Fractal Aspects of QCD

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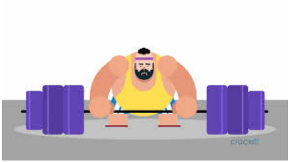
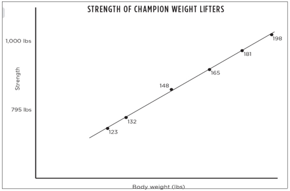
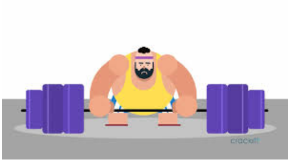
Scale and Self-Similarity



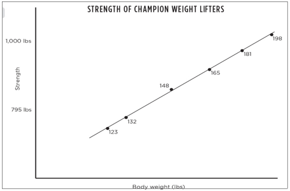
SCALING



Scale and Self-Similarity



SCALING

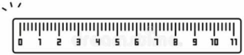


SELF-SIMILARITY

Scaling and dimensions

$$L_1 = 1.5\, m$$

$$L_2 = 152\, cm = 1.52 \times 10^2 \left(m/10^2\right)$$



$$L_3 = 1527\, mm = 1.527 \times 10^3 \left(m/10^3\right)$$

$$L_\lambda = L_o \times \lambda^1 \, (m/\lambda)$$

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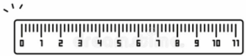
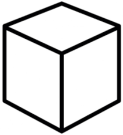


$L_3 = 1527\, mm = 1.527 \times 10^3 \left(m/10^3\right)$

$L_\lambda = L_o \times \lambda^1 \, (m/\lambda)$

$S_\lambda = S_o \times \lambda^3 \, (m/\lambda)^3$

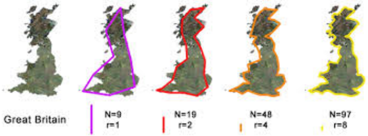
$S_\lambda = S_o \times \lambda^2 \, (m/\lambda)^2$



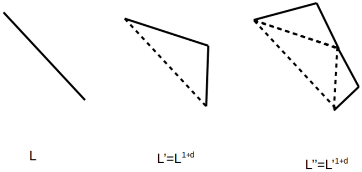
Fractal Borders

What are the lenghts of these coastlines?

Fractal Borders



Fractal Lenght



$$L' = L_o \lambda^{1+d} \left(l/\lambda\right)$$

Different shapes induces a fractal spectrum of dimensions

Renormalization of gauge fields

Yang-Mills theory is renormalizable:

$$\Gamma(p, m, g) = \lambda^{-D}\Gamma(p, \mu, \bar{g})$$

F. Dyson, PR 75 (1949) 1736

M. Gell-Mann and F.E. Low, PR 95 (1954) 1300

Renormalization group equation:

$$\left[M\frac{\partial}{\partial M} + \beta_g\frac{\partial}{\partial \bar{g}} + d\right]\Gamma = 0$$

Effective coupling constant $\bar{g}$

Effective mass μ

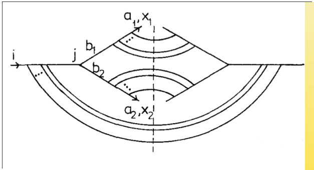
Callan-Symanzik Equation

C.G. Callan Jr., PRD 2 (1970) 1541

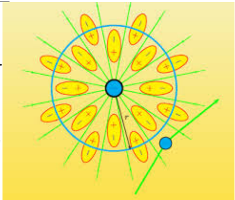
K. Symanzik, Comm. Math. Phys. 18 (1970) 227

Scaling properties are present in YMF

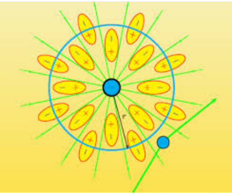
Is there internal structure?

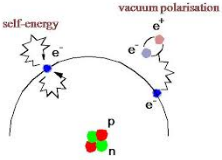


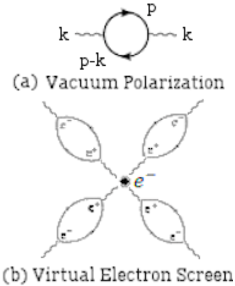
K. Konishi, Phys. Scr. 19 (1979) 195

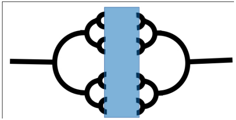


The effective parton

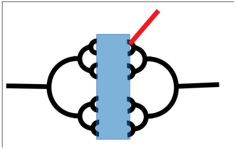








effective parton graph



proper vertex graph

Including fractal structure in YM fields

A.D., E..P.Menezes PRD 101, (2020) 034019

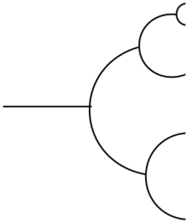
$$Z = Tr \left\langle \Psi_f | e^{-iHt} | \Psi \right\rangle_o$$

$$|\Psi\rangle = \sum_{\{n\}} \langle \Psi_n | \Psi \rangle \, |\Psi_n\rangle$$

n: order in pertubative calculation

$\{n\}$ the sum all graphs.

$$|\Psi_n\rangle = \frac{(-i)^n}{n!} e^{-iH_o(t_n - t_{n-1})} g \ldots e^{iH_o(t_2 - t_1)} g \, |\Psi_o\rangle$$



$$|\Psi_n\rangle = \sum_N \langle \psi_N | \Psi_n \rangle \, |\psi_N\rangle$$

N is the number of external lines

$$|\psi_N\rangle = \mathcal{S} \, |\gamma_1, m_1, p_1, \ldots, \gamma_N, m_N, p_N\rangle$$

$$\langle \psi_f | = \langle \gamma_o, m_o, p_o, \ldots |$$

$$\langle \gamma_o, m_o, p_o, \ldots | \Psi(t) \rangle = \sum_n \sum_N \langle \Psi_n | \Psi \rangle \, \langle \psi_N | \Psi_n \rangle \, \langle \gamma_o, m_o, p_o, \ldots | \psi_N \rangle$$

Including fractal structure in YM fields

A.D., E..P.Menezes PRD 101, (2020) 034019

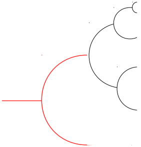
$$\langle \gamma_o, m_o, p_o, \ldots | \Psi(t) \rangle = \sum_n \sum_N \langle \Psi_n | \Psi \rangle \langle \psi_N | \Psi_n \rangle \langle \gamma_o, m_o, p_o, \ldots | \psi_N \rangle$$

$$\begin{cases} \langle \Psi_n | \Psi \rangle = G^n P(E) dE \\ \langle \psi_N | \Psi_n \rangle = A_N(n) \\ \langle \psi_f | = \langle \gamma_o, m_o, p_o, \ldots | \end{cases}$$

$$\langle \gamma_o, m_o, p_o, \ldots | \psi_N \rangle \to f(p_j) d^4 p_j$$

$$f(p_j) d^4 p_j = d^4 p_j \frac{1}{8\pi} \frac{\Gamma(4N)}{\Gamma(4(N-1))} E^{-4} \left(1 - \frac{p_j^0}{E}\right)^{4N-5}$$

$$\tilde{P}(p_o) = \langle \gamma_o, m_o, \ldots | \Psi \rangle = \sum_n \sum_N G^n \left(\frac{N}{n\tilde{N}}\right)^4 \left(1 - \frac{\varepsilon_j}{M}\right)^{4N-5} d^4 \left(\frac{p}{M}\right) P(E) dE$$



Introducing self-similarity

Parent parton is also a parton $\to P(E) \propto \tilde{P}(p_o)$.

Self-symmetry in gauge fields!

Scaling factor: $P(\frac{E}{\mathcal{E}}) = \tilde{P}(\frac{p_o}{E})$

$$\chi = \frac{\varepsilon}{\lambda} = \frac{p_j^o}{E} = \frac{E}{\mathcal{E}}$$

It can be show that $P(\mu)$ must be such that:

$$P(\varepsilon) = A[1 - (q-1)\frac{\varepsilon}{\lambda}]^{\frac{1}{q-1}}$$

AD, PRD (2016)

AD, E. Megías, D.P. Menezes, T. Frederico, Entropy 20 (2018) 633

Non extensivity in gauge field theory

$$P(\varepsilon) = G^n[1-(q-1)\tfrac{\varepsilon}{\lambda}]^{\frac{1}{q-1}}$$

Tsallis q-exponential function → Tsallis Statistics

AD, PRD (2016)

AD, E. Megías, D.P. Menezes, T. Frederico, Entropy 20 (2018) 633

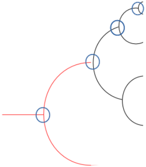
Thermofractals present similar behavior:

1) $\tilde{N}$ constituents and $U = F + E$

2) $\chi = E/F$ for type I or $\chi = E/U$ for type II, $P(\chi)$ is the same for all constituents

3) At some level n, $P(E_n)\, dE_n = \rho\, dE_n$

q is related to the number of internal degrees of freedom in the fractal structure that are relevant in the process of energy transfer to the effective parton



Suggest that at each vertex, momentum and effective

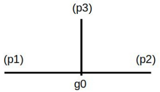
$$\bar{g} = \prod_i G\left[1-(q-1)\tfrac{\varepsilon_i}{k\tau}\right]^{\frac{1}{q-1}}$$

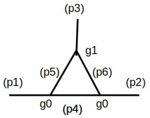
Effective coupling

Calculation of q from gauge field parameters

One loop calculation for QCD

Assume asymptotic limit and our ansatz for the effective coupling





From our ansatz we get: $\beta_g = -\frac{1}{q-1} g^{N'+1}$

From CS equation we get: $\frac{1}{q-1} = d - \gamma$

From QCD: $d - \gamma = \left[\frac{11}{3} c_1 - \frac{4}{3} c_2\right]$

And finally we get $q = 1.14$

Comparison with experiments

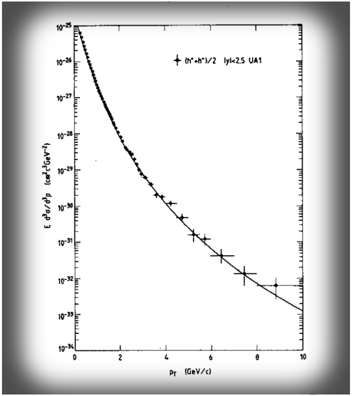
Extended Hagedorn theory to non extensive statistics: AD, Physica A 391 (2012) 6380

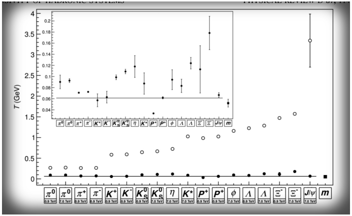
use of Tsallis factor: $P(\varepsilon) = A[1 + (q-1)\frac{\varepsilon}{k\tau}]^{-\frac{1}{q-1}}$

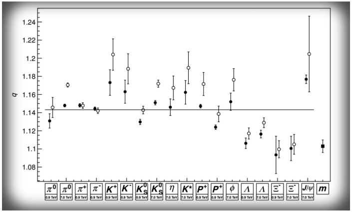
L. Marques, E. Andrade-II, AD, PRD 87 (2013) 114022

L. Marques, J. Cleymans, AD, PRD 91 (2015) 054025

Experimental value $q = 1.14 \pm 0.01$





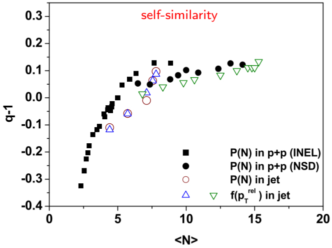


Experimental verification

Scale invariance of gauge theory

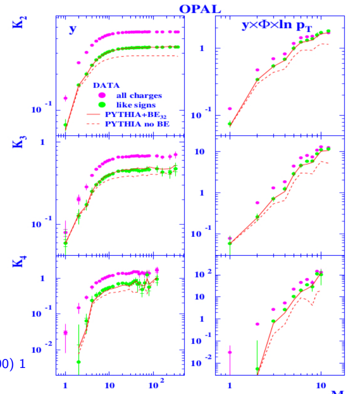
leads to fractal structure

fractal dimension in multiparticle production



G. Wilk and Z. Wlodarczyk, PLB 727 (2013) 163

fractal dimension - from intermittecy analysis



E. Sarkisyan-Grinbaum, PLB 477 (2000) 1

$D \sim 0.69$ AD PRD 93 (2016) 054001

Multiplicity and energy

Multiplicity as a manifestation of fractal aspects: AD, PRD 93 (2016) 054001

r is the scale in wich energies are measured.
$\varepsilon \sim r^{-D}$ is the scaling behavior of the individual parton energy.
$E \sim r^{-1}$ is the scaling behavior of the total energy.
$\mathcal{N}$ is the multiplicity.
R is the ratio between parton energy ε and its immediate parent energy
$\mathcal{N} r^{-D} \propto r^{-1} \rightarrow \mathcal{N} \propto r^{-1+D}$
$\mathcal{N} \propto E^{1-D}$
$D \sim 0.69$ from fractal dimension analysis and intermittence analysis

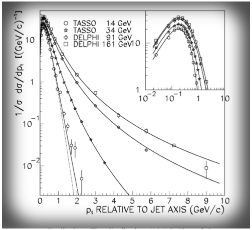
Theory: $1 - D \sim 0.31$

Experiment: $1 - D \sim 0.302$
E. Sarkisyan-Grinbaum et al. PRD 93 (2016) 054046

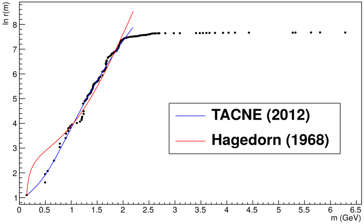
Experimental verification

Scale invariance of gauge theory

- leads to fractal structure
- fractal dimension in multiparticle production
- Tsallis statistics
- non extensive self-consistent thermodynamics



power-law distributions



non extensive mass spectrum

Conclusions:

Scale invariance in gauge fields leads to:

- Self-consistency and fractal struture
- Recursive calculations at any order
- Non extensive statistics
- Reconciles Hagedorn's theory with QCD
- Agreement with experimental data
- Short review is going to appear soon in
  - A.D, E. Megìas and D.P. Menezes, Physics (2020)

Thank you