Superluminality in beyond Horndeski theory with extra matter

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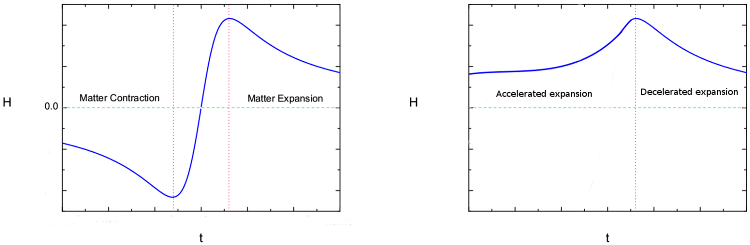
- **Motivation**: modelling the early Universe dynamics without the initial singularity (Big Bang), e.g. scenarios with a cosmological bounce or Genesis.



- **Specific properties**: both scenarios imply violation of the Null Energy condition (NEC)

$$\boxed{T_{\mu\nu}k^\mu k^\nu > 0 \quad (g_{\mu\nu}k^\mu k^\nu = 0) \qquad \longleftrightarrow \qquad p + \rho > 0}$$

$$\dot{H} = -4\pi G(p+\rho) + \frac{\kappa}{a^2}, \qquad \frac{d\rho}{dt} = -3H(\rho + p) < 0.$$

Generalized Galileon theories / Horndeski theories

G.Horndeski, Int.J.Theor.Phys. 10, 363 (1974)

Lagrangian of the theory

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

where π is a scalar field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $G_{iX} = \partial G_i/\partial X$.

- EOMs are second order (hence, no Ostrogradsky ghost) $\longrightarrow 2 + 1$ DOFs.
- NEC can be violated with no pathologies arising at the linearized level.

Beyond Horndeski (GLPV)

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}})\,,$$

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$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}$$
$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}.$$

M.Zumalacárregui, J.García-Bellido, Phys.Rev. D 89 (2014)
J.Gleyzes, D.Langlois, F.Piazza, F.Vernizzi, Phys.Rev.Lett. 114 (2015)

- EOMs are third order, BUT still no extra DOFs (Degeneracy Feature).
- Safe NEC violation is possible.
- Further extension to Degenerate Higher Order Scalar-Tensor (DHOST) theories *D. Langlois, K. Noui, JCAP 02 (2016)*.

Stability at the linearized level

- Background set-up: a homogeneous and isotropic setting $\pi = \pi_b(t)$ with a spatially-flat FLRW geometry.

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Quadratic action for dynamical DOFs

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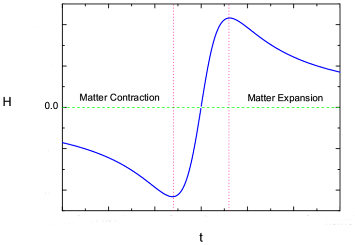
Stability and (sub)luminality conditions

$$\mathcal{G}_\mathcal{T}, \mathcal{F}_\mathcal{T} > \epsilon > 0, \quad \mathcal{G}_\mathcal{S}, \mathcal{F}_\mathcal{S} > \epsilon > 0, \quad \mathcal{F}_\mathcal{T} \leq \mathcal{G}_\mathcal{T}, \quad \mathcal{F}_\mathcal{S} \leq \mathcal{G}_\mathcal{S}$$

Stable non-singular cosmological solutions in beyond Horndeski theory: a construction technique

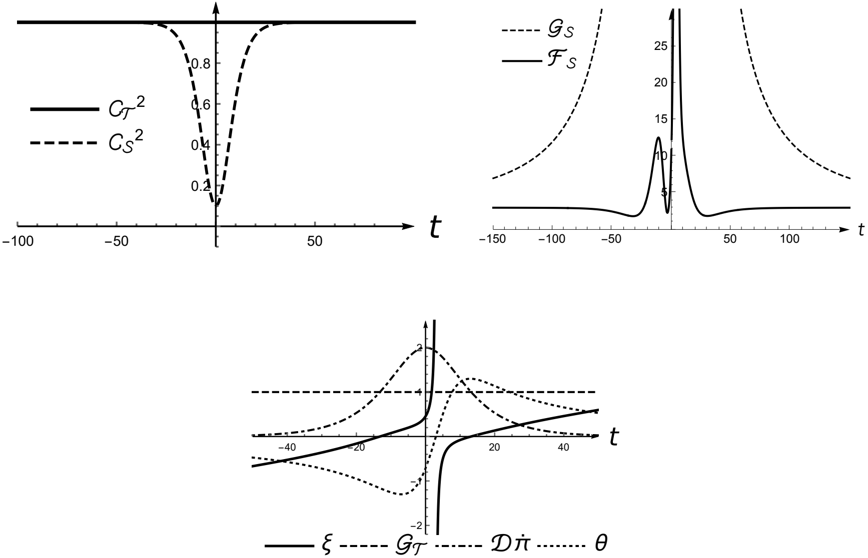
A construction technique for stable non-singular cosmologies:

- Explicit choice of the Hubble parameter $H(t)$ and $\pi_b(t) = t$:



- Reconstruction of the Lagrangian functions F, G_4, G_5, F_4 and F_5, which comply with the following conditions:
  (a) background EOM
  (b) stability and (sub)luminality conditions $(\mathcal{G}_\mathcal{T} \geq \mathcal{F}_\mathcal{T} > 0,\ \mathcal{G}_\mathcal{S} \geq \mathcal{F}_\mathcal{S} > 0)$
  (c) * specific form of the asymptotics of the theory

Bouncing solution in beyond Horndeski theory: an example



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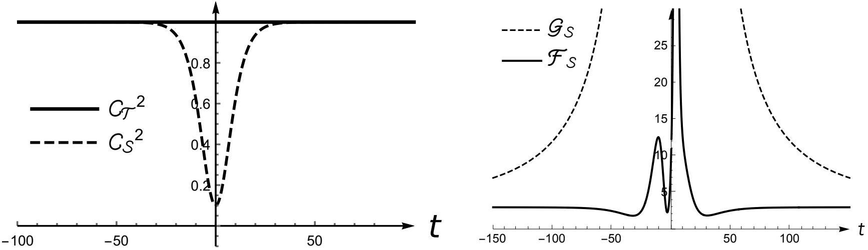

- Specific feature of the bouncing solution: we chose $\mathcal{G}_{\mathcal{T}} = 1$ and $\mathcal{F}_{\mathcal{T}} = 1$ at all times $\longrightarrow c_{\mathcal{T}}^2 = \dfrac{\mathcal{F}_{\mathcal{T}}}{\mathcal{G}_{\mathcal{T}}} = 1$ at all times.

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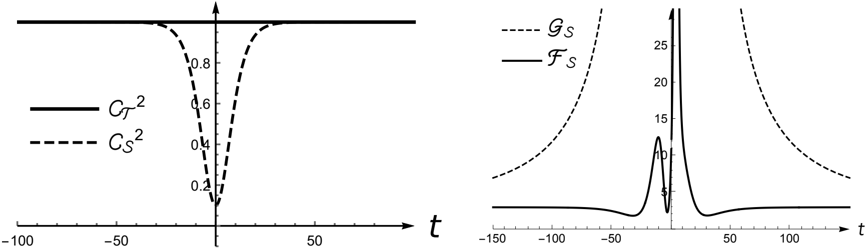

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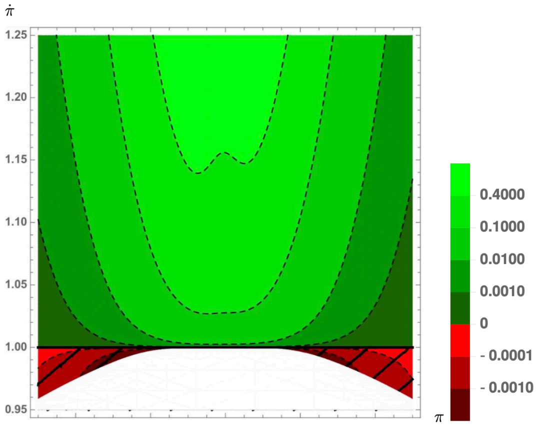
- The Lagrangian was constructed for $\pi_b(t) = t$ (which is always possible to achieve *on a single solution*).

Do tensor modes remain safely (sub)luminal even in the vicinity of the solution with $\pi_b(t) = t$?

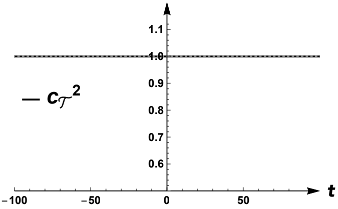
Phase space $(\pi, \dot{\pi})$ around the solution with $\pi_b(t) = t$

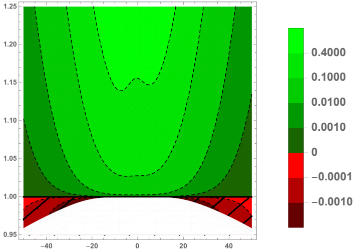
Variance $(1 - c_{\mathcal{T}}^2(\pi, \dot{\pi}))$ of the speed squared of tensor modes in the phase space $(\pi, \dot{\pi})$ for our bouncing model with $c_{\mathcal{T}}^2 = 1$ on the solution.

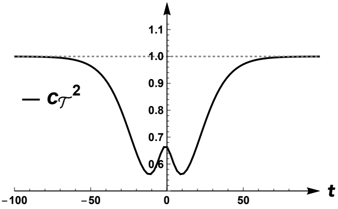


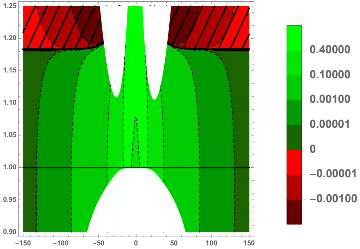
The bouncing solution with $\pi_b(t) = t$ lies right on the verge of the domain with superluminal tensor modes.

Phase space $(\pi, \dot{\pi})$ around the solution with $\pi_b(t) = t$









The modified bouncing solution is safely away from the superluminal domain.

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- Scalar sector: additional mode $\delta u_i = \partial_i \mathcal{V}$ (velocity potential)

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Stability conditions

$$\mathcal{G}_\mathcal{T}, \mathcal{F}_\mathcal{T} > \epsilon > 0, \quad \mathcal{G}_\mathcal{S} > \epsilon > 0, \quad \mathcal{F}_\mathcal{S} > \frac{\mathcal{G}_\mathcal{T} + \mathcal{D}\dot{\pi}}{2\Theta^2}$$

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$$c^2_{\mathcal{S}(1,2)} = \frac{1}{2}(u_s^2 + \mathcal{A}) \pm \frac{1}{2}\sqrt{(u_s^2 - \mathcal{A})^2 + \mathcal{B}}$$

$$\mathcal{A} = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}} - (\rho + p)\, \frac{\mathcal{G}_\mathcal{T}(\mathcal{G}_\mathcal{T} + 2\mathcal{D}\dot{\pi})}{2\mathcal{G}_\mathcal{S}\Theta^2}\,, \qquad \mathcal{B} = 4u_S^2(\rho + p)\frac{(\mathcal{D}\dot{\pi})^2}{2\mathcal{G}_\mathcal{S}\Theta^2}$$

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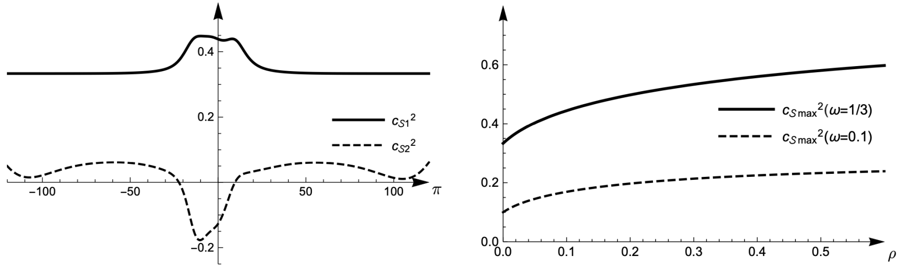
Horndeski case: $c^2_{\mathcal{S}(1)}|_{\mathcal{D}=0} = \dfrac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}} - \dfrac{(p+\rho)}{2\,\mathcal{G}_\mathcal{S}}\dfrac{{\mathcal{G}_\mathcal{T}}^2}{\Theta^2}, \quad c^2_{\mathcal{S}(2)}|_{\mathcal{D}=0} = u_s^2$

Adding extra matter: radiation $u_s^2 = 1/3$

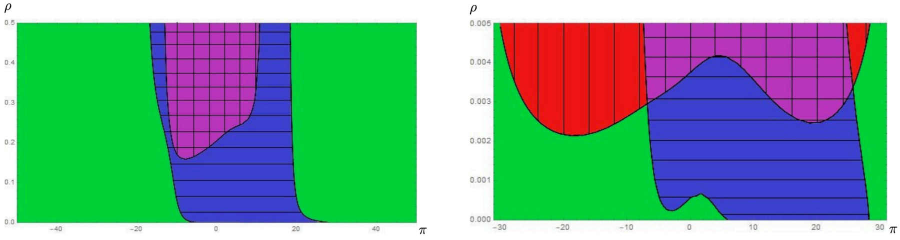


Left panel: Sound velocities squared at $\rho = 0.1$ (in Planck units) as functions of π for $w = u_s^2 = 1/3$ (radiation) at $\dot{\pi} = 1$.

Right panel: Maximum values of the larger sound velocity squared, $\max_\pi c^2_{\mathcal{S}(1)}(\pi, \rho)$, as function of ρ (in Planck units) for $w = u_s^2 = 1/3$ (radiation) and $w = u_s^2 = 0.1$, both at $\dot{\pi} = 1$.

Even though $c_{\mathcal{S}(1)}$ may be substantially larger than u_s at large ρ, it does not exceed 1 in entire phase space (π, ρ) provided that $w \equiv u_s^2$ is not too large.

Adding extra matter: $u_s^2 = 0.75$ and $u_s^2 = 0.99$



Left panel: Phase space (π, ρ) for $w = 3/4$, where ρ is in Planck units. Solid green region is subluminal and stable. Instability region ($c^2_{\mathcal{S}(2)} < 0$) is shown in blue, whereas superluminal region ($c^2_{\mathcal{S}(1)} > 1$) is shown in purple. The purple region is inside the blue one, so superluminality is actually not problematic.

Right panel: Same as in the left panel, but for $w = 0.99$. In the red region outside the blue one perturbations are stable and superluminal, $c^2_{\mathcal{S}(2)} > 0$, $c^2_{\mathcal{S}(1)} > 1$.

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- Adding small amount of matter does not ruin the stability ($c^2_{\mathcal{S}(2)} > 0$ by continuity)
- BUT since $\mathcal{B} > 0$, it leads to superluminality, $c^2_{\mathcal{S}(1)} > 1$

For $u_s^2 = 1$, adding even small amount of matter to stable beyond Horndeski cosmology makes one of the modes superluminal, $c^2_{\mathcal{S}(1)} > 1$, while keeping the setup stable, $c^2_{\mathcal{S}(2)} > 0$.

Conclusions

- The completely stable bouncing solution in beyond Horndeski theory remains safely stable and subluminal upon adding a small amount of matter like radiation ($\omega \leq 1/3$)
- Adding different types of matter with large enough ω and/or large enough amounts of extra matter results in the reappearance of superluminality
- Any beyond Horndeski model (in a cosmological setting) becomes superluminal upon adding even small energy density of extra matter with the luminal flat-space sound speed, $u_s = 1$ (or almost luminal u_s)

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Thank you for your attention!

No-go theorem in Horndeski theory

$$S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\; a^3\left[\frac{\mathcal{G_T}}{8}\left(\dot{h}_{ij}^T\right)^2 - \frac{\mathcal{F_T}}{8a^2}\left(\partial_k h_{ij}^T\right)^2 + \mathcal{G_S}\dot{\zeta}^2 - \mathcal{F_S}\frac{(\nabla\zeta)^2}{a^2}\right]$$

$$\mathcal{F_S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F_T} \qquad \longrightarrow \qquad \boxed{\frac{\mathrm{d}\xi}{\mathrm{d}t} = a\left(\mathcal{F_S} + \mathcal{F_T}\right) > 0}$$

$$\mathcal{F_S} > \epsilon > 0, \quad \mathcal{F_T} > \epsilon > 0 \quad \longrightarrow \quad \exists t_0,\ \xi(t_0) = 0$$

By definition:

$$\xi = \frac{a\mathcal{G_T}^2}{\Theta}$$

In Horndeski theories one cannot make $\xi(t)$ behave in a way suggested by stability conditions, hence, there are always gradient instabilities arising.

Appendix: coefficients in the quadratic action

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4G_{4X}X + G_{5\pi}X - 2HG_{5X}X\dot{\pi},$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4 - 2G_{5X}X\ddot{\pi} - G_{5\pi}X,$$

$$\mathcal{D} = -2F_4X\dot{\pi} - 6HF_5X^2,$$

$$\begin{aligned}\Theta = &-K_XX\dot{\pi} + 2G_4H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi}\\ &- 5H^2G_{5X}X\dot{\pi} - 2H^2G_{5XX}X^2\dot{\pi} + 3HG_{5\pi}X + 2HG_{5\pi X}X^2\\ &+ 10HF_4X^2 + 4HF_{4X}X^3 + 21H^2F_5X^2\dot{\pi} + 6H^2F_{5X}X^3\dot{\pi},\end{aligned}$$

$$\begin{aligned}\Sigma = &F_XX + 2F_{XX}X^2 + 12HK_XX\dot{\pi} + 6HK_{XX}X^2\dot{\pi} - K_\pi X - K_{\pi X}X^2\\ &- 6H^2G_4 + 42H^2G_{4X}X + 96H^2G_{4XX}X^2 + 24H^2G_{4XXX}X^3\\ &- 6HG_{4\pi}\dot{\pi} - 30HG_{4\pi X}X\dot{\pi} - 12HG_{4\pi XX}X^2\dot{\pi} + 30H^3G_{5X}X\dot{\pi}\\ &+ 26H^3G_{5XX}X^2\dot{\pi} + 4H^3G_{5XXX}X^3\dot{\pi} - 18H^2G_{5\pi}X - 27H^2G_{5\pi X}X^2\\ &- 6H^2G_{5\pi XX}X^3 - 90H^2F_4X^2 - 78H^2F_{4X}X^3 - 12H^2F_{4XX}X^4\\ &- 168H^3F_5X^2\dot{\pi} - 102H^3F_{5X}X^3\dot{\pi} - 12H^3F_{5XX}X^4\dot{\pi}.\end{aligned}$$