New Results in Models with Reduced Couplings

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9^{th} International Conference on New Frontiers in Physics 2020

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arXiv:2002.10983 (hep-ph)

10 September 2020

The ad hoc Yukawa and Higgs sectors of the Standard Model induce $\sim$ 20 free parameters. How can they be related to the gauge sector in a more *fundamental* level?

The straightforward way to induce relations among parameters is to add more symmetries.

$\rightarrow$ i.e. GUTs.

Another approach is to look for renormalization group invariant (RGI) relations among couplings at the GUT scale that hold up to the Planck scale.

$\rightarrow$ less free parameters $\rightarrow$ more predictive theories

Reduction of Couplings

About *dimensionless* couplings: an RGI expression among couplings $\mathcal{F}(g_1, ..., g_N) = 0$ must satisfy the pde

$$\mu \frac{d\mathcal{F}}{d\mu} = \sum_{a=1}^{N} \beta_a \frac{\partial \mathcal{F}}{\partial g_a} = 0$$

There are $(N-1)$ independent $\mathcal{F}s$ and finding them is equivalent to solve the ode

$$\beta_g \left(\frac{dg_a}{dg} \right) = \beta_a, \qquad a = 1, ..., N$$

where g is the primary coupling $\rightarrow$ *Reduction Equations (RE)*. *Zimmermann (1985)*

Ansatz: power series solutions to the REs (which preserve perturbative renormalizability):

$$g_a = \sum_n \rho_a^{(n)} g^{2n+1}$$

Examining in 1-loop is sufficient for uniqueness to all loops

Oehme, Sibold, Zimmermann (1984); (1985)

Finiteness

SM $\rightarrow$ quadratic divergences
SUSY $\rightarrow$ only logarithmic divergences
Finite theories $\rightarrow$ no divergences

For a chiral, anomaly free, $N = 1$ theory the superpotential is:

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k$$

$N = 1$ non-renormalization theorem $\rightarrow$ no mass and cubic-interaction-terms infinities $\rightarrow$ only wave-function infinities.

The 1-loop gauge β-functions are given by

$$\beta_g^{(1)} = \frac{g^3}{16\pi^2} \left[\sum_i T(R_i) - 3C_2(G) \right]$$

The Yukawa β-functions are related to the anomalous dimensions of the matter fields:

$$\beta_{ijk}^{(1)} = C_{ijl} \gamma_k^l + C_{ikl} \gamma_j^l + C_{jkl} \gamma_i^l \qquad \gamma_j^{i(1)} = \frac{1}{32\pi^2} \left[C^{ikl} C_{jkl} - 3g^2 C_2(R) \delta_j^i \right]$$

In 1-loop, all β-functions of the theory vanish if the 1-loop gauge β-functions and the anomalous dimensions of all superfields vanish, imposing the conditions:

$$\sum_i T(R_i) = 3C_2(G) \quad , \qquad C_{ikl}C^{jkl} = 2\delta^i_j g^2 C_2(R_i)$$

$\rightarrow$ The gauge and Yukawa sectors of the theory are now related (Gauge-Yukawa Unification - GYU).

- 1-loop finiteness is sufficient for 2-loop finiteness *Parkes, West (1984)*
    - 2-loop corrections for matter fields vanish if 1-loop finite $\rightarrow$ sufficient for $\beta_g^{(2)} = 0 = \beta_{ijk}^{(2)}$
- $C_2[U(1)] = 0 \rightarrow$ finiteness cannot be achieved in the MSSM $\rightarrow$ GUT
- $C_2[singlet] = 0 \rightarrow$ supersymmetry can be broken only softly.

All-loop Finiteness

Theorem — *Lucchesi, Piguet, Sibold (1988)*

Consider an N=1 supersymmetric Yang-Mills theory with simple gauge group. If:

1. There is no gauge anomaly
2. The gauge β-function vanishes at 1-loop $\beta_g^{(1)} = 0$
3. All superfield anomalous dimensions vanish at 1-loop $\gamma_j^{i(1)} = 0$
4. The *RE*s admit uniquely determined power series solution that in lowest order is a solution of the vanishing anomalous dimensions
   - $C_{ijk} = \rho_{ijk} g$
   - these solutions are *isolated and non-degenerate* when considered as solutions of vanishing one-loop Yukawa β-functions

then the associated Yang-Mills models depend on the single coupling constant g with a β-function which *vanishes at all orders*.

Soft supersymmetry breaking terms

What about *dimensionful* parameters?

The soft supersymmetry breaking sector introduces more than 100 new free parameters.

Reduction can be *extended* to the dimensionful sector.

Kubo, Mondragon, Zoupanos (1996)

$\rightarrow$ Consider a $N = 1$ supersymmetric gauge theory with soft terms:

$$-\mathcal{L}_{SSB} = \frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}(m^2)^j_i\phi^{*i}\phi_j + \frac{1}{2}M\lambda\lambda + h.c.$$

In addition to $\beta_g^{(1)} = 0 = \gamma_j^{i(1)}$, one-loop finiteness can be achieved if we demand:

$$h^{ijk} = -MC^{ijk} \qquad\qquad (m^2)^j_i = \frac{1}{3}MM^*\delta^j_i$$

Jones, Mezincescu, Yao (1984)

Like in the dimensionless case, the above one-loop conditions are also sufficient for 2-loop finiteness.

Jack, Jones (1994)

However, the soft scalar masses universal rule leads to phenomenological problems:

- charge and colour breaking vacua
- Incompatible with radiative electroweak breaking

Assuming

- 1-loop finiteness in the dimensionless sector $\beta_g^{(1)} = \gamma_j^{i(1)} = 0$
- the REs $\beta_C^{ijk} = \beta_g \frac{dC^{ijk}}{dg}$ admit power series solutions $C^{ijk} = g \sum \rho_{(n)}^{ijk} g^{2n}$
- the soft scalar masses satisfy the diagonality relation $(m^2)^i_j = m_j^2 \delta^i_j$

Kobayashi, Kubo, Mondragon, Zoupanos (1998)

then the universal rule can be "relaxed" to a (*2-loop*) soft scalar mass sum rule,

$$(m_i^2 + m_j^2 + m_k^2)/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)}$$

where the 2-loop correction $\Delta^{(2)}$ vanishes for the $N = 1$ $SU(5)$ FUTs.

- It is possible to upgrade the soft sector to all-loop finite!

Hisano, Shifman (1997); Kazakov (1999); Jack, Jones, Pickering (1998)
Kobayashi, Kubo, Zoupanos (1998); Kobayashi, Kubo, Mondragon, Zoupanos (2000)

The Finite $N = 1$ $SU(5)$ Model

1+2-loop finite — Hamidi, Schwarz; Jones, Raby (1984); Leon, Perez-Mercader, Quiros (1985);
all-loop finite — Kapetanakis, Mondragon, Zoupanos (1993)

We study an all-loop finite $N = 1$ supersymmetric $SU(5)$ model with content:

$$3\,\bar{\mathbf{5}} + 3\,\mathbf{10} + 4\,(\mathbf{5} + \bar{\mathbf{5}}) + \mathbf{24}$$

Heinemeyer, Mondragon, Zoupanos (2008)

Under GUT scale: broken $SU(5) \rightarrow$ MSSM; no longer finite.

In order for the model to become predictive, it should also have the following properties:

- Fermions do not couple to the adjoint rep **24**
- The two Higgs doublets of the MSSM are mostly made out of a pair of Higgs $(\mathbf{5} + \bar{\mathbf{5}})$ which couple to the third generation

The isolated and non-degenerate solutions to $\gamma_i^{(1)} = 0$ give:

$$(g_1^u)^2 = \frac{8}{5}\, g^2\,,\ (g_1^d)^2 = \frac{6}{5}\, g^2\,,\ (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5}\, g^2\,,$$
$$(g_2^d)^2 = (g_3^d)^2 = \frac{3}{5}\, g^2\,,\ (g_{23}^u)^2 = \frac{4}{5}\, g^2\,,\ (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5}\, g^2\,,$$
$$(g^\lambda)^2 = \frac{15}{7} g^2\,,\ (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2}\, g^2\,,\ (g_1^f)^2 = 0\,,\ (g_4^f)^2 = 0$$

From the sum rule we obtain:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$$
$$m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}$$
$$m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}$$

Only two free parameters ($m_{\mathbf{10}}$ and M) in the dimensionful sector.

Phenomenology

Gauge symmetry broken $\rightarrow$ MSSM $\rightarrow$ boundary conditions at M_{GUT} remain of the form:

(a) $C_i = \rho_i g$
(b) $h = -MC$
(c) sum rule

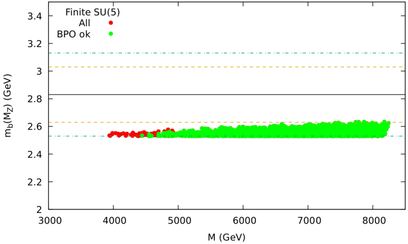
1-loop β-functions for the soft sector, everything else in 2 loops.

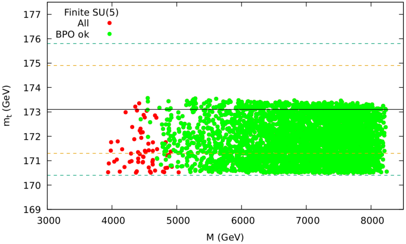
Input: The only value fixed is the one of m_τ.

Output:

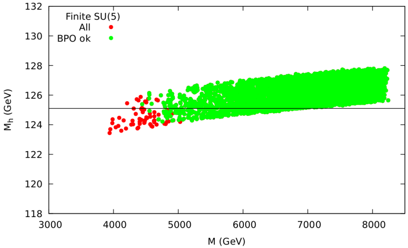
- solutions that satisfy m_t , m_b , m_h experimental constraints
- Only $\mu < 0$ phenomenologically acceptable choice.
- solutions that satisfy B physics observables
- neutral LSP
- no fast proton decay
- SUSY breaking scale and full SUSY spectrum

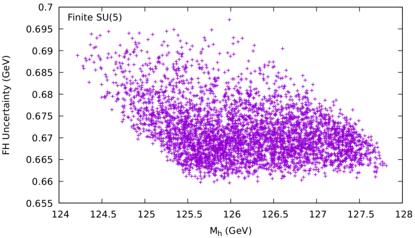
The Finite $SU(5)$ Model





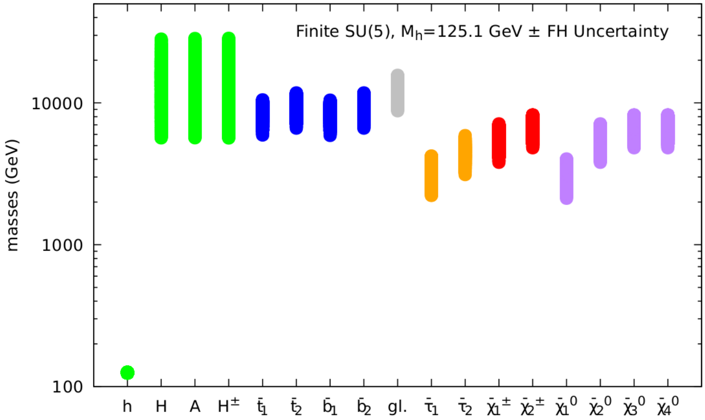
Points in green satisfy $b \to s\gamma$, $B_u \to \tau\nu$, $B_s \to \mu^+\mu^-$ and $\Delta M_{B_s}^{\rm exp}$





FeynHiggs 2.16.0 $\to$ uncertainty lowered by more than $\sim$ 1 *GeV*

Supersymmetric Spectrum



- $\tan\beta \sim 50$
- SUSY spectrum > 2000 *GeV*

The Reduced MSSM

There is a way to find RGI expressions among couplings without going to a GUT gauge structure.

Heinemeyer, Mondragon, Tracas, Zoupanos (2017)

- Gauge/gaugino unification $\rightarrow$ assumption of covering GUT
- relations among couplings serve as boundaries at M_{GUT}

$\rightarrow$ 2-loop *dimensionless* sector (3^{rd} gen only):

$$\alpha_i = G_i^2 \alpha_3 + J_i^2 \alpha_3^2, \qquad i = t, b$$

where

$$G_t^2 = \frac{1}{3} + \frac{71\rho_1}{525} + \frac{3\rho_2}{7} + \frac{\rho_\tau}{35}, \qquad G_b^2 = \frac{1}{3} + \frac{29\rho_1}{525} + \frac{3\rho_2}{7} - \frac{6\rho_\tau}{35}, \qquad J_t^2 = \frac{1}{4\pi}\frac{N_t}{D}, \qquad J_b^2 = \frac{1}{4\pi}\frac{N_b}{5D}$$

$\rightarrow$ 1-loop *dimensionful* sector:

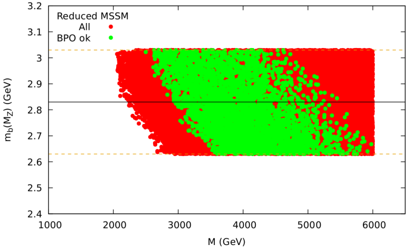
- Trilinear soft couplings: $h_i = c_i Y_i M_3 = c_i G_i M_3 g_3, \qquad i = t, b$
- Soft scalar masses: $m_i^2 = c_i M_3^2, \quad i = Q, u, d, H_u, H_d$

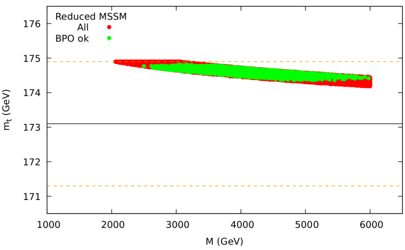
  For the completely reduced system: $c_Q = c_u = c_d = \frac{2}{3}, \quad c_{H_u} = c_{H_d} = -1/3$

$$\rightarrow \qquad m_Q^2 + m_u^2 + m_{H_u}^2 = M_3^2, \qquad m_Q^2 + m_d^2 + m_{H_d}^2 = M_3^2$$

- $M_{1,2} = b_{1,2} M_3 \qquad \rightarrow \qquad M_2 < M_1$

The Reduced MSSM





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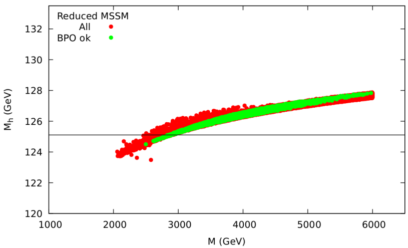
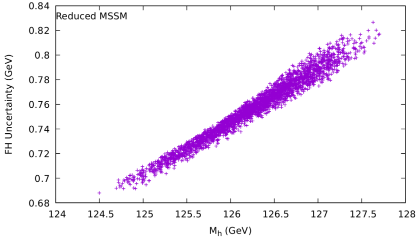
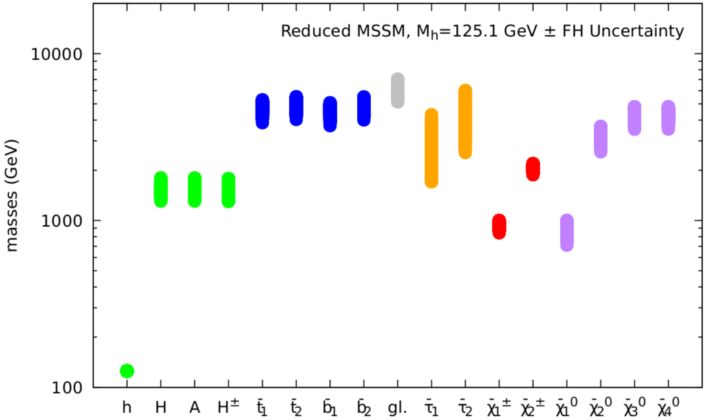




FeynHiggs 2.16.0 $\to$ uncertainty dropped below 1 *GeV*

Supersymmetric Spectrum



- $\tan\beta \sim 40 - 45$
- SUSY spectrum $> 700\ GeV$

Summary

- Reduction of Couplings: powerful tool that implies Gauge-Yukawa Unification
- Finiteness: old dream of HEP, very predictive models
- completely finite theories $\rightarrow$ both in dimensionless and dimensionful sector

Finite $SU(5)$:

- past analysis predicted the lightest Higgs boson mass
- re-examined in 2-loop (1-loop for the SSB sector) and with the new FeynHiggs code
- $\mu < 0$ survives phenomenological constraints
- SUSY spectrum too heavy for discovery at HL-LHC

Reduced MSSM:

- reduction of couplings below M_{GUT}
- examined in 2-loop (1-loop for the SSB sector) and with the new FeynHiggs code
- lightest scenaria: LSP < 1 *TeV* $\rightarrow$ discovery potential at HL-LHC

...Coming soon: discovery potential in (near) future colliders