Four-dimensional Noncommutative Gravity

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General Relativity

Second order formulation (Einstein gravity):

- independent field: metric tensor, $g_{\mu\nu}$
- curvature parametrized by Riemann tensor:
  $R^{\rho}_{\mu\nu\sigma} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$
- torsion: $T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}$
- action: $S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R$

First order formulation:

- independent fields: vierbein and spin connection, $e_\mu{}^a, \omega_\mu{}^{ab}$
- curvature parametrized by the curvature 2-form:
  $R_{\mu\nu ab} = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} - \omega_{\mu ac}\omega_\nu{}^c{}_b - \omega_{\nu ac}\omega_\mu{}^c{}_b$
- torsion: $T_{\mu\nu}{}^a = \partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a + \omega_\mu{}^a{}_b e_\nu{}^c - \omega_\nu{}^a{}_b e_\mu{}^c$ gr as gauge
- action: $S = \frac{1}{16\pi G}\int \frac{1}{2}\epsilon_{abcd}e^a \wedge e^b \wedge R^{cd}$ (Palatini action)

Einstein 4d gravity as a gauge theory

The algebra

- Employ the first order formulation of GR
- Gauge theory of Poincaré group ISO(1,3)
- Ten generators (Translations P_a & LT M_{ab})

see for details:
Utiyama '56, Kibble '61,
McDowell-Mansuri '77,
Kibble - Stelle '85

Generators satisfy the commutation relations:

$$[M_{ab}, M_{cd}] = \eta_{ac} M_{db} - \eta_{bc} M_{da} - \eta_{ad} M_{cb} + \eta_{bd} M_{ca}$$
$$[P_a, M_{bc}] = \eta_{ab} P_c - \eta_{ac} P_b \,, \qquad [P_a, P_b] = 0$$

where $\eta_{ab} = \mathrm{diag}(-1, +1, +1, +1)$ and $a, b, c, d = 1, \ldots, 4$.

The gauging procedure

- Intro of a gauge field for each generator: ${e_\mu}^a, {\omega_\mu}^{ab}$ (transl, LT) - mix of gauge and spacetime
- Define the covariant derivative → the $\mathfrak{iso}(1,3)$-valued 1-form gauge connection is:

$$A_\mu = {e_\mu}^a(x) P_a + \tfrac{1}{2}{\omega_\mu}^{ab}(x) M_{ab}$$

- Transforms in the adjoint rep, according to the rule:

$$\delta A_\mu = \partial_\mu \epsilon + [A_\mu, \epsilon]$$

- The gauge transformation parameter, $\epsilon(x)$ is expanded as:

$$\epsilon(x) = \xi^a(x) P_a + \tfrac{1}{2}\lambda^{ab}(x) M_{ab}$$

- *Combining* the above → transformations of the fields:

$$\delta {e_\mu}^a = \partial_\mu \xi^a - {e_\mu}^b {\lambda^a}_b + {\omega_\mu}^{ab}\xi_b$$
$$\delta {\omega_\mu}^{ab} = \partial_\mu \lambda^{ab} - {\lambda^a}_c {\omega_\mu}^{cb} + {\lambda^b}_c {\omega_\mu}^{ca}$$

- Gauge transf equivalent to diffeo transf (on-shell) → *gauge invariance* ⟷ *general covariance!*

Curvatures and action

- Curvatures of the fields are given by:

$$R_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

- Tensor $\mathrm{R}_{\mu\nu}$ is also valued in $\mathfrak{iso}(1,3)$:

$$R_{\mu\nu}(A) = T_{\mu\nu}{}^a P_a + R_{\mu\nu}{}^{ab} M_{ab}$$

- *Combining* the above → component tensor curvatures:

$$T_{\mu\nu}{}^a = \partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a + e_\mu{}^b \omega_{\nu b}{}^a - e_\nu{}^b \omega_{\mu b}{}^a$$
$$R_{\mu\nu}{}^{ab} = \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab} - \omega_\mu{}^{cb} \omega_\nu{}^a{}_c + \omega_\mu{}^{ac} \omega_{\nu c}{}^b$$

2nd order

- Dynamics of Einstein gravity cannot be obtained by an ISO(1,3) action of Yang-Mills type

Stelle-West '80, Kibble - Stelle '85

- E-H action from de Sitter group, consider the Pontryagin index + use an auxiliary field, gauge fix it → break the symmetry to the Lorentz subgroup → scalar curvature action + torsionless condition

Conformal 4d gravity as a gauge theory

- Group parametrizing the symmetry: SO(2,4)
- 15 generators: 6 LT M_{ab}, 4 translations, P_a, 4 conformal boosts K_a and the dilatation D
- Following the same procedure one calculates transf of the gauge fields and tensors after defining the gauge connection comm limit
- Action is taken of Yang-Mills form
- Initial symmetry breaks under certain constraints resulting to the *Weyl action* *Kaku-Townsend-Van Nieu/zen '77, Fradkin-Tseytlin '85*
- Alternative: Initial symmetry can break down to recover the *Einstein* action (two specific constraints are taken to hold simultaneously)

Chamseddine '02

The nc framework & gauge theories

The nc framework — *Szabo '01*

- Possible nc regime: Planck scale - coords may be considered non-commuting
- Late 40's: Nc structure of spacetime at small scales for an effective ultraviolet cutoff → control of divergences in qfts

  Snyder '47
- Ignored → success of renormalization programme
- 80's: nc geometry revived after the generalization of diff structure

  Connes '85, Woronowicz '87
- Along with the definition of a generalized integration → Yang-Mills gauge theories on nc spaces

  Connes-Rieffel '87
- Inspiration from qm: Operators instead of variables

- quantization of phase space of $x^i, p_j \rightarrow$ replace with Herm operators: $\hat{x}^i, \hat{p}_j$ satisfying:$[\hat{x}^i, \hat{p}_j] = i\hbar\delta^i_j$
- Noncommutative space $\rightarrow$ quantization of space: $x^i \rightarrow$ replace with operators $\hat{x}^i$ ($\in \mathcal{A}$) satisfying: $[\hat{x}^i, \hat{x}^j] = i\theta^{ij}(\hat{x})$

Connes '94, Madore '99

- Antisymmetric tensor $\theta^{ij}(\hat{x})$ - defines the nc of the space
  - Canonical case: $\theta^{ij}(\hat{x}) = \theta^{ij}, i, j = 1, \ldots, N$

    For $N = 2 \rightarrow$ *Moyal plane*
  - Lie-type case: $\theta^{ij}(\hat{x}) = C^{ij}{}_k \hat{x}^k, i, j = 1, \ldots, N$

    For $N = 3 \rightarrow$ Noncommutative *(fuzzy) sphere* (SU(2))
- nc framework admits a matrix representation (operators)
  - Derivation: $e_i(A) = [d_i, A]$, $d_i \in \mathcal{A}$
  - Integration $\rightarrow$ Trace
- Algebra of operators with normal multiplication $\leftrightarrow$ algebra of functions with different product - *star product*
- Weyl correspondence: 1-1 correspondence of operators and functions. *Sternheimer '98, Alvarez-Gaumez, Wadia '01*

The nc gauge theories

- Natural intro of nc gauge theories through *covariant nc coordinates*: $\mathcal{X}_\mu = X_\mu + A_\mu$ *Madore-Schraml-Schupp-Wess '00*
- Obeys a covariant gauge transformation rule: $\delta \mathcal{X}_\mu = i[\epsilon, \mathcal{X}_\mu]$
- A_μ transforms in analogy with the gauge connection: $\delta A_\mu = -i[X_\mu, \epsilon] + i[\epsilon, A_\mu]$, ($\epsilon$ - the gauge parameter) 4dncgravity
- Definition of a nc *covariant field strength tensor* depends on the space:
  - Canonical case: $F_{ab} = [\mathcal{X}_a, \mathcal{X}_b] - i\theta_{ab}$
  - Lie-Type case: $F_{ab} = [\mathcal{X}_a, \mathcal{X}_b] - iC_{abc}\mathcal{X}_c$
- Gauge theory could be abelian or non-abelian:
  - Abelian if ϵ is a function in $\mathcal{A}$
  - Non-abelian if ϵ is matrix valued $(\mathrm{Mat}(\mathcal{A}))$

Non-Abelian case

▷ *In nonabelian case, where are the gauge fields valued?*

- Let us consider the CR of two elements of an algebra:

$$[\epsilon, A] = [\epsilon^A T^A, A^B T^B] = \frac{1}{2}\{\epsilon^A, A^B\}[T^A, T^B] + \frac{1}{2}[\epsilon^A, A^B]\{T^A, T^B\}$$

- *Not* possible to restrict to a matrix algebra:
  last term neither *vanishes* in nc nor is an *algebra element*

- There are two options to overpass the difficulty:
  - Consider the universal enveloping algebra
  - Extend the generators and/or fix the rep so that the anticommutators close

▷ *We employ the second option*

(Note: replacing with the star-product $\rightarrow$ *nc gauge theory with functions)*

The 4d covariant noncommutative space

Motivation for a 4d covariant nc space

- Constructing gravitational models on nc spaces is non-trivial: nc deformations break Lorentz invariance
- Special: *covariant* noncommutative spaces, i.e. coords transform as vectors under rotations
- such an example is the fuzzy sphere (2d space) - coords are identified as rescaled SU(2) generators
- Previous work on 3d nc gravity on the (already existing) covariant spaces $R^3_\lambda, R^{1,2}_\lambda$

  L. Jonke, A. Chatz/dis, D. Jurman, GM, P. Manousselis, G. Zoupanos '18

  Hammou-Lagraa-Sheikh Jabbari '02

  Vitale-Wallet '13, Vitale '14
- Need of 4d covariant nc space to construct a gravity gauge theory...

...so let us build one! *Manousselis, G.M, Zoupanos '20*

Construction of the 4d covariant nc space

Kimura '02, Heckman-Verlinde '15

Sperling-Steinacker '17

- dS_4 : homogeneous spacetime with constant curvature (positive)
- Described by the embedding $\eta^{AB} X_A X_B = R^2$ into $\mathrm{M}^{1,4}$
- We aim for a nc version of dS_4
- Coords must satisfy $[X_a, X_b] = i\theta_{ab}$, with θ_{ab} to be determined
- Analogy to the fuzzy sphere case: identification of the coordinates with generators of the SO(1,4) (isometry group of dS_4)
- BUT: θ_{ab} cannot be assigned to generators of the algebra $\rightarrow$ covariance breaks
- Requiring covariance $\rightarrow$ use a group with larger symmetry $\rightarrow$ minimum extension: SO(1,5) (SO(6) in our language)

- The SO(6) generators, $J_{AB}, A, B = 1, \ldots, 6$, satisfy the commutation relation:

$$[J_{AB}, J_{CD}] = i(\delta_{AC} J_{BD} + \delta_{BD} J_{AC} - \delta_{BC} J_{AD} - \delta_{AD} J_{BC})$$

- After decomposition in SO(4) notation, $A = a, 5, 6$, we identify the component generators as:

$$J_{ab} = \frac{1}{\hbar}\Theta_{ab}, \quad J_{a5} = \frac{1}{\lambda} X_a, \quad J_{a6} = \frac{\lambda}{2\hbar} P_a, \quad J_{56} = \frac{1}{2}\mathrm{h}\,,$$

where X_a, P_a and Θ_{ab} are the coordinate, momentum and nc tensor, respectively

- The above generators satisfy the following comm relations (among others):

$$[X_a, X_b] = i\frac{\lambda^2}{\hbar}\Theta_{ab}, \quad [X_m, \Theta_{np}] = i\hbar(\delta_{mp} X_n - \delta_{mn} X_p)$$

- Therefore, coordinates are identified as SO(6) generators represented in a (large N) matrix representation
- Covariance ensured - coords transform as vectors under rotations

Noncommutative gauge theory of 4d gravity

- Formulation of gravity on the above space
- Noncommutative gauge theory toolbox + the procedure described in the Einstein gravity case

  Kimura '02, Heckman-Verlinde '15

- Gauge the isometry group of the space, SO(5) as seen as a subgroup of the SO(6) we ended up
- Anticommutators do not close → enlargement of the algebra + fix the representation *Aschieri-Castellani '09*

  L. Jonke, A. Chatz/dis, D. Jurman, GM, P. Manousselis, G. Zoupanos '18

- Noncommutative gauge theory of SO(6)xU(1) ($\sim$ U(4)) in the 4 representation
- The generators of the group are represented by combinations of the 4x4 gamma matrices in euclidean signature

- Specifically, the 4x4 generators of the SO(6) group are:
  - six Lorentz rotation generators:
    $\mathrm{M}_{ab} = -\frac{i}{4}[\Gamma_a, \Gamma_b] = -\frac{i}{2}\Gamma_a\Gamma_b \,, a < b$
  - four generators for conformal boosts: $\mathrm{K}_a = \frac{1}{2}\Gamma_a$
  - four generators for translations: $\mathrm{P}_a = -\frac{i}{2}\Gamma_a\Gamma_5$
  - one generator for special conformal transformations:
    $\mathrm{D} = -\frac{1}{2}\Gamma_5$

  Also the following is included
  - one U(1) generator: 1
- The above expressions of the generators allow the calculation of the algebra they satisfy:

$$[M_{ab}, M_{cd}] = i(\delta_{ac}M_{bd} + \delta_{bd}M_{ac} - \delta_{bc}M_{ad}\delta_{ad}M_{bc})\,, [K_a, P_b] = i\delta_{ab}D$$
$$[K_a, K_b] = iM_{ab}\,, [P_a, P_b] = iM_{ab}\,, [P_a, D] = iK_a\,, [K_a, D] = -iP_a$$
$$[K_a, M_{bc}] = i(\delta_{ac}K_b - \delta_{ab}K_c)\,, [P_a, M_{bc}] = i(\delta_{ac}P_b - \delta_{ab}P_c)\,, [D, M_{ab}] = 0$$

- Gauging procedure → definition of the covariant coordinate: $\hat{X}_m = X_m \otimes 1 + A_m(X)$
- Gauge connection $\mathrm{A}_m(X)$ taking values in the algebra:

$$A_m(X) = e_m{}^a(X) \otimes P_a + \omega_m{}^{ab}(X) \otimes M_{ab}(X) + b_m{}^a(X) \otimes K_a(X) + \tilde{a}_m(X) \otimes D + a_m(X) \otimes 1$$

- Introduced a gauge field for each generator
- Consider a gauge parameter $\epsilon(x)$:

$$\epsilon = \epsilon_0(X) \otimes 1 + \xi^a(X) \otimes K_a + \tilde{\epsilon}_0(X) \otimes D + \lambda^{ab}(X) \otimes M_{ab} + \tilde{\xi}^a(X) \otimes P_a$$

- Determine the field strength tensor:

$$\mathcal{R}_{mn} = [\hat{X}_m, \hat{X}_n] - \frac{i\lambda^2}{\hbar} \hat{\Theta}_{mn} \otimes 1 \,,$$

where $\hat{\Theta}_{mn} = \Theta_{mn} \otimes 1 + \mathcal{B}_{mn}$, where $\mathcal{B}_{mn}$ is a 2-form gauge field valued in U(4) transforming covariantly

- The field strength tensor is also valued in the algebra → it is spanned on the generators

$$\mathcal{R}_{mn}(X) = R_{mn}{}^{ab} \otimes M_{ab} + \tilde{R}_{mn}{}^a \otimes P_a + R_{mn}{}^a \otimes K_a + \tilde{R}_{mn} \otimes D + R_{mn} \otimes 1$$

- Generators satisfy the following anticommutation relations:

Smolin '03

$$\{M_{ab}, M_{cd}\} = \frac{1}{8}\left(\delta_{ac}\delta_{bd} - \delta_{bc}\delta_{ad}\right)\mathbb{1} - \frac{\sqrt{2}}{4}\epsilon_{abcd}D$$

$$\{M_{ab}, K_c\} = \sqrt{2}\epsilon_{abcd}P_d\,, \quad \{M_{ab}, P_c\} = -\frac{\sqrt{2}}{4}\epsilon_{abcd}K_d$$

$$\{K_a, K_b\} = \frac{1}{2}\delta_{ab}\mathbb{1}\,, \quad \{P_a, P_b\} = \frac{1}{8}\delta_{ab}\mathbb{1}\,, \quad \{K_a, D\} = \{P_a, D\} = 0$$

$$\{P_a, K_b\} = \{M_{ab}, D\} = -\frac{\sqrt{2}}{2}\epsilon_{abcd}M_{cd}\,.$$

- The necessary information for calculating the transformations of the fields and the component tensors is in these two slides!
- Along with the general treatment of nc gauge theories ncgt

The transformations of the fields:

$$\delta\omega_m{}^{ab} = -i[X_m, \lambda^{ab}] - i[a_m, \lambda^{ab}] + i[\epsilon_0, \omega_m{}^{ab}] - 2\{\xi^a, b_m{}^b\} - \frac{1}{2}\{\lambda^a{}_c, \omega_m{}^{bc}\}$$
$$- \frac{1}{2}\{\tilde{\xi}^a, e_m{}^b\} + i[\xi^c, e_m{}^d]\epsilon_{abcd} + \frac{i}{2}[\tilde{\epsilon}_0, \omega_m{}^{cd}]\epsilon_{abcd} + \frac{i}{2}[\lambda^{cd}, \tilde{a}_m]\epsilon_{abcd} - i[\tilde{\xi}^c, b_m{}^d]\epsilon_{abcd}$$

$$\delta e_m{}^a = -i[X_m, \tilde{\xi}^a] - i[a_m, \tilde{\xi}^a] + i[\epsilon_0, e_m{}^a] - \{\xi^a, \tilde{a}_m\} + \{\tilde{\epsilon}_0, b_m{}^a\} + \frac{1}{4}\{\lambda^a{}_b, e_m{}^b\}$$
$$- \frac{1}{4}\{\tilde{\xi}_b, \omega_m{}^{ab}\} + i[\xi^c, \omega_m{}^{bd}]\epsilon_{abcd} - i[\lambda^{cd}, b_m{}^b]\epsilon_{abcd}$$

$$\delta b_m{}^a = -i[X_m, \xi^a] - i[a_m, \xi^a] + i[\epsilon_0, b_m{}^a] - \{\xi_b, \omega_m{}^{ab}\} - 2\{\tilde{\epsilon}_0, e_m{}^a\} + \frac{1}{2}\{\lambda^a{}_b, b_m{}^b\}$$
$$+ \{\tilde{\xi}^a, \tilde{a}_m\} + i[\lambda^{bc}, e_m{}^d]\epsilon_{abcd} + i[\tilde{\xi}^b, \omega_m{}^{cd}]\epsilon_{abcd}$$

$$\delta a_m = -i[X_m, \epsilon_0] - i[a_m, \epsilon_0] + i[\xi^a, b_m{}^a] + i[\tilde{\epsilon}_0, \tilde{a}_m] + \frac{i}{2}[\lambda_{ab}, \omega_m{}^{ab}] + \frac{i}{2}[\tilde{\xi}_a, e_m{}^a]$$

$$\delta\tilde{a}_m = -i[X_m, \tilde{\epsilon}_0] - i[a_m, \tilde{\epsilon}_0] + i[\epsilon_0, \tilde{a}_m] + \{\xi_a, e_m{}^a\} - \{\tilde{\xi}_a, b_m{}^a\} + \frac{i}{2}[\lambda^{ad}, \omega_m{}^{bc}]\epsilon_{abcd}$$

(Transformations of the component of $\mathcal{B}_{mn}$ are calculated as well)

The component curvatures:

$$R_{mn} = [X_m, a_n] - [X_n, a_m] + [a_m, a_n] + [b_m^{\ a}, b_{na}] + [\tilde{a}_m, \tilde{a}_n] + \frac{1}{2}[\omega_m^{\ ab}, \omega_{nab}]$$
$$+ [e_{ma}, e_n^{\ a}] - \frac{i\hbar}{\lambda^2} B_{mn}$$
$$\tilde{R}_{mn} = [X_m, \tilde{a}_n] + [a_m, \tilde{a}_n] - [X_n, \tilde{a}_m] - [a_n, \tilde{a}_m] - i\{b_{ma}, e_n^{\ a}\} + i\{b_{na}, e_m^{\ a}\}$$
$$+ \frac{1}{2}\epsilon_{abcd}[\omega_m^{\ ab}, \omega_n^{\ cd}] - \frac{i\hbar}{\lambda^2}\tilde{B}_{mn}$$
$$R_{mn}^{\ \ a} = [X_m, b_n^{\ a}] + [a_m, b_n^{\ a}] - [X_n, b_m^{\ a}] - [a_n, b_m^{\ a}] + i\{b_{mb}, \omega_m^{\ ab}\} - i\{b_{nb}, \omega_m^{\ ab}\}$$
$$+ i\{\tilde{a}_m, e_n^{\ a}\} - i\{\tilde{a}_n, e_m^{\ a}\} + \epsilon_{abcd}([e_m^{\ b}, \omega_n^{\ cd}] - [e_n^{\ b}, \omega_m^{\ cd}]) - \frac{i\hbar}{\lambda^2} B_{mn}^{\ \ a}$$
$$\tilde{R}_{mn}^{\ \ a} = [X_m, e_n^{\ a}] + [a_m, e_n^{\ a}] - [X_n, e_m^{\ a}] - [a_n, e_m^{\ a}] + i\{b_m^{\ a}, \tilde{a}_n\} - i\{b_n^{\ a}, \tilde{a}_m\}$$
$$- ([b_m^{\ b}, \omega_n^{\ cd}] - [b_n^{\ b}, \omega_m^{\ cd}])\epsilon_{abcd} - i\{\omega_m^{\ ab}, e_{nb}\} + i\{\omega_n^{\ ab}, e_{mb}\} - \frac{i\hbar}{\lambda^2}\tilde{B}_{mn}^{\ \ a}$$
$$R_{mn}^{\ \ ab} = [X_m, \omega_n^{\ ab}] + [a_m, \omega_n^{\ ab}] - [X_n, \omega_m^{\ ab}] - [a_n, \omega_m^{\ ab}] + 2i\{b_m^{\ a}, b_n^{\ b}\} + ([b_m^{\ c}, e_n^{\ d}]$$
$$- [b_n^{\ c}, e_m^{\ d}])\epsilon_{abcd} + \frac{1}{2}([\tilde{a}_m, \omega_n^{\ cd}] - [\tilde{a}_n, \omega_m^{\ cd}])\epsilon_{abcd} + 2i\{\omega_m^{\ ac}, \omega_n^{\ b}{}_c\}$$
$$+ 2i\{e_m^{\ a}, e_n^{\ b}\} - \frac{i\hbar}{\lambda^2} B_{mn}^{\ \ ab}$$

The commutative limit

- U(1) gauge field decouples, $a_m = 0$
- 2-form gauge field decouples, $\mathcal{B}_{mn} = 0$
- commutators $\rightarrow$ partial derivatives
- anticommutators $\rightarrow$ multiplication
- up to some numerical coefficient
- Results of conformal gravity are recovered! conformal

The constraints for the symmetry breaking

- We want to result with SO(4) symmetry out of SO(6) part
- Employ the constraint of the torsionless condition: $\tilde{R}(P) = 0$
- We adopt: $e_m{}^a = b_m{}^a$ and fix $\tilde{a}_m = 0$
- Solving the constraint we obtain an expression $\omega_m{}^{ab} = \omega_m{}^{ab}(e, a)$:

$$\omega_n{}^{ac} = \frac{3}{4} e^m{}_b \, \epsilon^{abcd} [D_m, e_{nd}]$$

The action of the theory

- Choice of the action: SO(6)xU(1) invariant of *Yang-Mills* type:

$$\mathcal{S} = \mathrm{Trtr}\Gamma_5 \mathcal{R}_{mn} \mathcal{R}_{rs} \epsilon^{mnrs}$$

- (In the action we also include the kinetic term of the $\mathcal{B}$ field)
- Applying the constraints, the expressions of the tensors we obtain the action with reduced gauge symmetry SO(4)xU(1)

Conclusions & Future plans

- Successful construction of a 4d covariant noncommutative space
- Description of gravity in a regime where coords can be considered nc
- Study the Lorentz invariant action and try to relate it with the 4d Einstein-Hilbert - connect the large and low-energy regimes of the interaction
- Include matter fields and examine a spontaneous symmetry breaking of the initial action
- Obtain the equations of motion

Thank you for your attention!