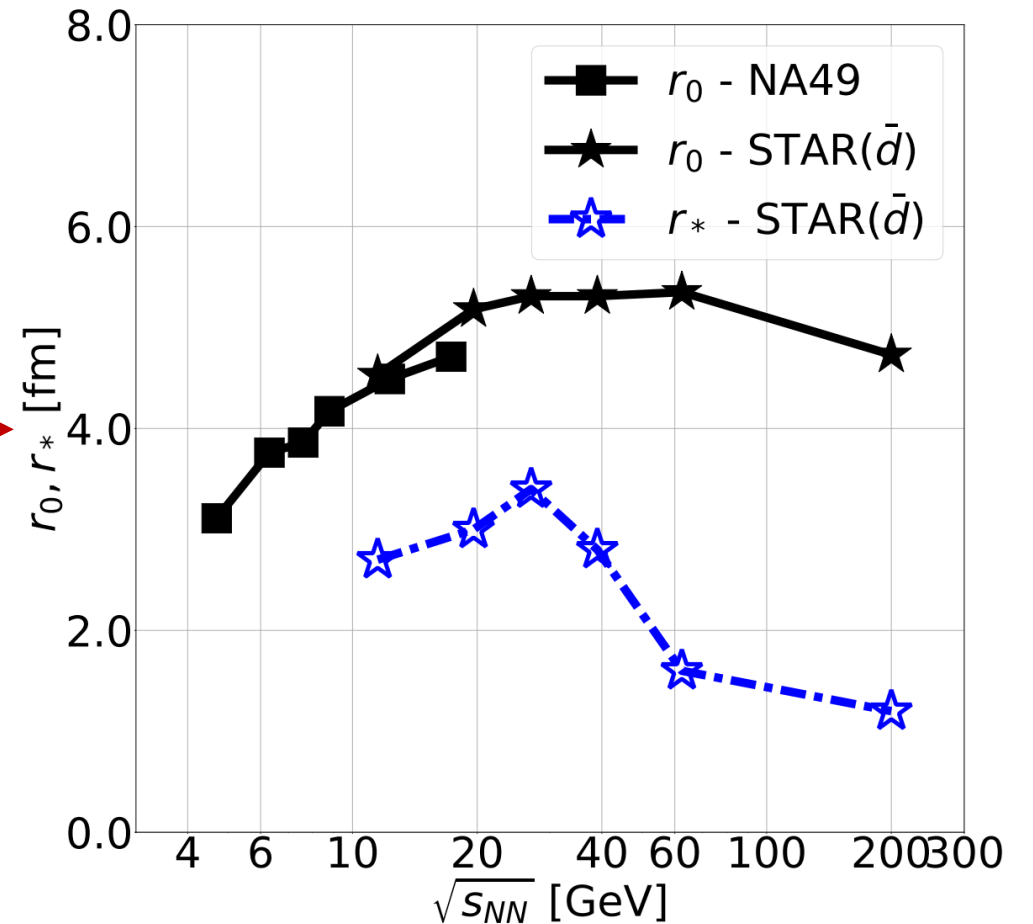
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## Summary

1. Convert Number of Charged-particle to RMS

2. Choose the scaling factor for  $const = 0.9$  such that  $r^*$  is zero at 200 GeV

3. Extract  $(r_0, r_*)$  pair simultaneously from

$$\mathcal{A}(r_0, r_*) \propto B_2$$

$$\langle r^2 \rangle(r_0, r_*) = Const. N_{ch}^{\frac{1}{3}}$$

$$\langle r^2 \rangle \propto N_{ch}^{\frac{1}{3}} \text{ where } \langle \bar{r}^2 \rangle = \sqrt{\frac{3(r_0^5 - r_*^5)}{(r_0^3 - r_*^3)}}$$





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