

# Multiplicity dependence of quarkonia and open heavy flavor mesons

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This talk is partially based on materials published in

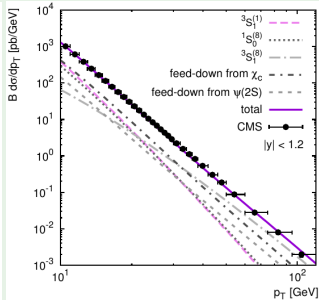
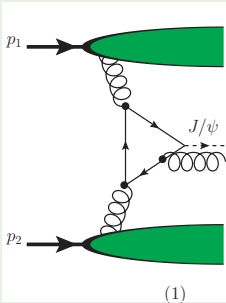
Eur.Phys.J. C79 (2019) no.5, 376, Phys.Rev.D 101 (2020) 9, 094020

Eur.Phys.J.C 80 (2020) 6, 560.

# $J/\psi$ production in $pp$ collisions (short summary)

(see QWG review, Eur.Phys.J. C71 (2011) 1534)

## Color Singlet Model & NRQCD corrections



## CSM (alone):

- Reasonable description for:
  - small- $p_T$  observables
  - $p_T$ -integrated observables

- Wrong behavior for

$$p_T \gg m_{J/\psi}$$

(EPJC 79 (2019) 241)

## NRQCD:

- Description for large- $p_T$

(Reggeized) gluon=(cut) pomeron

## Other mechanisms at large $p_T$ ?

- Co-production ( $J/\psi + \bar{Q}Q, \dots$ )

(PRL 101 (2008) 152001)

- Quark and gluon fragmentation

(EPJC 79 (2019) 241)

- ... ?

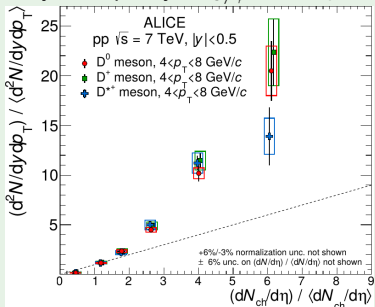
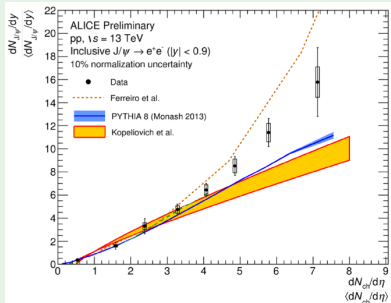
[Phys. Rev. D 96, no. 3, 034019 (2017)]:

## Challenge for NRQCD: LDMEs not universal ?!

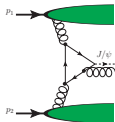
	$\langle \mathcal{O}^v [^3S_1^{(1)}] \rangle / \text{GeV}^3$	Color octet contributions		
		$\langle \mathcal{O}^v [^1S_0^{(8)}] \rangle / \text{GeV}^3$	$\langle \mathcal{O}^v [^3S_1^{(8)}] \rangle / \text{GeV}^3$	$\langle \mathcal{O}^v [^3P_0^{(8)}] \rangle / \text{GeV}^3$
<b>kT-factorization</b>				
A0	1.97	0.0	$9.01 \times 10^{-4}$	0.0
JH	1.62	$1.71 \times 10^{-2}$	$2.83 \times 10^{-4}$	0.0
KMR	1.58	$8.35 \times 10^{-3}$	$2.32 \times 10^{-4}$	0.0
<b>Collinear factorization</b>				
[11]	1.32	$3.04 \times 10^{-2}$	$1.68 \times 10^{-3}$	$-9.08 \times 10^{-3}$
[15]	1.16	$9.7 \times 10^{-2}$	$-4.6 \times 10^{-3}$	$-2.14 \times 10^{-2}$

# Multiplicity dependence of charmonia production [ALICE, 1811.01535]

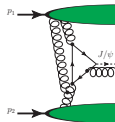
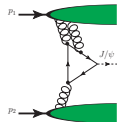
- Observable:  $J/\psi$  + charged hadrons, study multiplicity  $dN_{J/\psi}$  vs.  $dN_{ch}$



- Enhancement (deviation from linear) seen both for  $J/\psi$  and  $D$  mesons
- So far not clear if the effect exists for  $\psi(2S)$ ,  $\chi_c$ ,  $\Upsilon$ ?



- Difficult to explain in terms of gluon-gluon fusion approach: if each reggeized gluon (cut pomeron) contributes approx. equal number  $\bar{n}$  of charged hadrons, expect milder dependence
- Data hints that multipomeron mechanisms are pronounced (usually discarded as corrections).
- Production on nuclei:  $\sim A^{1/3}$  enhancement

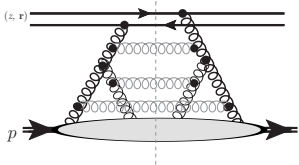


- We used CGC/Sat (color dipole) framework for our evaluations.
  - ▶ In the literature it is frequently assumed that the dipole cross-sections are universal object which already takes into account all possible interactions with the target.
  - ▶ Before we continue discussion, we'll stop briefly on what is included into dipole cross-section and what is not.
  - ▶ Also we'll consider how the heavy dipole scattering cross-section depends on the number of (charged) particles  $dN_{\text{ch}}$  produced per rapidity bin  $d\eta$  during the scattering.

# Dipole approach: what it includes and what doesn't

- BK equation for the rapidity evolution

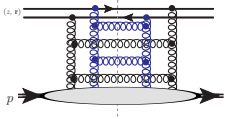
► effectively resums the fan-like diagrams as shown in the Figure



## b-CGC parametrization:

► Constructed as interpolation of asymptotic solutions of BK equation

► extra gluon attachment to quarks which are **not** included in *b*-CGC:



## High multiplicity events:

► Relation of saturation scale  $Q_s$  to observed multiplicity dependence

$$\frac{dN_{\text{ch}}}{dy} \approx \text{const } N_P \frac{Q_s^2}{\bar{\alpha}_S(Q_s)}$$

▷  $N_P$ -number of cut pomerons

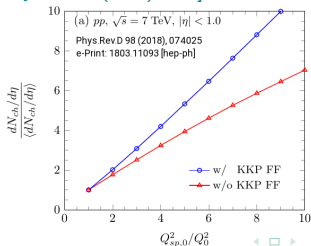
⇒ Modification of saturation scale:

$$Q_s^2(x, b; n) \approx n Q^2(x, b),$$

$$n = dN_{\text{ch}} / \langle dN_{\text{ch}} \rangle$$

(modulo small logarithmic corrections)

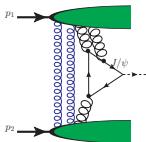
[Phys.Rev.D 98 (2018) 7, 074025, Phys.Lett.B 710 (2012) 125, Eur.Phys.J.C 71 (2011) 1699]



# Multiplicity enhancement mechanisms

● Local Parton Hadron Duality:  
 $dN_{\text{ch}} \sim N_{\text{partons}} \sim N_{IP}$  (number of cut pomerons)

► Pomeron disconnected from hard process (blue) after resummation give a common factor  $P(n)$ , same as in inclusive production without  $J/\psi$



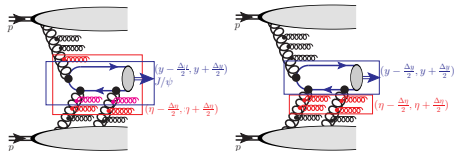
► Experimental results are given for self-normalized ratio

$$\frac{dN_{\mathbf{M}}/dy}{\langle dN_{\mathbf{M}}/dy \rangle} = \frac{d\sigma_{\mathbf{M}}(y, \eta, \sqrt{s}, n)/dy}{d\sigma_{\mathbf{M}}(y, \eta, \sqrt{s}, \langle n \rangle = 1)/dy} \left( \frac{d\sigma_{\text{ch}}(\eta, \sqrt{s}, Q^2, n)/d\eta}{d\sigma_{\text{ch}}(\eta, \sqrt{s}, Q^2, \langle n \rangle = 1)/d\eta} \right)^{-1}$$

so  $P(n)$  cancels

⇒ should focus only on cut pomerons connected to hard amplitude

The result for multiplicity depends if the bin used to collect charged particles (red in plot) overlaps with a bin used to collect quarkonia (blue in plot):



► If bins overlap, enhanced multiplicity is shared by all pomerons (summation over all possible partitions is implied)

► If bins are separated, enhanced multiplicity is shared by all pomerons

► If  $n$  is not very large, dipole size  $\langle r \rangle \sim 1/m_Q$ , so each cut pomeron contributes a factor  $\sim (r Q_s)^{\gamma_{\text{eff}}} \sim n_i^{\gamma_{\text{eff}}}$

► If  $n \gg 1$ , dipole size  $\langle r \rangle \sim 1/Q_s$ , so the multiplicity dependence saturates

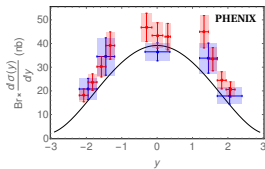
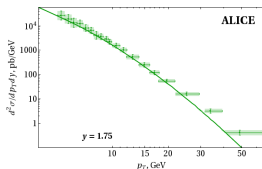
► Coupling of pomeron to heavy quarks  $\sim \alpha_s(m_Q)/m_Q$ , suppressed in  $m_Q \rightarrow \infty$  limit

# $J/\psi$ production through 2-pomeron fusion

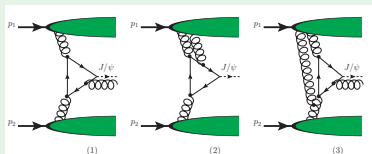
[PRC95 (2017), 065203]

$$\begin{aligned}
 \frac{d\sigma_{pp}}{dy d^2p_T} &= \frac{9}{8} g(x_1(y)) \int d^2\mathbf{b} e^{i\mathbf{p}_T \cdot \mathbf{b}} \int d\alpha_G d\alpha_1 d^2r_1 d\alpha_2 d^2r_2 d^2\rho \\
 &\times \underbrace{\Psi_M^*(\alpha_1 r_1) \Psi_M(\alpha_2 r_2)}_{\text{Meson WFs}} \sum_{n,n'=1}^6 \underbrace{\eta_n \eta_{n'}}_{\text{numerical factor}} \underbrace{N(y, r_n - r_{n'} + \mathbf{b})}_{\text{dipole cross-section}} \\
 &\times \text{Tr} \left[ \Lambda_M \Phi_{g \rightarrow \bar{Q}Q}(\epsilon_n, \vec{r}_n^{(1)}) \Phi_{Q \rightarrow Qg}(\delta_n, \vec{\rho}_n) \right] \\
 &\times \text{Tr} \left[ \Lambda_M \Phi_{g \rightarrow \bar{Q}Q}(\epsilon_{n'}, \vec{r}_{n'}^{(2)}) \Phi_{Q \rightarrow Qg}(\delta_{n'}, \vec{\rho}_{n'}) \right]^*
 \end{aligned}$$

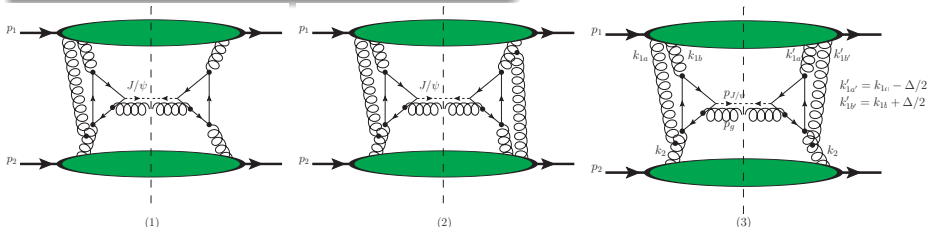
- $\Lambda_M$ -spin projector on meson WF
- $\Phi_{g \rightarrow \bar{Q}Q}$ ,  $\Phi_{Q \rightarrow Qg}$  are evaluated perturbatively ( $m_c \rightarrow \infty$  limit)
- $\vec{r}_n^{(1,2)} \approx \vec{r}_{1,2} + \delta\vec{r}_n(\alpha, \alpha_G, r, \rho)$ ,  $\vec{\rho}_n \approx \vec{\rho} + \delta\vec{\rho}_n(\alpha, \alpha_G, r, \rho)$
- Sum over all possible gluon attachments in amplitude and its conjugate is implied
- Reasonable description of  $pp$  data:



# 3-pomeron contributions



- Diagram (2) contributes additively to  $d\sigma$
- Diagrams (1) and (3) might interfere:

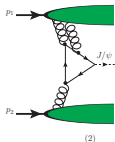


- Diagrams (1) and (2) include Parton Distributions with odd number of gluons
- ... such distributions are sensitive if the proton is polarized [PLB 668 (2008), 216, PRD 78 (2008), 014024 , PRD 78 (2008), 114013].
- ... corresponding asymmetries are small, might neglect them [PHENIX Collaboration, PRD 82, 112008 (2010), PRD 86 (2012), 099904]

⇒ Additive contribution to the LO cross-section, as shown in (3).



## 3-gluon fusion in dipole approach



$$\begin{aligned}
 \frac{d\sigma(Y, Q^2)}{dy d^2q_T} &= 4 \int \frac{d^2 Q_T}{(2\pi)^2} S_h^2(Q_T) x_g G(x_g, M_{J/\psi}) \\
 &\times \int_0^1 dz \int_0^1 dz' \int \frac{d^2 r}{4\pi} \frac{d^2 r'}{4\pi} d^2 b e^{-iq_T \cdot b} \\
 &\times \langle \Psi_g(r, z) \Psi_M(r, z) \rangle \langle \Psi_g(r', z') \Psi_M(r', z') \rangle \left( N\left(y; \mathbf{b} - \frac{1}{2}(\mathbf{r} - \mathbf{r}')\right) \right. \\
 &\left. + N\left(y; \mathbf{b} + \frac{1}{2}(\mathbf{r} - \mathbf{r}')\right) - N\left(y; \mathbf{b} + \frac{1}{2}(\mathbf{r} + \mathbf{r}')\right) - N\left(y; \mathbf{b} - \frac{1}{2}(\mathbf{r} + \mathbf{r}')\right) \right)^2 \\
 &+ (y \rightarrow Y - y)
 \end{aligned}$$

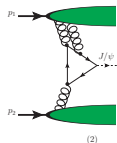
- **Note:** Results are defined up to nonperturbative normalization factor  $\sim \int \frac{d^2 Q_T}{(2\pi)^2} S_h^2(Q_T)$ , related to Fourier image of cut pomeron Green function. This factor can be related to  $\sigma_{\text{eff}}$  which appears in models of DPDF & “pocket formula” for Double Parton Scattering,

$$\mathcal{F}(x_{1a}, k_{1a, T}, x_{1b}, k_{1b, T}) \sim \frac{1}{\sigma_{\text{eff}}} \mathcal{F}(x_{1a}, k_{1a, T}) \mathcal{F}(x_{1b}, k_{1b, T}),$$

$$\Rightarrow \int \frac{d^2 Q_T}{(2\pi)^2} S_h^2(Q_T) \sim (\text{const}) \sigma_{\text{eff}}^{-1}$$

- **Caveat:** the “universal” value of  $\sigma_{\text{eff}}$  is not that universal; typical values are between 5 mb and 20 mb.

# Fixing uncertainty from diffractive photoproduction



$$\frac{d\sigma(Y, Q^2)}{dy} = \left( \int \dots \right) \langle \Psi_g(r, z) \Psi_{J/\psi}(r, z) \rangle \langle \Psi_g(r', z') \Psi_{J/\psi}(r', z') \rangle$$

$$\times \left( N\left(y; \frac{r-r'}{2}\right) - N\left(y; \frac{r+r'}{2}\right) \right)^2$$

- Use diffractive photoproduction ( $\gamma^* p \rightarrow J/\psi X$ ) data to fix normalization:

$$\frac{d\sigma_{\text{diff}}(Y, Q^2)}{dy} = \left( \int \dots \right) \langle \Psi_{\gamma^*}(r, z) \Psi_{J/\psi}(r, z) \rangle \langle \Psi_{\gamma^*}(r', z') \Psi_{J/\psi}(r', z') \rangle$$

$$\times N(y; r) N(y; r')$$

For  $Q^2 \rightarrow \infty$ ,  $m_c \rightarrow \infty$ ,  $r \rightarrow 0$ , can use pQCD expression for both  $\Psi_{\gamma^*} \sim \Psi_g$ .

- The dipole amplitude  $N(y; r) \sim (Q_s^2 r^2)^{\bar{\gamma}}$  at small  $r$ , so cross-sections become proportional to each other  $\Rightarrow$  Can fix *hadro*production cross-section  $d\sigma$  from diffractive *photoproduction*  $d\sigma_{\text{diff}}$  studied at HERA:

$d\sigma(y, Q^2)/dy$	Theoretical estimates	Experiment
$\sqrt{s} \approx 1.96$ TeV	2.1-2.6 $\mu\text{b}$	2.38 $\mu\text{b}$ [67]
$\sqrt{s} \approx 7$ TeV	3.8-5.6 $\mu\text{b}$	5.8 $\mu\text{b}$ [68]

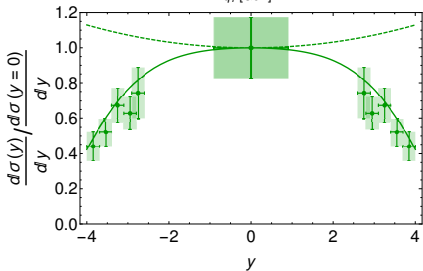
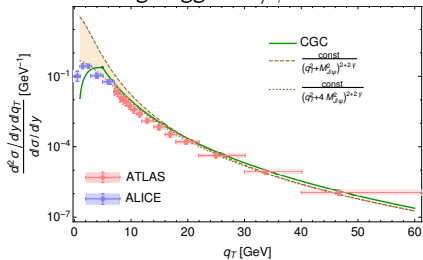
- Theoretical estimates: three-pomeron fusion gives sizeable contribution

- [Eur.Phys.J. C75 (2015) no.5, 213] (in  $k_T$ -fact): qualitatively agree; smaller due to larger  $\sigma_{\text{eff}}$

- In what follows will focus on shapes for  $p_T$ , rapidity, multiplicity dependence

# 3-pomeron $J/\psi$ production with gluon emission

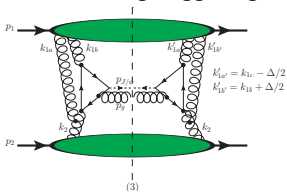
## ● Result for $g + gg \rightarrow J/\psi$



● BFKL: each gluon  $\sim x^\alpha$  (dashed)

● Experiment: add endpoint suppression factor  $\sim (1-x)^5$  (solid)

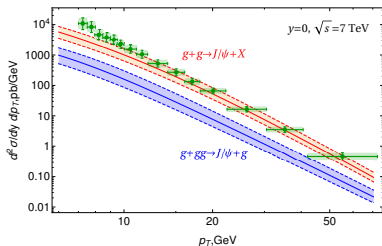
## ● Result for $g + gg \rightarrow g + J/\psi$



▶ Formally suppressed by  $\alpha_s$

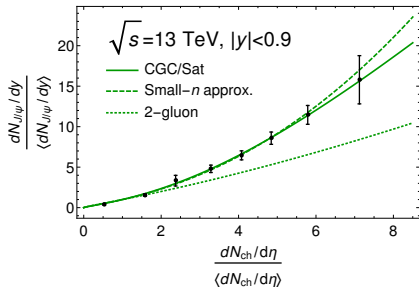
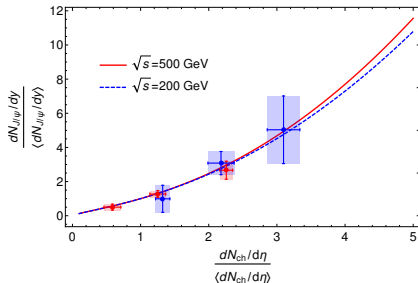
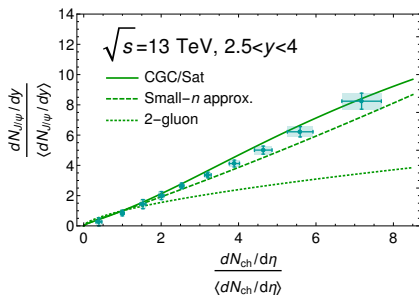
▶ See [J.Phys.G 46 (2019) 6, 065002] for details

● Result for the cross-section:



●  $\Rightarrow$  Contribution is sizeable, yet smaller than  $g+gg \rightarrow J/\psi$  (due to  $\mathcal{O}(\alpha_s(m_c))$ )

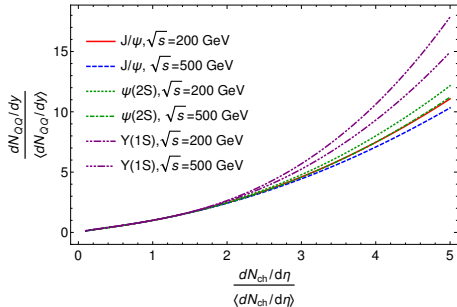
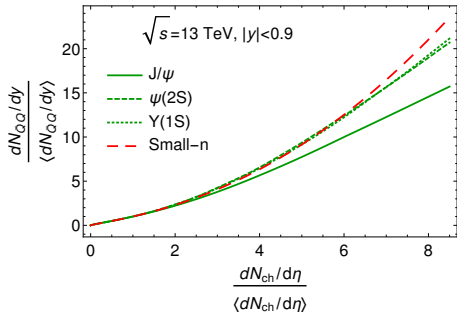
# Multiplicity dependence for $J/\psi$



- Reasonable description of available data on multiplicity dependence from ALICE (both central & forward multiplicities) as well as STAR data at lower energies

- We believe that similar measurements of multiplicity dependencies might be done for  $\psi(2S)$ ,  $\Upsilon(nS)$ . These data would help to confirm the role of the 3-pomeron mechanism in their formation.

# Multiplicity dependence for other quarkonia (predictions)



Expected multiplicity dependence of other hadrons is similar to that of  $J/\psi$ :

► The typical dipole size  $\langle r \rangle \sim \min(m_Q^{-1}, Q_s^{-1}(x, n))$ , significantly smaller than the quarkonium size  $\langle r_M \rangle \sim (m_Q \alpha_s(m_Q))^{-1}$ .

⇒ The quarkonium wave function might be approximated as  $\psi_M(r) \approx \psi_M(0) + \mathcal{O}(r^2)$ , cancels in self-normalized ratio

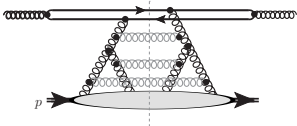
► The residual dependence on quarkonia is due to  $\sim \mathcal{O}\left(\frac{\langle r^2 \rangle}{\langle r_M^2 \rangle}\right)$  corrections beyond the heavy quark mass limit

▷ At small  $n$ ,  $\langle r^2 \rangle \sim m_Q^{-2}$ , so  $\langle r^2 \rangle / \langle r_M^2 \rangle \sim \alpha_s^2(m_Q)$ -no energy dependence, mild log-dependence on  $m_Q$

▷ At large  $n$ ,  $\langle r^2 \rangle \sim 1/Q_s^2$ , so  $\langle r^2 \rangle / \langle r_M^2 \rangle \sim (m_Q^2 \alpha_s^2(m_Q) / Q_s^2)$ -pronounced dependence on  $m_Q$  and energy (through  $Q_s$ )

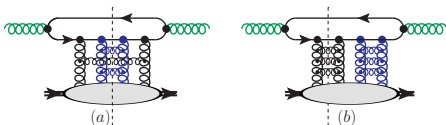
## D- and B-mesons

- Leading order (2-Pomeron fusion):



- ▶ CGC/Sat (dipole) approach: the bCGC takes into account certain fan-like topologies

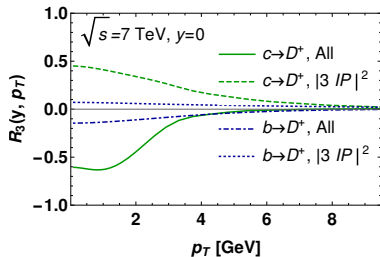
- 3-pomeron corrections:



- suppressed by  $\sim \alpha_s (m_Q)$
- Two types of contributions:
  - (a) “genuine” three-pomeron fusion, same as for  $J/\psi$
  - (b) interference diagram
- Have opposite signs, comparable magnitude. Different number of cut pomerons is relevant for multiplicity dependence.

- Ratio of 3-pomeron to 2-pomeron contributions:

$$R^{(3)}(y, p_T) = \frac{d\sigma^{(3)}/dy dp_T}{d\sigma^{(2)}/dy dp_T}$$

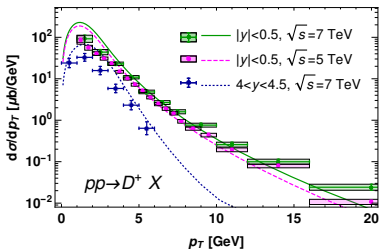


- “3IP” is “genuine” contribution
- “All” includes also interference diagrams

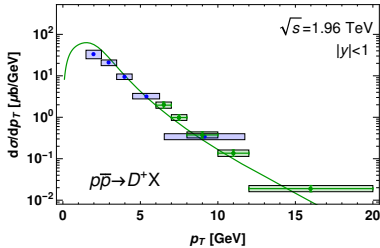
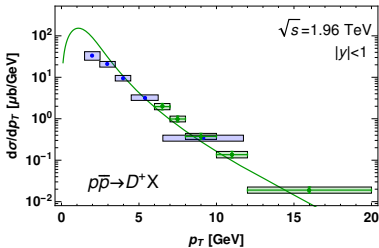
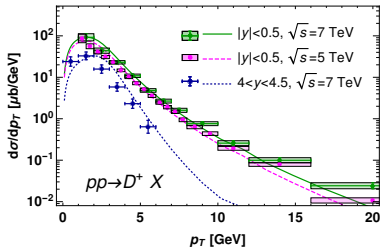
- Effect is sizeable for charm at small  $p_T$ 
  - ▶ Decreases at large  $p_T$
  - ▶ Decreases as a function of mass  $m_Q$  (so for bottom quarks the 3-pomeron contribution is small)

# 3-pomeron contributions vs data

Results w/o 3-pomeron contribution:



Results with 3-pomeron contribution:

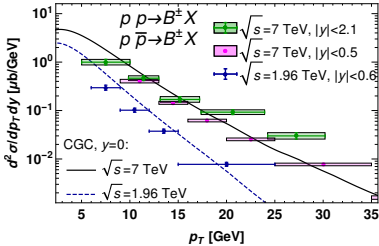


● overestimate the data by factor  $\sim 2$  at  $p_T \lesssim 3$  GeV (dominant in  $p_T$ -integrated cross-section)

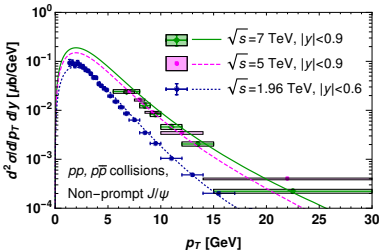
● Inclusion of 3-pomeron contribution definitely improves small- $p_T$  description

# Comparison with inclusive data

## B-meson production



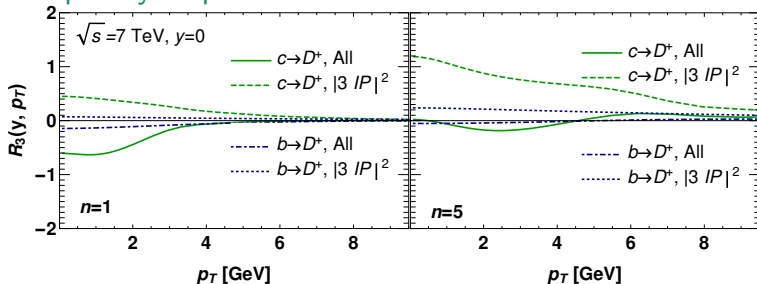
## Non-prompt charmonia production



- Very reasonable description for  $B$ -mesons and non-prompt  $J/\psi$  achieved



# Multiplicity dependence



- Ratio of 3-pomeron to 2-pomeron contributions which we defined earlier:

$$R^{(3)}(y, p_T) = \frac{d\sigma^{(3)}/dy dp_T}{d\sigma^{(2)}/dy dp_T}.$$

- ▶ "3IP" is "genuine" contribution
- ▶ "All" includes also interference diagram
- ▶  $n = (dN_{\text{ch}}/dy)/\langle dN_{\text{ch}}/dy \rangle$ -relative enhancement of multiplicity

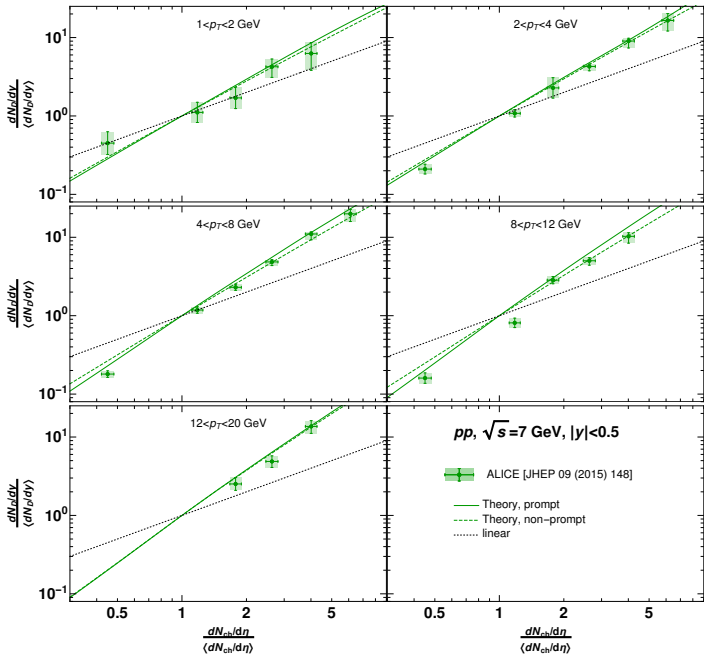
## Multiplicity dependence

(overlapping bins @central rapidity):

- ▶ The leading twist contribution  $\sim n^{2\gamma}$
- ▶ The "genuine" tw-3 contribution grows as  $\sim n^{3\gamma}$
- ▶ The interference term grows as  $\sim n^{2\gamma}$ . It is *negative*, for  $n \sim 1$  is twice larger than genuine contribution
- ⇒ The last two mechanisms partially cancel, for  $n \sim 5$  almost complete cancellation
- ▶ For  $n \gtrsim 6$  the "genuine" contribution becomes dominant, grows rapidly with  $n$ .

● Reasonable description of multiplicity dependence from ALICE

► Theoretical curves include both 2- and 3-pomeron contributions



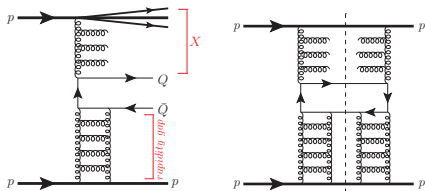
# Summary

- We found that the three-pomeron terms give sizeable contribution in LHC kinematics, especially for charm in small- $p_T$  kinematics.

The inclusion of three-pomeron contributions:

- ▶ Improves description of inclusive data
- ▶ Allows to explain the multiplicity dependence seen both in open heavy flavor and quarkonia production, without additional model assumptions

- Possible way to measure the 3-pomeron fusion mechanism only: Single Diffractive production,  $pp \rightarrow p + (\text{rapidity gap}) + \{D, B, J/\psi\} X$



- ▶ Evaluation is similar to inclusive production via 3-pomeron fusion (though differs in kinematical factors)

▷ See our recent [arXiv:2008.12446](https://arxiv.org/abs/2008.12446) for details & predictions

*Thank You for your attention!*