

PHQMD

(Parton-Hadron-Quantum-Molecular-Dynamics)

- a microscopic transport approach to study cluster production in heavy ion reactions

J. Aichelin

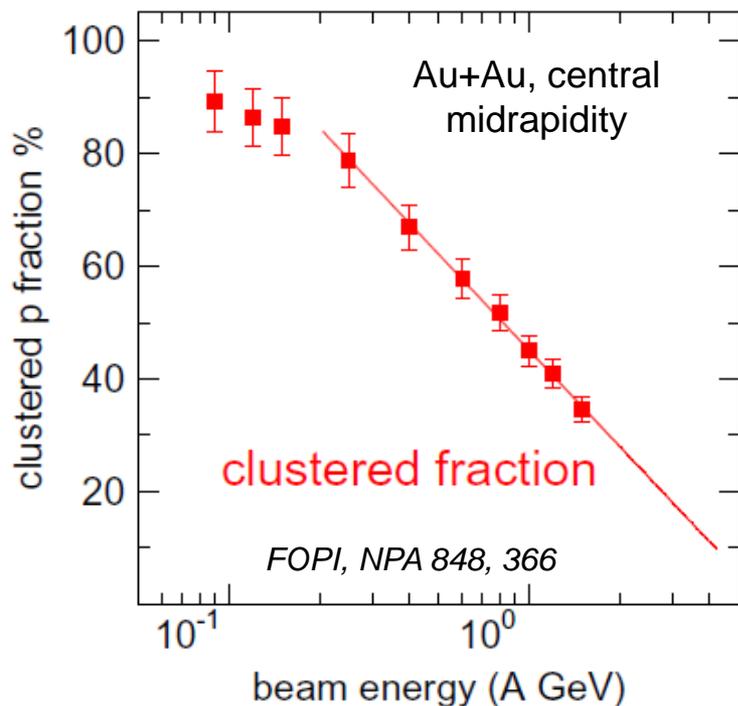
(S. Glässel, V. Kireyev, E. Bratkovskaya, M. Winn, A. Le Fèvre, Y. Leifels, V. Kolesnikov, V. Voronyuk, C. Blume)

In heavy ion collisions a fireball is observed at midrapidity. Multiplicity and particle spectra suggest a temperature of $T \approx 100$ MeV. Nevertheless clusters (d, t, ^3He , ^4He) and hyperclusters (with Λ 's) are seen with a binding of < 10 MeV.

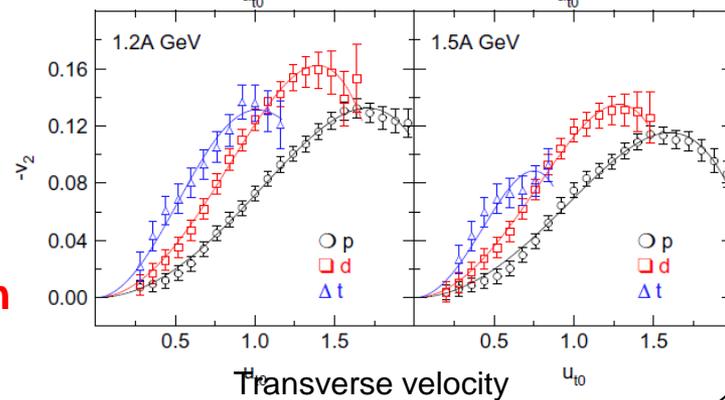
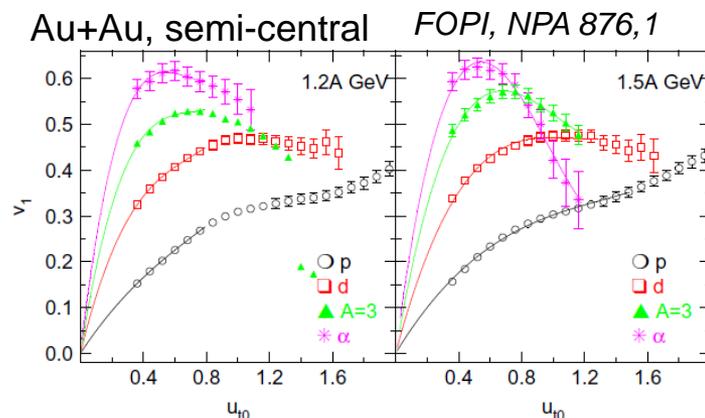
Ice in fire or snowball in hell.

- PHQMD transport theory (which produces clusters)
- Cluster identification
- Comparison with existing data
- Perspectives for BMN/NICA/FAIR/RHIC/LHC

Clusters in (low energy) HICs



- Clusters are very **abundant at low energy**;
- **at 3 AGeV in central Au+Au collisions ~20% of the baryons are in clusters!**
- ... and baryons in clusters have quite different properties ($v_1, v_2, dn/dp_T$)



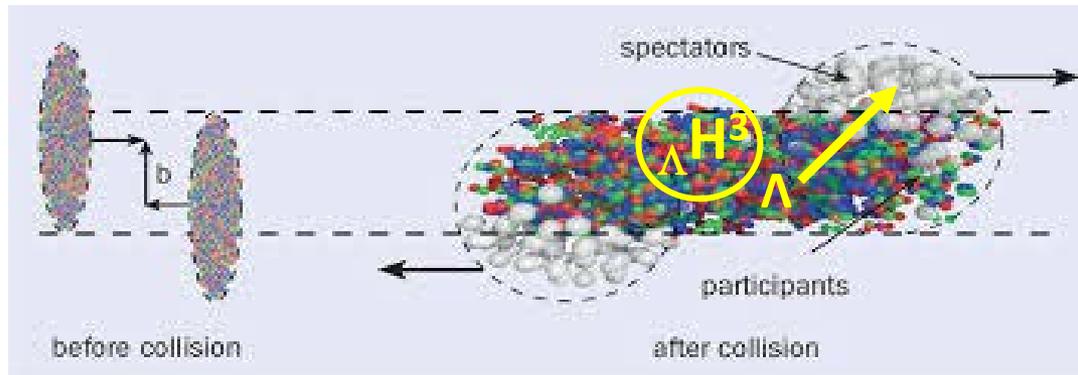
➔ Understanding of cluster formation is needed

□ for proper description of nucleon observables ($v_1, v_2, dn/dp_T$)

□ to explore new physics opportunities like

- **hyper-nucleus formation**
- **possible signals of the 1st order phase transition**
- **cluster formation at midrapidity (RHIC, LHC)**

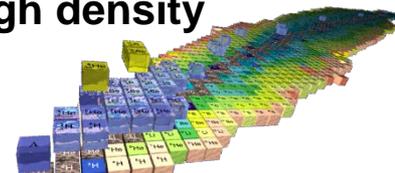
Why do we study especially hypermatter production?



- **Access to the nuclear dynamics:**
different mechanisms for hypernucleus production vs. rapidity:
 - at mid-rapidity : **Λ -coalescence** - hypernuclei test the phase-space distribution of baryons in the expanding participant matter
 - at target/projectile y: **Λ -absorption** by spectators - elucidates the physics at the interface between spectator and projectile matter

Hypernuclei as bound objects:

- give access to the **third dimension** of the nuclear chart (strangeness)
- give information on **hyperon-nucleon and hyperon-hyperon interactions**
- important e.g. for **neutron stars** (production of hypermatter at high density and low temperature)
- new field of hyperon spectroscopy



Present transport approaches are not designed for this

PHQMD

The goal: to develop a **unified n-body microscopic transport approach** for the description of heavy-ion dynamics and dynamical cluster formation from low to ultra-relativistic energies

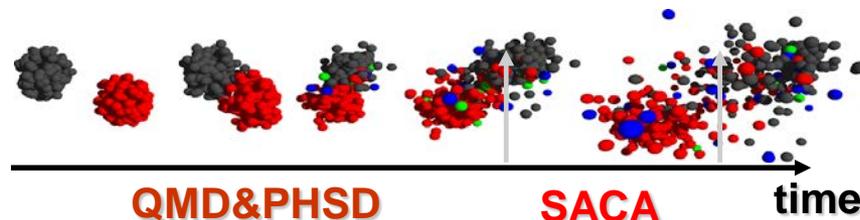
Realization: combined model **PHQMD = (PHSD & QMD) & SACA**

Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

Propagation of partons (quarks, gluons) and mesons
+ **collision integral** = interactions of hadrons and partons (QGP)
from **PHSD (Parton-Hadron-String Dynamics)**

Clusters recognition:
SACA (Simulated Annealing Clusterization Algorithm)
vs. **MST (Minimum Spanning Tree)**



Transport eqs. for N-body theories like (PH)QMD

Roots in Quantum Mechanics

Remember QM cours when you faced the problem

- we have a Hamiltonian $\hat{H} = -\frac{\hbar^2 \nabla^2}{2m} + V$
- the Schrödinger eq.

$$\hat{H}|\psi_j\rangle = E_j|\psi_j\rangle$$

has no analytical solution

- we look for the ground state energy



Walther Ritz

Ritz variational principle:

Assume a **trial function** $\psi(q, \alpha)$ which contains one **adjustable parameter** α , which is varied to find the lowest energy expectation value:

$$\frac{d}{d\alpha} \langle \psi | \hat{H} | \psi \rangle = 0 \rightarrow \alpha_{min}$$

determines α for which $\psi(q, \alpha)$ is **closest to the true ground state** and $\langle \psi(\alpha_{min}) | \hat{H} | \psi(\alpha_{min}) \rangle = E_0(\alpha_{min})$ **closest to true ground state E**

Extended Ritz variational principle (Koonin, TDHF)

Take trial wavefct with time dependent parameters and solve

$$\delta \int_{t_1}^{t_2} dt \langle \psi(t) | i \frac{d}{dt} - H | \psi(t) \rangle = 0. \quad (1)$$

QMD trial wavefct for particle i with $p_{oi}(t)$ and $q_{oi}(t)$

$$\psi_i(q_i, q_{oi}, p_{oi}) = C \exp[-(q_i - q_{oi} - \frac{p_{oi}}{m}t)^2 / 4L] \cdot \exp[ip_{oi}(q_i - q_{oi}) - i \frac{p_{oi}^2}{2m}t]$$

For N particles: $\psi_N = \prod_{i=1}^N \psi_i(q_i, q_{oi}, p_{oi})$ QMD

$$\psi_N^F = \text{Slaterdet} \left[\prod_{i=1}^N \psi_i(q_i, q_{oi}, p_{oi}) \right] \quad \text{AMD/FMD}$$

For the QMD trial wavefct eq. (1) yields

$$\frac{dq}{dt} = \frac{\partial \langle H \rangle}{\partial p} \quad ; \quad \frac{dp}{dt} = - \frac{\partial \langle H \rangle}{\partial q}$$

For Gaussian wavefct
eq. of motion very similar
to Hamilton's eqs.
(but only for Gaussians !!)

Realisation

Potential in PHQMD

above $\varepsilon=0.5 \text{ GeV/fm}^3$ transition to QGP like in PHSD

Below:

Relativistic molecular dynamics (PRC 87, 034912) too time consuming

The **potential interaction** is most **important in two rapidity intervals**:

- ❑ at **beam and target rapidity** where the fragments are **initial - final state correlations** and created from spectator matter
- ❑ at **midrapidity** where - at the late stage - the phase space density is sufficiently high that small fragments are formed

In both situations: the **relative momentum between neighboring nucleons is small** and therefore **nonrelativistic kinematics can be applied (in the proper reference system)**. Potential interaction between nucleons

$$\begin{aligned} V_{i,j} &= V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t) = V_{\text{Skyrme}} + V_{\text{Coul}}(+V_{\text{mom}}) \\ &= \frac{1}{2} t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{\gamma + 1} t_2 \delta(\mathbf{r}_i - \mathbf{r}_j) \rho^{\gamma-1}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t) \\ &\quad + \frac{1}{2} \frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|} (+V_{\text{mom}}), \end{aligned}$$

$$\langle V(\mathbf{r}_i, t) \rangle = \sum_{j \neq i} \int d^3 r d^3 r' d^3 p d^3 p'$$

$$V(\mathbf{r}, \mathbf{r}', \mathbf{r}_i, \mathbf{r}_j) f(\mathbf{r}, \mathbf{p}, \mathbf{r}_i, \mathbf{p}_i, t) f(\mathbf{r}', \mathbf{p}', \mathbf{r}_j, \mathbf{p}_j, t)$$

$$\langle V_i^{Skyrme}(\mathbf{r}_i, t) \rangle = \alpha \left(\frac{\rho_{int}(\mathbf{r}_i, t)}{\rho_0} \right) + \beta \left(\frac{\rho_{int}(\mathbf{r}_i, t)}{\rho_0} \right)^\gamma$$

To describe the potential interactions in the **spectator matter** we transfer the Lorentz-contracted nuclei back into the **projectile and target rest frame**, neglecting the small time differences

$$\rho_{int}(\mathbf{r}_i, t) \rightarrow C \sum_j \left(\frac{4}{\pi L} \right)^{3/2} e^{-\frac{4}{L} (\mathbf{r}_i^T(t) - \mathbf{r}_j^T(t))^2} \cdot e^{-\frac{4\gamma_{cm}^2}{L} (\mathbf{r}_i^L(t) - \mathbf{r}_j^L(t))^2} .$$

For the midrapidity region $\gamma \rightarrow 1$. and we can apply nonrelativistic kinematics as well

All elastic and inelastic cross sections from PHSD - therefore at high energy the spectra of produced particles are similar to PHSD results (however initial distribution is different in PHSD and PHQMD)

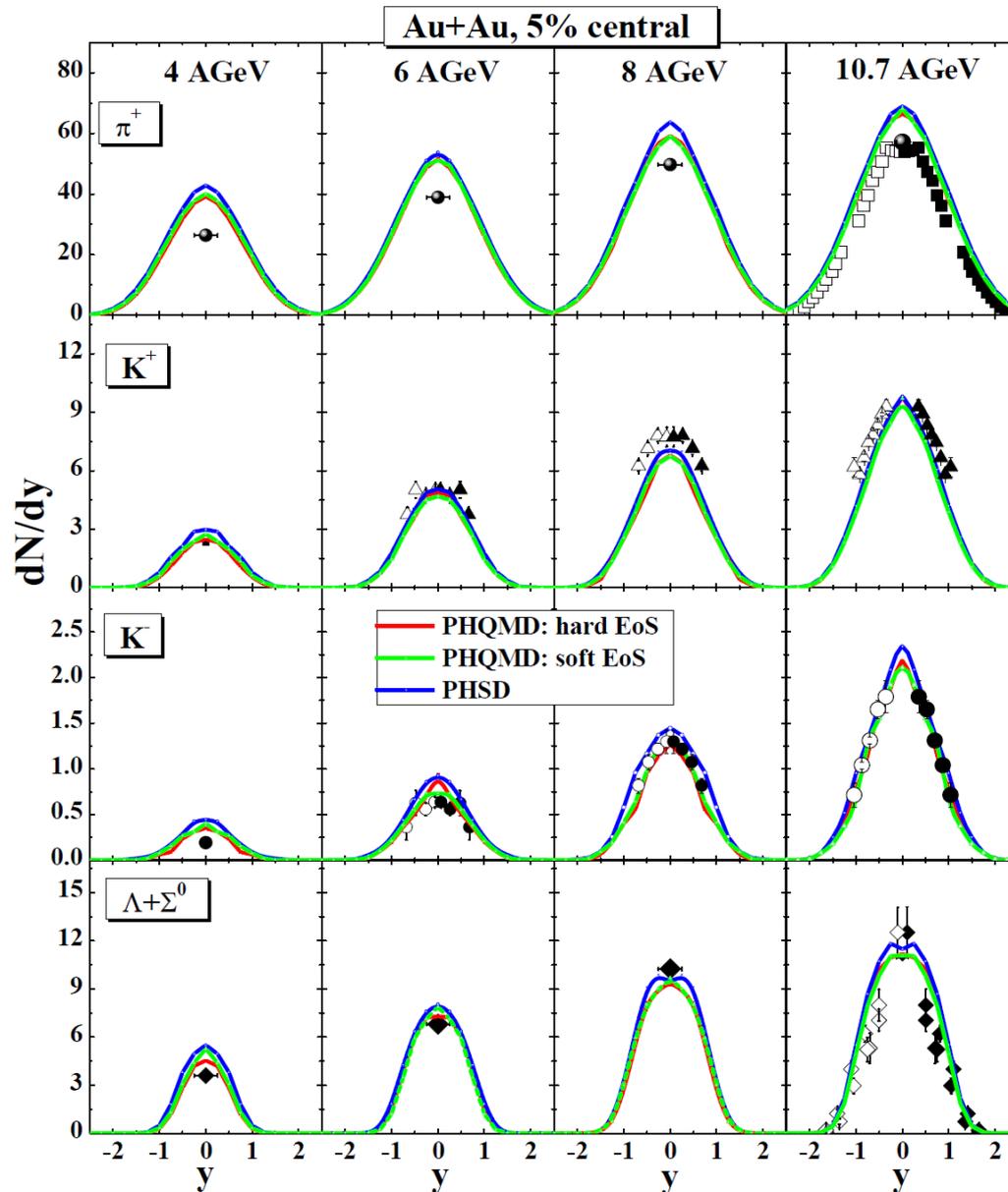
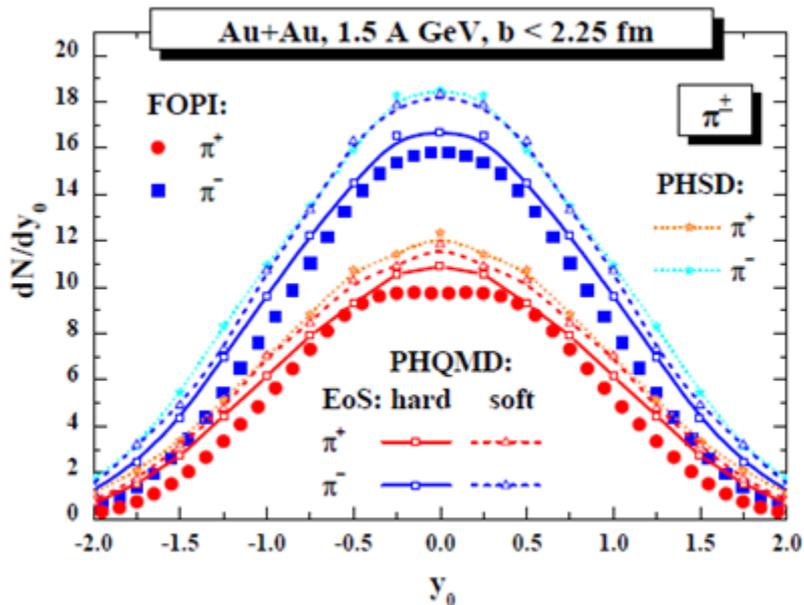
Results

Results of PHQMD

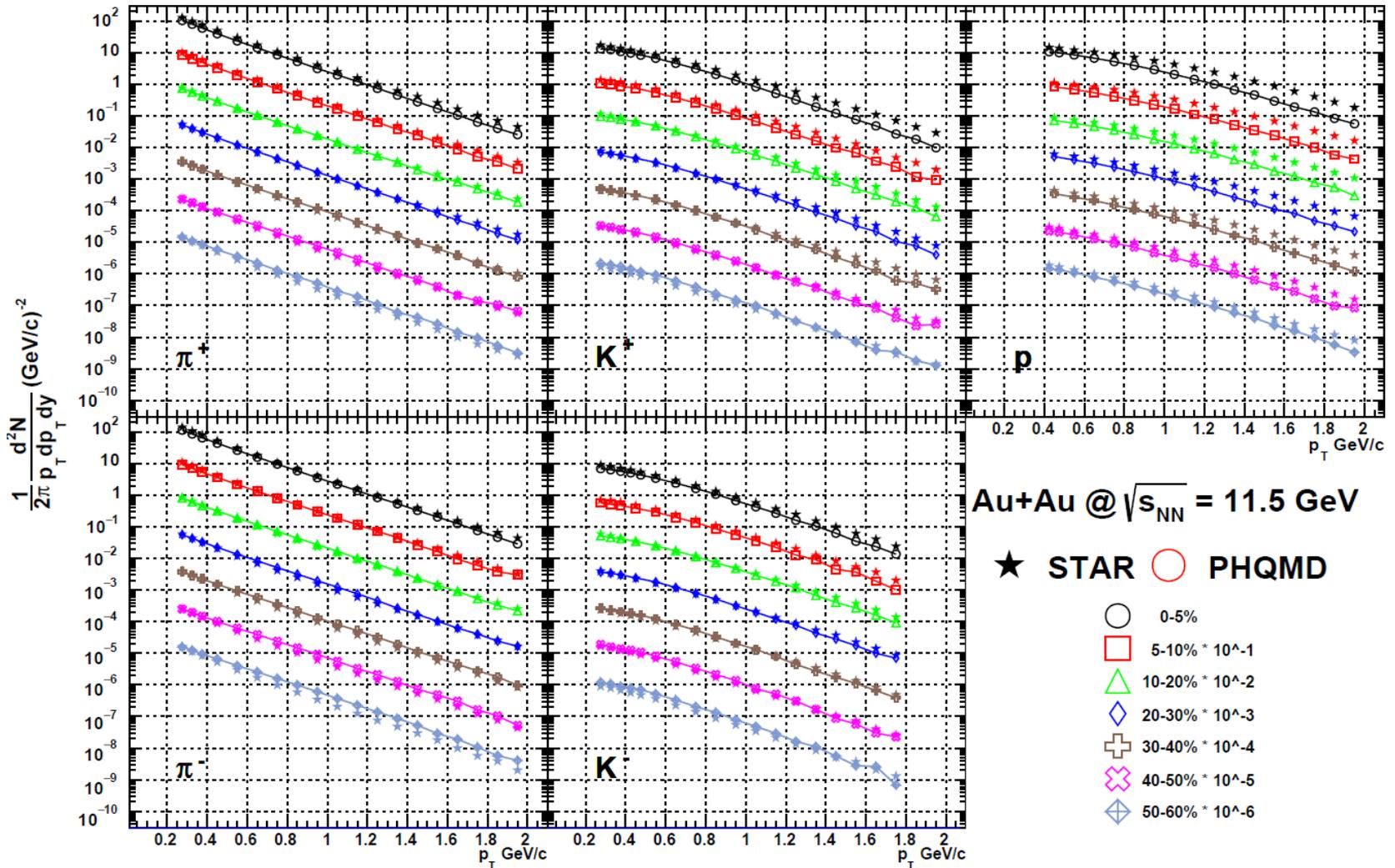
Produced particles

are well reproduced
at SIS/NICA/FAIR energies

(dominated by collisions)



.. And also the most recent STAR data at 11.5 AGeV



Methods to identify fragments in theories which propagate nucleons:

Static approaches:

means fragment multiplicity determined at a fixed time point

- coalescence (early, assumption: no coll. later)
- statistical model (V,T,N) very late $\rho \ll \rho_0$

Dynamical approaches:

means fragment multiplicity is fct. of time

- minimum spanning tree (correlation in coord. space)
- simulated annealing (correlation in mom and coord. space)
- time dep. perturbation theory using Wigner densities



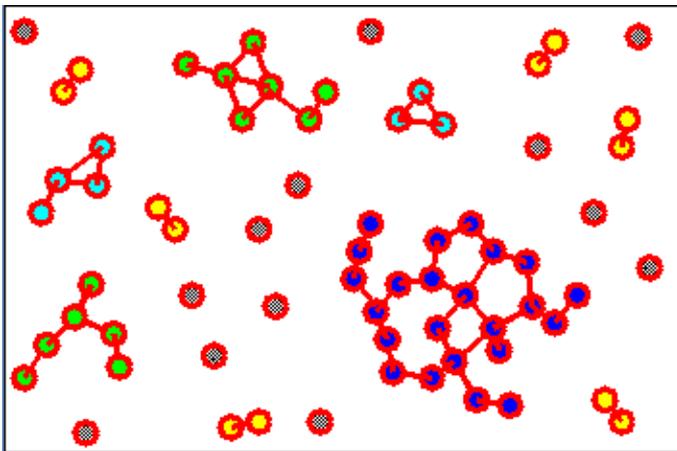
1. **Minimum Spanning Tree (MST)** is a **cluster recognition** method applicable for the (asymptotic) **final state** where coordinate space **correlations may only survive for bound states**.

The MST algorithm searches for accumulations of particles in coordinate space:

1. Two particles are **bound** if their distance in coordinate space fulfills

$$|\vec{r}_i - \vec{r}_j| \leq 2.5 fm$$

2. A particle is **bound to a cluster** if it is **bound with at least one particle** of the cluster.



Additional momentum cuts (coalescence) change little:
large relative momentum
-> finally not at the same position

II.SACA or ECRA now FRIGA

If we want to identify fragments earlier one has to use momentum space info as well as coordinate space info

Idea by Dorso et al. (Phys.Lett.B301:328,1993) :

- a) Take the positions and momenta of all nucleons at time t .
- b) Combine them in all possible ways into all kinds of fragments or leave them as single nucleons
- c) Neglect the interaction among clusters
- d) Choose that configuration which has the highest binding energy

Simulations show: Clusters chosen that way at early times are the **preclusters** of the final state clusters.

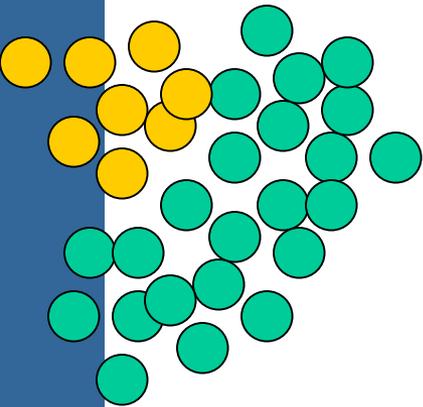
(large persistent coefficient)

How does this work?

Simulated Annealing Procedure: PLB301:328,1993

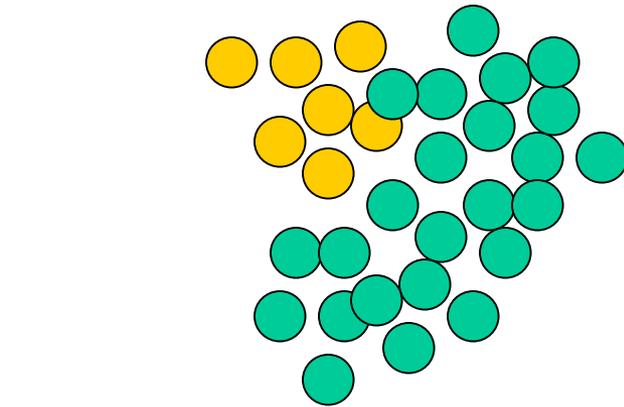
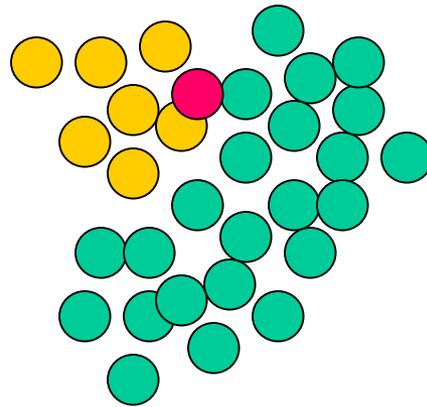
later SACA , now FRIGA :Nuovo Cim. C39 (2017) 399

Take randomly 1 nucleon
out of a fragment



$$E = E_{kin}^1 + E_{kin}^2 + V^1 + V^2$$

Add it randomly to another
fragment



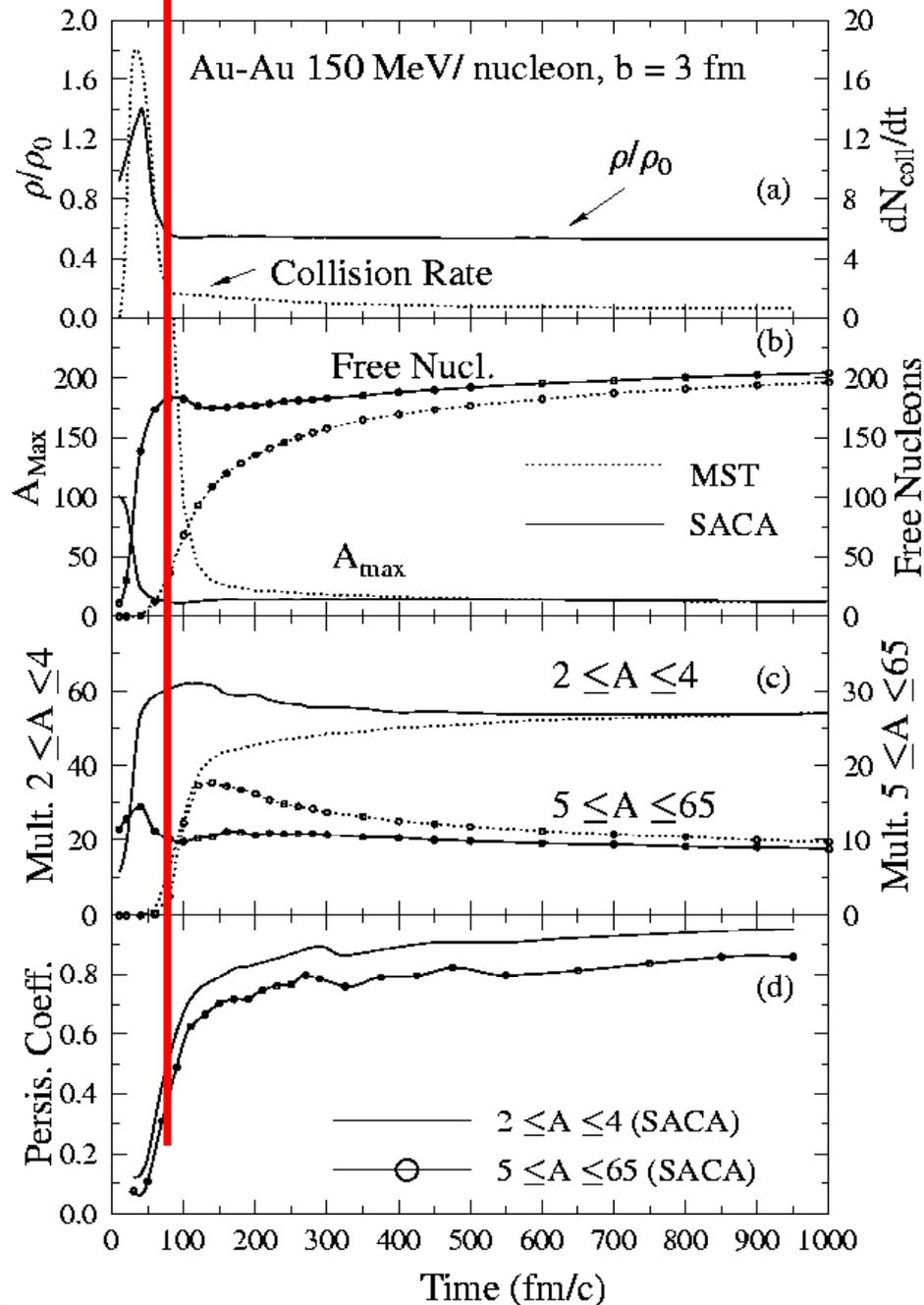
$$E' = E_{kin}^1 + E_{kin}^2 + V^1 + V^2$$

If $E' < E$ take the new configuration

If $E' > E$ take the old with a probability depending on $E' - E$

Repeat this procedure very many times

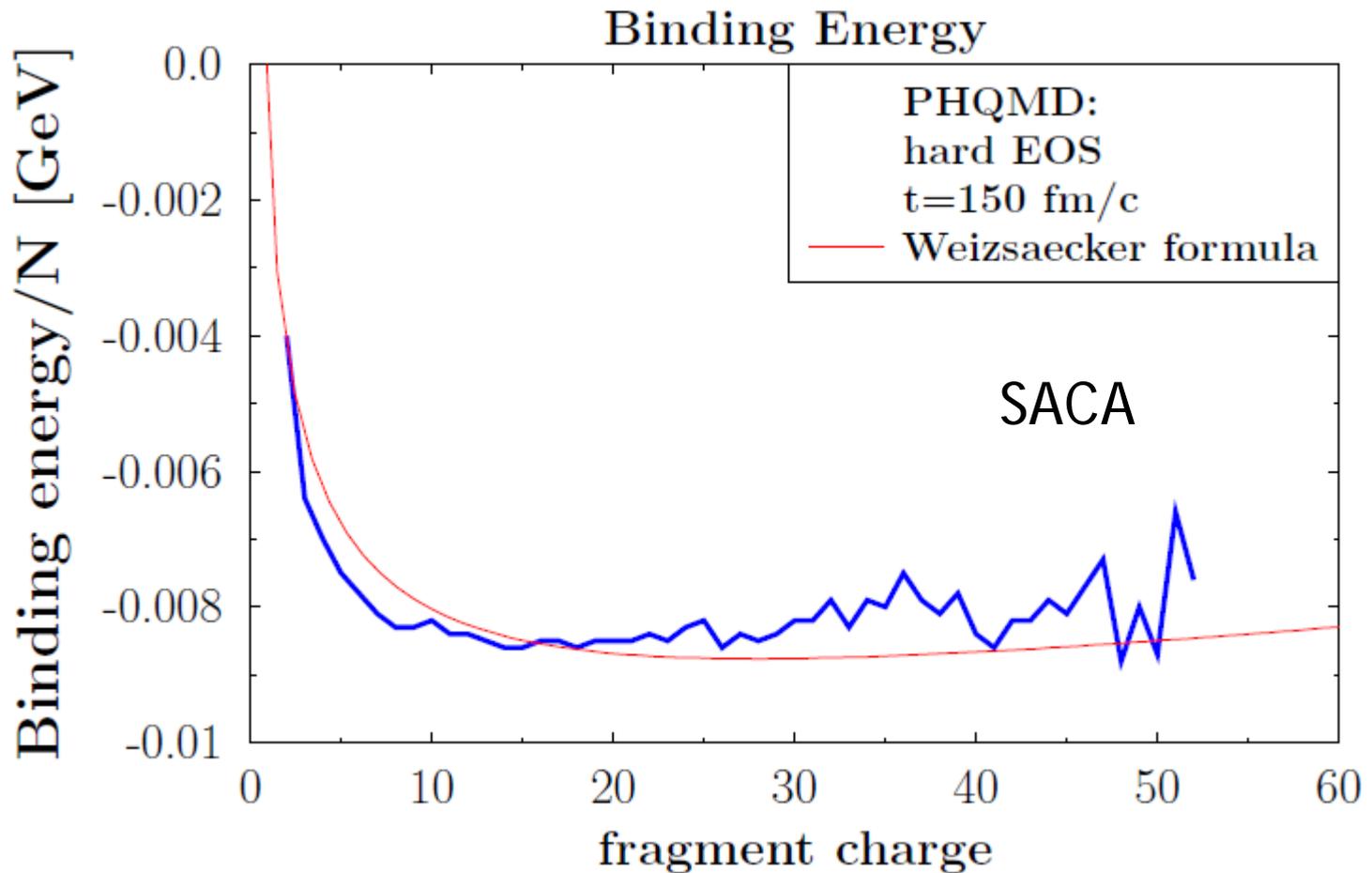
→ Leads automatically to the most bound configuration



SACA can really **identify the fragment pattern very early** as compared to the Minimum Spanning Tree (MST) which requires a maximal distance in coordinate space between two nucleons to form a fragment

At $1.5t_{\text{pass}}$ **A_{max} and multiplicities of intermediate mass fragments are determined**

Cluster formation in PHQMD



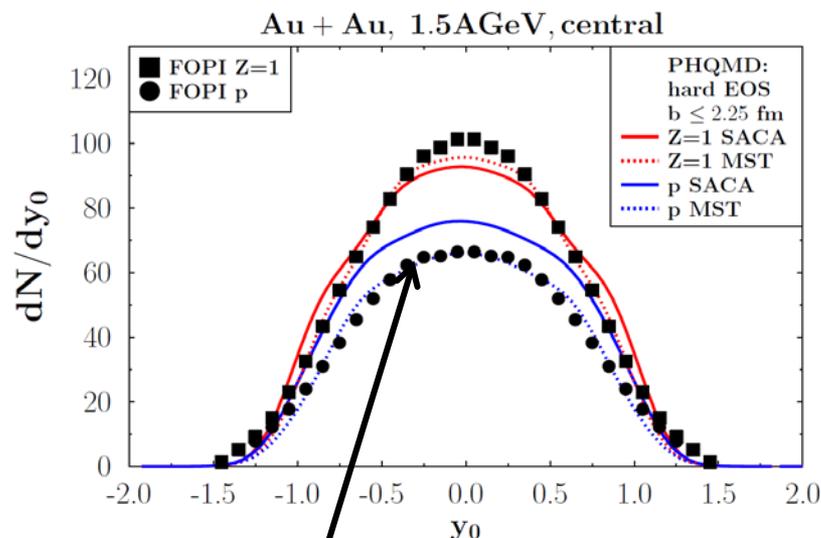
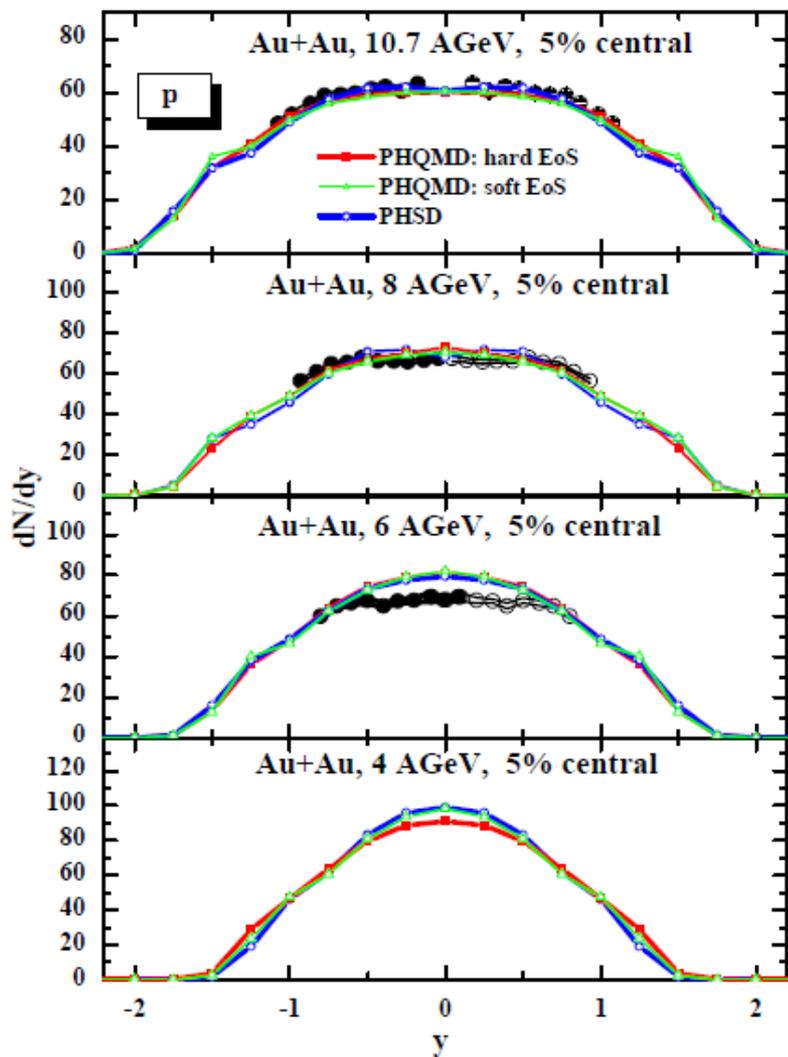
There are two kinds of fragments

- formed from **spectator matter**
 close to beam and target rapidity
 initial-final state correlations
 HI reaction makes spectator matter unstable
- formed from **participant matter**
 created during the expansion of the fireball
 “ice” ($E_{\text{bind}} \approx 8 \text{ MeV/N}$) in “fire” ($T \geq 100 \text{ MeV}$)
 origin not known yet
 seen from SIS to RHIC
 (quantum effects may be important)

First Results of PHQMD

Protons at **midrapidity** well described

midrapidity fragment production increases with decreasing energy

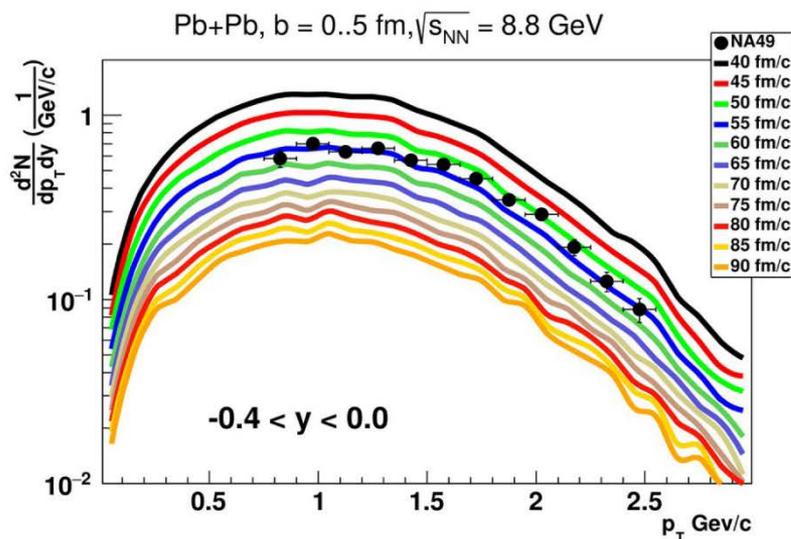
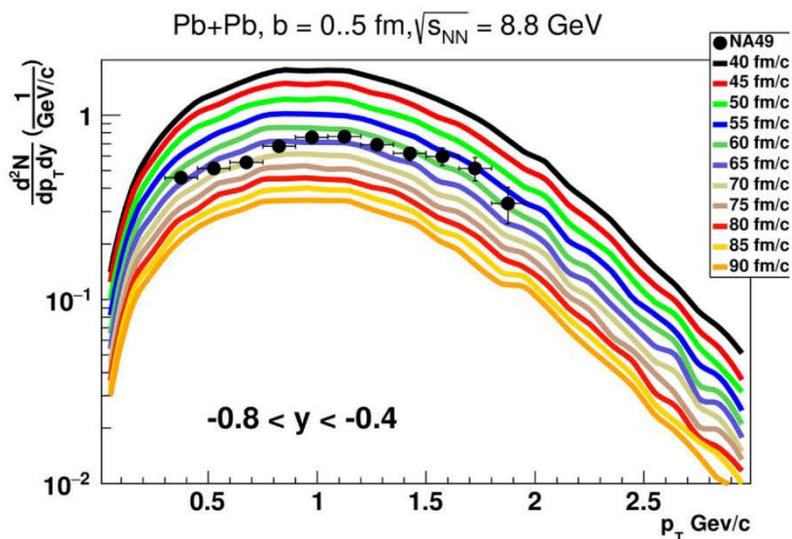
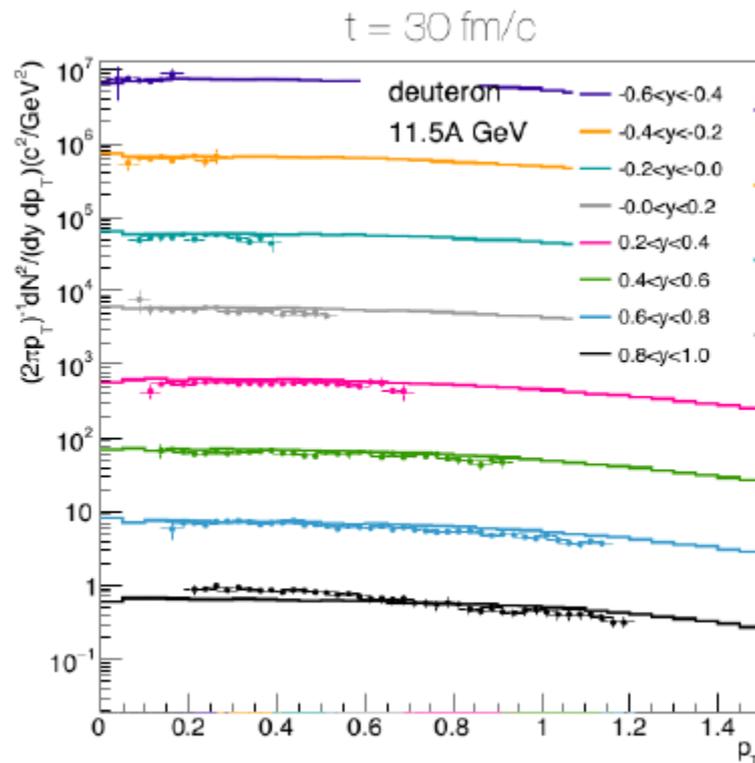


1.5 AGeV **central**

➤ 30% of protons bound in cluster
To improve: better potential for small clusters

At higher energies

- ❑ Multiplicity depends on extraction time
- ❑ p_T spectra well reproduced and does not change with the extraction time

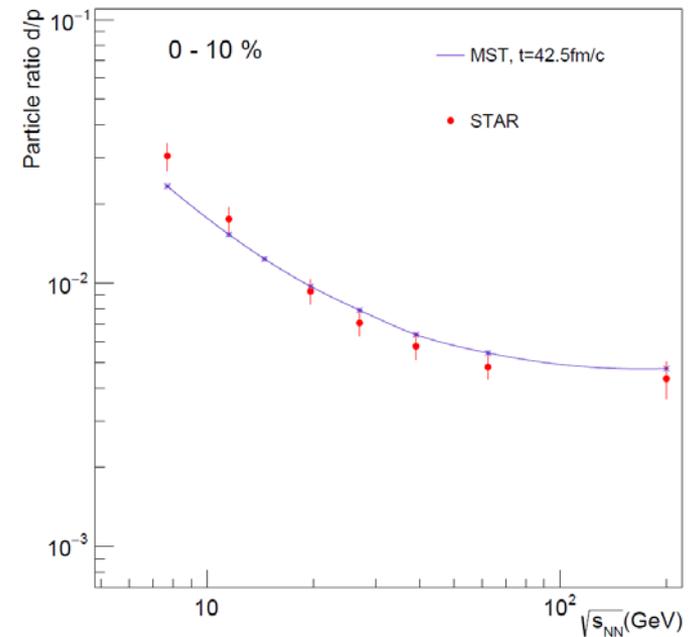
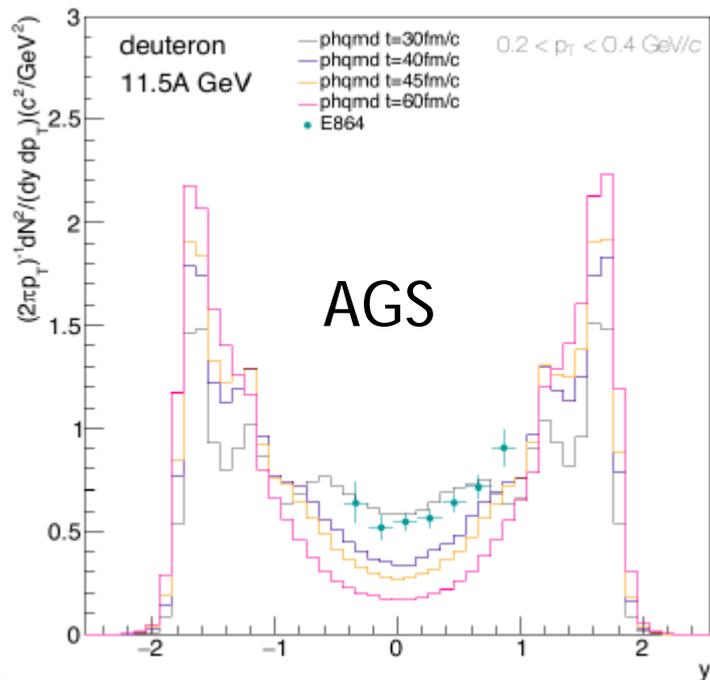


Why clusters are not stable in semi-classical approaches ?

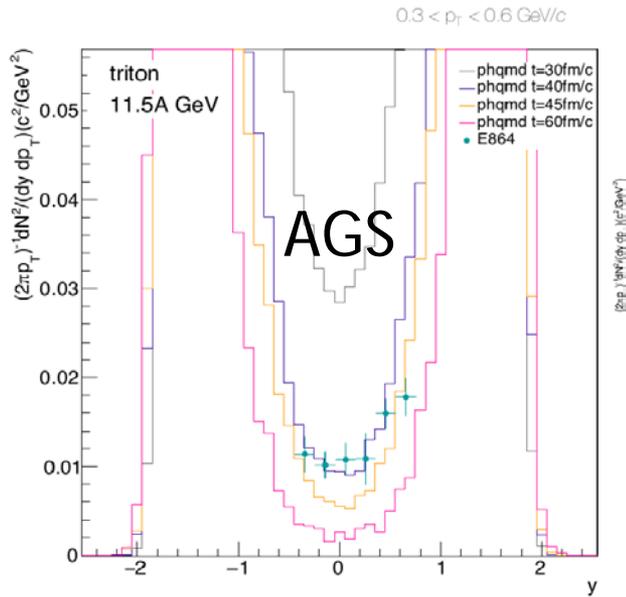
Take a ${}^4\text{He}$:

QM: all 4 nucleons are in the ground state with $\langle p^2 \rangle \neq 0$ determined by wave fct

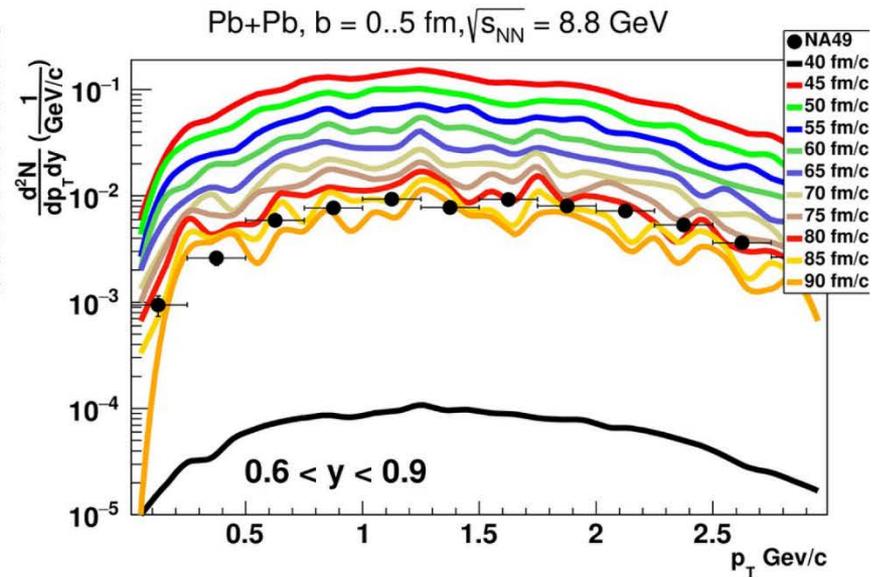
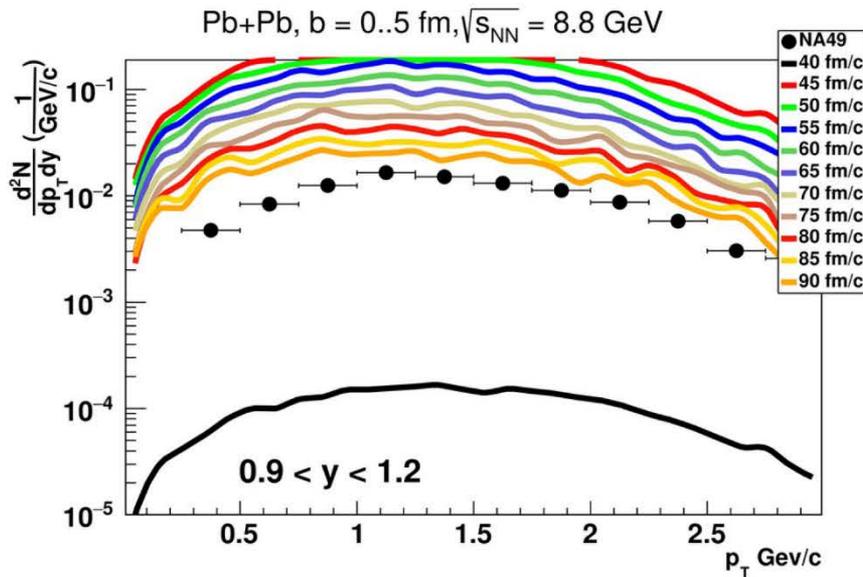
SC: for the same binding energy: one nucleon can get more momentum on the expense of the others -> not bound anymore -> escapes from cluster



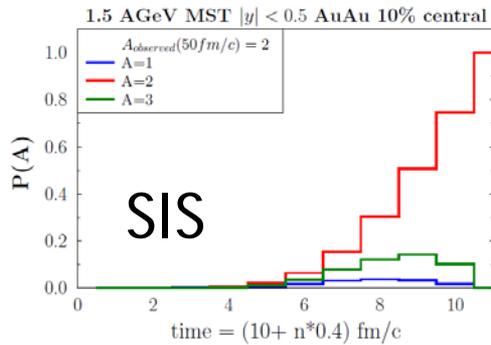
^3He and t



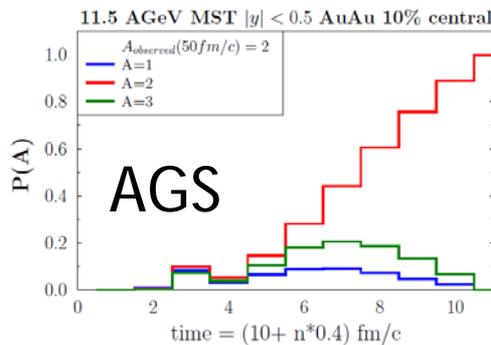
^3He



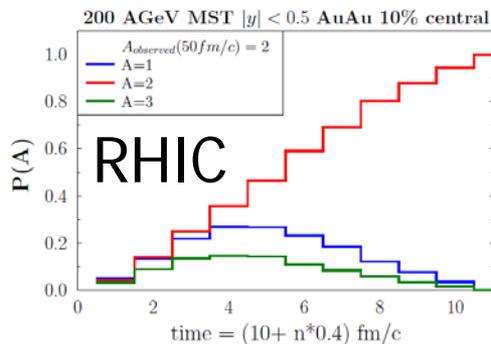
When are the fragments formed ?



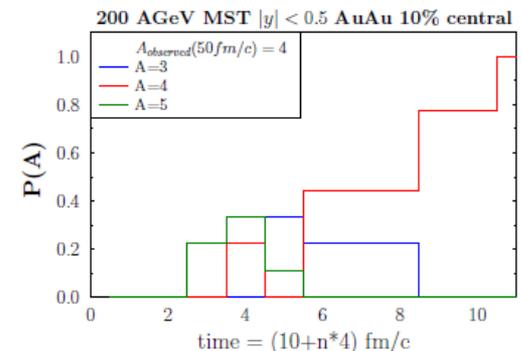
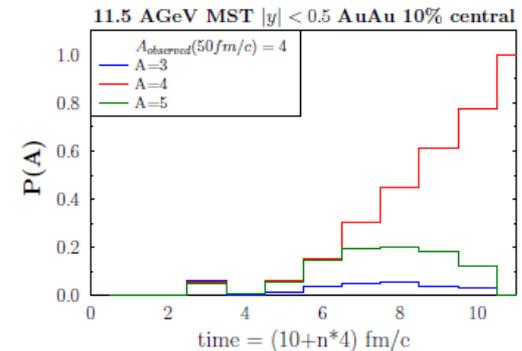
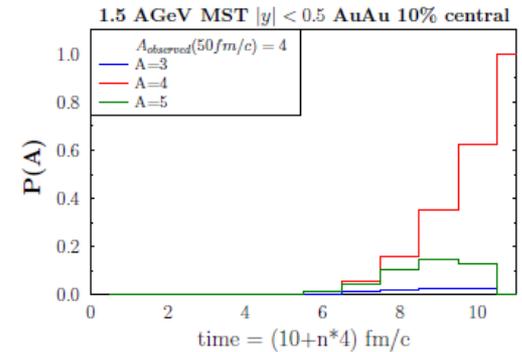
red:
fraction of final
clusters already formed
at time step n



green/blue
fraction of nucleons of the
final fragment A which have
been at this time step part
of a A+1 or A-1 cluster



A=2

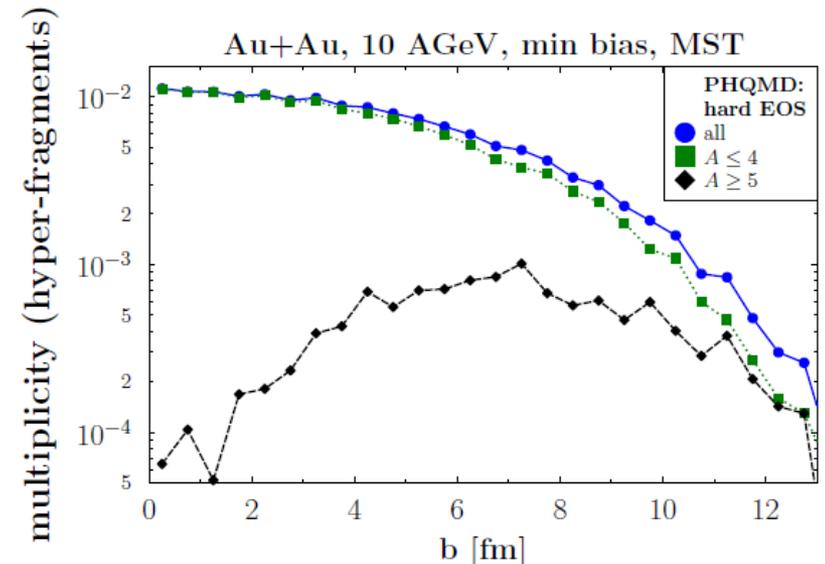
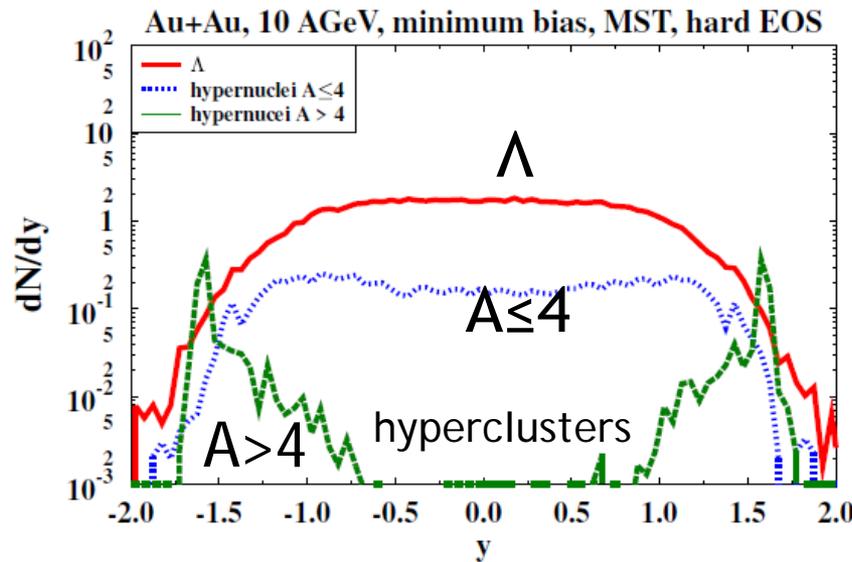
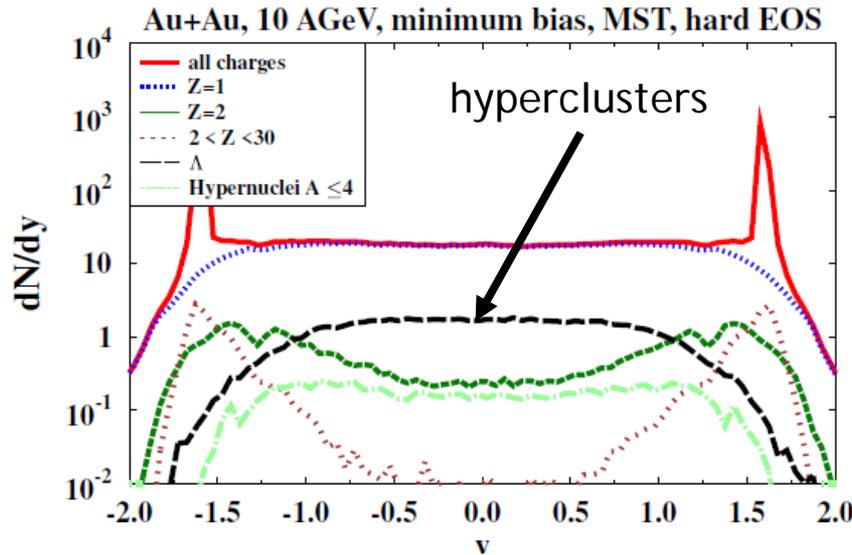


A=4

.. and what about hyper-nuclei ?

There are hyper-nuclei
 - at midrapidity (A small)
 - at beam rapidity (A large)
 few in number but
 more than in other reactions
 to create hyper-nuclei

Central collisions \rightarrow light hyper-nuclei
 Peripheral collisions \rightarrow heavy hyper-nuclei



Conclusions

We presented a new model, PHQMD, for the NICA/CBM energies which allows - **in contrast to all other models** - to predict the

dynamical formation of fragments

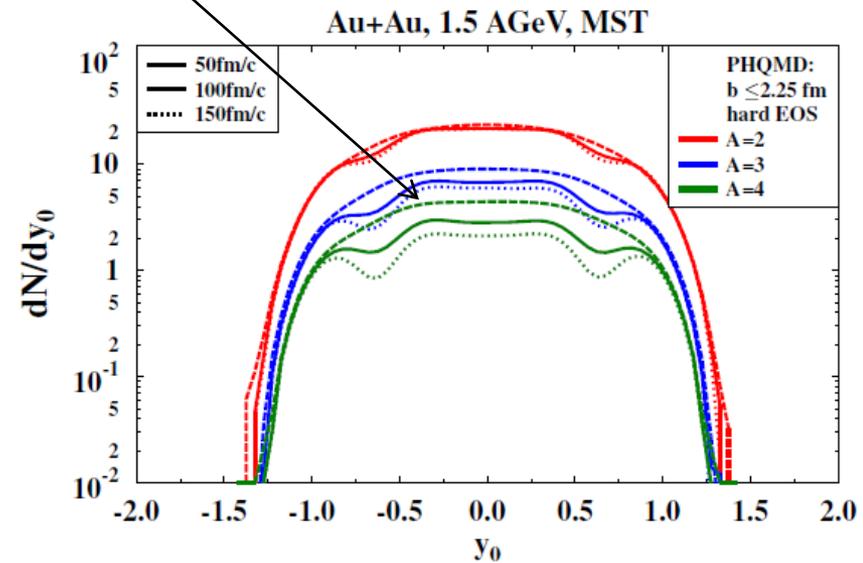
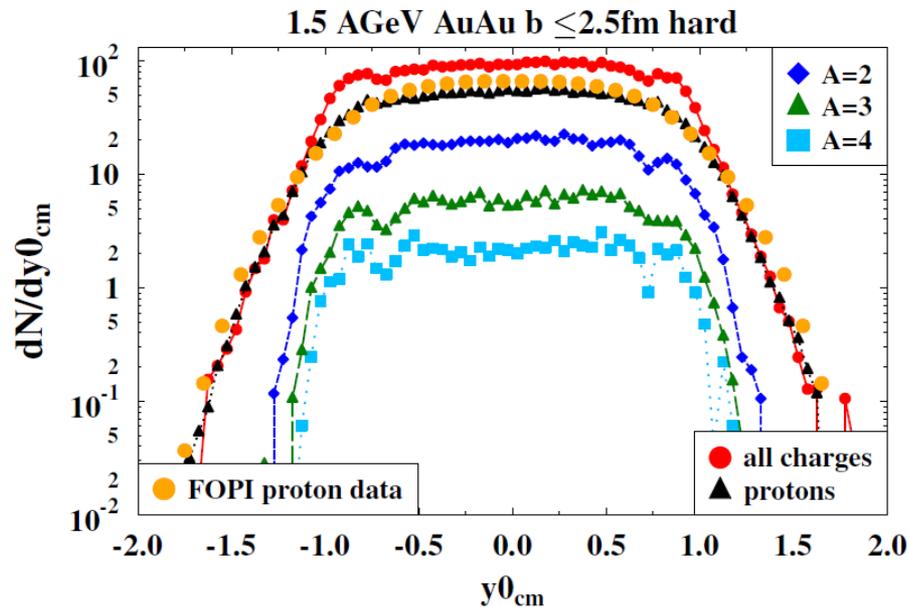
- allows to understand the proton spectra and the **properties of light fragments** ($dn/dp_T dy$, v_1, v_2 , fluctuations)
- allows to understand **fragment formation in participant and spectator region**
- allows to understand the formation of **hypernuclei**
- should allow to understand fragment formation at RHIC/LHC

Good agreement with the presently available fragment data and single particle spectra

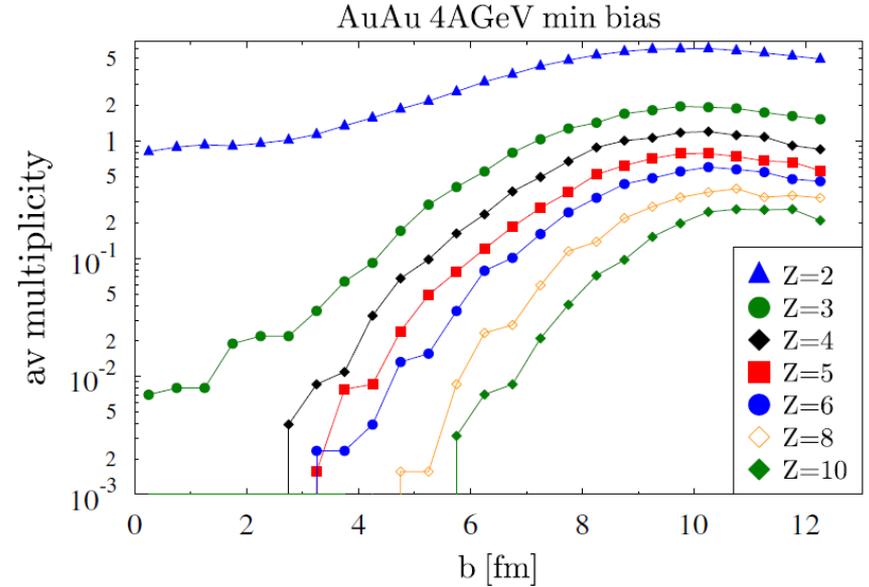
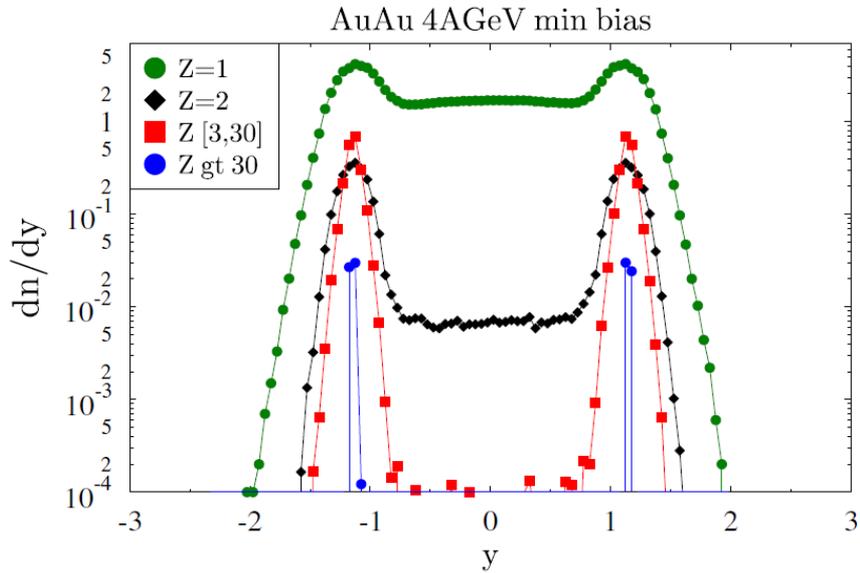
But a lot has still to be done!!

First Results of PHQMD

There are all kinds of dynamically produced fragments at midrapidity and **they are stable**
(MST finds at 50fm/c almost the same fragments as at 150fm/c)



First Results of PHQMD



- Only for most central events fragments do not play a big role
- Heavy fragments appear only in the residue rapidity range
- Complicated fragment pattern for larger impact parameters
- $M_z(b)$ is different for each fragment charge

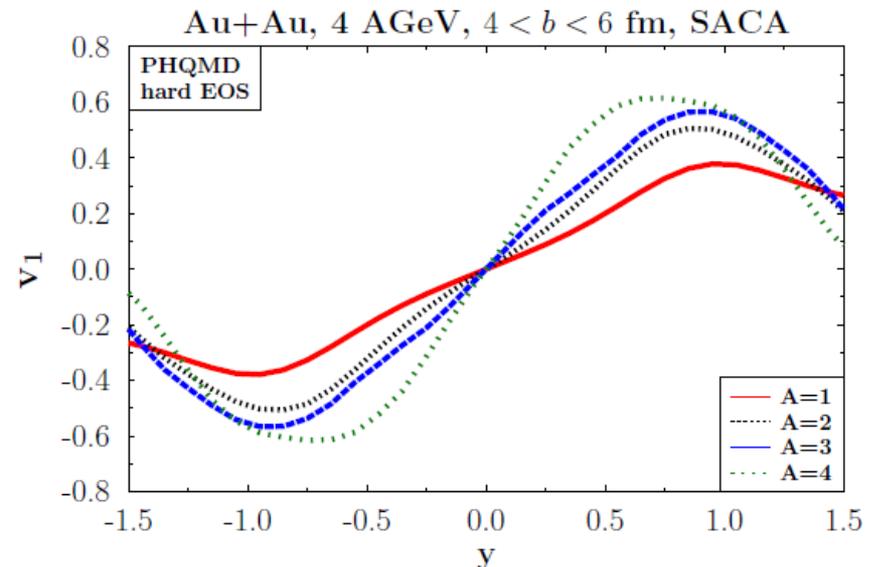
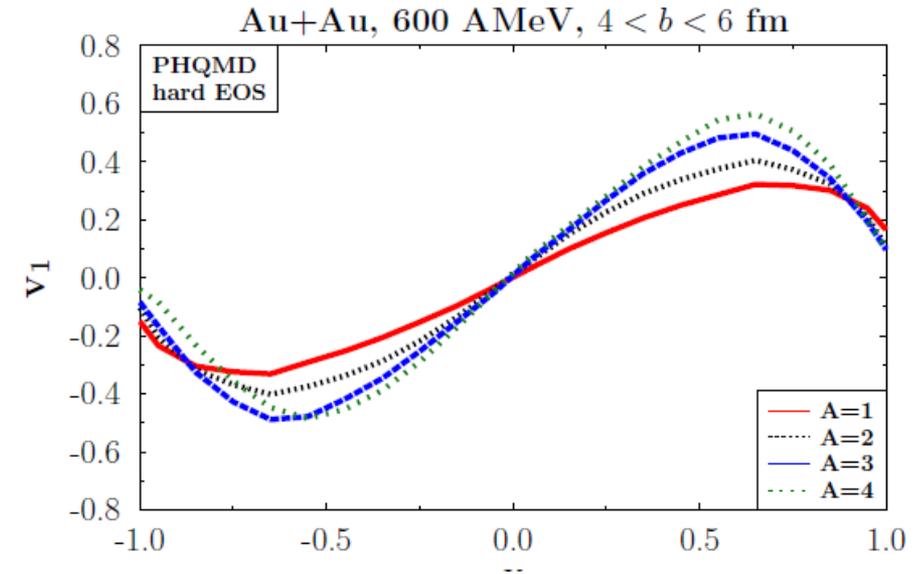
Dynamical variables - v_1

v_1 quite different for nucleons and fragments (as seen in experiments)

nucleons come from participant regions (-> small density gradient)

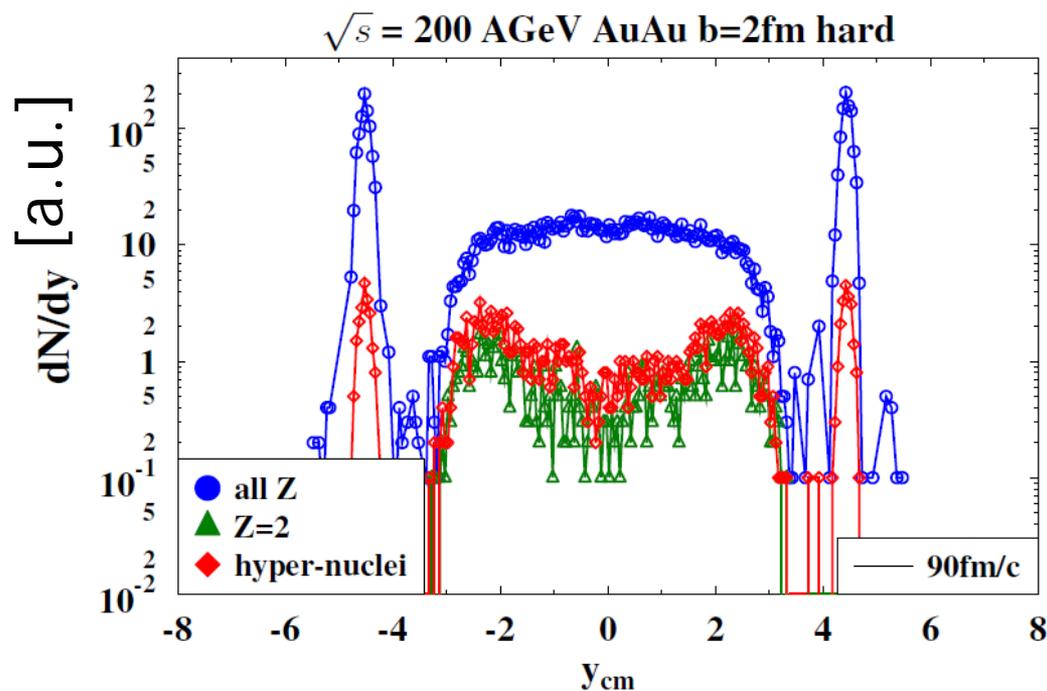
fragments from interface spectator-participant (strong density gradient)

v_1 increases with E_{beam}
larger density gradient
 $\rightarrow F_T t_p = p_T$ larger



At RHIC

hyper-nuclei also from spectator matter
Z=2 fragments at midrapidity
very preliminary



Back up

Proposals have been made to form clusters in the mean field approaches (which uses test particle method)

using a **coalescence** description for test particles

$$P_d(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2, t) = \underbrace{\rho_d^W(\mathbf{p}_1 - \mathbf{p}_2, \mathbf{r}_1 - \mathbf{r}_2)}_{\text{deuteron Wigner density}}$$

One can argue that this is theoretically not consistent because 1 and 2 are test particles, not nucleons.

In addition:

- result depends on the **number of test particles**
- result depends on **time** t when nucleons coalesce
- time is **different for different particles**: PRC56,2109
- no information about the formation process**

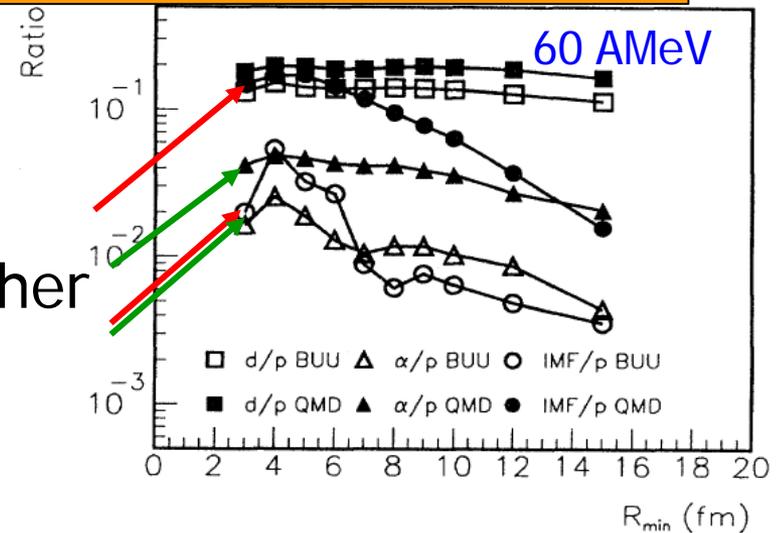
Importance of correlations and fluctuations

MST analysis with variable R_{\min}

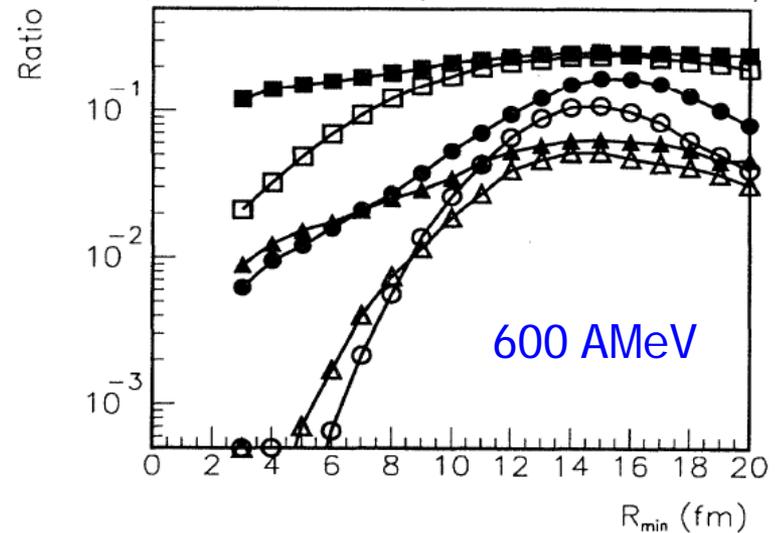
Two particles i, j are bound together

$$\text{if } |\vec{r}_i - \vec{r}_j| \leq R_{\min}$$

BUU obtained by event mixing
of QMD events



(a) Au+Au(60 A MeV), $B=0-3\text{Fm}$, $t=175\text{fm}/c$



(b) Au+Au(600 A MeV), $B=0-3\text{Fm}$, $t=175\text{fm}/c$

Gossiaux, Keane (EOS coll) et al
PRC51 (1995) 3357

Bi+Xe, 28 AMeV

b=5fm

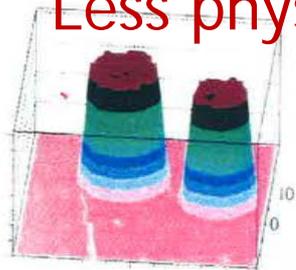
25 test particles/N

W. Bauer
U.Schröder

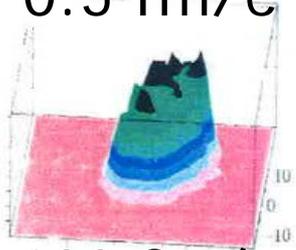
275 test particles/N

Less physical^M

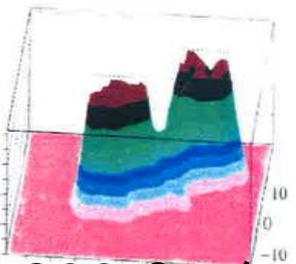
More physical



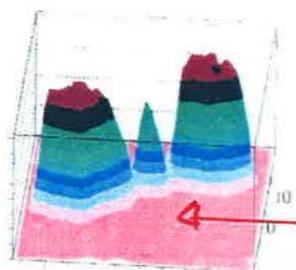
0.5 fm/c



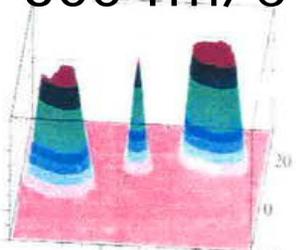
100 fm/c



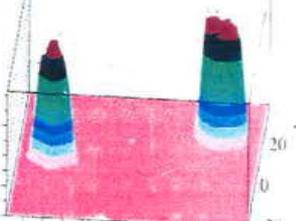
200 fm/c



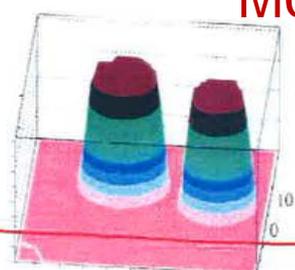
300 fm/c



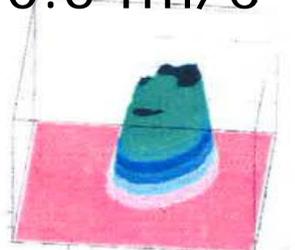
400 fm/c



500 fm/c



0.5 fm/c



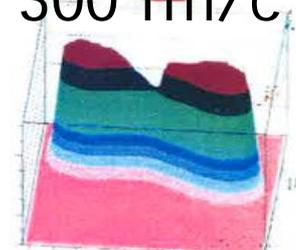
100 fm/c



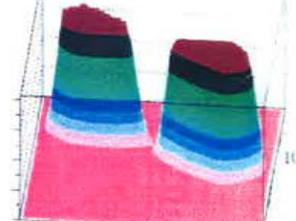
200 fm/c



300 fm/c



400 fm/c



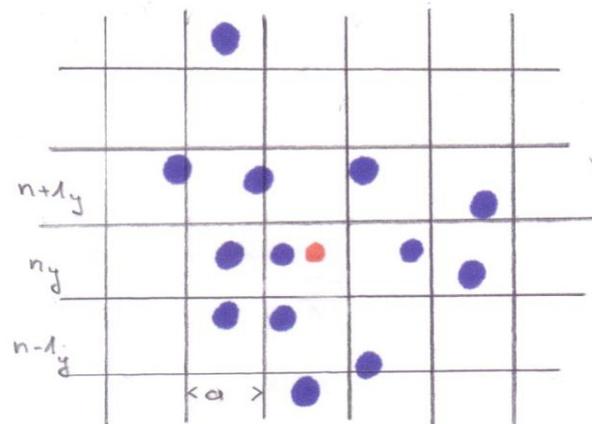
500 fm/c

Numbers of test particles must be large enough

When is N sufficiently large?

One uses delta like forces: $F(r) = \delta(r)$ (Skyrme) but then point-like test particles $f = \sum \delta(r-r_i(t))$ do **almost never interact**. Solution: one uses grids (and introduces the grid size a which plays a similar role as the width in QMD).

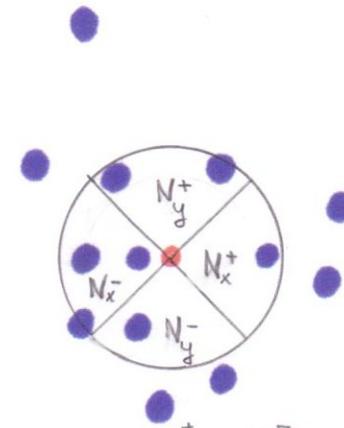
Euler



$$F_x = \frac{U_{n_x+1}(\rho) - U_{n_x-1}(\rho)}{2a}$$

Result
different
if number
of test
particles
is finite
(usually
 $N=100$)

Lagrange



$$F_x = \frac{U_{x \text{ right}}(\rho) - U_{x \text{ left}}(\rho)}{2a}$$

Average distance between nucleons 2fm. Grid size \approx 1fm (surface).

Therefore **very many test particles necessary** to **avoid numerical**

fluctuations: 100tp \rightarrow 12 in a cell \rightarrow 30% fluctuation

VUU, BUU, HSD, SMASH solve a Boltzmann type eq.

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}}$$

Same interaction, not possible classically

$$\left(\frac{\partial f}{\partial t} \right)_{\text{coll}} = \iint \underbrace{gI(g, \Omega)}_{\mathbf{v} \cdot \text{differential cross section}} [f(\mathbf{p}'_A, t)f(\mathbf{p}'_B, t) - f(\mathbf{p}_A, t)f(\mathbf{p}_B, t)] d\Omega d^3\mathbf{p}_A d^3\mathbf{p}_B.$$

$\mathbf{v} \cdot$ differential cross section

Only the **test particle method** made it possible to solve the BUU equations in complex situations

Test particle method -> replace integrals by sums (MC) integration

$$f(\mathbf{r}, \mathbf{p}, t) = \sum_{i=1}^{N \rightarrow \infty} \delta(\mathbf{r} - \mathbf{r}_i(t)) \delta(\mathbf{p} - \mathbf{p}_i(t)) \quad \text{test particle} \neq \text{nucleon}$$

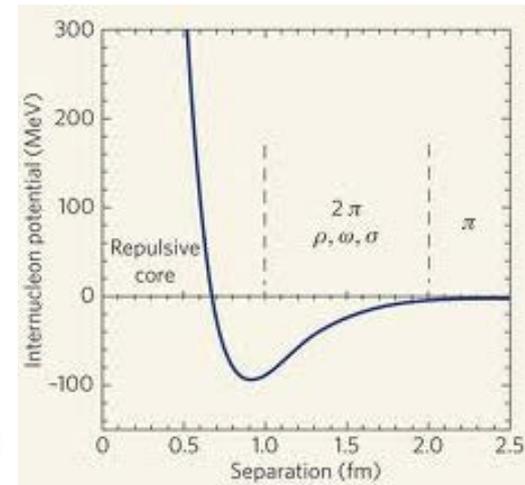
If N small unphysical fluctuations

What means $N \rightarrow \infty$ in reality ?

How does a collision term appear?

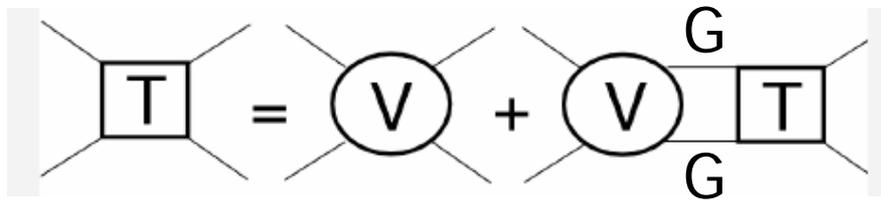
The Hamiltonian (Schrödinger and Boltzmann eq.) contains $V = NN$ potential

The **NN potential has a hard core**, would make **transport calculations very unrealistic** (Bodmer 75) (independent of the beam energy the participants would **thermalize** like In a cascade calculation without Pauli blocking)



Solution (taken over from TDHF):

Replace the NN potential V_{NN} by the solution of the Bethe-Salpeter eq. in T-matrix approach (Brueckner)



$$T_{\alpha}(E; q, q') = V_{\alpha}(q, q') + \int k^2 dk V_{\alpha}(q, k) G_{Q\bar{Q}}^0(E, k) T_{\alpha}(E; k, q')$$

$$T_{\alpha}(E; q, q') = V_{\alpha}(q, q') + \int k^2 dk V_{\alpha}(q, k) G_{Q\bar{Q}}^0(E, k) T_{\alpha}(E; k, q')$$

Consequences:

V_{NN} is real \rightarrow **T is complex** = **ReT** + **i Im T**

\swarrow \searrow
 corresponds to V_{NN} σ_{elast}
 in Hamiltonian **collisions**
 (Skyrme) done identically
BUU (test-particles)
and QMD (particles)

To this one adds inelastic collisions

(BUU, HSD, SMASH and QMD - the same way)!

\rightarrow Therefore in BUU and QMD the spectra of produced particles are (almost) identical (intensively checked in the past)

- take a small number of test particles (N_1):
 - mathematically this is then **not a correct solution** of the differential (BUU) equation
 - in practise problems with **energy and momentum conserv.**
 - assumes relations between physical (σ, T, ρ) and mathematical fluctuations ($1 = \overline{N}$) which are difficult to justify
- add a fluctuating force to the BUU equation
Colonna, Suraud, Ayik.....
 - mathematically correct
 - difficult to determine these fluctuations
size in Δr and Δp , dependence of T, ρ , (as effectively in QMD)..???
- move in BUU several testparticles simultaneously (Bertsch..)
 - how many and which ones?
 - in which way?

Question: Why not start directly from a N-body theory where fluctuations are (better) under control ?
(Width L fixed by nucl. density profile etc.)

How to determine the width L?

- **surface** of the nucleus -> L not too large
- **correlations** of the relative 2-part. wavefct in a nucleus (healing distance) $\approx 2\text{fm}$
- **range** of nuclear potential $\approx 2\text{ fm}$

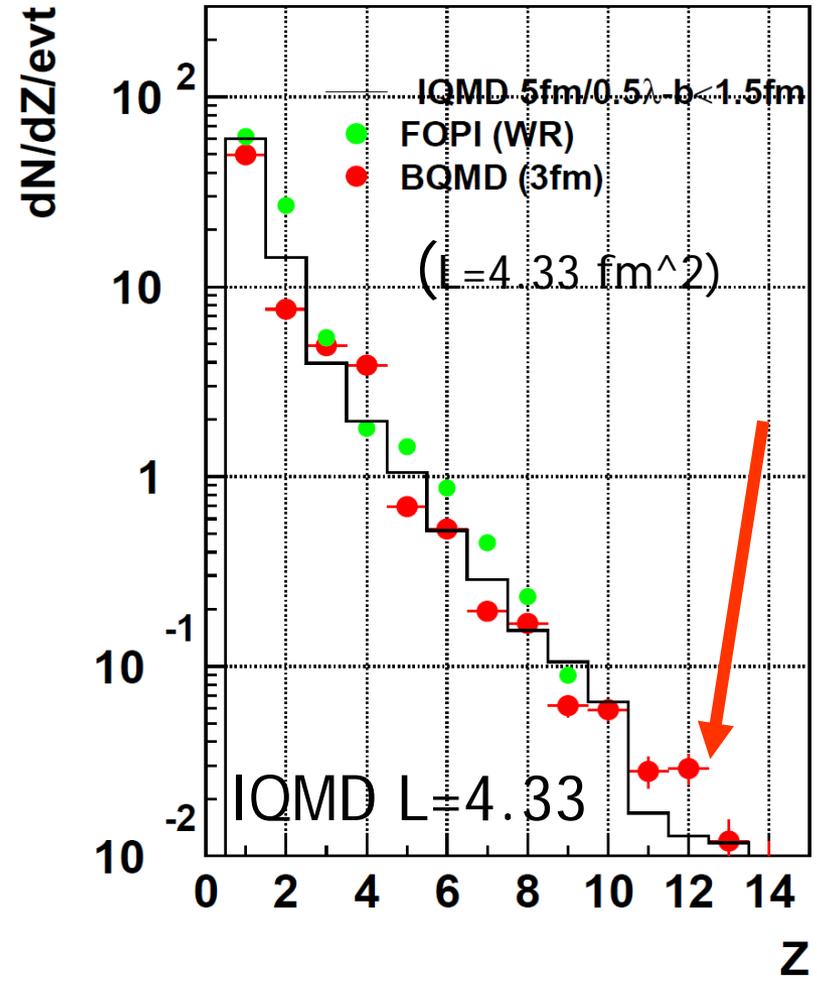
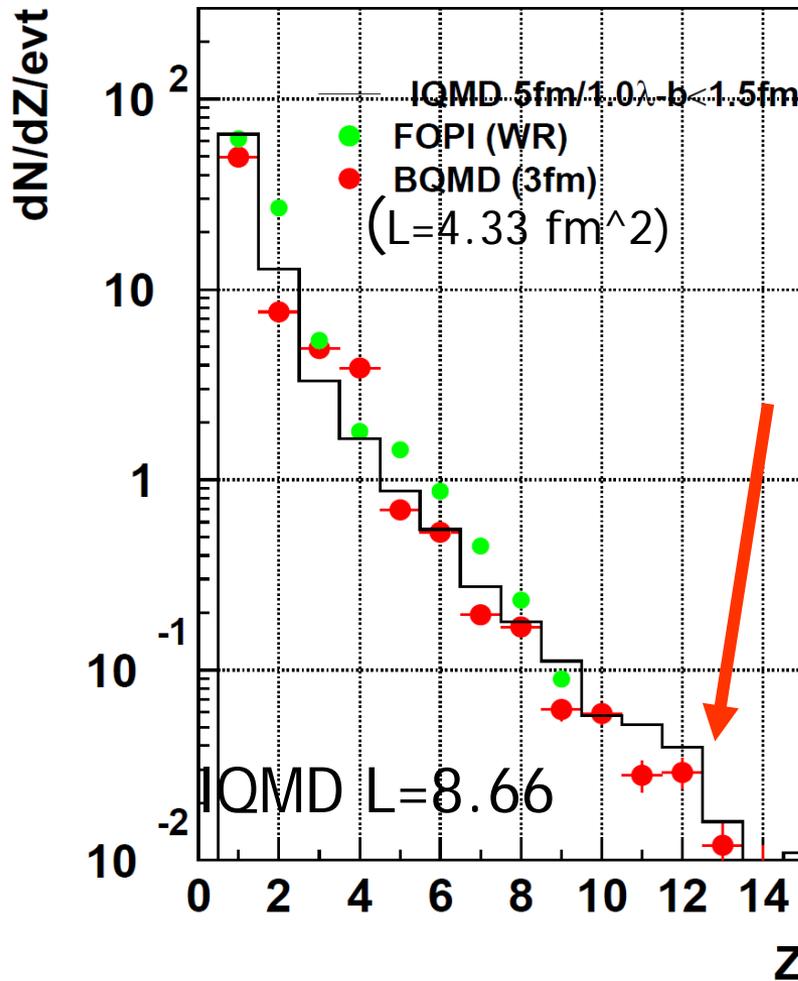
$$L = 4.33 \text{ fm}^2$$

Where L shows up in the observables?

- initially the **average over many simulations** gives the same $\rho(r)$ as BUU $\int d^3p f(r; p; t)$
but the density in each simulation fluctuates around $\rho(r)$
Initial state fluctuations depend on L
- L determines the local density change if a nucleon is kicked out by a hard collision (spectator fragmentation)
L influences spectator fragmentation
- L plays also a role when fragments are formed from prefer.
in participant fragmentation (via binding energies)

Influence of L on fragment yield (Y. Leifels)

AuAu 150 AMeV

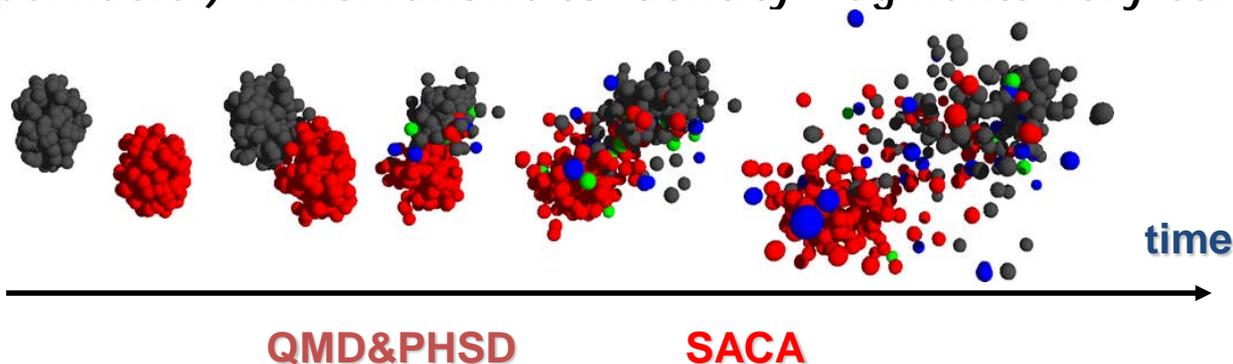


There are differences but they are modest

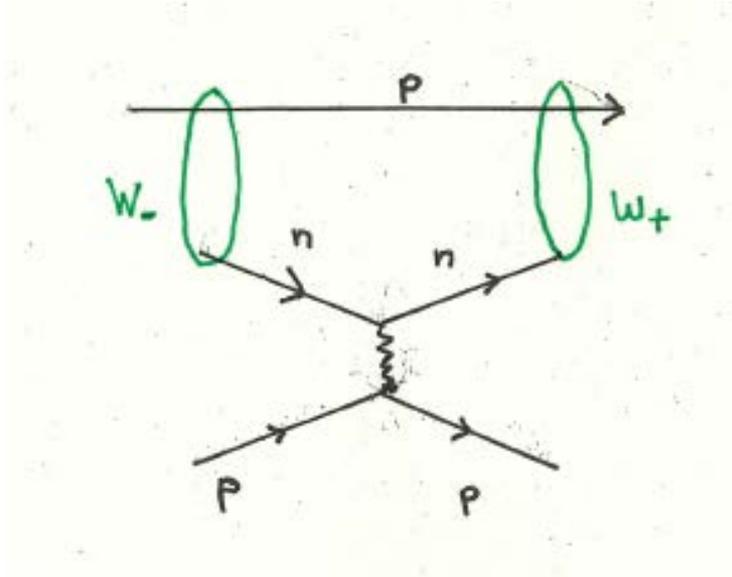
Modeling of fragment and hypernucleus formation

The goal: Dynamical modeling of cluster formation by a combined model
PHQMD = (QMD & PHSD) & SACA (FRIGA)

- ❑ **Parton-Hadron-Quantum-Molecular-Dynamics** - a non-equilibrium microscopic transport model which describes **n-body dynamics** based on **QMD propagation** with **collision integrals from PHSD** (Parton-Hadron-String Dynamics) and **cluster formation by the SACA model** or by the Minimum Spanning Tree model (MST).
- ❑ MST can determine clusters only at the end of the reaction.
- ❑ **Simulated Annealing Clusterization Algorithm** - cluster selection according to the largest binding energy (**extension of the SACA model** -> **FRIGA** which includes hypernuclei). FRIGA allows to identify fragments very early during the reaction.



III. Wigner density formalism (Remler (NPA 402, 596))



d-wave function

$$\Psi_d(\mathbf{r}, \mathbf{R}) \propto \exp^{-(\mathbf{r}-\mathbf{r}_0)^2 L} \exp^{-(\mathbf{R}-\mathbf{R}_0)^2 L/4}$$

d-Wigner density

$$\rho_d^W(\mathbf{r}, \mathbf{p}) \propto \exp^{-(\mathbf{r}-\mathbf{r}_0)^2 L} \exp^{-(\mathbf{p}-\mathbf{p}_0)^2 / L\hbar}$$

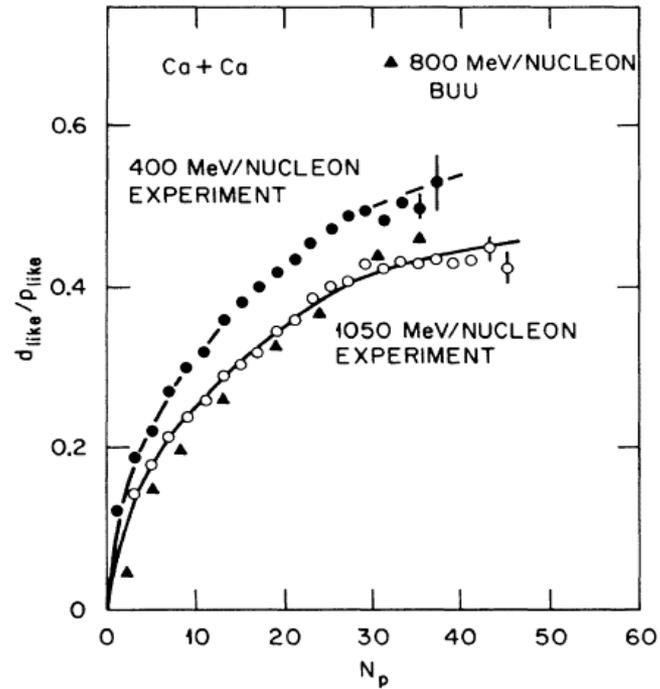
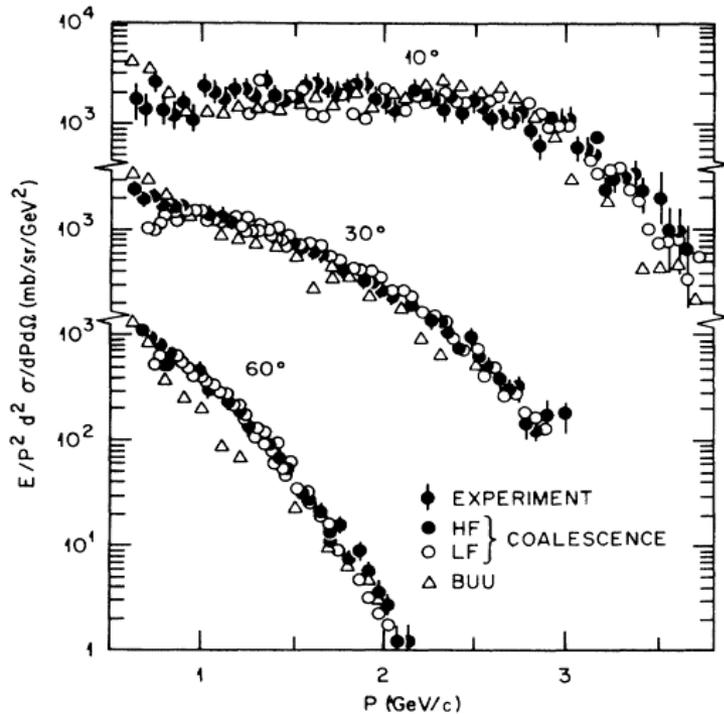
Yields for the rate

$$\begin{aligned} \Gamma(t) &= \sum_{\substack{i=1,2 \\ j \geq 3}} \sum \delta(t - t_{ij}(\nu)) \int \prod_i \frac{d^3 p_i d^3 x_i}{h^3} \overbrace{\rho_d^W(\mathbf{p}_1, \mathbf{x}_1, \mathbf{p}_2, \mathbf{x}_2)}^{W^+} [\rho_N^W(t + \epsilon) - \rho_N^W(t - \epsilon)] \\ &\quad \text{coll between } n \text{ or } p \text{ and rest} \\ &= \sum_{i=1,2} \sum_{i>3} \delta(t - t_{ij}(\nu)) [\rho_d^W(t + \epsilon) - \rho_d^W(t - \epsilon)] \end{aligned}$$

Wigner density of d

Easy to apply at lower energies

Ca+Ca 800 AMeV (PRC35,1291)



At higher energies: role of resonances ?
In PHQMD under construction

QMD (like AMD and FMD) are true N-body theories.

N-body theory: Describe the exact time evolution of a system of N particles. All correlations of the system are correctly described and fluctuations correctly propagated.

Roots in classical physics:

A look into textbooks on classical mechanics:
If one has a given Hamiltonian

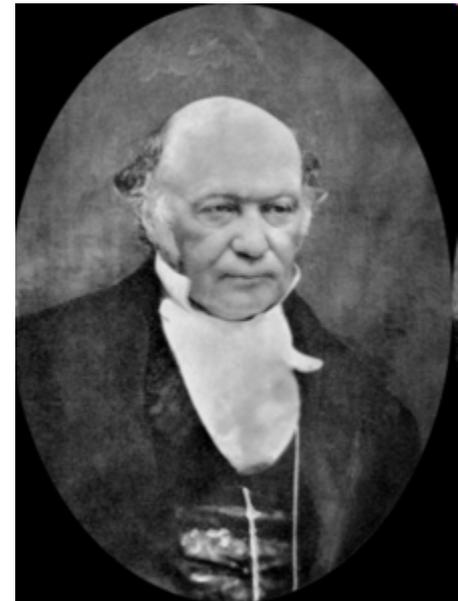
$$H(\mathbf{r}_1, \dots, \mathbf{r}_N, \dots, \mathbf{p}_1, \dots, \mathbf{p}_N, t)$$

$$\frac{d\mathbf{r}_i}{dt} = \frac{\partial H}{\partial \mathbf{p}_i}; \quad \frac{d\mathbf{p}_i}{dt} = -\frac{\partial H}{\partial \mathbf{r}_i}$$

For a given initial condition

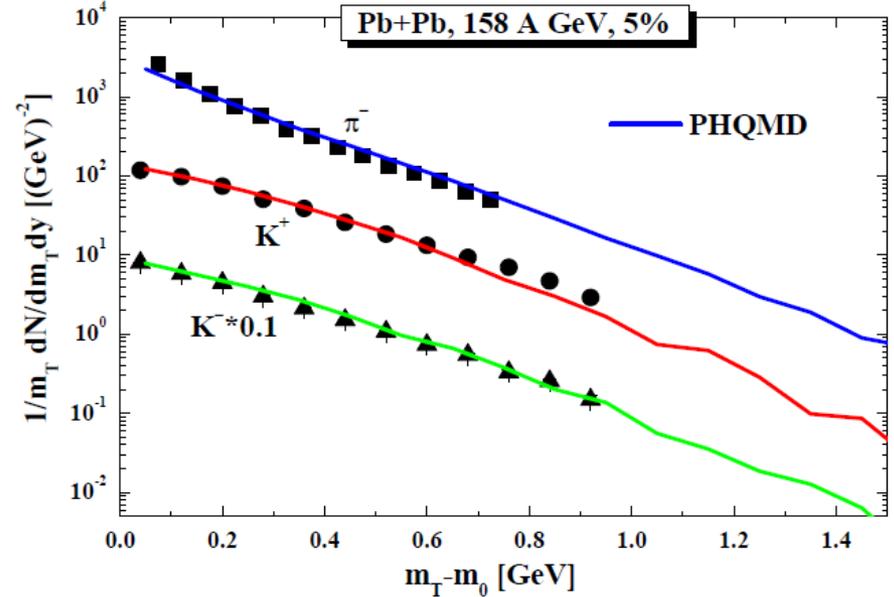
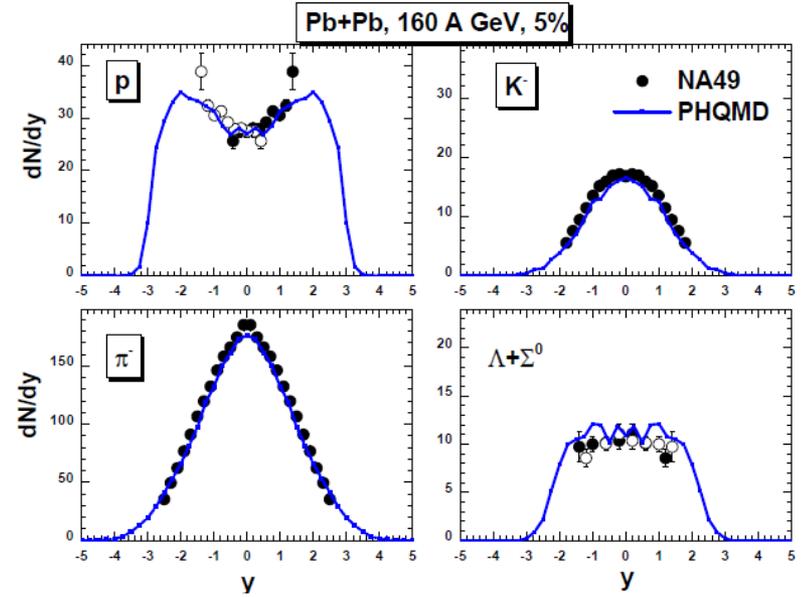
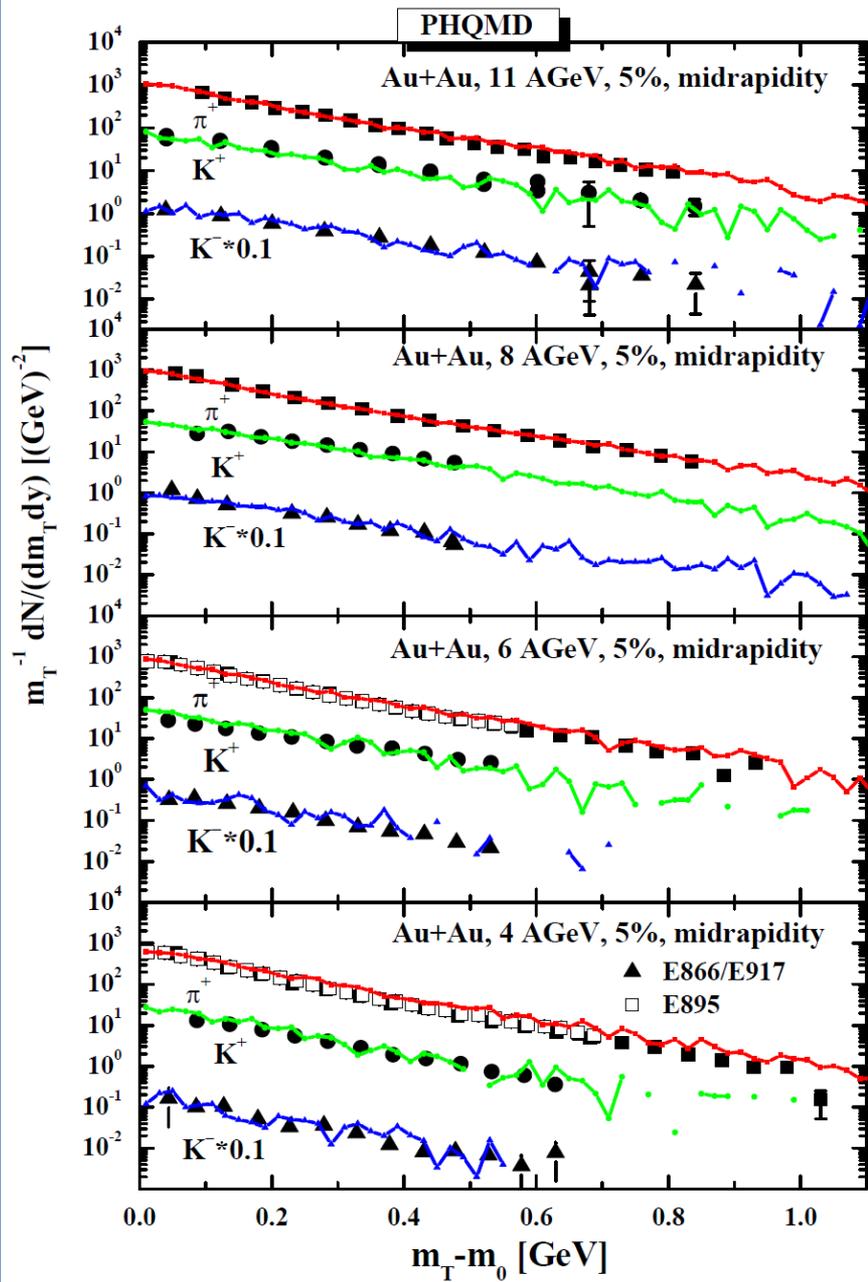
$$\mathbf{r}_1(t = 0), \dots, \mathbf{r}_N(t = 0), \mathbf{p}_1(t = 0), \dots, \mathbf{p}_N(t = 0)$$

the positions and momenta of all particles
are predictable for all times.



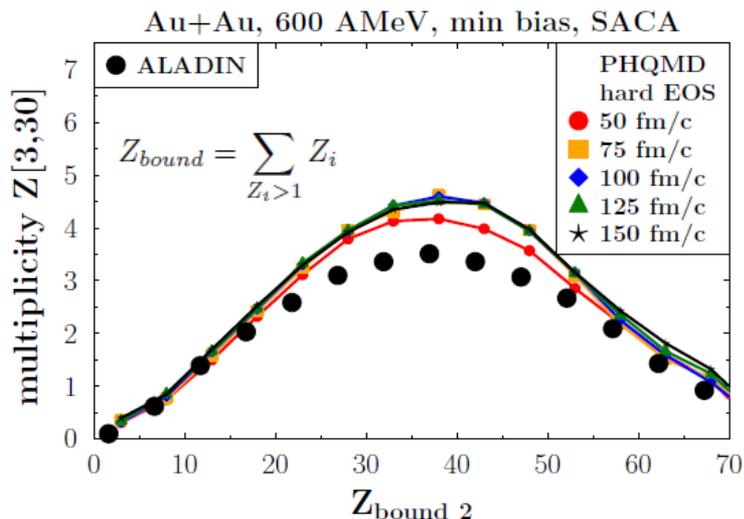
William Hamilton

As well as at SPS energies



Spectator Fragments

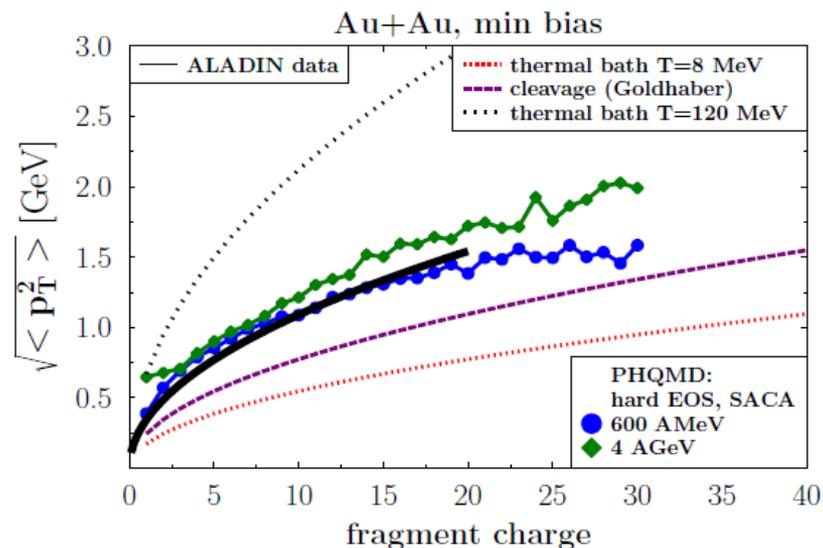
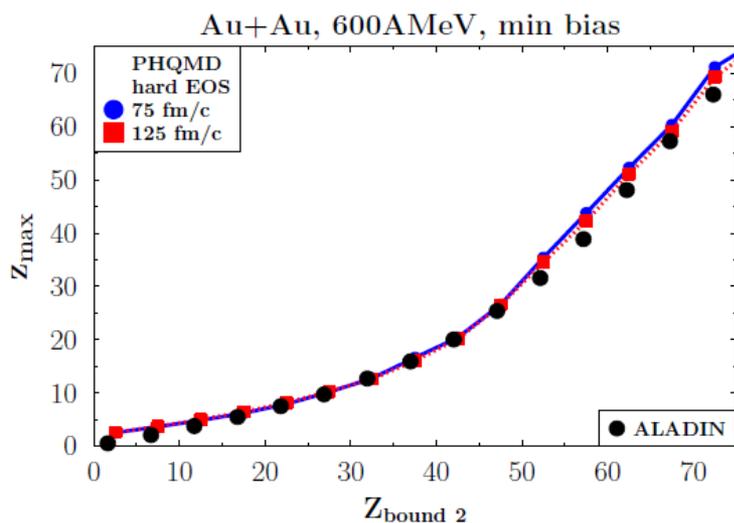
experim. measured up to $E_{\text{beam}} = 1 \text{ AGeV}$ (ALADIN)



agreement for **very complex fragment observables** like the

- energy independent “rise and fall”
- largest fragment (Z_{bound})

rms(p_t) shows \sqrt{Z} dependence





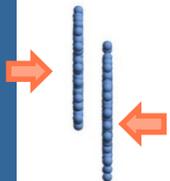
Parton-Hadron-String-Dynamics (PHSD)

collision integral
→ PHQMD

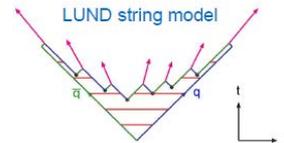
PHSD is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

Dynamics: based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

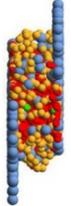
Initial A+A collision



Initial A+A collisions :
N+N → **string formation** → decay to pre-hadrons + leading hadrons

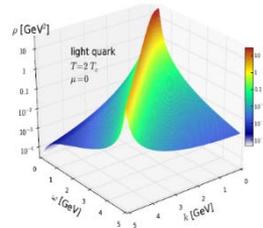


Partonic phase

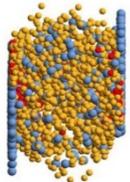


Formation of QGP stage if local $\varepsilon > \varepsilon_{\text{critical}}$:
dissolution of **pre-hadrons** → partons

Partonic phase - QGP:
QGP is described by the **Dynamical QuasiParticle Model (DQPM)** matched to reproduce **lattice QCD EoS** for finite T and μ_B (crossover)

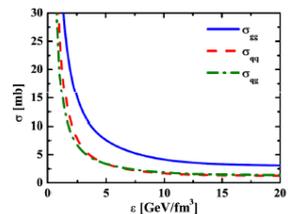


Hadronization

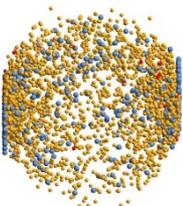


- **Degrees-of-freedom:** strongly interacting quasiparticles: **massive quarks and gluons (g, q, q_{bar})** with sizeable collisional widths in a self-generated mean-field potential

- **Interactions:** (quasi-)elastic and inelastic collisions of partons



Hadronic phase



Hadronization to colorless **off-shell mesons and baryons:**
Strict 4-momentum and quantum number conservation

Hadronic phase: **hadron-hadron interactions – off-shell HSD**

