Thank for the invitation

QUIVER THEORY

AND

GRAVITATIONAL WAVES

Paul H. Frampton

Culture Della materia, University of Salento.
Day Visitor, Oxford University (90%)

REFERENCE

C. Coriano, P.H. Frampton and A. Tatullo,

Conformal Unification in a Quiver Theory and Gravitational Waves.

`arXiv:2005.12216[hep-ph]`.

Introduction

Since the discovery of gravitational waves from the merger of two black holes, each with mass $M_{BH} \sim 30\ M_{\odot}$, announced as event GW150914 in 2016 by the LIGO-Virgo Collaboration, it has become clear that this provides a new and invaluable window into the early universe. Many subsequent similar observations have occurred and of special interest is one where two neutron stars merger where the event was shortly thereafter observed electromagnetically, thereby giving birth to multi-messenger astronomy.

The conventional way of seeking new physics at the highest possible energies has, since the invention of the cyclotron ninety years ago, been by using particle colliders. The highest energy of any active collider is at the LHC (Large Hadron Collider) with center of mass (com) energy 14 TeV. Possible colliders with com energies up to 100 TeV are under discussion. In the early universe, such energy / temperature existed at cosmic times with $t < 10^{-16}$ s. To study higher energies or shorter cosmic times colliders could become prohibitively expensive and a better method may be provided by gravitational wave detectors (GWDs) which can, in principle, be sensitive to signals generated from all cosmic times back to the Planck time $t \sim 10^{-44}$ s which could lay bare 14 more orders of magnitude in energies up to the Planck mass $M_{Planck} \sim 10^{19}$ GeV.

In the present talk, we more conservatively study energies up to a few TeV which may overlap with accessible collider energies yet where GWDs could give additional information about the type of phase transitions which occurred in the early universe.

Although such experiments could eventually investigate phase transitions up the a GUT scale *e.g.* 10^{16} GeV, the earliest such linkage is likely to come at a much lower energy.

A paper by Maldacena in 1997 considered the correspondence between string theory and quantum field theory by studying a Type IIB superstring compactified on a manifold $AdS_5 \times S^5$. The author showed that the superstring is dual to a maximally supersymmetric $\mathcal{N} = 4$ gauge theory with gauge group $SU(N)$ in a limit where $N \to \infty$. It followed that weakly coupled string theory is equivalent to strongly coupled gauge field theory, a hitherto unexpected connection which provided a number of insights into solution of problems in a broad range of theoretical physics.

To make a connection to particle phenomenology, it was then proposed that a generalisation of Maldacena which broke supersymmetry completely from $\mathcal{N} = 4$ to $\mathcal{N} = 0$ with finite N should be considered. This was attained by using a generalised manifold $AdS_5 \times S^5/Z_p$, an orbifold, leading to a gauge group $SU(N)^p$ and matter fields most conveniently characterised as bifundamental and adjoint representations in a quiver diagram; hence the name quiver theory.

One especially interesting example was discussed over a decade later. It used the values $p = 12$ and $N = 3$ and gives rise to a theory which unifies at an unusually low energy scale $E \simeq 4$ TeV. Proton decay is absent due to the quiver construction.

Quiver Model

We use a different strategy for unification of electroweak theory with QCD than in GUTs based on $SU(5)$ or $SO(10)$. The choice of quiver is motivated by bottom-up considerations. The desert with logarithmic running of couplings is abandoned. Instead, the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group is embedded in a semi-simple gauge group such as $SU(3)^p$ as suggested by gauge theories arising from compactification of the IIB superstring on an orbifold $AdS_5 \times S^5/\Gamma$ where Γ is the abelian finite group Z_p. In such non-supersymmetric quiver gauge theories the unification of couplings occurs abruptly at $\mu = M$ through the diagonal embeddings of 321 in $SU(3)^p$. The key prediction of such unification shifts from proton decay to additional particle content, in the present model at ~ 4 TeV.

We use the well-known RG β-functions. Taking the values at the Z-pole $\alpha_Y(M_Z) = 0.0101$, $\alpha_2(M_Z) = 0.0338$ and $\alpha_3(M_Z) = 0.118$, they are taken to run between M_Z and M according to the SM equations

$$\begin{aligned}
\alpha_Y(M) &= (0.01014)^{-1} - (41/12\pi)\ln(M/M_Z) \\
&= 98.619 - 1.0876y \qquad (0.1)\\
\alpha^{-1}(M) &= 0.0338)^{-1} + (19/12\pi)\ln(M/M_Z) \\
&= 29.586 + 0.504y \qquad (0.2)\\
\alpha^{-1}(M) &= (0.118)^{-1} + (7/2\pi)\ln(M/M_Z) \\
&= 8.474 + 1.114y
\end{aligned}$$

$$(0.3)$$

where $y = \log(M/M_Z)$.

The scale at which

$$\sin^2\theta(M) = \alpha_Y(M)/(\alpha_2(M) + \alpha_Y(M)) \quad (0.4)$$

satisfies $\sin^2\theta(M) = 1/4$ is at a value $M \simeq 4$ TeV. We focus on the ratio

$$R(M) \equiv \alpha_3(M)/\alpha_2(M). \quad (0.5)$$

We find that $R(M_Z) \simeq 3.5$ while

$$R(M_3) = 3, \ R(M_{5/2}) = 5/2, \ R(M_2) = 2 \quad (0.6)$$

occur respectively at the scales

$$M_3 \simeq 400\,\text{GeV}, \ M_{5/2} \simeq 4\,\text{TeV}, \ M_2 = 140\,\text{TeV}. \quad (0.7)$$

The proximity of $M_{5/2}$ and M, accurate to a few percent, suggests strong-electroweak unification at ~ 4 TeV. There remains the question of embedding such unification in an $SU(3)^p$ of the quiver type discussed in the Introduction.

Since the required embeddings of $SU(2)_L \times U(1)_Y$ into an $SU(3)$ necessitates $3\alpha_Y = \alpha_H$, the ratios of couplings at $\simeq 4$ TeV is

$$\alpha_{3C} : \alpha_{3W} : \alpha_{3H} :: 5 : 2 : 2 \tag{0.8}$$

and thus it is natural to examine $p = 12$ with diagonal embeddings of Colour (C), Weak (W) and Hypercharge(H) in $SU(3)^2, SU(3)^5, SU(3)^5$, respectively.

To accomplish this we specify the embedding of $\Gamma = Z_{12}$ in the global $SU(4)$ R-parity of the $\mathcal{N} = 4$ supersymmetry of the underlying theory. Defining $\alpha = \exp(2\pi i/12)$, this specification can be made by $\mathbf{4} \equiv (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ with $\sum A_\mu = 0 \pmod{12}$ and all $A_\mu \neq 0$ so that all four supersymmetries are broken from $\mathcal{N} = 4$ to $\mathcal{N} = 0$.

Having specified A_μ we calculate the content of complex scalars by investigating in $SU(4)$ the $\mathbf{6} \equiv (\alpha^{a_1}, \alpha^{a_2}, \alpha^{a_3}, \alpha^{-a_3}, \alpha^{-a_2}, \alpha^{-a_1})$ with $a_1 = A_1 + A_2$, $a_2 = A_2 + A_3$ and $a_3 = A_3 + A_1$ where all quantities are defined (mod 12). Finally we identify the nodes as C, W or H on the dodecahedral quiver such that the complex scalars

$$\sum_{i=1}^{3} \sum_{\alpha=1}^{12} \left(N_\alpha, \bar{N}_{\alpha+a_i}\right) \tag{0.9}$$

are adequate to allow the required symmetry breaking to the $SU(3)^3$ diagonal subgroup.

The chiral fermions given by

$$\sum_{\mu=1}^{4}\sum_{\alpha=1}^{12}\left(N_\alpha, \bar{N}_{\alpha+A_\mu}\right) \tag{0.10}$$

will be able to include the thee generations of fermions. These constraints are nontrivial but remarkably there exists a solution.

The unique solution is to adopt $A_\mu \equiv (1,2,3,6)$ and for the quiver nodes take the ordering:

$$-C-W-H-C-W^4-H^4- \tag{0.11}$$

with the two ends of Eq.(0.11) identified to form a dodecahedral quiver.

With this choice the scalars are provided by $A_I = (3,4,5)$ and are sufficient to break to

$$SU(3)_C \times SU(3)_W \times SU(3)_H \tag{0.12}$$

The choice of quiver nodes in Eq. (0.11) generates precisely three quark lepton families which transform under trinification as

$$3\left[(3,\bar{3},1)+(1,3,\bar{3})+(\bar{3},1,3)\right] \qquad (0.13)$$

The ordering of the quiver nodes merits further explication. The point is that breaking to a diagonal subgroup $SU(3)$ from $SU(3)^r$ is possible if and only if all the r nodes are connected by bifundamental scalars and no node is isolated. By trial and error, a physicist can become convinced that Eq.(0.11) is the unique choice which satisfies this highly restrictive constraint.

Once the number of C, W and H nodes has been chosen in order that the three couplings accurately unify, there is usually no quiver diagram which will allow the required symmetry breaking. We found one successful examples which is studied assiduously in this talk. The choice of gauge group and matter fields is less arbitrary than it may seem at first sight. The choice is unique.

Anomaly freedom of the superstring guarantees that the only possible combination of chiral fermions is as in Eq. (0.13). This fact makes it easier to confirm the occurrence of three familes in the complicated quiver diagram because one needs to check only one of the three representations, for example the colour triplets which all originate from C nodes.

Further breaking to the SM group gives the correct light chiral states. The couplings run up to $E = M$ and then become frozen for at least a finite energy range provided conformal invariance sets in as expected by analogy with the supersymmetric case of Maldacena. At $M \sim 4$TeV, there are many new particles awaiting discovery: gauge bosons, fermions and scalars. These are necessary to satisfy the conformal constraints.

This quiver model is interesting because it ameliorates the hierarchy problem in $SU(5)$ and $SO(10)$ GUTs between the weak / Higgs mass scale and the GUT scale. It predicts correctly the value of $\sin^2\theta(M_Z)$, of $\alpha_3(M_Z)$ and the appearance of exactly three families.

One final advantage is that the unification of the three SM couplings at $M \sim 4$TeV is very precise, more accurate even than in SusyGUTs. This has been calculated, with the robustness of the model.

We believe grand unification at 4 TeV has no disadvantage relative to unification at a trillion times higher scale, and has the advantage of avoiding the dubious desert hypothesis.

To clarify the quiver theory construction, we explain in more detail the case of the Z_{12} orbifold by exhibiting the relevant quiver diagrams. In this case the quiver diagram is a dodecahedron, like a clock face.

Certain shortcuts make use of the symmetries of the quiver diagram and obviate including every possible link which will make the diagram very dense with links and more difficult to understand. Let us begin with the chiral fermions which are denoted by oriented arrows between two nodes. The quarks can be counted by examining the $C \rightarrow W$ links and subtracting the $W \rightarrow C$ links, noting that anomaly freedom dictates that the chiral fermions will necessarily appear only in the specific combination of Eq. (0.13) and so no other C links need to be checked. The relevant quiver diagram is shown in Fig. 1.

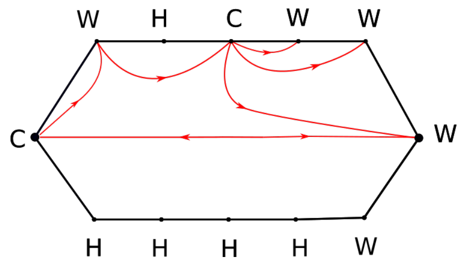

Figure 1 Quiver diagram for chiral fermions

We see that there are five families and two antifamilies, resulting in precisely three light chiral families as required. The family-antifamily pairs are not chiral, but vector-like, and can therefore acquire Dirac masses.

Next we exhibit in Fig. 2 and Fig. 3 two further Z_{12} quiver diagrams which illustrate the scalar sector. Complex scalars are denoted by unoriented dashed lines. We must ensure and check that there are sufficient scalars whose VEVs can spontaneously break the $SU(3)^2_C$ down to $SU(3)_C$, the $SU(3)^5_W$ down to $SU(3)_W$ and finally $SU(3)^5_H$ down to $SU(3)_H$.

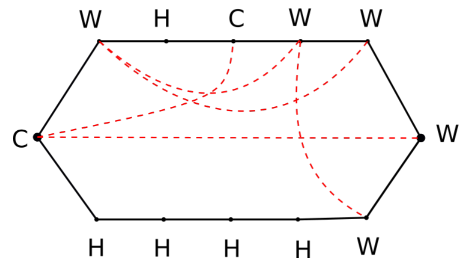

Figure 2 Scalars breaking symmetry to $SU(3)_C$ and $SU(3)_W$

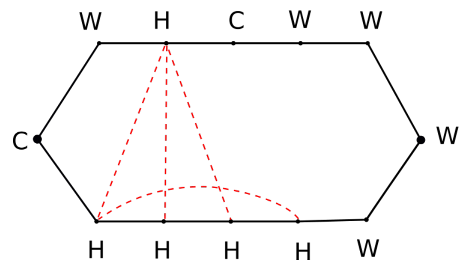

Figure 3 Scalars breaking symmetry to $SU(3)_H$

To break any $SU(3)^n$ down to the diagonal subgroup, a necessary and sufficient condition is that the bifundamental scalars link all n of the original gauge groups together. The $SU(3)$ gauge groups cannot be disconnected into subgroups nor can the bifundamental scalars separate into disconnected parts. In Fig. 2 the breakings of $SU(3)^2_C$ down to $SU(3)_C$ and of $SU(3)^5_W$ down to $SU(3)_W$ are shown to satisfy all these tight constraints so that the required spontaneous symmetry breaking is possible.

In Fig. 3 it is shown that the symmetry breaking $SU(3)^5_H$ down to $SU(3)_H$ also satisfies the same unforgiving connectivity requirements.

We note that this symmetry breaking is very non-trivial and is what underlies the correct identification of the nodes which is unique in allowing the required outcomes for both chiral fermions and complex scalars.

In fact, for 4 TeV grand unification without a desert, the $SU(3)^{12}$ construction appears to be unique when one insists that one arrives at three chiral families under trinification, and hence under the standard model, as well as ensuring that correct symmetry breaking is permitted.

The GUT gauge group $SU(3)^{12}$ has dimension 96 which is bigger than the dimensions 24 and 45 of $SU(5)$ and $SO(10)$ respectively. This can be regarded as the price to pay to avoid the desert. The wealth of additional state at 4TeV also changes the nature of the phase transition $SU(3)^{12} \rightarrow SU(3)^3$ which can generate the gravitational waves studied in the next section.

Gravitational Waves

In the present quiver model, there is only one cosmological phase transition at a scale above the electroweak scale. It is at an energy/temperature of 4 TeV. Henceforth, without justification, we shall assume this to be strongly FOPT(First Order Phase Transition).

We shall need g_*, the equivalent number of massless degrees degrees of freedom for the quiver theory, defined by

$$g_* = n_B + \frac{7}{8} n_F \qquad (0.14)$$

where n_B, n_F is the number for bosons, fermions respectively. It is easier to count g_* before spontaneous symmetry breaking, although of course the result is the same.

In the standard model with three families we have

$$\begin{aligned} n_B(\text{spin} = 1) &= 12 \times 2 = 24 \\ n_B(\text{spin} = 0) &= 4 \\ n_F(\text{spin} = 1/2) &= 3 \times 15 \times 2 = 90 \end{aligned} \tag{0.15}$$

so that in this case

$$g_* = 28 + \frac{7}{8}(90) = 106.75 \tag{0.16}$$

which will also be g_* for the quiver theory at energies $E < 4$ TeV.

In our present $SU(3)^{12}$ quiver theory we recall from the previous section that the scalars are in the bifundamental representations

$$\sum_{i=1}^{3}\sum_{\alpha=1}^{12}\left(3_\alpha, \bar{3}_{\alpha\pm a_i}\right) \qquad (0.17)$$

with $a_1 = 3, 4, 5$.

The chiral fermions are in bifundamentals

$$\sum_{\mu=1}^{4}\sum_{\alpha=1}^{12}\left(3_\alpha, \bar{3}_{\alpha+A_\mu}\right) \qquad (0.18)$$

with $A_\mu = 1, 2, 3, 6$.

The equivalent massless degrees of freedom are

$$\begin{aligned} n_B(\text{spin} = 1) &= 96 \times 2 = 192 \\ n_B(\text{spin} = 0) &= 12 \times 9 \times 3 = 324 \\ n_F(\text{spin} = 1/2) &= 12 \times 18 \times 4 = 864 \end{aligned} \tag{0.19}$$

so that for the full quiver theory

$$g_* = 516 + \frac{7}{8}(864) = 1,272 \tag{0.20}$$

which is the number of effective massless degrees of freedom for $E \geq 4$ GeV.

To discuss gravitational waves emitted during the phase transition at $T = 4$ TeV in the early universe, we focus initially on the gravitational radiation from bubble collisions, assuming that we are dealing with a FOPT (First Order Phase Transition).

The nature of the phase transition depends on the effective potential of the theory. Eq.(0.17) exhibits the scalars present in the quiver and the twelve nodes of the quiver are identified in Eq.(0.11). The dodecahedral quiver has nodes which we label clockwise by 1 to 12 by Color(C), Weak (W) and Hypercharge (H) as follows: (1) C, (2) W, (3) H, (4) C, (5 - 8) W, (9-12) H.

We are initially concerned with the breaking $SU(3)^{12} \rightarrow SU(3)^3$ at scale $E = 4$ TeV. This can be studied separately for C, W and H. Let us define lower-case Greek indices $\alpha_i, \beta_i, \gamma_i, \delta_i .... = 1, 2, 3$ for the SU(3) group of the i^{th} node and discriminate between subscripts which represent defining representations and superscripts which denote anti-defining representations. The SM color gauge group arises from the diagonal subgroup of the SU(3)'s at nodes 1 and 4 respectively and this symmetry breaking is achieved by VEVs of the complex scalar bifundamentals:

$$\Phi_{\alpha_1}^{\beta_4} \text{ and } \Phi_{\alpha_4}^{\beta_1} \qquad (0.21)$$

In the effective potential at tree level there are quadratic and quartic terms involving the 1 to 4 bifundamentals as follows

$$\mathcal{V}_{Eff}^{(C,Tree)} = \mathcal{C}_2^{(14)} \left(\Phi_{\alpha_1}^{\beta_4} \Phi_{\alpha_1}^{\beta_4}\right) + \mathcal{C}_4^{(14)} \left(\Phi_{\alpha_1}^{\beta_4} \Phi_{\alpha_1}^{\beta_4}\right)^2 + \mathcal{C}_4^{(14)'} \left(\Phi_{\alpha_1}^{\beta_4} \Phi_{\beta_4}^{\gamma_1} \Phi_{\gamma_1}^{\delta_4} \Phi_{\delta_4}^{\alpha_1}\right) \quad (0.22)$$

To break to the trinification group

$$SU(3)_C \times SU(3)_W \times SU(3)_H \quad (0.23)$$

a similar combination of bifundamental scalars conspire to arrive at diagonal subgroups for both the five $SU(3)_W$ nodes and the five $SU(3)_H$ nodes respectively.

Another intermediate symmetry-breaking stage is where $SU(3)_W$ in Eq.(0.23) breaks to the $SU(2)_L$ of the SM, also $SU(3)_W \times SU(3)_H$ breaks to the $U(1)_Y$ of the SM but for our analysis of gravitational radiation we shall focus only on a FOPT where the quiver group $SU(3)^{12}$ breaks at $E = 4$ TeV to the trinification group in Eq.(0.23).

For W we use scalars connecting nodes 2-5-6-7-8 and the relevant scalar bifundamentals in Eq.(0.17) are

$$\Phi_{\alpha_2}^{\beta_5} \;\; , \;\; \Phi_{\alpha_2}^{\beta_6} \;\; , \;\; \Phi_{\alpha_2}^{\beta_7} \;\; \text{and} \;\; \Phi_{\alpha_5}^{\beta_8} \qquad (0.24)$$

The corresponding quadratic and quartic terms in the tree-level effective potential composed of the scalars in Eq.(0.24) are as follows (next slide).

$$
\begin{aligned}
&\mathcal{C}_2^{(25678)} \left(\Phi_{\alpha_2}^{\beta_5} \Phi_{\beta_5}^{\alpha_2} + \Phi_{\alpha_2}^{\beta_6} \Phi_{\beta_6}^{\alpha_2} + \Phi_{\alpha_2}^{\beta_7} \Phi_{\beta_7}^{\alpha_2} + \Phi_{\alpha_5}^{\beta_8} \Phi_{\beta_8}^{\alpha_5} \right) \\
&\mathcal{C}_4^{(25678)} \left(\Phi_{\alpha_2}^{\beta_5} \Phi_{\beta_5}^{\alpha_2} + \Phi_{\alpha_2}^{\beta_6} \Phi_{\beta_6}^{\alpha_2} + \Phi_{\alpha_2}^{\beta_7} \Phi_{\beta_7}^{\alpha_2} + \Phi_{\alpha_5}^{\beta_8} \Phi_{\beta_8}^{\alpha_5} \right)^2 \\
&\mathcal{C}_4^{(25678)'} \left(\Phi_{\alpha_2}^{\beta_5} \Phi_{\beta_5}^{\gamma_2} \Phi_{\gamma_2}^{\delta_5} \Phi_{\delta_5}^{\alpha_2} + \Phi_{\alpha_2}^{\beta_6} \Phi_{\beta_6}^{\gamma_2} \Phi_{\gamma_2}^{\delta_6} \Phi_{\delta_6}^{\alpha_2} \right. \\
&\qquad \left. + \Phi_{\alpha_2}^{\beta_7} \Phi_{\beta_7}^{\gamma_2} \Phi_{\gamma_2}^{\delta_7} \Phi_{\delta_7}^{\alpha_2} + \Phi_{\alpha_5}^{\beta_8} \Phi_{\beta_8}^{\gamma_5} \Phi_{\gamma_5}^{\delta_8} \Phi_{\delta_8}^{\alpha_5} \right)
\end{aligned} \tag{0.25}
$$

For H we use scalars connecting nodes 3-9-10-11-12 and the relevant scalar bifundamentals in Eq.(0.17) are

$$
\Phi_{\alpha_3}^{\beta_{10}} \ , \ \Phi_{\alpha_3}^{\beta_{11}} \ , \ \Phi_{\alpha_3}^{\beta_{12}} \ \text{and} \ \Phi_{\alpha_9}^{\beta_{12}} \tag{0.26}
$$

The corresponding quadratic and quartic terms in the tree-level effective potential composed of the scalars in Eq.(0.26) are as follows.

$$\left(\Phi_{\alpha_3}^{\beta_{10}}\Phi_{\beta_{10}}^{\alpha_3}+\Phi_{\alpha_3}^{\beta_{11}}\Phi_{\beta_{11}}^{\alpha_3}+\Phi_{\alpha_3}^{\beta_{12}}\Phi_{\beta_{12}}^{\alpha_3}+\Phi_{\alpha_9}^{\beta_{12}}\Phi_{\beta_{12}}^{\alpha_9}\right)$$
$$\left(\Phi_{\alpha_3}^{\beta_{10}}\Phi_{\beta_{10}}^{\alpha_3}+\Phi_{\alpha_3}^{\beta_{11}}\Phi_{\beta_{11}}^{\alpha_3}+\Phi_{\alpha_3}^{\beta_{12}}\Phi_{\beta_{12}}^{\alpha_3}+\Phi_{\alpha_9}^{\beta_{12}}\Phi_{\beta_{12}}^{\alpha_9}\right)^2.$$
$$\left(\Phi_{\alpha_3}^{\beta_{10}}\Phi_{\beta_{10}}^{\gamma_3}\Phi_{\gamma_3}^{\delta_{10}}\Phi_{\delta_{10}}^{\alpha_3}+\Phi_{\alpha_3}^{\beta_{11}}\Phi_{\beta_{11}}^{\gamma_3}\Phi_{\gamma_3}^{\delta_{11}}\Phi_{\delta_{11}}^{\alpha_3}\right.$$
$$\left.+\Phi_{\alpha_3}^{\beta_{12}}\Phi_{\beta_{12}}^{\gamma_3}\Phi_{\gamma_3}^{\delta_{12}}\Phi_{\delta_{12}}^{\alpha_3}+\Phi_{\alpha_9}^{\beta_{12}}\Phi_{\beta_{12}}^{\gamma_9}\Phi_{\gamma_9}^{\delta_{12}}\Phi_{\delta_{12}}^{\alpha_9}\right) \tag{0.27}$$

Because the quiver theory above 4 TeV is conformal we must impose $C_2 = 0$ in all the quadratic terms. Next, before adding the three $\mathcal{V}_{Eff}^{Tree}$ expressions, let us examine the symmetries of the dodecahedral quiver which imply that

$$C_4^{(2,5678)} = C_4^{(3,9101112)} \equiv D_4$$
$$C_4^{(2,5678)'} = C_4^{(3,9101112)'} \equiv D_4' \tag{0.28}$$

whereupon, suppressing superscripts, the most general tree-level effective potential is

$$\mathcal{C}_4\left(\Phi^{\beta_4}_{\alpha_1}\Phi^{\beta_4}_{\alpha_1}\right)^2+\mathcal{C}'_4\left(\Phi^{\beta_4}_{\alpha_1}\Phi^{\gamma_1}_{\beta_4}\Phi^{\delta_4}_{\gamma_1}\Phi^{\alpha_1}_{\delta_4}\right)$$
$$+\mathcal{D}_4\left(\Phi^{\beta_5}_{\alpha_2}\Phi^{\alpha_2}_{\beta_5}+\Phi^{\beta_6}_{\alpha_2}\Phi^{\alpha_2}_{\beta_6}+\Phi^{\beta_7}_{\alpha_2}\Phi^{\alpha_2}_{\beta_7}+\Phi^{\beta_8}_{\alpha_5}\Phi^{\alpha_5}_{\beta_8}\right)^2 \tag{0.29}$$

Because of conformal invariance, we must add a Coleman-Weinberg term $\mathcal{V}^{CW}_{Eff}$ at the one-loop level. Finally, also at one-loop, we add a finite-temperature potential term $\mathcal{V}^{T}_{Eff}$ to arrive at an effective potential

$$\mathcal{V}_{Eff}=\mathcal{V}^{Tree}_{Eff}+\mathcal{V}^{CW}_{Eff}+\mathcal{V}^{T}_{Eff} \tag{0.30}$$

within which all available information about possible early universe phase transitions is encoded and is the most general class of effective potential.

Results

In order to compute the gravitational waves radiated at the quiver-GUT phase transition, we employ the PTPlot software available. To make our plots, we needed the values of five parameters: Wall velocity v_W; Phase transition strength α_θ; Inverse phase transition duration β/H_*; Transition temperature T_*; and Degrees of freedom g_*.

The last two parameters have been calculated earlier in this paper as $T_* = 4000$ GeV and $g_* = 1,732$. We use $v_W = 0.7$ as a typical value and find our results are quite insensitive to v_W in the range $0.1 < v_W < 1.0$. This leaves only the two parameters α_θ and β/H_* related to the details of the FOPT which depend on the effective potential calculated in the previous section.

For the two FOPT parameters, we use $\alpha_\theta = 0.1$ and find our results are quite insensitive to a wide range of α_θ. The key FOPT parameter is β/H_* for which our results are the most sensitive so we display a range of eight values: $\beta/H_* = 1, 2, 5, 10, 15, 20, 25, 30$. We do not display results for larger values because for none of them is LISA capable of detecting the FOTP gravitational waves.

Thus, we are assuming that the duration of the phase transition is short enough that the cosmic expansion can be neglected but is not extremely short.

The following are the plots obtained from using PTPlot.

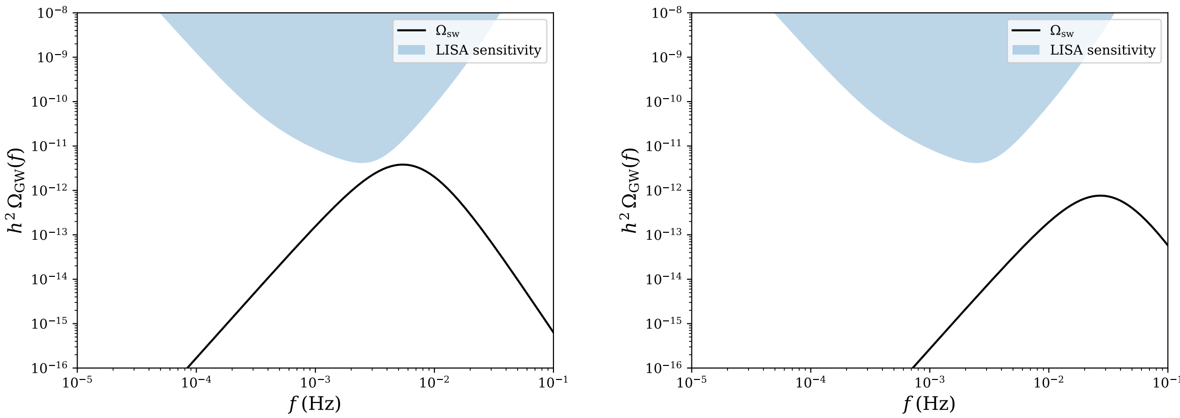

Figure 4 GW emission for $\alpha_\theta = 0.1, v_w = 0.1$,(left) $\beta/H_* = 1$; (right) $\beta/H_* = 5$

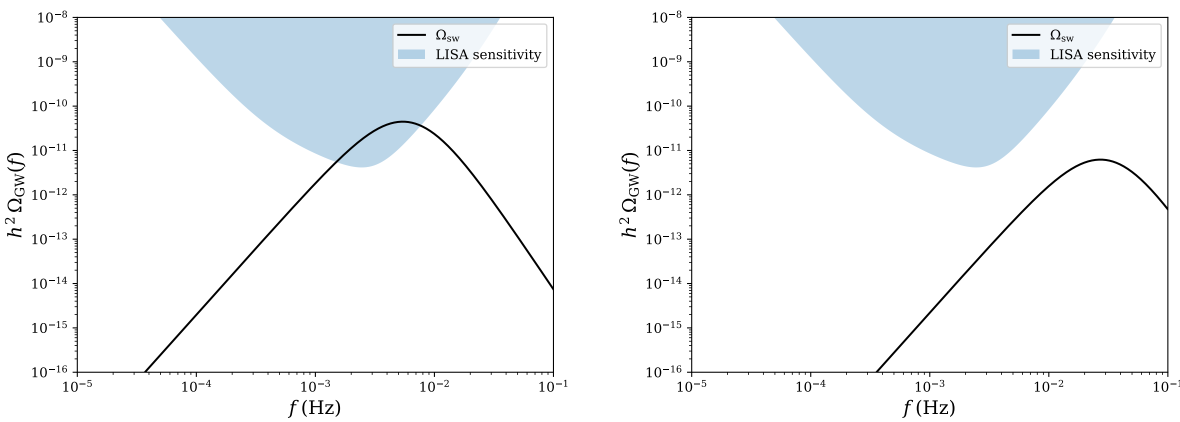

Figure 5 GW emission for $\alpha_\theta = 0.2, v_w = 0.1$,(left) $\beta/H_* = 1$; (right)$\beta/H_* = 5$

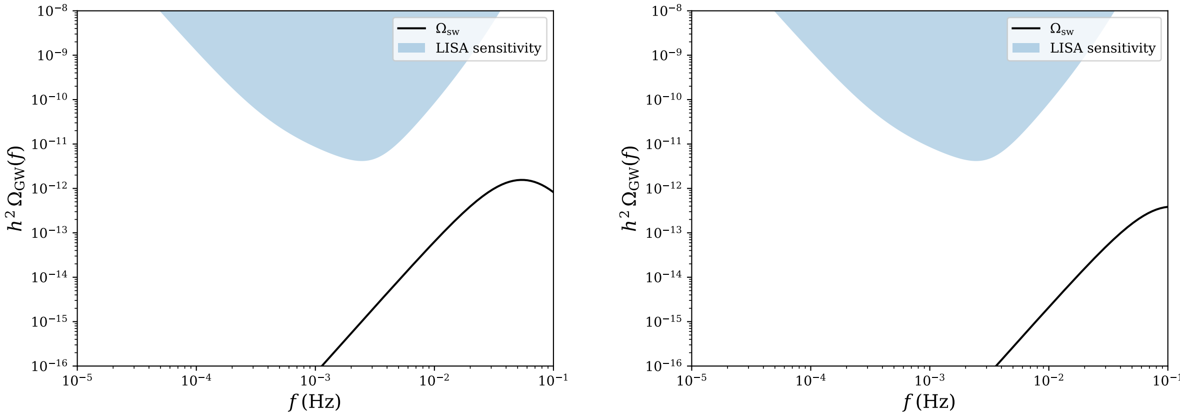

Figure 6 GW emission for $\alpha_\theta = 0.2, v_w = 0.1$,(left) $\beta/H_* = 10$; (right)$\beta/H_* = 20$

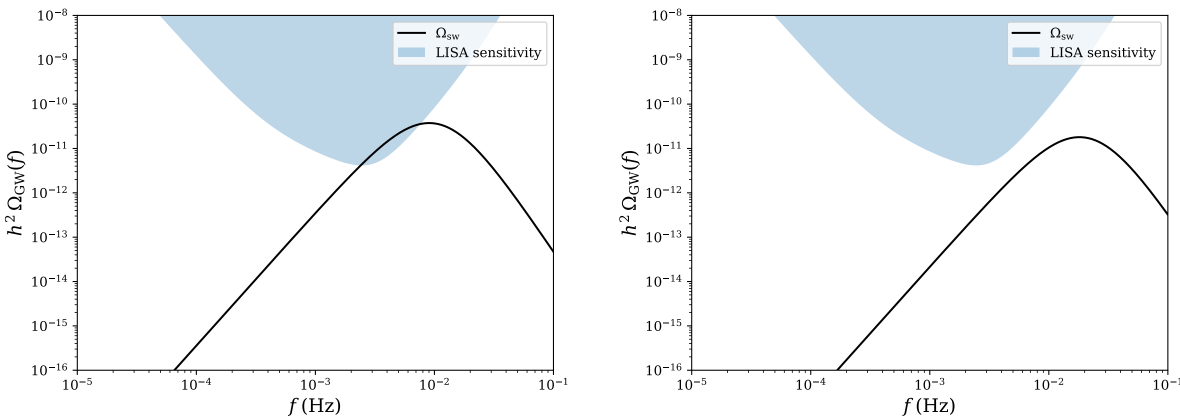

Figure 7 GW emission for $\alpha_\theta = 0.1, v_w = 0.3$,(left) $\beta/H_* = 5$; (right)$\beta/H_* = 10$

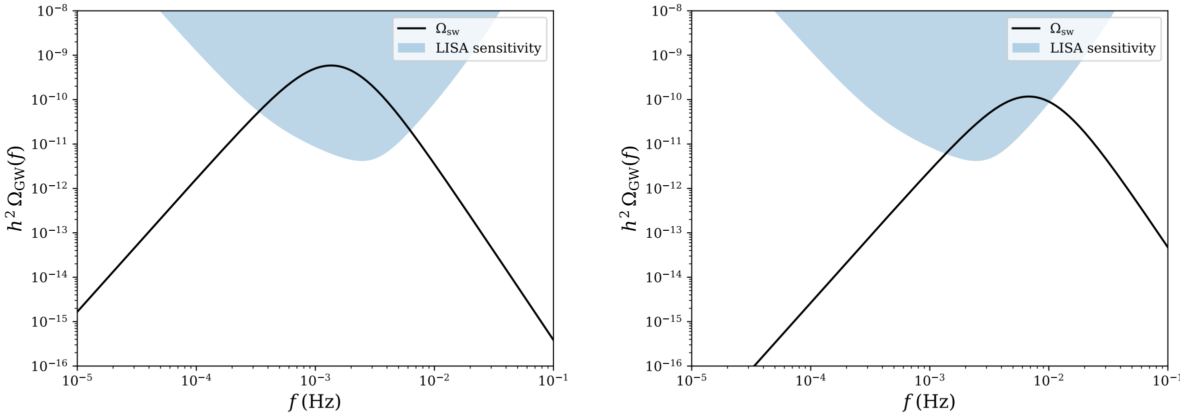

Figure 8 GW emission for $\alpha_\theta = 0.1, v_w = 0.4$,(left) $H/\beta = 1$; (right)$H/\beta = 5$

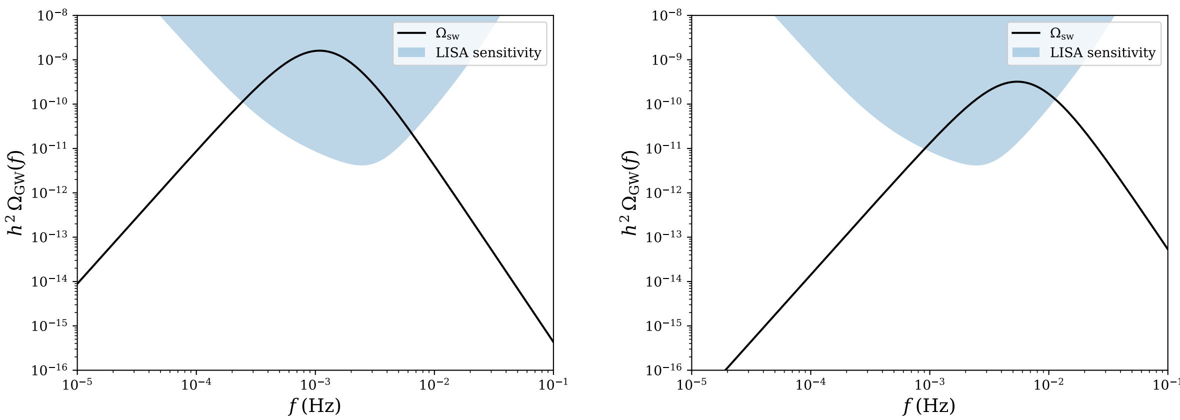

Figure 9 GW emission for $\alpha_\theta = 0.1, v_w = 0.5$,(left) $H/\beta = 1$; (right)$H/\beta = 5$

Discussion

Back forty years in 1980 it seemed more likely than not, and supported by 90% of the physics community, that minimal $SU(5)$ grand unification would agree with experiment and that proton decay would soon be observed with a lifetime about 10^{30} years. In 1984 by contrast the theory went down in flames when the proton lifetime was found experimentally to be 100 times too long, now known to be 10,000 times too long, to agree.

The reason that $SU(5)$ theory fails is presumably because of the desert hypothesis that there exists no new physics between the weak scale and the GUT scale.

In the present talk, therefore, we have avoided the desert hypothesis in a quiver GUT and makes no assumption about new physics at scales above 4 TeV except that the theory is expected to become conformally invariant up to much higher scales. Proton decay is absent at three level because of the quiver inspired assignments of the quarks and leptons

We have suggested that a phase transition in the early universe, expected by quiver GUT theory, could be detectable by the LISA gravitational wave detector. It will be interesting to know whether our suggestion can be realised.

`Thank you for your attention`