Measuring Axial Gauge Fields with a Calorimeter in a Weyl semimetal

2004.02473
1904.09113

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Partially supported by grant No. 0657-2020-0015 of the Ministry of Science and Higher Education of Russia.

INSTITUT DENIS POISSON ORLÉANS • TOURS Mathématiques & Physique Théorique

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Massless Dirac fermions

A generic system in particle physics, cosmology, solid state …

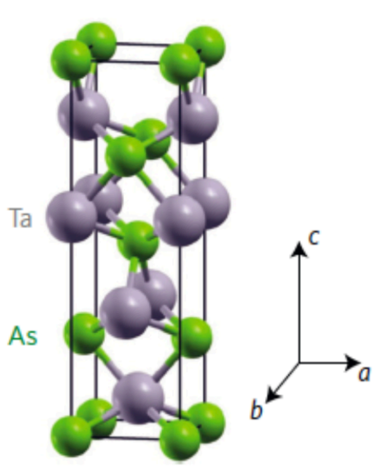
Covariant formulation (quantum field theory)

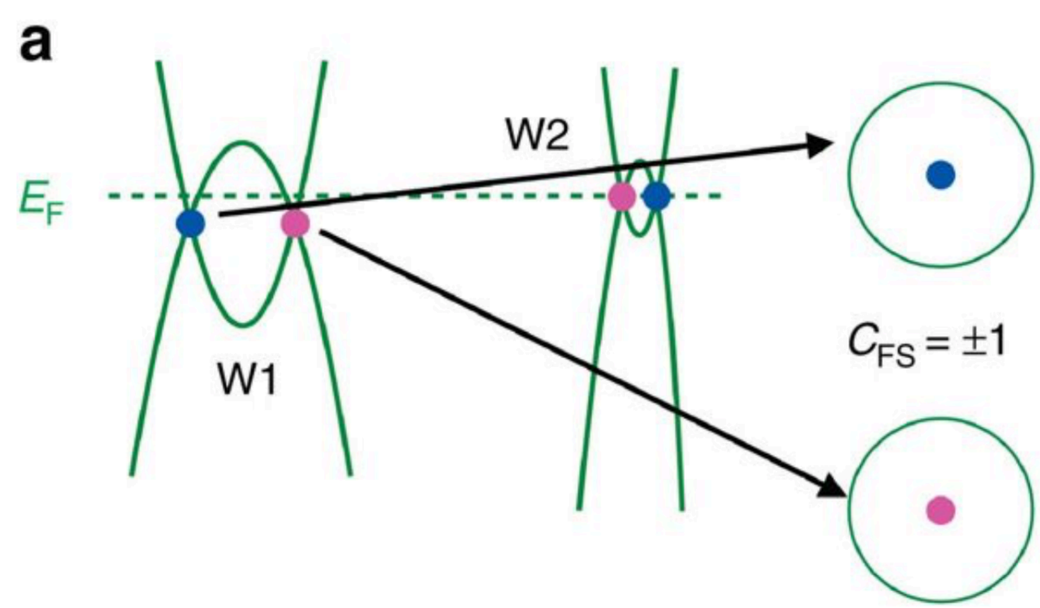
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{\bar{\psi} i \not{D} \psi} \longrightarrow \boxed{\bar{\psi}\left[i\gamma^0\hbar\frac{\partial}{\partial t} + v_F\boldsymbol{\gamma}(i\hbar\boldsymbol{\nabla} - e\boldsymbol{A})\right]\psi}$$

Dirac semimetals (solid state):

$$\not{D} = \gamma^\mu D_\mu \qquad D_\mu = \partial_\mu + ieA_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Weyl semimetals





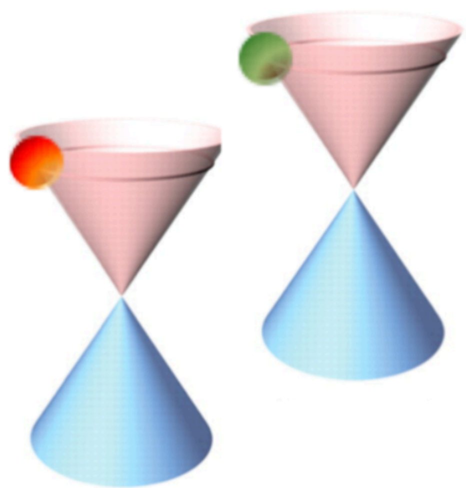
nature › nature materials › volumes › volume 15 › issue 11

Effective low energy description around band crossings in 3D crystals.

* Many thanks to María Vozmediano for sharing some slides

Weyl semimetals

Weyl semimetal
(non-degenerated bands)

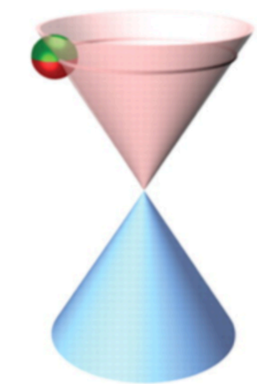


TaAs
NbAs
NbP
TaP

The nodes separation acts as a (constant) axial gauge field

$$b^\mu = (b_0, \mathbf{b})$$

Dirac semimetal
(doubly degenerated bands)



$ZrTe_5$
Na_3Bi,
Cd_3As_2

Weyl semimetals without (background) electromagnetic fields (not essential for this talk)

Fermionic action in the momentum representation:

$$S = \int d^4k \bar{\psi}_k (\gamma^\mu k_\mu - b_\mu \gamma^\mu \gamma^5) \psi_k$$

Separation between two Weyl cones in energy and momentum $b^\mu = (b_0, \mathbf{b})$

Electronic current

$$J^\mu = (\bar{\psi}\gamma^0\psi, v_F \bar{\psi}\gamma^i\psi)^T$$

$$b^\mu \to A_5^\mu(x)$$

spacetime dependence with smooth deformations

Axial gauge fields

Strained lattice

$$A_i^5 = \beta u_{ij} b^j$$

Axial gauge field

$$B^{i5} = \frac{1}{2}\epsilon^{ijk}\partial_j A_k^5$$

Axial magnetic field

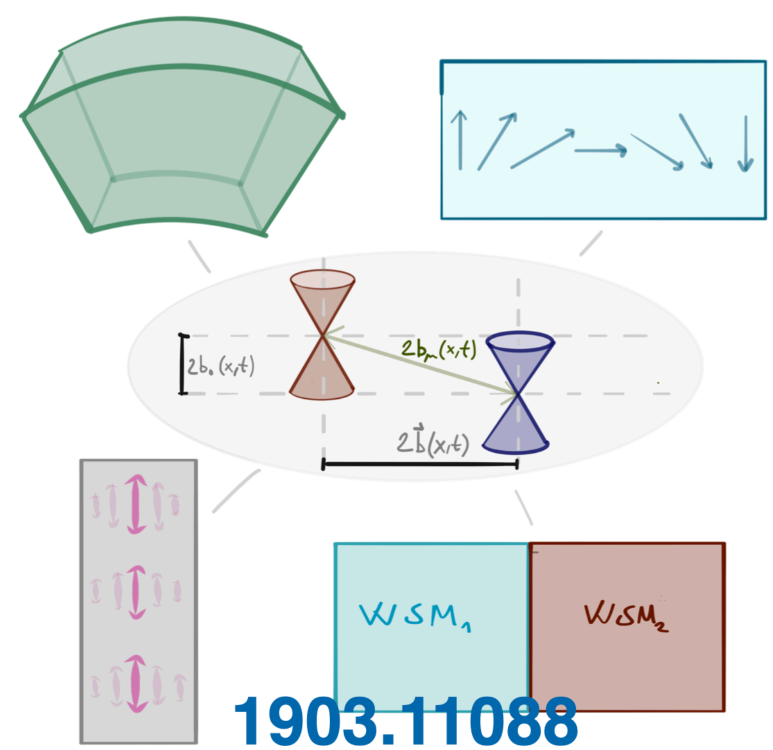
$$E_i^5 = -\partial_t A_i$$

Axial electric field

β = Grüneisen parameter

Elastic strain tensor:

$$u_{ij}(x) = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$$



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a nice review

AAA anomaly

Axial-Axial-Axial anomaly

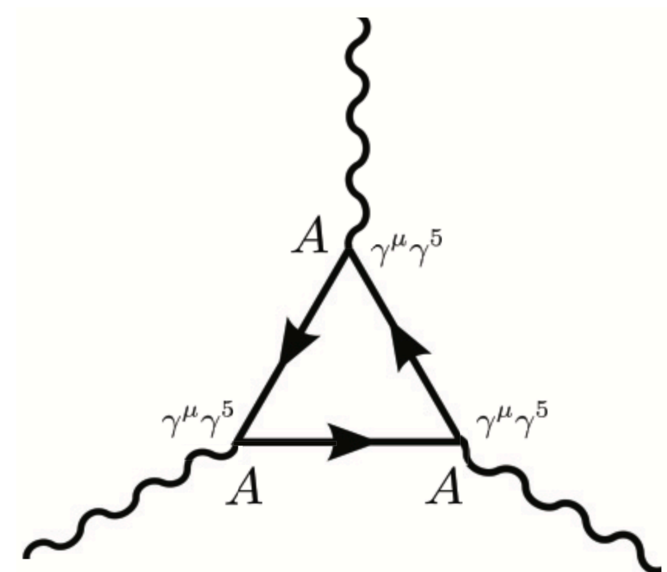
$$\partial_\mu j_5^\mu = \frac{1}{24\pi^2} F_{5,\mu\nu} \widetilde{F}_5^{\mu\nu} \equiv \boxed{\frac{1}{3}} \frac{1}{2\pi^2} \boldsymbol{E}_5 \cdot \boldsymbol{B}_5$$

$$\widetilde{F}_5^{\mu\nu} = 1/2 \epsilon^{\mu\nu\alpha\beta} F_{5,\alpha\beta}$$

Three times stronger than the usual axial (axial-vector-vector, AVV) anomaly.

$$j_5^\mu = j_L^\mu - j_R^\mu$$

axial current = left - right



Anomalous transport:

$$\underset{\text{axial}}{\boldsymbol{j}_5} = \frac{\overset{\text{axial}}{\mu_5}}{2\pi^2} \underset{\text{axial}}{\boldsymbol{B}_5}$$

chemical potentials

$$\mu_5 = (\mu_L - \mu_R)/2$$

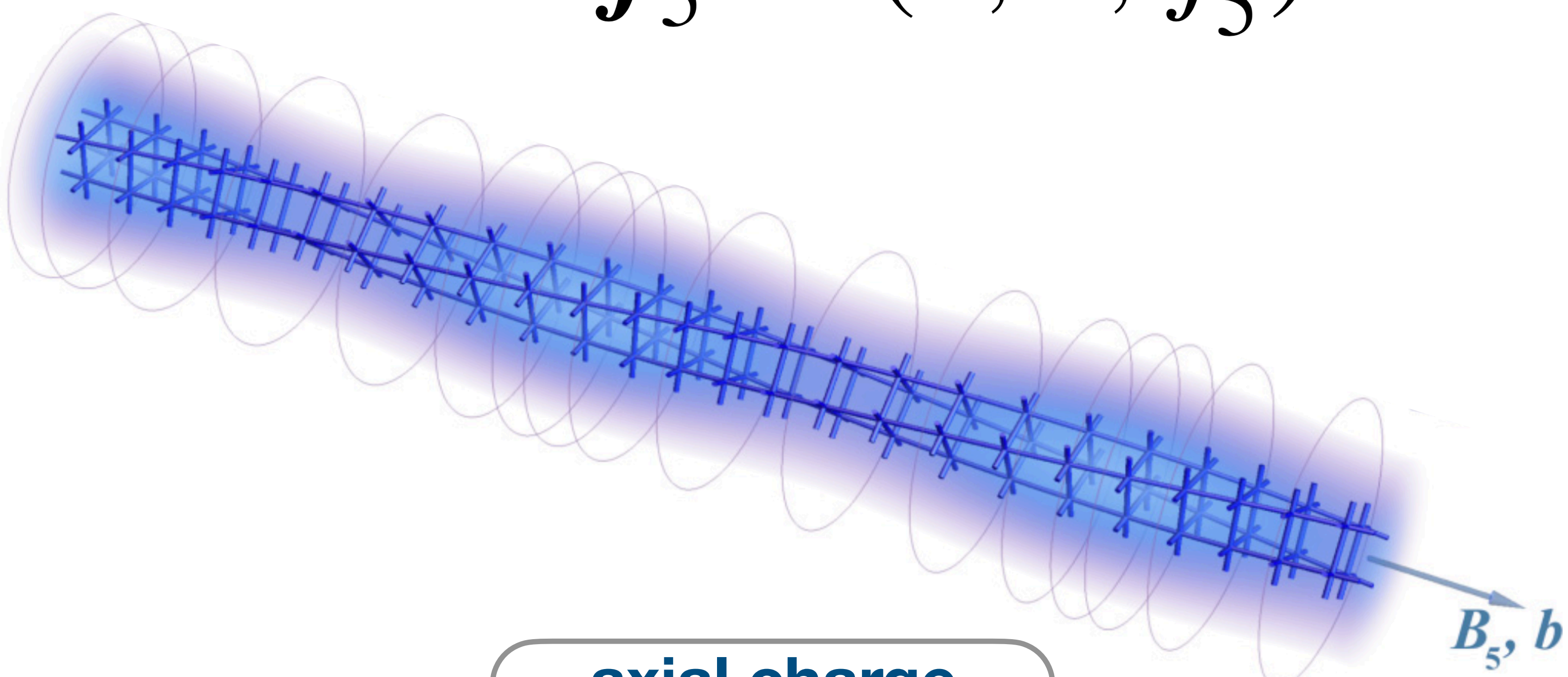
$$\mu = (\mu_L + \mu_R)/2$$

Twisted crystal

We twist a rod of a Weyl semimetal along its axis

$$\boldsymbol{B}_5 = (0, 0, B_5^z) \quad \text{axial magnetic field}$$

$$\boldsymbol{j}_5 = (0, 0, j_5^z) \quad \text{axial current}$$



$$A_i^5 = \beta u_{ij} b^j$$

Axial gauge field

$$B^{i5} = \tfrac{1}{2}\epsilon^{ijk}\partial_j A_k^5$$

Axial magnetic field

Elastic strain tensor:

$$u_{ij}(x) = \tfrac{1}{2}(\partial_i u_j + \partial_j u_i)$$

axial charge conservation

$$\frac{\partial \rho_5}{\partial t} + \frac{\partial j_5^z}{\partial z} = 0$$

axial charge density

$$\rho_5 \equiv j_5^0$$

no electric field

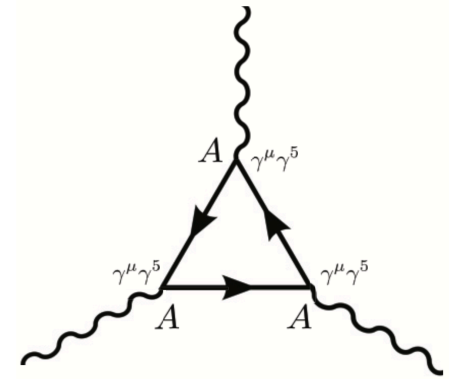
$$\boldsymbol{E}_5 = \boldsymbol{0}$$

no anomaly

$$\partial_\mu j_5^\mu = \frac{1}{24\pi^2} F_{5,\mu\nu}\widetilde{F}_5^{\mu\nu} \equiv \frac{1}{3}\frac{1}{2\pi^2}\boldsymbol{E}_5 \cdot \boldsymbol{B}_5$$

Chiral sound wave

axial sound



Axial charge conservation

$$\frac{\partial \rho_5}{\partial t} + \frac{\partial j_5^z}{\partial z} = 0$$

+

AAA-Anomalous transport

$$\boldsymbol{j}_5 = \frac{\mu_5}{2\pi^2}\boldsymbol{B}_5$$

+

Thermodynamics

$$\rho_5 = \frac{|B_5|}{2\pi^2}\frac{\mu_5}{v_F}$$

High-field limit

$$\rho_5 = \frac{\mu_5}{3v_F^3}\left(T^2 + \frac{3\mu^2}{\pi^2}\right) + \frac{\mu_5^3}{3\pi^2 v_F^3}$$

Low-field limit

=

Wave equation:

$$\left(\frac{\partial}{\partial t} + v_{\mathrm{CSW}}\frac{\partial}{\partial z}\right)\rho_5 = 0$$

One-way ~~ticket~~ propagation

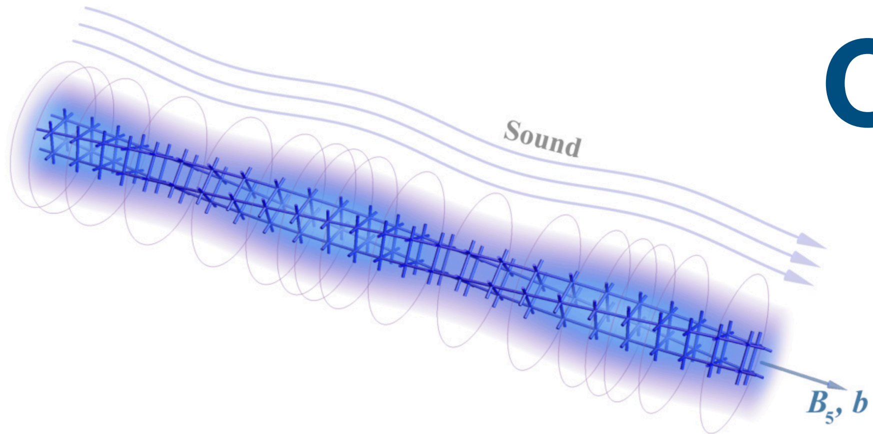
velocity

$$v_{\mathrm{CSW}} = \frac{3B_5 v_F^3}{2(\pi^2 T^2 + 3\mu^2)}$$

Low-field limit

$$v_{\mathrm{CSW}} = (\operatorname{sgn} B_5)v_F$$

High-field limit



Chiral sound

TABLE I. Typical reference parameters for the TaAs semimetal: Shown are the Fermi velocity $v_F \equiv v_z$ in the W1 pocket along the z axis and the $v_s \equiv v_{zz}$ velocity for the longitudinally polarized ultrasound along the same z axis.

TaAs					
Quasiparticles		Phonons			
v_F (m/s)	ν_5 (1/s)	v_s (m/s)	ν_s (1/s)	v_s/v_F	ν_s/ν_5
3×10^5	2×10^9	4.8×10^3	2.6×10^6	1.6×10^{-2}	1.3×10^{-3}

Velocity of CSW of TaAs

$$v_{\mathrm{CSW}} \simeq 225 \text{ m/s}$$

Chiral electronic sound mixes with acoustic phonons

electrons

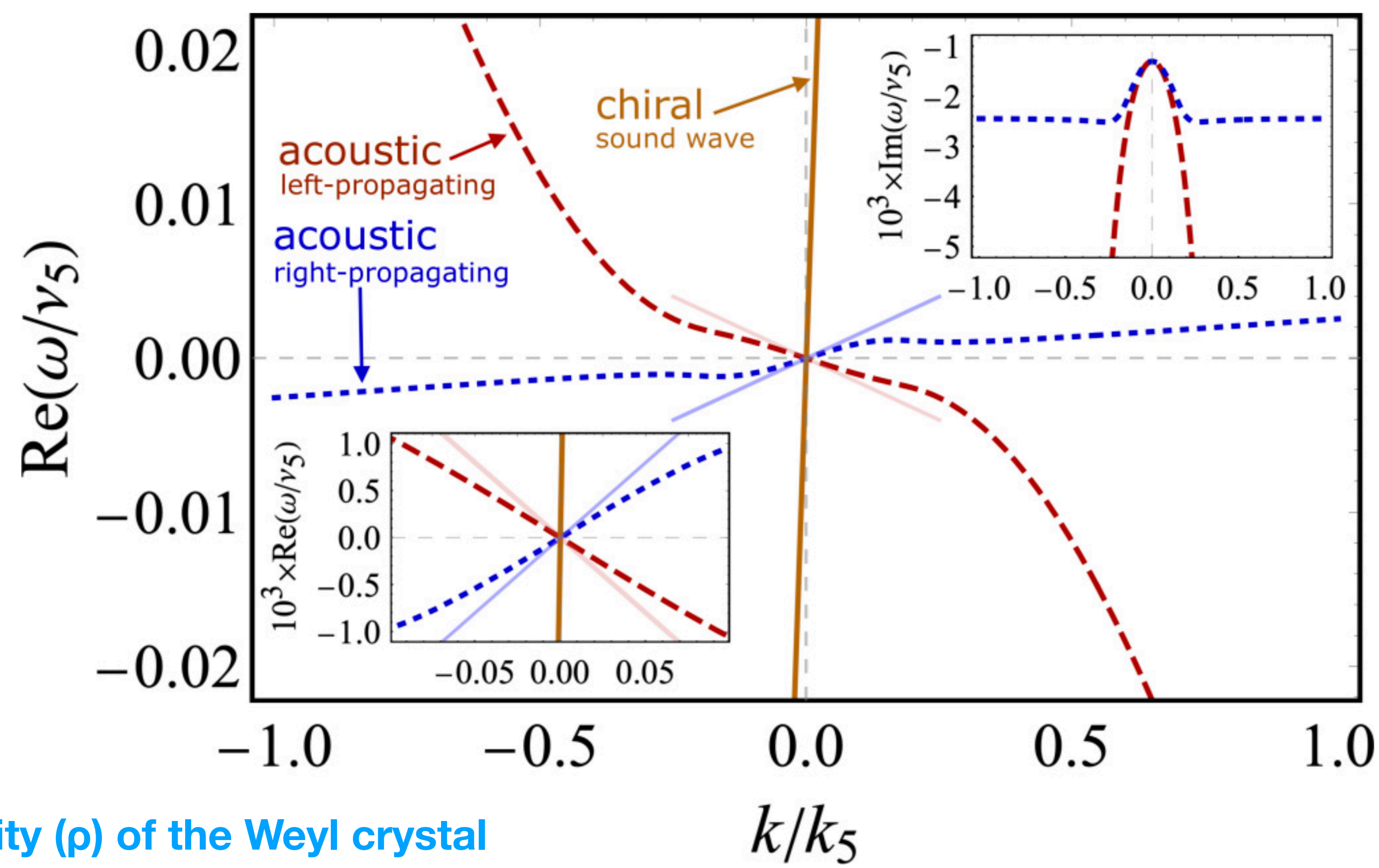
$$\frac{\partial \rho_5}{\partial t} + \frac{\partial j_5^z}{\partial z} = -\frac{\theta b^2}{6\pi^2}\frac{\partial^2 u^z}{\partial t \partial z} - \frac{\rho_5}{\tau_5}$$

phonons

$$\left(\frac{1}{v_s^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)u^z + \kappa b \frac{\partial j_5^z}{\partial z} = 0,$$

$$v_s = \sqrt{\frac{3B+4G}{3\rho}}, \quad \kappa = \frac{3\beta}{3B+4G}$$

bulk (B) and shear (G) moduli and density (ρ) of the Weyl crystal



Chiral sound and thermodynamics

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Standard contribution of phonons to specific heat:

$$c_v^{(ph)} = \frac{12\,\pi^4}{5} k_B \left(\frac{T}{\Theta}\right)^3 \quad \text{for} \quad T \ll \Theta$$

Debye temperature:

$$\Theta = \frac{\hbar\,\omega_D}{k_B} = \frac{\hbar\,v_s}{k_B}\left(6\,\pi^2\,\frac{N}{V}\right)^{1/3}$$

Chiral sound wave contributes to the thermal energy:

cutoff

$$U(T)_{\mathrm{CSW}} = V \int_0^{\tilde\Lambda} \frac{2\pi k_\perp dk_\perp}{(2\pi)^2} \int_0^{\tilde\Lambda} \frac{dk}{2\pi} \frac{\hbar v_{\mathrm{CSW}} k}{e^{\hbar v_{CSW} k/k_B T} - 1}$$

comes from

1) density of states:

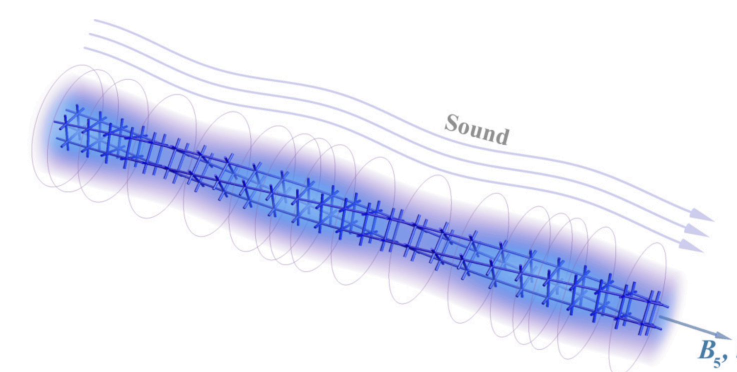
$$D(\omega) = \frac{V\,\omega^2}{2\,\pi^2\,v_s^3}$$

2) thermal energy:

$$U(T) = \frac{3\,V\,\hbar}{2\,\pi^2\,v_s^3} \int_0^{\omega_D} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}\,d\omega$$

3) specific heat:

$$c_V = \frac{1}{N}\frac{\partial U(T)}{\partial T}$$



cutoff $\tilde\Lambda = 2\pi/a$

Chiral sound and thermodynamics

The specific heat of the twisted sample at low temperature:

$$c_v(T) = \left(\frac{\Lambda}{v_{\rm CSW}}\right)\left(\frac{k_B^2}{\hbar}\right) T + \frac{12\,\pi^4}{5} k_B \left(\frac{T}{\Theta}\right)^3 + \dots$$

chiral sound phonons $\Lambda = a^3 \tilde{\Lambda}^2/24$

For TaAs: $v_F \simeq 3 \times 10^5\,{\rm m/s}\,, \quad b \simeq 0.06\,\pi/a\,, \quad a \simeq 3 \times 10^{-10}\,{\rm m},$

$\hbar/k_B \simeq 7.6 \times 10^{-12}\,{\rm s\,K}\,, \quad \Theta \simeq 341\,{\rm K}.$

Comparable contributions at the "crossover" temperature $T^* \sim 6\,K$

Diffusive/dissipative effects, the dispersion:

$$\omega + i/\tau_5 - v_{\rm CSW} k_z + i D k_z^2 = 0$$

Dissipation is negligible:

$\omega > 2\pi/\tau_5$

Diffusion is negligible:

$v_{\rm CSW} \gg D|k_z|$

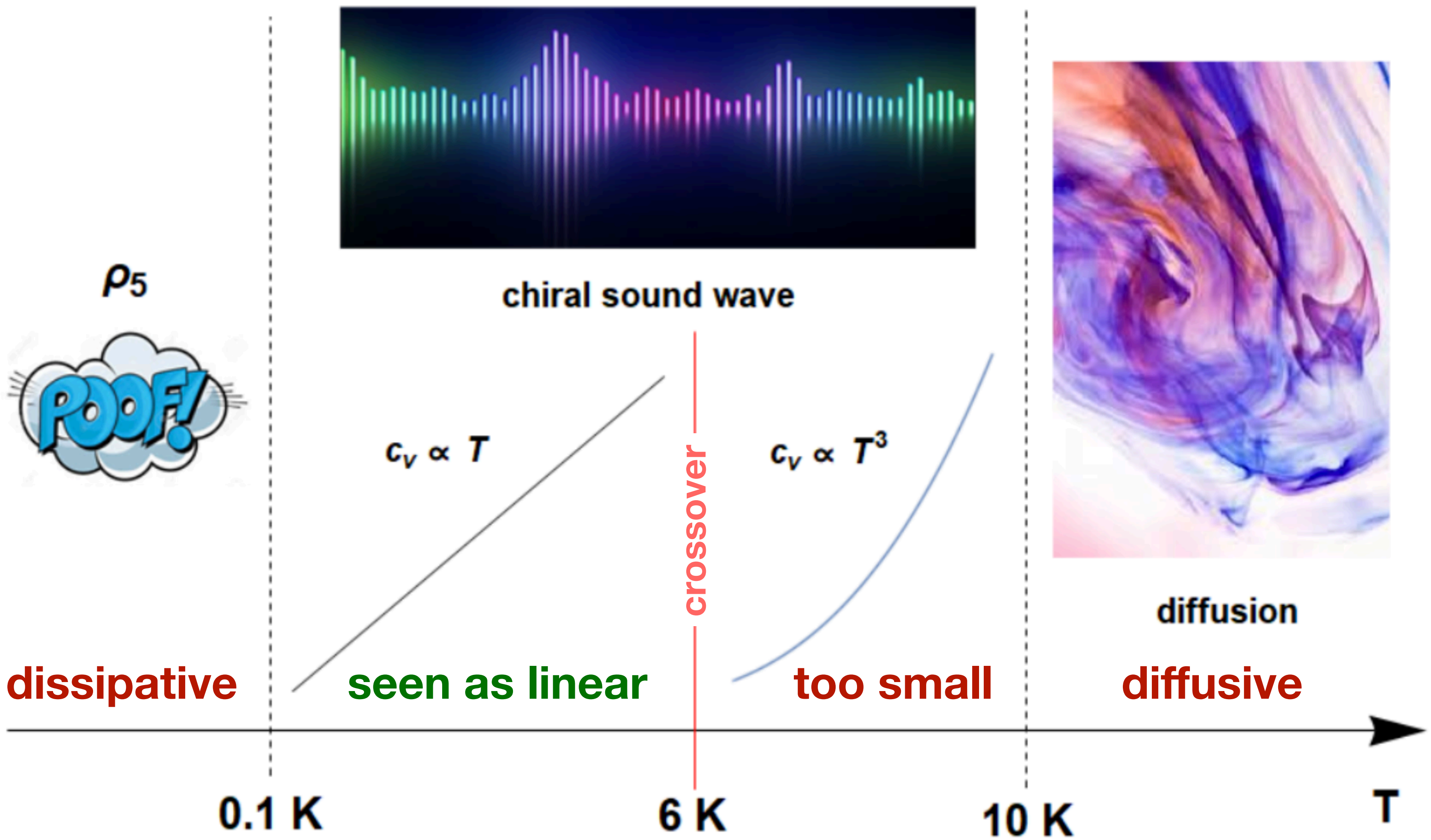
Maximal momentum $k_z^{\rm max} = k_B T/v_{\rm CSW}$

Diffusion constant $D \simeq v_F^2 \tau$

Kinetic relaxation rate τ

Chiral sound and thermodynamics

specific heat: general picture for twisted TaAs



$$\omega < 2\pi/\tau_5$$

dissipative regime

wave decays in half-period

chiral sound dominance

phonon dominance

diffusive regime

$$\cancel{v_{\mathrm{CSW}} \gg D|k_z|}$$

reference twist of 1 degree in a rod of length L = 1 μm

elasto-calorific effect

twist cools:

1. specific heat increases
2. temperature drops down
3. a few Kelvin effect at the crossover temperature

Chiral sound and thermal conductivity

Phonon conductivity:

$$\kappa^{(ph)} = \frac{1}{3} v_s \, l^{(ph)} \, \frac{12\,\pi^4}{5} k_B \left(\frac{T}{\Theta}\right)^3$$

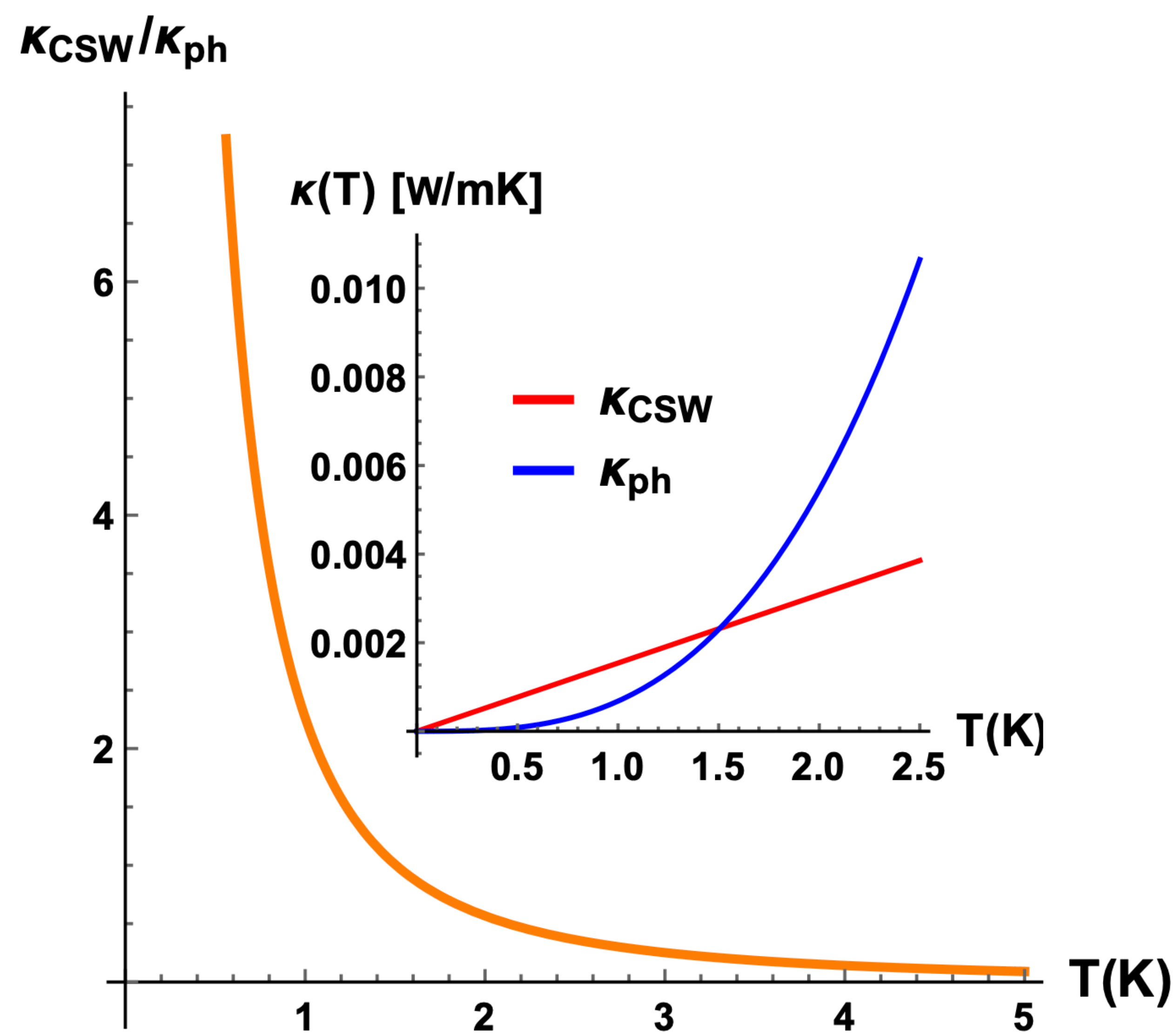
general expression

$$\kappa = \frac{1}{3} v \, l \, c_v$$

Chiral sound conductivity:

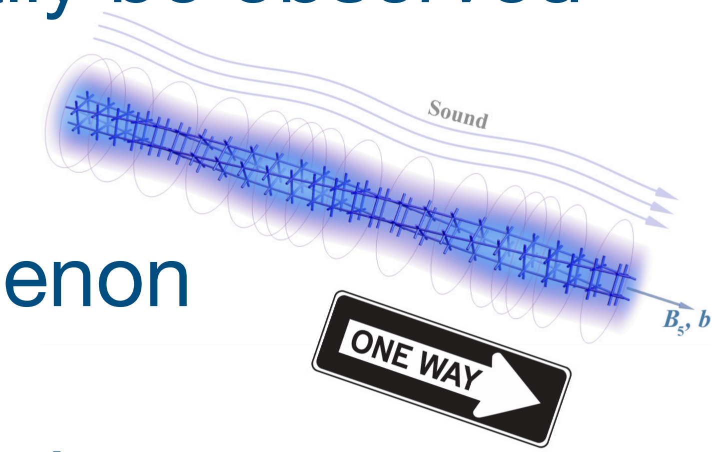
$$\kappa^{CSW} = \frac{1}{3} v_F \, \tau_5 \, \Lambda \frac{k_B^2 T}{\hbar}$$

reference twist of 1 degree
in a rod of length L = 1 μm



Summary

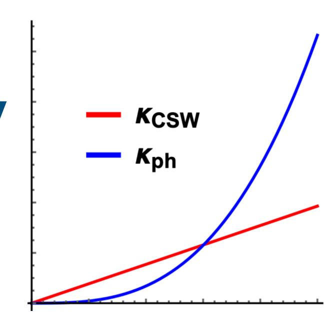
— Axial-Axial-Axial (AAA) anomaly leads to the emergence of the electronic chiral sound that can potentially be observed in torsionally strained Weyl semimetals

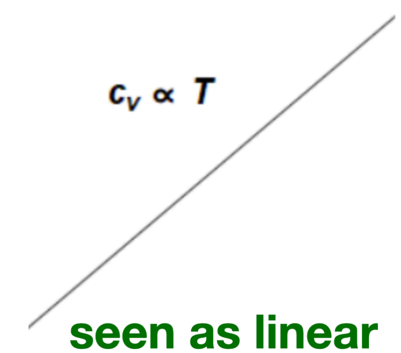


— the chiral sound is a unidirectional phenomenon

— leads to a linear behavior of the specific heat

— enhances of the thermal conductivity





Experimental verification of the exotic triple-axial anomaly and the reality of the elastic pseudomagnetic fields in Weyl semimetals.