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Chiral separation effect for spin 3/2 fermions.

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The chiral separation effect(CSE) is an emergence of an equilibrium axial current directed along an external magnetic field in the presence of a chemical potential

$$\mathbf{j}^5 = \frac{e^2\mu}{2\pi^2}\mathbf{B} \quad (1)$$

Main statements

We discuss Chiral Separation Effect in case of fermions with spin-3/2. We discuss two types of fermions - relativistic Rarita-Schwinger fermions and quasispin 3/2 fermions in Rarita-Schwinger-Weyl semimetals. In all cases coefficients in the conductivity of the chiral separation effect and in front of the axial anomaly coincide

Motivation of investigations

1.From the experimental point of view, interest in this phenomenon is associated with the possibility of observing it both in high energy physics and in solid state physics.
2. From the theoretical point of view special emphasis is usually placed on the relationship between CSE and the axial anomaly.The last one is protected from renormalization via interactions.
(Non-renormalization of the coefficient was observed on the lattice!)

Let's begin from QFT case and discuss extended Rarita-Schwinger model

The original equation was formulated in 1941 to describe hypothetical relativistic fermions with spin 3/2. We start with the Lagrangian for the Rarita-Schwinger fermions.

$$L_{cl} = \frac{1}{2}\bar{\psi}_\mu i\epsilon^{\mu\nu\lambda\rho}\gamma_5\gamma_\nu D_\lambda \psi_\rho \tag{2}$$

In this Lagrangian, in the massless case, additional symmetry arises, which is associated with the transformation:

$$\psi_\rho \to \psi_\rho + D_\rho \epsilon \tag{3}$$

ϵ - some spinor.This is similar to the situation with Maxwell field. And also as in the case of photons for fermions moving with the speed of light only two states are physically realized with highest and lowest projection of the spin onto the momentum.

Inclusion of interaction in the original Rarita - Schwinger field leads to difficulties. Problems can arise either when quantizing the field or even at the classical level where paradoxes with superluminal velocities arise. But even in this case one can calculate the anomaly. The method was given by Alvares-Gaume and Witten they found that the coefficient is 3 times larger than for ordinary fermions. For its calculation one must add ghosts contribution (bosons with spin one-half) $3 = 4 \oplus -2 \oplus +1$. Very interesting but it seems very hard to make predictions in this case. So we will not follow this way!

A very beautiful way to include interaction with abelian field for fermions with $\frac{3}{2}$-spin has proposed by Adler recently. He extended Rarrita-Schwinger model and included an interaction with spin 1/2 fermions in the following way:

$$S = S(\psi_\mu) + S(\lambda) + S_{int}, S = \int d^4x \bar{\psi}_\mu R^\mu, R^\mu = i\epsilon^{\mu\eta\nu\rho}\gamma_5\gamma_\eta D_\nu \psi_\rho$$

$$D_\nu \psi_\rho = (\partial_\nu + gA_\nu)\psi_\rho \tag{4}$$

$$\bar{\psi}_\mu = \psi^\dagger_\mu i\gamma_0 \tag{5}$$

$$S(\lambda) = -\int d^4x \bar{\lambda}\gamma_\nu D_\nu \lambda \tag{6}$$

$$D_\nu \lambda = (\partial_\nu + gA_\nu)\lambda, \bar{\lambda} = \lambda^\dagger i\gamma^0 \tag{7}$$

$$S_{interaction} = m \int d^4x(\bar{\lambda}\gamma^\nu \psi_\nu - \bar{\psi}_\nu \gamma^\nu \lambda) \tag{8}$$

here ψ is fermionic field with spin 3/2, λ is fermionic field with spin 1/2, m is intensity of interaction in this model.

We can find Feynman rules:

$$N = \begin{pmatrix} N^{\frac{3}{2}}_{\rho\sigma} & 0 \\ 0 & 0 \end{pmatrix}, N^{\frac{3}{2}}_{\rho\sigma} = \frac{-i}{2k^2}(\gamma_\sigma \not{k} \gamma_\rho - \frac{4}{k^2} k_\rho k_\sigma \not{k}) \tag{9}$$

$$V^\nu = \begin{pmatrix} -ie\gamma^{\mu\nu\rho} & 0 \\ 0 & -ie\gamma^\nu \end{pmatrix}, A^\nu = \begin{pmatrix} -ie\gamma^{\mu\nu\rho}\gamma_5 & 0 \\ 0 & -ie\gamma^\nu\gamma^5 \end{pmatrix} \tag{10}$$

$$\gamma^{\mu\nu\rho} = \frac{1}{2}(\gamma^\mu\gamma^\nu\gamma^\rho - \gamma^\rho\gamma^\nu\gamma^\mu) \tag{11}$$

Here N describes propagator of the system , V^ν is the vector and A^ν is the axial vertex

As we show, the expression for the CSE conductivity in static limit in case of fermions with spin 3/2 is five times larger than the expression for the ordinary fermions . The chemical potential is included in a standard way as the zero component of the vector gauge field ,this inclusion leads to the replacement $k_0 \to i(\omega_n - i\mu)$. We calculate the conductivity in the linear response theory:

$$\sigma_{cse} = \lim_{p_i \to 0} \lim_{\omega \to 0} \frac{i}{2p_i} \epsilon^{ijk} \Pi^{AV}_{jk} \tag{12}$$

$$\Pi^{AV}_{jk} = \int d^4 x e^{ik_\mu x_\mu} < j^A_i j^V_j >_\mu \tag{13}$$

$$\sigma_{cse3/2} = \lim_{p_i \to 0} \lim_{\omega \to 0} \frac{i\epsilon^{ijk}}{2p_i} T \sum_{n=2\pi(n+1/2)T} e^2 Tr \int \frac{d^3 r}{(2\pi)^3} N_{\rho\eta}(r) A^{\rho\eta}_j N_{\eta\mu}(r+p)$$

$$V^{\mu\rho}_k |_{i\omega_n \to p_0 + i0} \tag{14}$$

$$T \sum_{n=2\pi(n+1/2)T} Tr \int \frac{d^3 r}{(2\pi)^3} N_{\alpha\beta}(r)\gamma^j\gamma^5 N_{\beta\alpha}(p+r)\gamma^k =$$

$$= T \sum_{n=2\pi(n+1/2)T} Tr \int \frac{d^3 r}{(2\pi)^3} \frac{A}{(r)^2(r+p)^2} \tag{15}$$

where:

$$A = [1 + 4\frac{(r\cdot(r+p))^2}{r^2(r+p)^2}] tr(\not r + \not p)\gamma^k(\not r)\gamma^j\gamma^5 \tag{16}$$

So we have two contributions.

The first term in (16) after insertion in (14) gives

$$\sigma^1_{cse} = \lim_{p_i\to 0}\lim_{\omega\to 0}\frac{i\epsilon^{ijk}}{2p_i}T\sum_n Tr\int\frac{d^3r}{(2\pi)^3}\frac{tr(\not r+\not p)\gamma^k(\not r)\gamma^j\gamma^5}{(r)^2(r+p)^2}|_{i\omega_n\to p_0+i0}$$

$$\sigma^1_{cse} = \frac{e^2\mu}{2\pi^2} \qquad (17)$$

Obtaining of the second part of the conductivity is a more complicated.

$$\sigma^2_{cse} = \lim_{p_i\to 0}\lim_{\omega\to 0}\frac{i\epsilon^{ijk}}{2p_i}4T\sum_n Tr\int\frac{d^3r}{(2\pi)^3}\frac{tr(\not r\gamma^j\gamma^5)(\not r+\not p)\gamma^k}{r^4}|_{i\omega_n\to p_0+i0}$$

After some efforts we obtain the following expression:

$$e^2\lim_{p_i\to 0}\frac{i\epsilon^{ijk}4i\epsilon^{bk0j}p_b}{2p_i}\frac{\mu}{2\pi^2} = \frac{4e^2\mu}{2\pi^2}, \epsilon^{ijk}\epsilon^{bk0j} = -2\delta^{ib} \qquad (18)$$

$$\sigma^2_{cse} = \frac{4e^2\mu}{2\pi^2} \qquad (19)$$

all other terms are of $O(p^2)$ and we can neglect them

We have calculated the conductivity in the extended Rarita-Schwigner model. In the leading order in momentum this expression is five times larger than conductivity of ordinary fermions (but can differ for non-zero frequencies)

$$\sigma_{cse3/2} = \sigma^1_{cse} + \sigma^2_{cse} = \frac{5e^2\mu}{2\pi^2} \tag{20}$$

We need to discuss ghost contributions. They arise due to contraints in the theory In Adler's three possible ways of ghost inclusion is discussed, one of them leads to non-propagating ghost with zero contribution to the anomaly, the second one leads to propagating ghost but with extremely high mass $\sim \lim_{\delta \to 0} \frac{m}{\delta}$ and -1 contribution, and the third one is an exclusion of contribution 1 in Alvares-Gaume-Witten's manner.

For ghost roles in CSE we offer two arguments. The first one: very heavy particles (mass of non-propagating ghost is proportional to m) cannot give non-zero contribution to the thermodynamics domain because they are suppressed by distribution function and in the limit $T << m$ their contribution must be negligible. The second one is more ellegant: from Zakharov's arguments it follows that without interparticle interaction the chemical potential plays the same role as scalar potential, and we must reproduce the anomaly with the substitution $\mu \to \mu + \phi(z)$.

Let's return from relativistic physics to more realistic questions. We see that for charged particles with a spin 3/2 in high-energy physics some analogue of chiral separation effect can exist. In this part we want to test our reasoning in solid systems. The situation is simplified because we do not have problems of physical/non-physical degrees of freedom, as well as problems with superluminal velocities. A very good test is connected with so-called Rarita-Schwinger-Weyl Semimetals. Recently, first experimental evidences of emergent spin-3/2 fermions have been reported in CoSi, RhSi , AlPt , and PdBiSe . For their description a whole series of Hamiltonians is usually considered. We decide to focus on two of them.

The simplest choice is:

$$H_{s1} = v \sum_{i=1..3} p_i \sigma_0 \otimes \sigma_i \tag{21}$$

We are dealing with a doublet of Weyl fermions in this case . The second possible choice is :

$$H_{3/2RS} = v \sum_{i=1..3} p_i S_i \tag{22}$$

It is obvious that the Hamiltonian acts on fermion field with index$(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2})$.

matrix S_i are:

$$S_1 = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix}, S_2 = \begin{pmatrix} 0 & \frac{-i\sqrt{3}}{2} & 0 & 0 \\ i\frac{\sqrt{3}}{2} & 0 & -i & 0 \\ 0 & i & 0 & -i\frac{\sqrt{3}}{2} \\ 0 & 0 & i\frac{\sqrt{3}}{2} & 0 \end{pmatrix},$$

$$S_3 = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$

We use Kubo formulae for calculation of the conductivity in the first case. The current vertex is $A_i = v_f \sigma_0 \otimes \sigma_i$ and we need to calculate the following expression:

$$G(p) = \frac{1}{i(\omega_n - i\mu) - H_{s1}(p)} \tag{23}$$

$$\sigma_{s1} = \lim_{p_i \to 0} \lim_{\omega \to 0} \sum_{n=2\pi T(n+1/2)} \frac{i\epsilon^{ijk}}{4p_i} Tr \int \frac{d^3 r}{(2\pi)^3} G(r) A_j G(r+p) A_k |_{i\omega_n \to \mathbf{p_0} + i\epsilon} \tag{24}$$

$$\sigma_{cse} = \lim_{p_i \to 0} \lim_{\omega \to 0} \sum_n \frac{i\epsilon^{ijk}}{4p_i} I \tag{25}$$

$$I = Tr \int \frac{d^3 r}{(2\pi)^3} \frac{r_l \sigma_0 \otimes \sigma^l}{r^2} (v_f \sigma_0 \otimes \sigma_j) \frac{(r_a + p_a)\sigma_0 \otimes \sigma^a}{(p+r)^2} (v_f \sigma_0 \otimes \sigma_k)$$

As for the integral we can write:

$$I = \sigma_0 \otimes \int \frac{d^3 r}{(2\pi)^3} \frac{r_l \sigma^l}{r^2} (v_f \sigma_j) \frac{(r_a + p_a)\sigma^a}{(p+r)^2} (v_f \sigma_k) \tag{26}$$

After substitution $r \rightarrow v_f r$ we find:

$$\sigma_{s1} = Tr \sigma_0 \otimes \sigma_{cse} = 2\sigma_{cse} \tag{27}$$

In the second case we can use different strategies. The easiest way is the following: in the complete model there are monopoles in momentum space with charges $\pm 3, \pm 1$.We use semiclassical approximation that describes electrons in crystals with inclusion of the Berry curvature. Equations of motions in this case read as:

$$\dot{\mathbf{r}}^a = \frac{\partial \epsilon^a}{\partial \mathbf{p}^a} + \dot{\mathbf{p}}^a \times \Omega_p^a \tag{28}$$

$$\dot{\mathbf{p}}^a = \dot{\mathbf{r}}^a \times \mathbf{B} \tag{29}$$

where $i\mathbf{A}_p = \mathbf{u}_p^\dagger \nabla_p u_p$, u_p is 4-component spinor. We discuss only "left" subspace.

Using connection between the Berry flux and monopoles:

$$N^a = \int \frac{d^3p}{2\pi} \partial_{p_i} \Omega_i^a, a = 1, 2 \tag{30}$$

$$N^1 = 3, N^2 = 1 \tag{31}$$

One can construct the current in magnetic field:

$$\mathbf{j}^a = -\int \frac{d^3p}{(2\pi)^3} \left(\epsilon_{\mathbf{p}} \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} + (\Omega_p^a \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}) \epsilon_{\mathbf{p}} \mathbf{B} \right) \tag{32}$$

We are interested only in the response to an external magnetic field and we consider only the second term:

$$\mathbf{j}^a = -\mathbf{B} \int \frac{d^3p}{(2\pi)^3} \Omega^a \cdot [\mu \partial_{\mathbf{p}} n_p] \tag{33}$$

$$\mathbf{j} = \frac{\mu(N^1 + N^2)}{4\pi^2} \mathbf{B} \tag{34}$$

The other limit is connected with a strong magnetic field. We can calculate the spectrum directly from the second Hamiltonian, after that we can calculate the current using Fermi-Dirac distribution. This case is more illustrative. We have gauged partial derivatives via:

$$\hat{p}_i \to \hat{p}_i + A_i = P_i \tag{35}$$

$$\hat{P}_i \hat{P}_j - \hat{P}_j \hat{P}_i = iF_{ij} \tag{36}$$

We choose vector-potential as $A_x = By$ and we can write:

$$a = \frac{\hat{p}_x + By + i\hat{p}_y}{\sqrt{2B}} \tag{37}$$

$$a^+ = \frac{\hat{p}_x + By - i\hat{p}_y}{\sqrt{2B}} \tag{38}$$

$$H_{3/2} = v_f \sqrt{2B}(S_+ a + S_- a^+) + v_f p_z S_z \tag{39}$$

$$\tag{40}$$

The current in z-direction is $j^z = v_f \psi^\dagger S_z \psi$.

$$J^z = \int d^2x \sum_E (n(E) v_f \psi^\dagger S_z \psi) \tag{41}$$

$$n(E) = \frac{sign(E)}{exp(\beta(E-\mu)sign(E)) + 1}, \beta = \frac{1}{T} \tag{42}$$

The second equation implies:

$$n(-|E|) = -\frac{1}{exp(\beta(|E|+\mu)) + 1} \tag{43}$$

with this fact in mind we can rewrite expression for the current as:

$$J^z = v_f \int \frac{dp}{2\pi}(n_f(v_f|p| - \mu)\psi^\dagger S_z \psi - n_F(v_f|p| + \mu)\psi^\dagger S_z \psi) \tag{44}$$

$$n_f(v_f p \mp \mu) = \frac{1}{exp(\frac{v_f p \mp \mu}{T}) + 1} \tag{45}$$

We try to find solutions in the form $\chi(x, y, z) = exp(ip_z z) exp(ip_x)\psi(y)$, where $\psi(y) = (< y|n >, < y|m >, < y|l >, < y|k >)$.

The first point is $\psi(y) = (0, 0, 0, < y|0 >)$, with dispersion relation $\epsilon = -\frac{3}{2} v_f p_z$. There is a degeneracy connected with magnetic flux, we can calculate it in Landau's manner. As for the current, it is equal to:

$$J^z = v_f \sum_{deg} \int \frac{dp}{2\pi} \left(\frac{3}{2} n_f(\frac{3 v_f p}{2} - \mu) - \frac{3}{2} n_f(\frac{3 v_f p}{2} + \mu) \right) = -\frac{\mu}{4\pi^2} BS$$
$$\mu = V_0 \tag{46}$$

summation is over all degenerate modes, S is the area of the sample, V_0 is a voltage, and we use relation:

$$\int dp(n_f(v_f p - \mu) - n_f(v_f p + \mu)) = \frac{\mu}{v_f} \tag{47}$$

Another two modes are $\psi(y) = (0, 0, c_1 < y|0 >, c_2 < y|1 >)$. Zeroes of the energy are at $p_z = \pm 2\sqrt{2B}$ with coefficients $(0, 0, c_1, c_2) = (0, 0, \pm\frac{\sqrt{3}}{2}, \frac{1}{2}) + ...$. Where "..." means small corrections from non-linearity of dispersion relation. In both cases Fermi velocity is $\frac{3v_f}{4}$. And, thus, currents are:

$$J^z = -v_f \frac{\mu}{\frac{3v_f}{4}} \frac{3}{4} \frac{BS}{4\pi^2} = -\frac{\mu}{4\pi^2} BS \tag{48}$$

The last mode is $\psi = (0, c_1 < y|0 >, c_2 < y|1 >, c_3 < y|2 >)$. Zero of the energy are at $p_z = 0$, $(0, c_1, c_2, c_3) = (0, \sqrt{\frac{3}{5}}, 0, \sqrt{\frac{2}{5}}) + ...$. Fermi velocity is $\frac{3v_f}{10}$. After summation over all contributions we have

$$J^z = -\frac{\mu}{\pi^2} BS \tag{49}$$

We have obtained results both in relativistic physics and in solid state physics.
In case of the extended Rarita Schwinger model we have obtained the chiral conductivity from the Kubo formula. In the relativistic case the coefficient in the chiral separation effect in the static limit (i.e $p \to 0, \omega \to 0$) is five times larger than for ordinary fermions and coincides with the coefficient in front of the anomaly. In case of the Rarita-Schwinger-Weyl semimetals we have calculated the CSE conductivity for two different Hamiltonians. For the first one (21) we used the Kubo formula, for the second Hamiltonian we used two methods: the kinetic equation as well as direct calculations of the current from system's spectrum in uniform magnetic field. In first case conductivity is two times larger than for ordinary Weyl fermions (27), for the second case it is four times larger, and in both cases it doesn't depend neither of Fermi velocity nor of the temperature. But the last statement doesn't seem universal, because existence of zero energy points affect by the temperature. Different coefficients in CSE conductivity in all cases are connected with different symmetries of systems.

In relativistic case we did not derive the conductivity explicitly for non-zero frequencies and one can investigate it in the future. Also the resulting expression implies a very naive way to prove that the anomaly is independent of the temperature and chemical potential. but this question itself requires further investigations.

Consideration of CSE for spin 3/2 fermions raised the question of the ghost contribution (bosons with spin 1/2) in the presence of a non zero chemical potential. In the model under consideration these degrees of freedom do not affect physics, but in general this statement seems to be non-universal.

We also want to mention that it will be very interesting to calculate the chiral vortical conductivity in extended Rarita-Schwinger model, because there are discussions about correct calculations of this quantity (validity of the Kubo formulae, correct definition of the axial current and infrared regularization) for higher spin theories.

As for Rarita-Schwinger-Weyl semimetals, the main question here is related to the inclusion of interaction. First of all, we need to take into account the Coulomb interaction and the effect of impurities.
Also it is easy to include the four-fermion vertex, which in this case can lead to changes in the ground state in comparison with the noninteracting theory. Despite the fact that the theory is non-renormalizable, we can use natural ultraviolet cutoff associated with the interatomic distance or some ultraviolet scale can be generated by interaction. In this case the question about corrections to the effect is closely connected with the question about stability of monopoles.

Thank you for your attention!