Nonperturbative Casimir Effects in Lattice Gauge Theories

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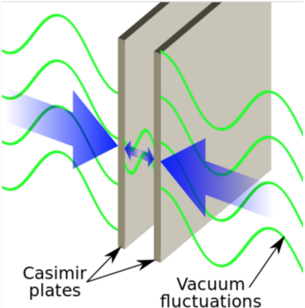
A short review of the Casimir effect

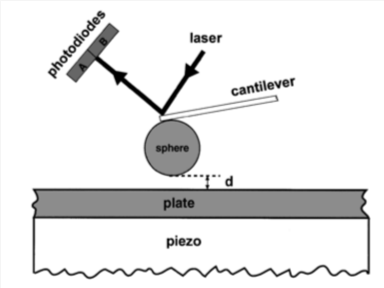
Casimir effect: Named after Dutch physicist Hendrik Casimir

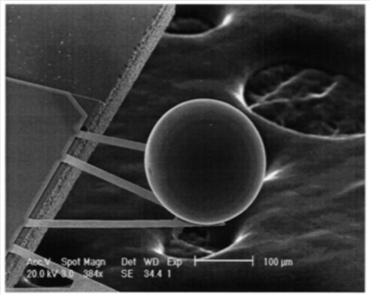
[H.B.G. Casimir, Proc. K. Ned. Acad. Wet. 51, 793 (1948)]



Simplest setup: Two parallel perfectly conducting plates at finite distance R.

Experimentally CE confirmed in plate-sphere geometries. [S. K. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997)]:





A very small force at human scales. However, at R=10 nm the pressure is about 1 atmosphere. (From U. Mohideen and A. Roy, Phys. Rev. Lett. 81, 4549 (1998), down to 100 nm scale.)

Apart from simplest geometries (plates, spheres, ...),

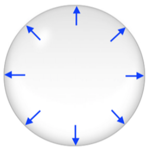
- the simples, tree-level Casimir effect (no radiative/loop corrections)
- for simples boundary conditions (ignore frequency dependent ϵ and μ of a real material)

an analytical calculation of Casimir effect is a very difficult task because it required calculation of a full eigenspectrum of a (usually) simple operator (say, Laplacian) subjected to some boundary conditions. Finally, one should calculate, regularize and find a finite part of the sum over all energies.

A popular and simple explanation of the effect: *boundaries restrict the number of virtual photons inside a cavity so that the pressure of the virtual photons from outside prevails* is, actually, incorrect.

For example, a spherical geometry the Casimir force is acting outwards:

$$< E >_{sphere} = + \frac{0.0461765}{R}$$



[T. H. Boyer, Phys. Rev. 174, 1764 (1968)]

This property is used in the bag models of hadrons:
[A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974); A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974); T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975); K. A. Milton, Phys. Rev. D 22, 1441 (1980)]

Unexpected qualitative phenomenon: between the plates, light travels faster than light outside the plates (the Scharnhorst effect) [G. Barton, K. Scharnhorst, J. Phys. A 26, 2037 (1993); K. Scharnhorst, Annalen Phys. 7, 700 (1998)].
The Sharnhorst effect is a purely renormalisation effect that comes from the two-loop corrections of the photon propagator.

Since the vacuum EM spectrum is reduced the renormalisation point is shifted to $q^2 < 0$ leading to the space-like photons.
Is it a perturbation theory artefact?

The question of negative mass and negative energy

Qualitative effect:

The Casimir energy between two perfectly conducting plates is negative, thus:

$$M^{Cas}_{Inertial} = M^{Cas}_{Gravitational} = \frac{E}{c^2} < 0$$

A pure Casimir cavity would levitate in a gravitational field following a quantum "Archimedes principle" [K. A. Milton et al; J.Phys.A41, 164052 (2008); G. Bimonte et al., Phys.Rev. D76, 025008 (2007); V. Shevchenko, E. Shevrin, Mod.Phys.Lett. A31 (2016) no.29, 1650166]

Counter argument: "...the mass energy of the cavity structure necessary to enforce the boundary conditions must exceed the magnitude of the negative vacuum energy, so that all systems of the type envisaged necessarily have positive mass energy." [J.D. Bekenstein, PRD 88, 125005 (2013)]

Non-perturbative Casimir effect within Lattice gauge theories

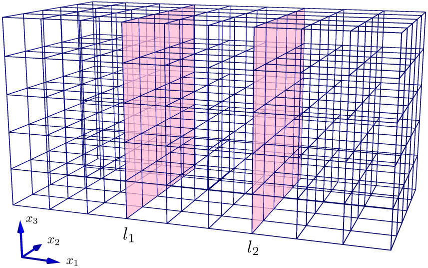


Figure: The Casimir plates l_1 and l_2 are separated by the distance R. The two-dimensional space is compactified into a torus due to periodic boundary conditions.

Electric-type boundary conditions for U(1), which, in two spatial dimensions, are given by the following local condition:

$$\epsilon^{\mu\alpha\beta} n_\mu(x) f_{\alpha\beta}(x) = 0\,, \tag{1}$$

The corresponding boundary condition is given by the lattice version of the action:

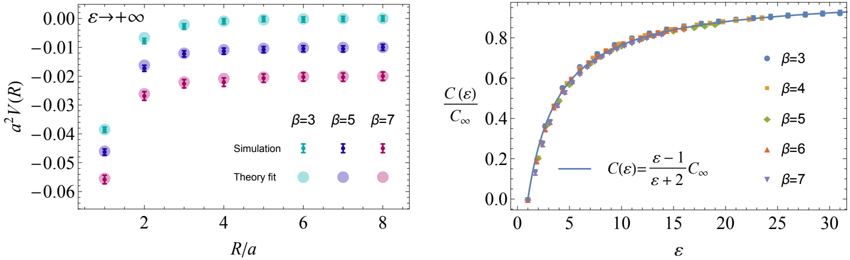
$$\cos\theta_{x,23} = 1\,, \tag{2}$$

To this end the lattice $U(1)$ action can be changed as follows:

$$S_\varepsilon[\theta;\mathcal{P}_\mathcal{S}] = \sum_P \beta_P(\varepsilon)\cos\theta_P\,, \tag{3}$$

where the plaquette-dependent gauge coupling is

$$\beta_{P_{x,\mu\nu}}(\varepsilon) = \beta\big[1 + (\varepsilon - 1)(\delta_{\mu,2}\delta_{\nu,3} + \delta_{\mu,3}\delta_{\nu,2}) \cdot (\delta_{x,l_1} + \delta_{x,l_2})\big]\,.$$



Fits of the numerical data for the zero-point energy by the lattice potential (M. N. Chernodub, V. A. Goy and A. V. Molochkov, Phys. Rev. D 94, no. 9, 094504 (2016)):

$$V_{\rm Cas}(R,\varepsilon) = -\frac{C(\varepsilon)}{R^2}\,. \quad C(\varepsilon) = \frac{\varepsilon-1}{\varepsilon+2}C_\infty \tag{4}$$

Corresponds to the tree-level Casimir effect in QED.

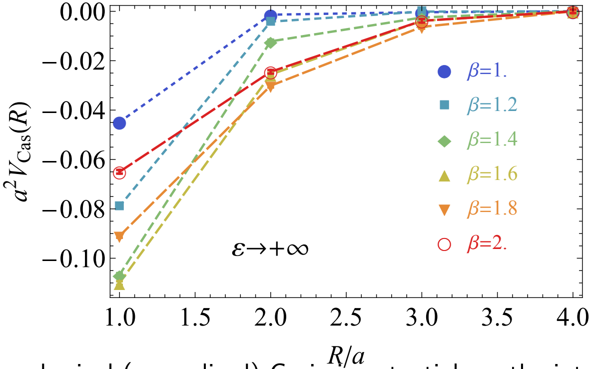


Figure: The physical (normalised) Casimir potential vs. the interwire distance R at various values of the lattice coupling constant β (in lattice units).

Vacuum restructuring in the strong coupling regime

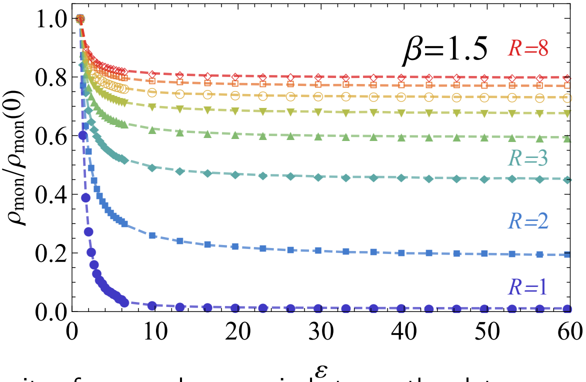


Figure: Density of monopoles $\rho_{\rm mon}$ in between the plates vs. permittivity ε for various values of the lattice gauge coupling $\beta = 1.0, 1.1, 1.2, 1.3$ and distances between the plates $R = 1, 2, \ldots, 8$ (in lattice units).

Example of the monopole distribution

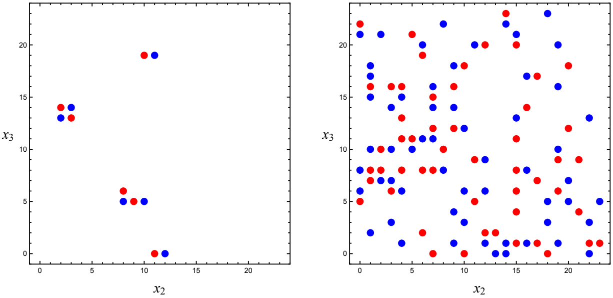
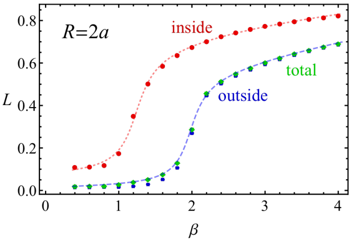


Figure: Example of configurations of monopoles (the red dots) and anti-monopoles (the blue dots) in the space between the wires (the left plot) and outside the wires (the right plot) at strong coupling regime ($\beta = 1$) and at large permittivity ($\varepsilon = 59$). (M. N. Chernodub, V. A. Goy and A. V. Molochkov, Phys. Rev. D 95, no. 7, 074511 (2017).)



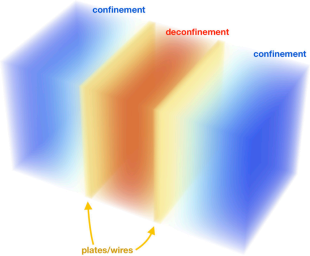


Figure: The expectation values of the Polyakov loop: in the whole space as well as inside and outside of the wires for a large permittivity ϵ with the next-to-minimal separation between the wires, $R = 2a$.

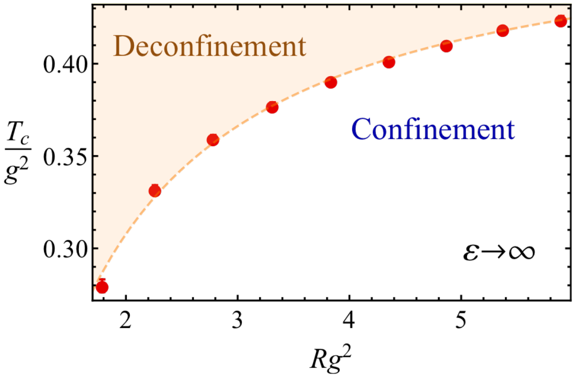


Figure: The critical temperature T_c of the confinement-deconfinement transition as the function of the separation between the plates R in the ideal-metal limit ($\epsilon \to \infty$) in physical units. (M.N. Chernodub, V.A.Goy and A.V.Molochkov, Phys. Rev. D 96, no. 9, 094507 (2017))

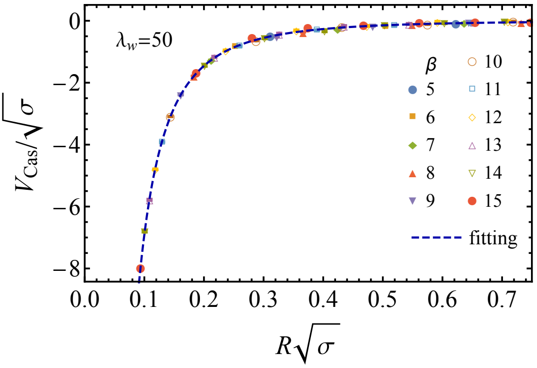


Figure: The SU(2) Casimir potential V_{Cas} as the function of the distance R between the wires in units of the string tension σ. The potential is given for the large excess of wire's coupling ($\lambda_w = 50$) and at various bulk lattice couplings β. Perfectly fits to calculation within effective theory [Dimitra Karabali, V.P.Nair, Phys. Rev. D 98, 105009 (2018)]

$$V_{\mathrm{Cas}}^{\mathrm{fit}}(R) = 3\frac{\zeta(3)}{16\pi}\frac{1}{R^{\nu}\sigma^{(\nu-2)/2}}e^{-M_{\mathrm{Cas}}R}, \tag{5}$$

The asymptotic value of the Casimir mass M_{Cas}, corresponding to the energy of the vacuum fluctuations of non-Abelian gauge field between perfect wires ($\lambda_w \to \infty$),

$$M_{\mathrm{Cas}}^{\infty} = 1.38(3)\sqrt{\sigma}\,, \tag{6}$$

Surprisingly, the Casimir mass M_{Cas}, Eq. (6), turns out to be substantially smaller than the mass

$$M_{0^{++}} \approx 4.7\sqrt{\sigma}\,, \tag{7}$$

of the lowest 0^{++} glueball in SU(2) gauge theory (calculated numerically in Refs. M. J. Teper,Phys. Rev. D **59**, 014512 (1999), A. Athenodorou and M. Teper, JHEP **1702**, 015 (2017)).

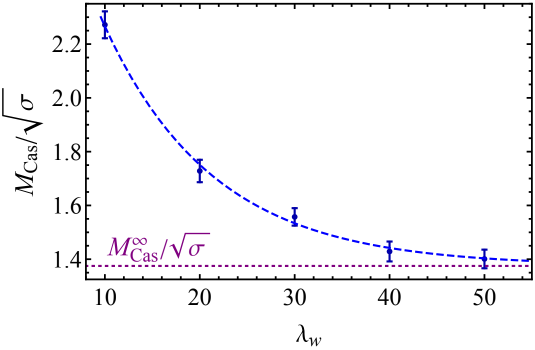


Figure: The Casimir mass $M_{\rm Cas}$ as the function of the strength of the wire λ_w. The thick dashed lines denotes the best fit by function $\mathcal{O}(\lambda_w) = \mathcal{O}^{\infty} + \alpha_{\mathcal{O}} e^{-\lambda_w/\lambda_w^{\mathcal{O}}}$, and the short-dashed horizontal line denotes the asymptotic value $M^{\infty}_{\rm Cas} = 1.38(3)\sqrt{\sigma}$. (M. N. Chernodub, V. A. Goy, A. V. Molochkov and H. H. Nguyen, Phys. Rev. Lett. 121, no. 19, 191601 (2018))

Conclusions:

The Casimir effect leads to vacuum restructuring of a finite system:

- Low-temperature deconfinement via modification of quantum fluctuations in a confining field theory.
- Emergence of a new scale, the Casimir mass, which is unexpectedly three times lighter than the mass of the lowest glueball. As the chromometallic wires become more opaque, the Casimir mass increases, presumably towards the lowest glueball mass.