On duality between quantum statistical and field-theoretic approaches

1911.04545, 1906.03529, 2003.11119

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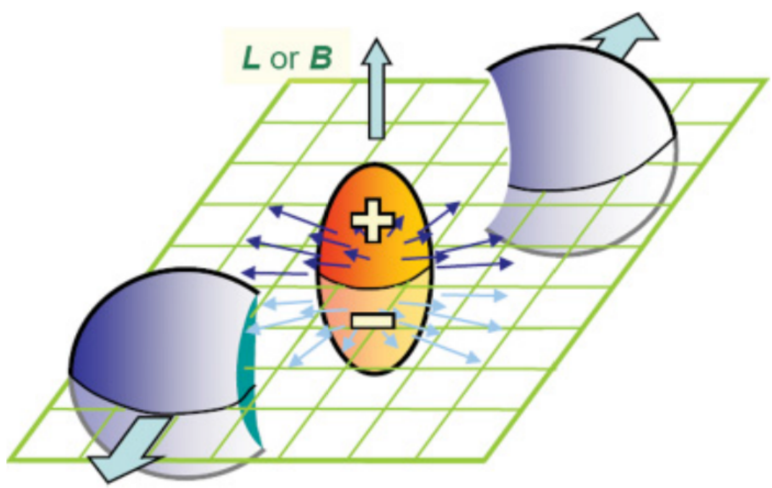
Introduction

Introduction: motivation from phenomenology

In noncentral collisions of heavy ions, **huge magnetic fields** and a **huge angular momentum** arise. *Differential* rotation - different at different points: **vorticity** and vortices.

– Rotation 25 orders of magnitude *faster* than the rotation of the Earth:

the vorticity is about 10^{22} sec^{-1}



- **Acceleration** is of the **same order** of magnitude as vorticity (as the components of the same tensor): contributes to polarization.

[I. Karpenko and F. Becattini, Nucl. Phys., A982:519–522, 2019]

- *Another mechanism*: accelerations due to the tension of the hadron string (acceleration is much higher – of the order of Λ_{QCD})

$$e^+e^- \to \gamma^* \to q\bar{q} \to \text{ hadrons}$$

[P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J., C52:187–201, 2007]

Introduction: (some) methods for studying vorticity and acceleration effects

Flat space

1. Kubo formulas.
[K. Landsteiner, et al. Lect. Notes Phys., 871:433–468, 2013], [S. Golkar, et al. JHEP, 02:169, 2015]

2. Zubarev quantum-statistical density operator.
[M. Buzzegoli, et al. JHEP, 10:091, 2017], [G. P., O. V. Teryaev, and V. I. Zakharov. JHEP, 03:137, 2020]

3. Wigner function for a medium with thermal vorticity.
[F. Becattini, et al. Annals Phys., 338:32–49,2013], [W. Florkowski, et al. Prog.Part.Nucl.Phys.,108:103709, 2019]

Curved space

4. Field theory in curved space (with a conical singularity).
[V. P. Frolov, et al. Phys. Rev., D35:3779–3782, 1987], [J. S. Dowker. Class. Quant. Grav., 11:L55–L60, 1994]

5. Field theory in space of a (rotating) black hole and gravitational anomaly on the horizon of a black hole.
[M. Stone, et al. Phys. Rev., D98(2):025012, 2018], [S. P. Robinson, et al. Phys. Rev. Lett., 95:011303, 2005]

6. Other methods (holography, quantization in cylindrical coordinates ...)
[M.N.Chernodub, et al. Phys.Rev.D98(2018)no 6,065016], [K. Landsteiner, et al. Phys.Rev.Lett.,107:021601, 2011]

Introduction: the density operator for a medium with a thermal vorticity tensor

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta\widehat{Q}\right]$$

Generators of Lorentz transformations

$$\widehat{J}^{\mu\nu} = \int_\Sigma \mathrm{d}\Sigma_\lambda \left(x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu}\right)$$

$$\varpi_{\mu\nu}\hat{J}^{\mu\nu} = -2\alpha^\rho \hat{K}_\rho - 2w^\rho \hat{J}_\rho$$

$\hat{K}^\mu$ – boost (associated with **acceleration**)

$\hat{J}^\mu$ – angular momentum (associated with **vorticity**)

Temperature and **acceleration** are **independent** parameters: different temperature ranges can be considered in an accelerated medium

in contrast of:

Unruh effect: if a Minkowski vacuum is created, then medium temperature **is related** to acceleration

$$T_U = \frac{|a|}{2\pi}$$

Emergent conical geometry in the Zubarev density operator

Effects of acceleration from Zubarev operator

Perturbation theory in **boost generator** was constructed to describe **acceleration** effects

$$\langle \hat{O}(x) \rangle = \langle \hat{O}(0) \rangle_{\beta(x)} + \sum_{N=1}^{\infty} \frac{(-1)^N a^N}{N!} \int_0^{|\beta|} d\tau_1 d\tau_2 \ldots d\tau_N \langle T_\tau \hat{K}_{-i\tau_1 u} \ldots \hat{K}_{-i\tau_N u} \hat{O}(0) \rangle_{\beta(x),c} \,.$$

In the fourth order of perturbation theory, the following terms are possible:

$$\begin{aligned} \langle \hat{T}^{\mu\nu} \rangle &= (\rho_0 + A_1 T^2 |a|^2 + A_2 |a|^4) u^\mu u^\nu - (p_0 + A_3 T^2 |a|^2 + A_4 |a|^4) \Delta^{\mu\nu} \\ &\quad + (A_5 T^2 + A_6 |a|^2) a^\mu a^\nu + \mathcal{O}(a^6) \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \,, \end{aligned}$$

Second-order terms were found in
[M. Buzzegoli, E. Grossi, and F. Becattini, JHEP, 10: 091, 2017]

- We have found the **4th order** terms for **Dirac** fields, for **scalar** fields – the calculation of 5-point correlators.

[G. Y., O. V. Teryaev, and V. I. Zakharov. JHEP, 03:137, 2020]

Effects of acceleration from Zubarev operator: results

$$\langle \hat{T}^{\mu\nu} \rangle^0_{\text{fermi}} = \Big(\frac{7\pi^2 T^4}{60} + \frac{T^2|a|^2}{24} - \frac{17|a|^4}{960\pi^2}\Big) u^\mu u^\nu - \Big(\frac{7\pi^2 T^4}{180} + \frac{T^2|a|^2}{72} - \frac{17|a|^4}{2880\pi^2}\Big) \Delta^{\mu\nu} + \mathcal{O}(a^6)$$

$$\langle \hat{T}^{\mu\nu} \rangle^0_{\text{real}} = \Big(\frac{\pi^2 T^4}{30} + \frac{T^2|a|^2}{12} - \frac{11|a|^4}{480\pi^2}\Big) u^\mu u^\nu - \Big(\frac{\pi^2 T^4}{90} - \frac{T^2|a|^2}{18} + \frac{19|a|^4}{1440\pi^2}\Big) \Delta^{\mu\nu} + \Big(\frac{T^2}{12} - \frac{|a|^2}{48\pi^2}\Big) a^\mu a^\nu + \mathcal{O}(a^6)\,.$$

The energy-momentum tensor **vanishes** at the **Unruh temperature**

$$\boxed{\langle \hat{T}^{\mu\nu} \rangle = 0 \qquad (T = T_U)}$$

Thus, a consequence of the **Unruh effect** is **justified**.

[G. Y., O. V. Teryaev, and V. I. Zakharov. JHEP, 03:137, 2020]

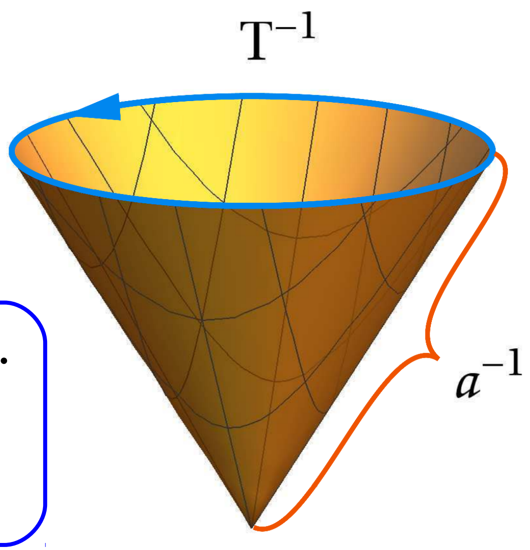
Space-time with conical singularity

The effects of acceleration can also be investigated from the point of view of an **accelerated observer**. In this case, the **Rindler coordinates** are to be used:

$$ds^2 = -r^2 d\eta^2 + dr^2 + d\mathbf{x}_\perp^2$$

Passing to imaginary time: $$ds^2 = \boxed{r^2 d\eta^2 + dr^2} + d\mathbf{x}_\perp^2$$

It describes a flat two-dimensional cone with an angular deficit $2\pi - a/T$. This metric contains a **conical singularity** at $r = 0$.



Dictionary for translation
Thermodynamic characteristics in *Geometrical*:

Inverse **acceleration** $\Longleftrightarrow$ **distance** from the **vertex**.

Inverse proper **temperature** $\Longleftrightarrow$ **circumference**.

[G. Y., O. V. Teryaev, and V. I. Zakharov. JHEP, 03:137, 2020]

The duality of statistical and geometric approaches

The following expressions were obtained for the vacuum value of T_2^2 in spacetime with a cosmic string [V. P. Frolov and E. M. Serebryanyi, Phys. Rev. D 35, 3779 (1987)]

String:

$$\langle T_2^2 \rangle_{s=0} = \frac{\nu^4}{480\pi^2 r^4} + \frac{\nu^2}{48\pi^2 r^4} - \frac{11}{480\pi^2 r^4}$$
$$\langle T_2^2 \rangle_{s=1/2} = \frac{7\nu^4}{960\pi^2 r^4} + \frac{\nu^2}{96\pi^2 r^4} - \frac{17}{960\pi^2 r^4}$$

The coincidence will be for **massive** fields as well

Passing to the Euclidean Rindler spacetime, we obtain

Rindler:

$$\rho_{s=0} = \frac{\pi^2 T^4}{30} + \frac{T^2 |a|^2}{12} - \frac{11|a|^4}{480\pi^2},$$
$$\rho_{s=1/2} = \frac{7\pi^2 T^4}{60} + \frac{T^2 |a|^2}{24} - \frac{17|a|^4}{960\pi^2}$$

= Zubarev

The obtained expressions **correspond** to the energy calculated using the **Zubarev density operator** in inertial frame.

[G. Y., O. V. Teryaev, and V. I. Zakharov. JHEP, 03:137, 2020]

The duality of statistical and geometric approaches

The duality of two approaches has been discovered: statistical and geometrical.

Despite the correspondence of the results, theories are ***different***: in statistics, the perturbation theory was considered for **effective interaction with the boost** generator, and the **curvilinear** coordinates were **not used** in any way.

[G. Y., O. V. Teryaev, and V. I. Zakharov. JHEP, 03:137, 2020]

Instability below Unruh temperature

Instability below Unruh temperature: acceleration as imaginary chemical potential

The density operator and the covariant Wigner function [F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, Annals Phys., 338:32-49, 2013] lead to the **integral representation** of energy density

$$\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} = 2\int \frac{d^3p}{(2\pi)^3}\left(\frac{|\mathbf{p}| + ia}{1+e^{\frac{|\mathbf{p}|}{T}+\frac{ia}{2T}}} + \frac{|\mathbf{p}| - ia}{1+e^{\frac{|\mathbf{p}|}{T}-\frac{ia}{2T}}}\right)$$

$$+4\int \frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}}-1} \qquad (T > T_U)$$

in red: *modifications* compared to the *Wigner function*

- In the first integral, the **acceleration** enters as an **imaginary chemical potential** $\pm\frac{ia}{2}$ [G.P., O. Teryaev, V. Zakharov, Phys. Rev. D 98, no. 7, 071901 (2018)].

Motivation:

- **Exact match** with the fundamental result from the density operator at $T > T_U$.
- True limit at $a \to 0$.
- Some terms can be obtained directly from the Wigner function.

[G. P., O. V. Teryaev, V. I. Zakharov, Phys. Rev., D100(12):125009, 2019]

Instability below Unruh temperature: jump of the derivative

Comparing the two formulas

$$\rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} \qquad T > T_U$$

$$\rho = \frac{127\pi^2 T^4}{60} - \frac{11 T^2 a^2}{24} - \frac{17 a^4}{960\pi^2} - \pi T^3 a + \frac{T a^3}{4\pi} \qquad T < T_U$$

we see that the values in $T = T_U$ coincide, also the first derivatives turn out to be the same, however, a **jump** occurs in the **second derivative**:

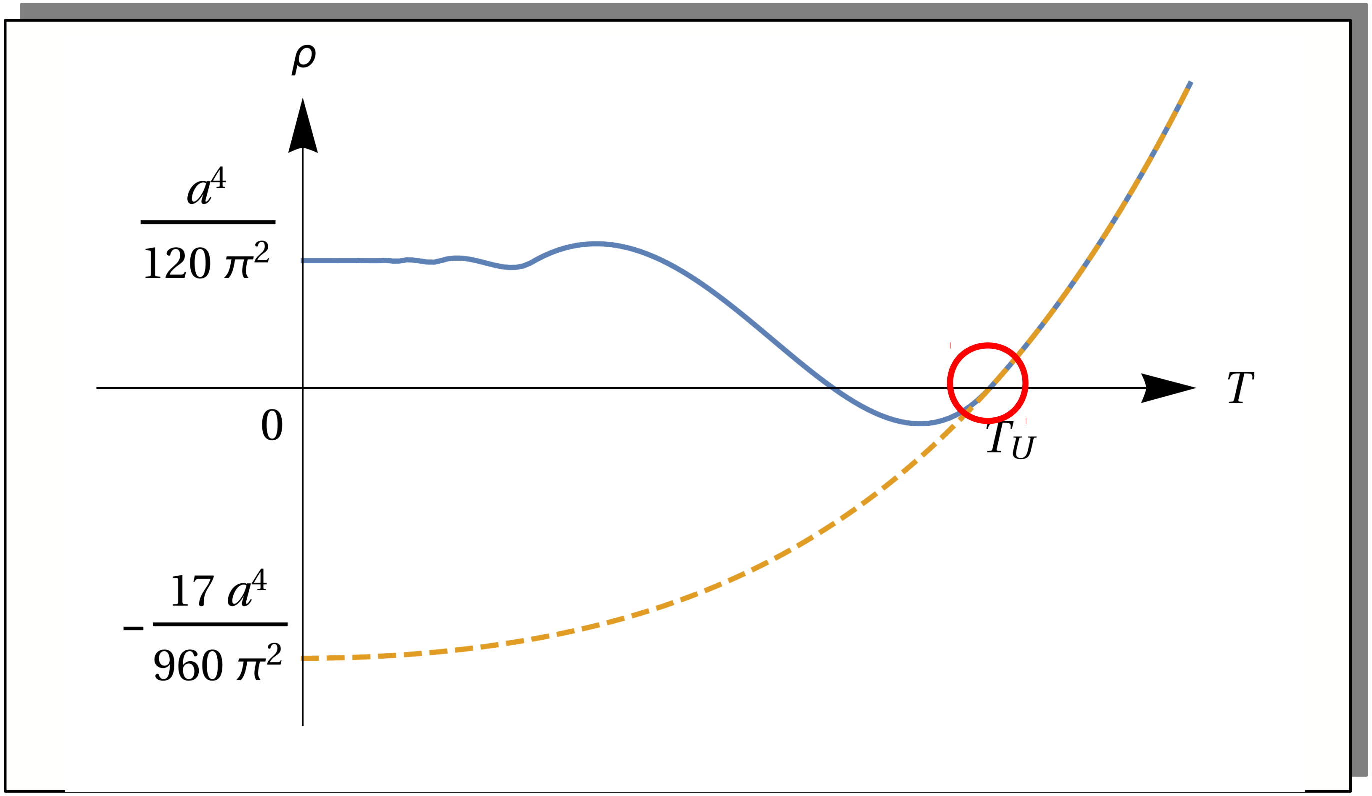
$$\rho_{T>T_U}(T \to T_U) = \rho_{T<T_U}(T \to T_U) = 0\,,$$

$$\frac{\partial}{\partial T}\rho_{T>T_U}(T \to T_U) = \frac{\partial}{\partial T}\rho_{T<T_U}(T \to T_U) = \frac{a^3}{10\pi}\,,$$

$$\frac{\partial^2}{\partial T^2}\rho_{T>T_U}(T \to T_U) \neq \frac{\partial^2}{\partial T^2}\rho_{T<T_U}(T \to T_U)\,,$$

[G. P., O. V. Teryaev, V. I. Zakharov, Phys. Rev., D100(12):125009, 2019]

Instability below Unruh temperature: jump of the derivative

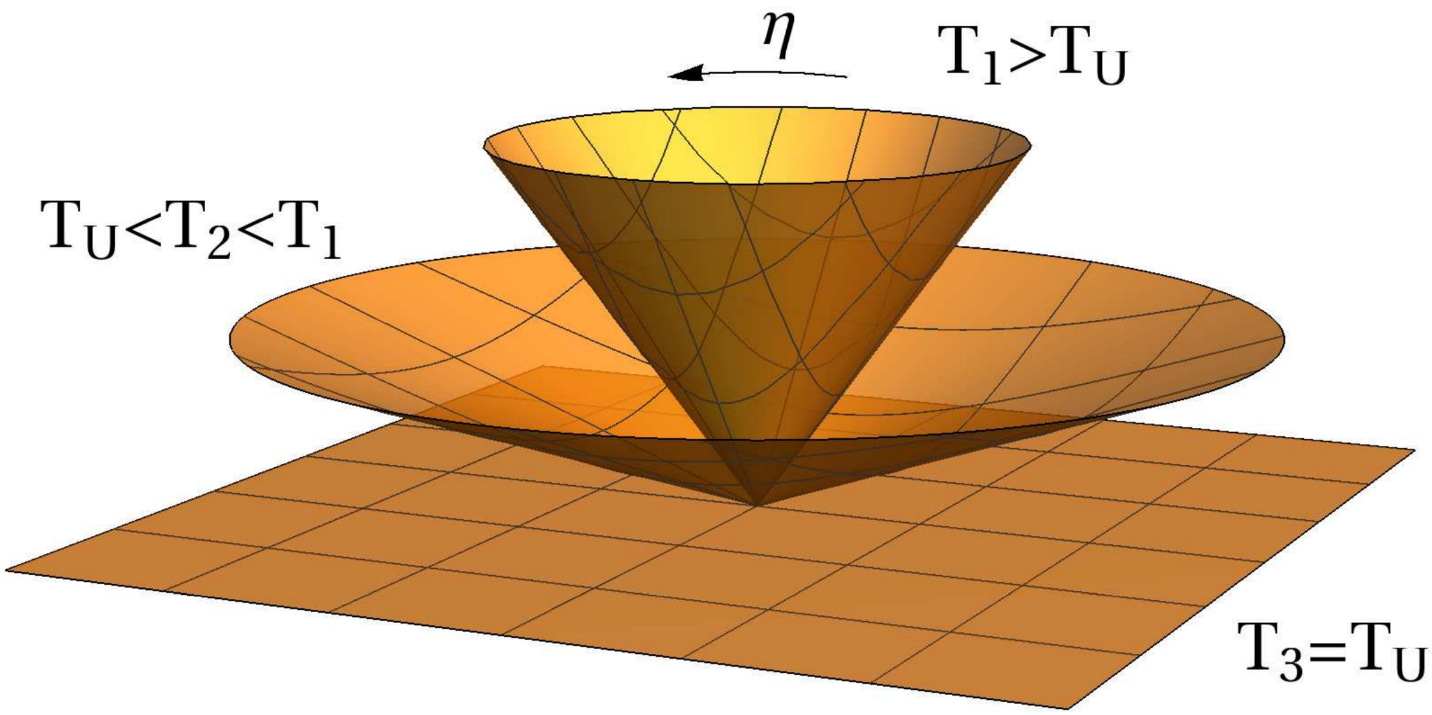


The solid blue line is the energy density as a function of temperature, corresponding to the integral representation.
The dashed orange line - the result of the fundamental calculation based on the density operator.

[G. P., O. V. Teryaev, V. I. Zakharov, Phys. Rev., D100(12):125009, 2019]

Instability at Unruh temperature: source in geometry

In the geometrical approach at $T = T_U$ the **cone** transforms into a **plane**.
Statistical instability is accompanied by a qualitative **change** in **geometry**.



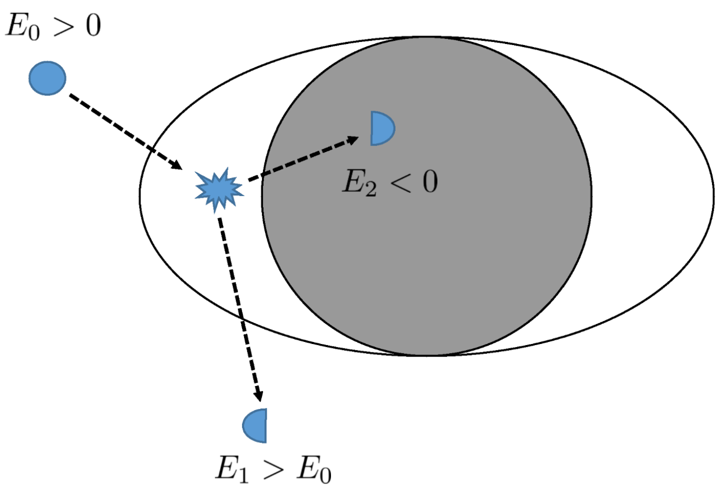
[G. P., O. V. Teryaev, V. I. Zakharov, Phys. Rev., D100(12):125009, 2019]

Instability at Unruh temperature: parallels

At $T = T_U$, the energy density is zero as an evidence of the Unruh effect, while at $T < T_U$ it becomes **negative** *(F. Becattini, Phys. Rev. D 97, no. 8, 085013 (2018))* - *evidence of* ***instability***.

Negative energy leads to instability: **Superradiance**
(R. Penrose Nuovo Cimento.J. Serie 1 (1969) 252.)

One of the two particles (photons) into which the particle decays will have an energy **greater** than the original particle, since the second of the particles has gone to a level with **negative energy** in the *ergosphere* and then moved into BH.



Brito, Richard et al. Lect.Notes Phys. 906 (2015) pp.1-237

Investigation of vacuum stability *(G. L. Pimentel, A. M. Polyakov and G. M. Tarnopolsky, Rev. Math. Phys. 30, no. 07, 1840013 (2018))*.

Analytical continuation into instability region allowed to show the **smoothness** of the transition

$$S_{eff}(v) = mT\sqrt{1 - V^2}$$

At $T < T_U$ We also: construct analytical continuation, have discontinuty at the point, show smoothness.

[G. P., O. V. Teryaev, V. I. Zakharov, Phys. Rev., D100(12):125009, 2019]

Instability below Unruh temperature and Unruh effect

Effect of instability below the Unruh temperature appears since temperature and acceleration can be considered as independent parameters, unlike **Unruh effect**.

So these two effects are different (though being related).

Phenomenological consequences?

Hypothetical decay of unstable state (below the Unruh temperature) as a source of hadronisation → similar to the picture

[P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J., C52:187–201, 2007]

Higher spins and gravitational anomaly

CVE for spin ½: duality

Gravitational anomaly

$$\nabla_\mu j_5^\mu = -\frac{Q^2e^2}{16\pi^2\sqrt{-g}}\varepsilon^{\mu\mu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\mu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

Holography

- *[K. Landsteiner, E. Megias, F. Pena-Benitez. Phys. Rev. Lett. 107, 021601 (2011).]*
- *[M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).]*
- *[S. P. Robinson, F. Wilczek Phys.Rev.Lett. 95 (2005) 011303 MIT-CTP-3561 gr-qc/0502074.]*

Although the gravitational chiral anomaly **is not important in volume**, it is **significant** at the boundary or at the **horizon**.

Gauge anomaly

$$j_5^\mu = n_5 u^\mu + \frac{\mu^2+\mu_5^2}{2\pi^2}\omega^\mu + \frac{\mu}{2\pi^2}B^\mu$$

[D. T. Son, et al. Phys. Rev. Lett., 103:191601, 2009]
[A. V. Sadofyev, et al. Phys. Rev., D83:105025, 2011]

Gravitational anomaly

$$j_5^\lambda = \left(\frac{T^2}{6} + \frac{\mu^2+\mu_5^2}{2\pi^2}\right)\omega^\lambda$$

CVE for spin ½: duality

Gravitational anomaly for spin ½:

[Luis Alvarez-Gaume, et al. Nucl. Phys., B234:269, 1984]

$$\boxed{\nabla^\alpha j_\alpha^N = \frac{1}{768\pi^2\sqrt{-g}}\epsilon^{\alpha\beta\gamma\delta}R_{\alpha\beta\rho\sigma}R^{\rho\sigma}_{\gamma\delta}}$$

According to *[M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018)]* the coefficient before the anomaly determines the coefficient T^2 :

Weyl fermions:

$$\vec{j}^N_{CVE} = \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)\vec{\Omega}$$

Correspondence of **statistical** and **gravitational** approaches (with a **gravitational anomaly** on the horizon)!

CVE for spin 1: problem of the factor 2

- Chiral current for **spin 1** (*magnetic helicity*):

$$K^\mu = \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma$$

Other definitions of chirality:

[M. N. Chernodub, et al. Phys.Rev. D98 (2018) no.6, 065016]

- **Chiral anomaly for spin 1:**

[A. I. Vainshtein, A. D. Dolgov, V. I. Zakharov, and I. B. Khriplovich, Sov. Phys. JETP 67 (1988) 1326, Zh. Eksp. Teor. Fiz. 94 (1988) 54]

$$< \nabla_\mu K^\mu > = \frac{1}{96\pi^2} R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma}$$

- According to *[M. Stone and J. Kim, Phys. Rev. D98, no. 2, 025012 (2018).]* the coefficient before the anomaly determines the coefficient T^2

$$\frac{(CVE)_{photons}}{(CVE)_{Weyl\ \ fermions}}\Big|_{black\ \ hole} = 4$$

$$\frac{(CVE)_{photons}}{(CVE)_{Weyl\ \ spinor}}\Big|_{Kubo\ \ relation} = 2$$

[A. Avkhadiev, et al. Phys.Rev. D96 (2017) no.4, 045015]

CVE for spin 1: sources of the problem of the factor 2 and possible solutions

- **Infrared** effects and **zero mode**?

Landau levels in *magneic* field:

$$\boxed{E_n = \pm\sqrt{2H(n+1/2) + P_3^2 + H\sigma_3}}$$

In the gravitational case the gyromagnetic ratio is **two times smaller** and there is **no zero mode for spin ½.** *However it may exist for spin 1:*

$$\boxed{E_{min} = 0\,, \quad spin\ 1\,,\ gravity}$$

- **Regularization** method?

$$1/R \ll \Omega \ll T$$

one may consider *another* case:

$$\boxed{\Omega \ll 1/R \ll T}$$

[M. N. Chernodub, et al. Phys.Rev. D98 (2018) no.6, 065016]

- The **duality** $T \leftrightarrow \frac{a}{2\pi}$ of temperature and acceleration?

$$\boxed{J_{CVE} \sim c_1 T^2\Omega + c_2 a^2 \Omega}$$

may also give *cubic dependence* on spin S^3 (next slide)

Higher spins

Gravitational anomaly for **arbitrary** spin:

[M. J. Duff, Cambridge Univ. Press, 1982, preprint Ref.TH.3232-CERN]

[A.I. Vainshtein, A.D. Dolgov, V. I. Zakharov, and I.B. Khriplovich, Sov. Phys. JETP 67 (1988) 1326, Zh. Eksp. Teor. Fiz. 94 (1988) 54]

$$\nabla_\mu K^\mu_S = \frac{(-1)^{2S}(2S^3 - S)}{192\pi^2\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

- Used in supersymmetry
- Checked in other approaches for 3/2

Prediction for **vortical** chiral **current** (CVE) of **arbitrary spin**:

$$\vec{K}_S = \frac{(-1)^{2S}(2S^3 - S)}{3}T^2\vec{\Omega}\ (gravitational\ anomaly)$$

The statistical approach **does not** reproduce the **cubic dependence** S^3

[X. G. Huang, et al. JHEP 03, 084 (2019)]

Conclusions

Conclusions

1. The **duality** of the statistical approach (the **Zubarev** density operator) and the geometrical one (field theory in space with a conical singularity) is shown.

2. The existence of **instability** in the accelerated medium is shown at temperatures **below the Unruh temperature**.

3. A problem in establishing the **duality** between the **statistical approach** in flat space and the **geometrical** one (radiation from the horizon of a rotating black hole due to the **gravitational anomaly**) in the case of **spin 1** is revealed. *Sources* and possible *solutions* are investigated.

4. A quantitative prediction is made for the **CVE of an arbitrary spin** based on the connection with the **gravitational anomaly for an arbitrary spin**.

Conclusions

An accelerated medium has **instability**, as indicated by both statistics and geometry, at temperatures **below the Unruh temperature** (*phase transition?*), which may have important consequences for the physics of heavy ions.

Thank you for the attention!