

# Parton Distributions and the Higgs Cross-section

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## Obtaining PDF sets – General procedure.

Start parton evolution at low scale  $Q_0^2 \sim 1\text{GeV}^2$ . In principle 11 different partons to consider.

$$u, \bar{u}, \quad d, \bar{d}, \quad s, \bar{s}, \quad c, \bar{c}, \quad b, \bar{b}, \quad g$$

$m_c, m_b \gg \Lambda_{\text{QCD}}$  so heavy parton distributions determined perturbatively. Leaves 7 independent combinations, or 6 if we assume  $s = \bar{s}$  (just started not to).

$$u_V = u - \bar{u}, \quad d_V = d - \bar{d}, \quad \text{sea} = 2 * (\bar{u} + \bar{d} + \bar{s}), \quad s + \bar{s} \quad \bar{d} - \bar{u}, \quad g.$$

Input partons parameterised as, e.g. **MSTW**, – much more general form for **NNPDF**, but same limits as  $x \rightarrow 0, 1$ .

$$xf(x, Q_0^2) = (1 - x)^\eta (1 + \epsilon x^{0.5} + \gamma x) x^\delta.$$

Evolve partons upwards using **LO**, **NLO** (or **NNLO**) **DGLAP** equations.

$$\frac{df_i(x, Q^2, \alpha_s(Q^2))}{d \ln Q^2} = \sum_j P_{ij}(x, \alpha_s(Q^2)) \otimes f_j(x, Q^2, \alpha_s(Q^2))$$

Fit data for scales above  $2 - 5 \text{GeV}^2$ . Need many different types for full determination.

- Lepton-proton collider HERA – (DIS)  $\rightarrow$  small- $x$  quarks (best below  $x \sim 0.05$ ). Also gluons from evolution (same  $x$ ), and now  $F_L(x, Q^2)$ . Also, jets  $\rightarrow$  moderate- $x$  gluon. Charged current data some limited info on flavour separation. Heavy flavour structure functions – gluon and charm, bottom distributions and masses.
- Fixed target DIS – higher  $x$  – leptons (BCDMS, NMC, ...)  $\rightarrow$  up quark (proton) or down quark (deuterium) and neutrinos (CHORUS, NuTeV, CCFR)  $\rightarrow$  valence or singlet combinations.
- Di-muon production in neutrino DIS – strange quarks and neutrino-antineutrino comparison  $\rightarrow$  asymmetry . Only for  $x > 0.01$ .
- Drell-Yan production of dileptons – quark-antiquark annihilation (E605, E866) – high- $x$  sea quarks. Deuterium target –  $\bar{u}/\bar{d}$  asymmetry.
- High- $p_T$  jets at colliders (Tevatron) – high- $x$  gluon distribution –  $x > 0.01$  .
- $W$  and  $Z$  production at colliders (Tevatron) – different quark contributions to DIS.

## Different PDF sets

- **MSTW08** – fit all previous types of data. Most up-to-date **Tevatron** jet data. Not most recent **HERA** combination of data. PDFs at **LO**, **NLO** and **NNLO**.
- **CTEQ6.6** – very similar. Not quite as up-to-date on **Tevatron** data. PDFs at **NLO**.
- **NNPDF2.0** – include all above except **HERA** jet data (not strongest constraint) and heavy flavour structure functions. Include **HERA** combined data. PDFs at **NLO**.
- **HERAPDF2.0** – based entirely on **HERA** inclusive structure functions, neutral and charged current. Use combined data. PDFs at **LO**, **NLO**.
- **ABKM09** – fit to **DIS** and fixed target **Drell-Yan** data. PDFs at **NLO** and **NNLO**.
- **GJR08** – fit to **DIS**, fixed target **Drell-Yan** and **Tevatron** jet data. PDFs at **NLO** and **NNLO**.

Use of **HERA** combined data instead of original data  $\rightarrow$  **1 – 2.5%** increase in quarks at low  $x$  (depending on procedure), similar on  $\alpha_S(M_Z^2)$  if free (**MSTW** prelim.), and somewhat less on gluon. More stable at **NNLO** (**MSTW** prelim.).

## Determination of best fit and uncertainties

All but **NNPDF** minimise  $\chi^2$  and define eigenvectors of parameter combinations expanding about best fit.

**NNPDF** create many replicas of data and obtain PDF replicas in each case by fitting to training set and comparing to validation set.

- **MSTW08** – 20 eigenvectors. Due to incompatibility of different sets and (perhaps to some extent) parameterisation inflexibility (little direct evidence for this) have inflated  $\Delta\chi^2$  of 5 – 20 for eigenvectors.
- **CTEQ6.6** – 22 eigenvectors. Inflated  $\Delta\chi^2$  of 50 for 1 sigma for eigenvectors (no normalization uncertainties in **CTEQ6.6**).
- **NNPDF2.0** – uncertainty determined by spread of replicas. Direct relationship to  $\Delta\chi^2$  in global fit not trivial.
- **HERAPDF2.0** – 9 eigenvectors. Use “ $\Delta\chi^2 = 1$ ”. Additional model and parameterisation uncertainties.

- ABKM09 – 21 parton parameters. Use  $\Delta\chi^2 = 1$ .
- GJR08 – 12 parton parameters. Use  $\Delta\chi^2 \approx 20$ . Impose strong theory constraint on input form of PDFs.

Perhaps surprisingly all get rather similar uncertainties for PDFs and predicted cross-sections.

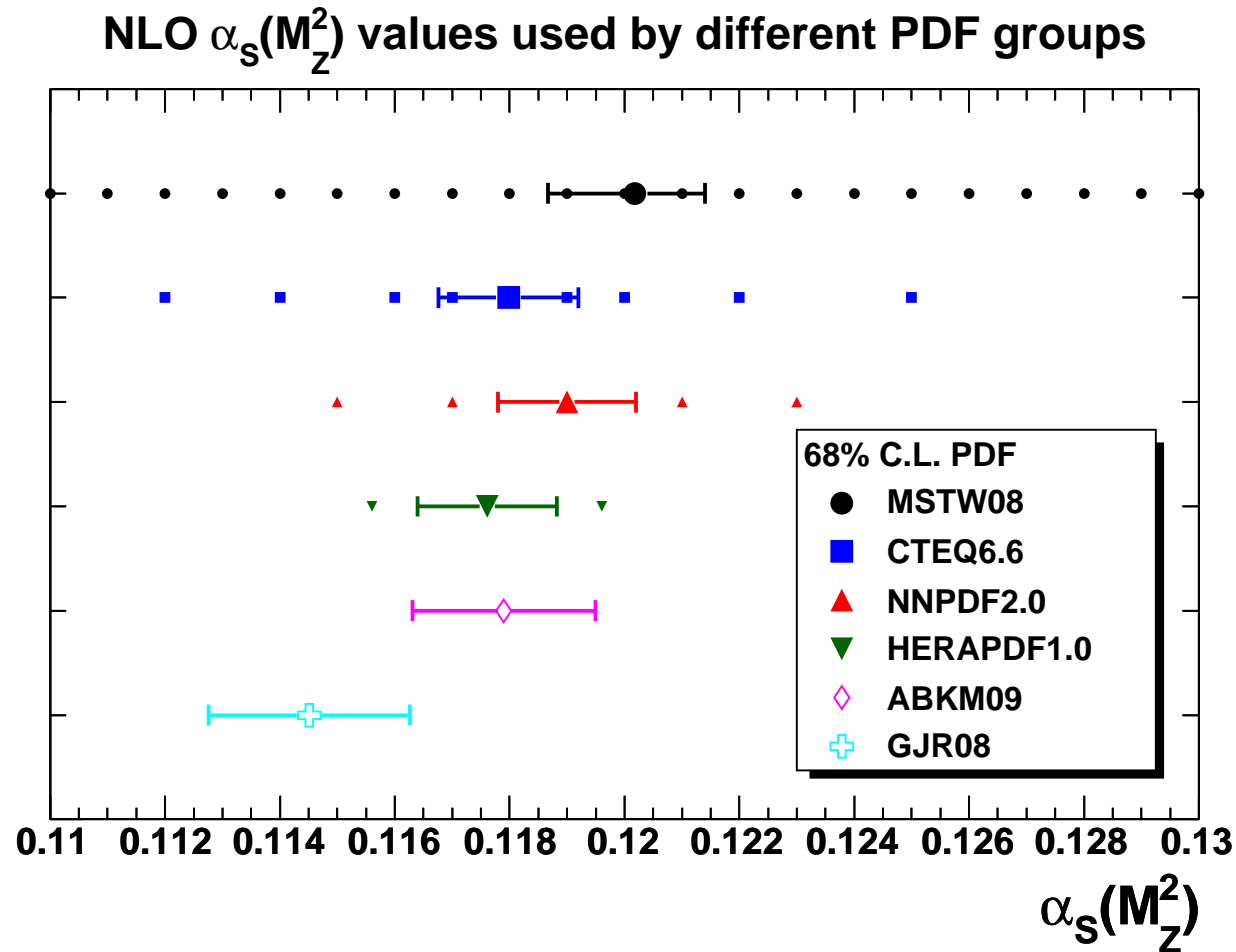
Some exceptions (more details later)

NNPDF and due to extra parameters MSTW have more complicated shape for gluon at smaller  $x$  and bigger small- $x$  uncertainty.

Choice of parameterisation leads to bigger very high- $x$  gluon uncertainty for CTEQ.

Different theory assumptions in strange leads to vastly different uncertainties – MSTW small  $\rightarrow$  NNPDF large. Feeds into other “light” quarks.

# Choices of $\alpha_S(M_Z^2)$



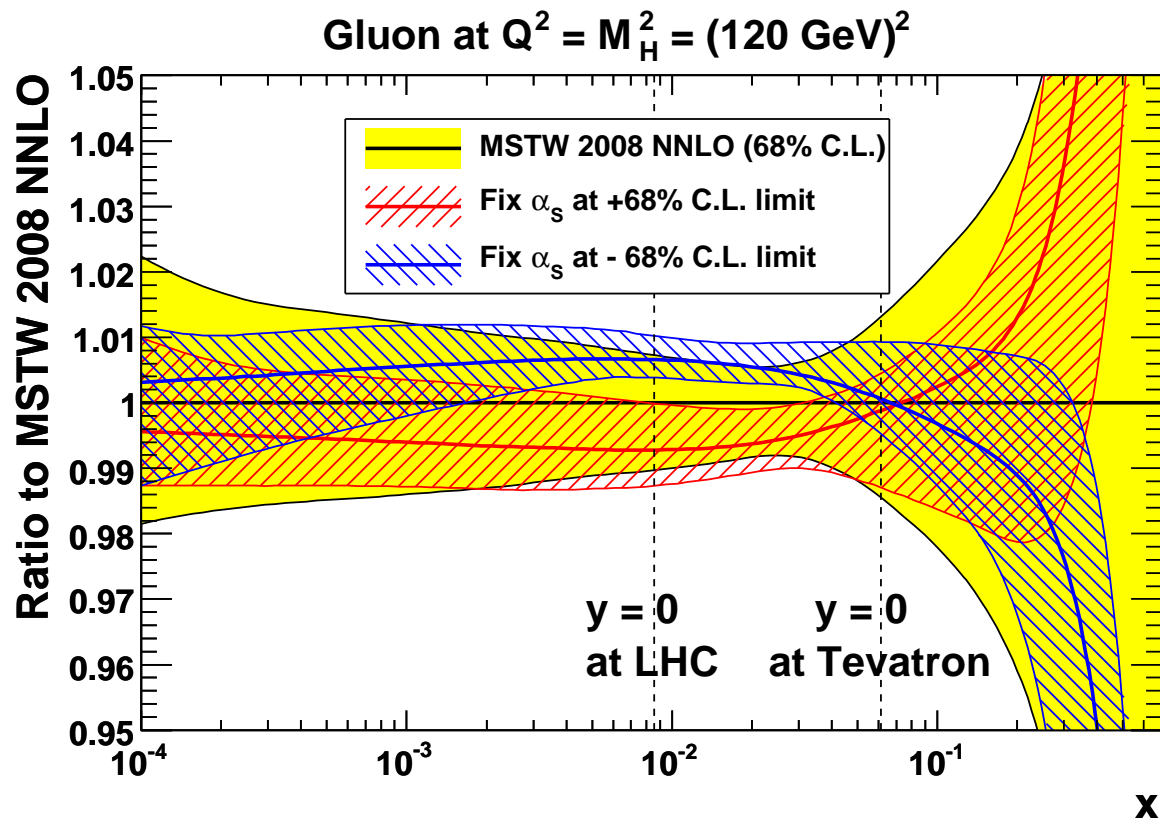
$\alpha_S(m_Z^2)$  values and uncertainty determined by fit for MSTW08, ABKM09 and GJR08. In each case NNLO value about 0.003 – 0.004 lower than NLO value.

Others pick *standard* values and uncertainties.

## PDF correlation with $\alpha_S$ .

Can also look at PDF changes and uncertainties at different  $\alpha_S(M_Z^2)$ . Latter usually only for one fixed  $\alpha_S(M_Z^2)$ . Can be determined from fit, e.g.  $\alpha_S(M_Z^2) = 0.1202^{+0.0012}_{-0.0015}$  at NLO and  $\alpha_S(M_Z^2) = 0.1171^{+0.0014}_{-0.0014}$  at NNLO from MSTW.

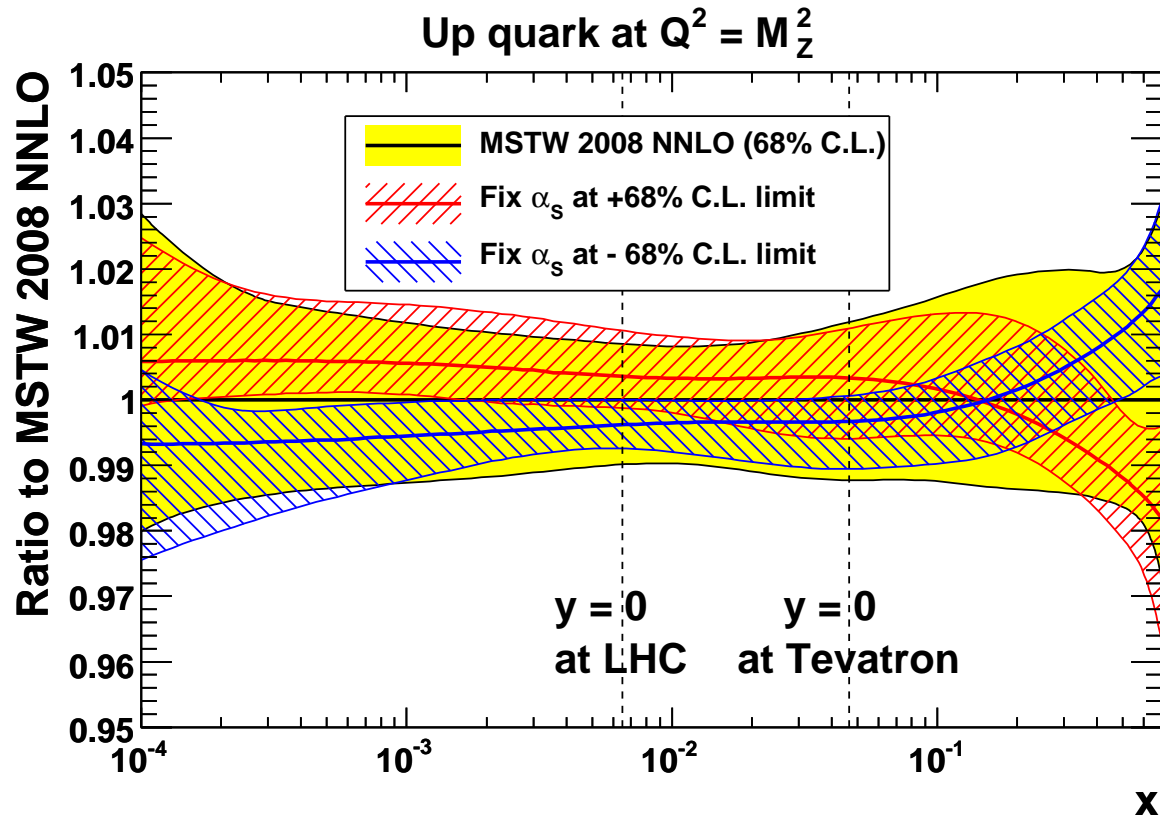
PDF uncertainties reduced since quality of fit already worse than best fit.



Expected gluon- $\alpha_S(M_Z^2)$  small- $x$  anti-correlation  $\rightarrow$  high- $x$  correlation from sum rule.



Gluon feeds into evolution of quarks, but change in  $\alpha_S(M_Z^2)$  just outweighs gluon change, i.e. larger  $\alpha_S(M_Z^2) \rightarrow$  slightly more evolution.

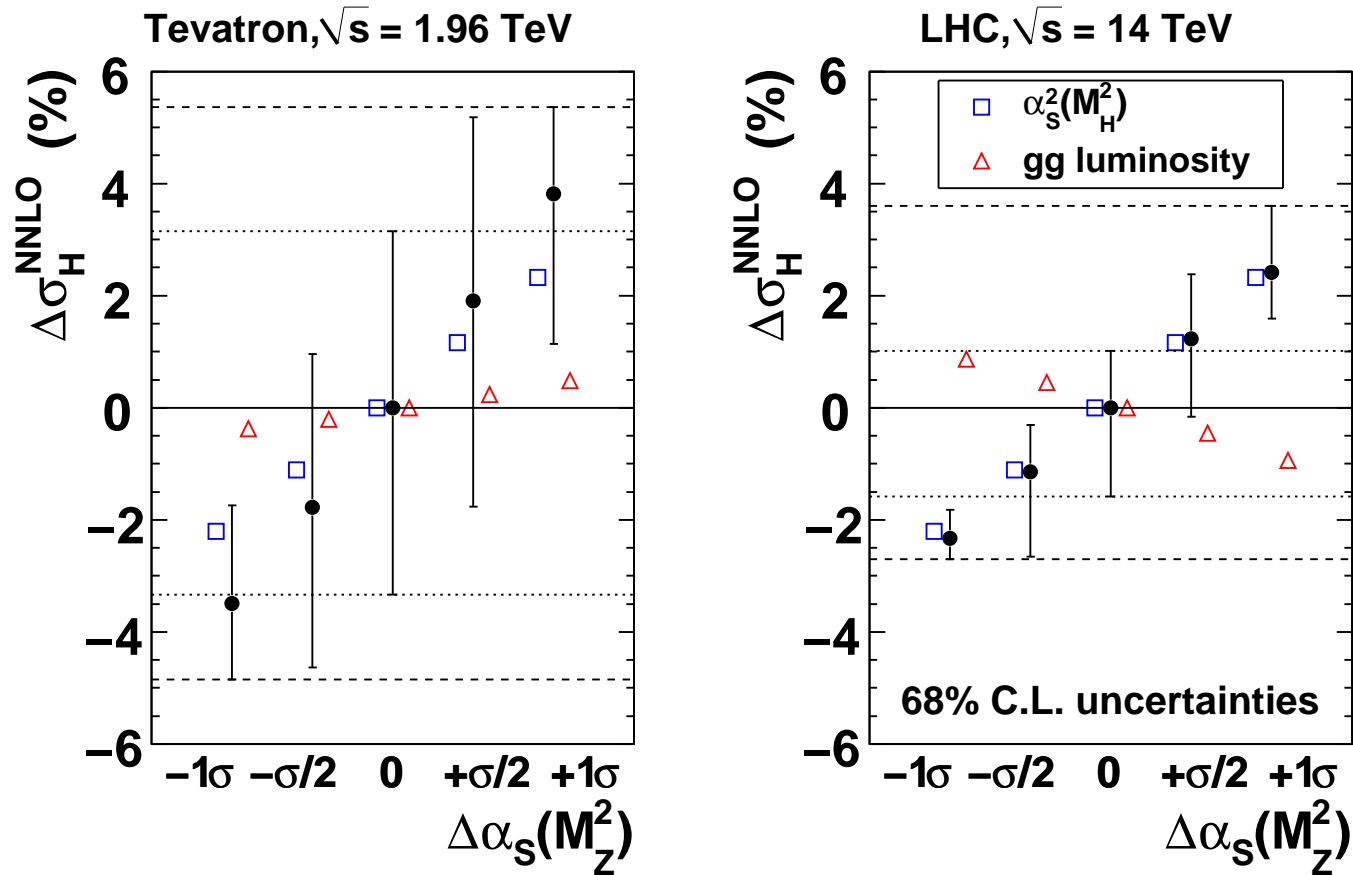


Strong anti-correlation at high- $x$  due to evolution and positive coefficient functions.

Quarks roughly opposite to gluons.

NNLO predictions for Higgs (120 GeV) production for different allowed  $\alpha_S(M_Z^2)$  values and their uncertainties.

### Higgs ( $M_H = 120$ GeV) with MSTW 2008 NNLO PDFs



Increases by a factor of 2–3 (up more than down) at LHC. Direct  $\alpha_S(M_Z^2)$  dependence mitigated somewhat by anti-correlated small- $x$  gluon (asymmetry feature of *minor* problems in fit to HERA data). At Tevatron intrinsic gluon uncertainty dominates.

## Consideration of NNLO

Very good evidence that one should use NNLO if possible rather than NLO – many physical cross-sections, particularly  $gg \rightarrow H$ , not very convergent.

Fewer PDF sets available, can study differences between them better at NLO, but for central prediction need NNLO.

Related to issue of use and uncertainty of  $\alpha_S(M_Z^2)$ . Noted systematic change in value from fit as one goes from NLO to NNLO. Also highlighted in stability of predictions.

Consider percentage change from NLO to NNLO in MSTW08 predictions for best fit  $\alpha_S$  compared to fixed  $\alpha_S(M_Z^2) = 0.119$ .

	$\sigma_{W(Z)}$ 7TeV	$\sigma_{W(Z)}$ 14TeV	$\sigma_H$ 7TeV	$\sigma_H$ 7TeV
MSTW08 best fit $\alpha_S$	3.0	2.6	25	24
MSTW08 $\alpha_S = 0.119$	5.3	5.0	32	30

$\alpha_S(M_Z^2)$  is not a physical quantity. In (nearly) all PDF related quantities (and many others) shows tendency to decrease from order to order. Noticeable if one has fit at NNLO. Any settling on, or near common  $\alpha_S(M_Z^2)$  has to take this into account.

Note that like  $\alpha_S$  the PDFs are systematically different at NNLO compared to NLO. At high scales more so for quarks than gluon.

**Heavy Quarks** – Essential to treat these correctly. Two distinct regimes:

Near threshold  $Q^2 \sim m_H^2$  massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme (FFNS)**.

$$F(x, Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Does not sum  $\ln^n(Q^2/m_H^2)$  terms, and not calculated for many processes beyond **LO**. Still occasionally used. Sometimes final state details in this scheme only.

Alternative, at high scales  $Q^2 \gg m_H^2$  heavy quarks like massless partons. Behave like **up, down, strange**. Sum  $\ln(Q^2/m_H^2)$  terms via evolution. **Zero Mass Variable Flavour Number Scheme (ZM-VFNS)**. Normal assumption in calculations. Ignores  $\mathcal{O}(m_H^2/Q^2)$  corrections.

$$F(x, Q^2) = C_j^{ZMVF} \otimes f_j^{n_f+1}(Q^2).$$

Can devise a **General Mass Variable Flavour Number Scheme (GM-VFNS)** interpolating between the two well-defined limits of  $Q^2 \leq m_H^2$  and  $Q^2 \gg m_H^2$ . Used by **MRST/MSTW** and more recently (as default) by **CTEQ**, and now also more regularly by **H1, ZEUS**.

General result, evolution at small  $x$  quicker using ZM-VFNS than using GM-VFNS which is quicker than using FFNS.

→ small- $x$  gluon and consequently light quarks smaller in ZM-VFNS than GM-VFNS and largest in FFNS.

Details follow, but to summarise

- Small- $x$  quarks can be up to 8% smaller at electroweak scale in ZM-VFNS than in GM-VFNS – CTEQ, similar for MSTW. Slightly smaller effect in gluon. Similar size effects in LHC cross-sections. (Only twice PDF change for distinct rapidity.)
- Various definitions of GM-VFNS possible. Versions used by MSTW (RT) and CTEQ (ACOT) have converged somewhat.
- Variation in vaguely *sensible* definitions of GM-VFNS lead to changes of maximum 3% in LHC cross-sections – MSTW study.
- Use of ZM-VFNS gives about 0.0015 lower value of  $\alpha_S(M_Z^2)$ . Basis of existing CTEQ study on  $\alpha_S(M_Z^2)$  dependence.

Related issue. Can be 1 – 2% variation in predicted cross-sections from variations in charm mass of 0.15GeV. Perhaps more like 1% at 7TeV.

## Different PDF sets

- **MSTW08** – use definition of **GM-VFNS** at **LO**, **NLO** and **NNLO**. (Have done since **MRST98**, but details changed in **2006** – pre-**2006 NNLO** prescription incomplete).
- **CTEQ6.6** – use **GM-VFNS** at **NLO** as default. Only used as special case in pre-**CTEQ6.5** sets.
- **NNPDF2.0** – currently use **ZM-VFNS**. Have version of **GM-VFNS** bench-marked along with **MSTW** and **CTEQ** ready to use.
- **HERAPDF2.0** – use same **GM-VFNS** as **MSTW**.
- **ABKM09** – perform fit using **FFNS**. Claim insensitivity to using **GM-VFNS**. Currently heavy quark treatment same at **NNLO** as at **NLO**.
- **GJR08** – use **FFNS**, again same at **NNLO** as at **NLO**.

# Predictions at the LHC

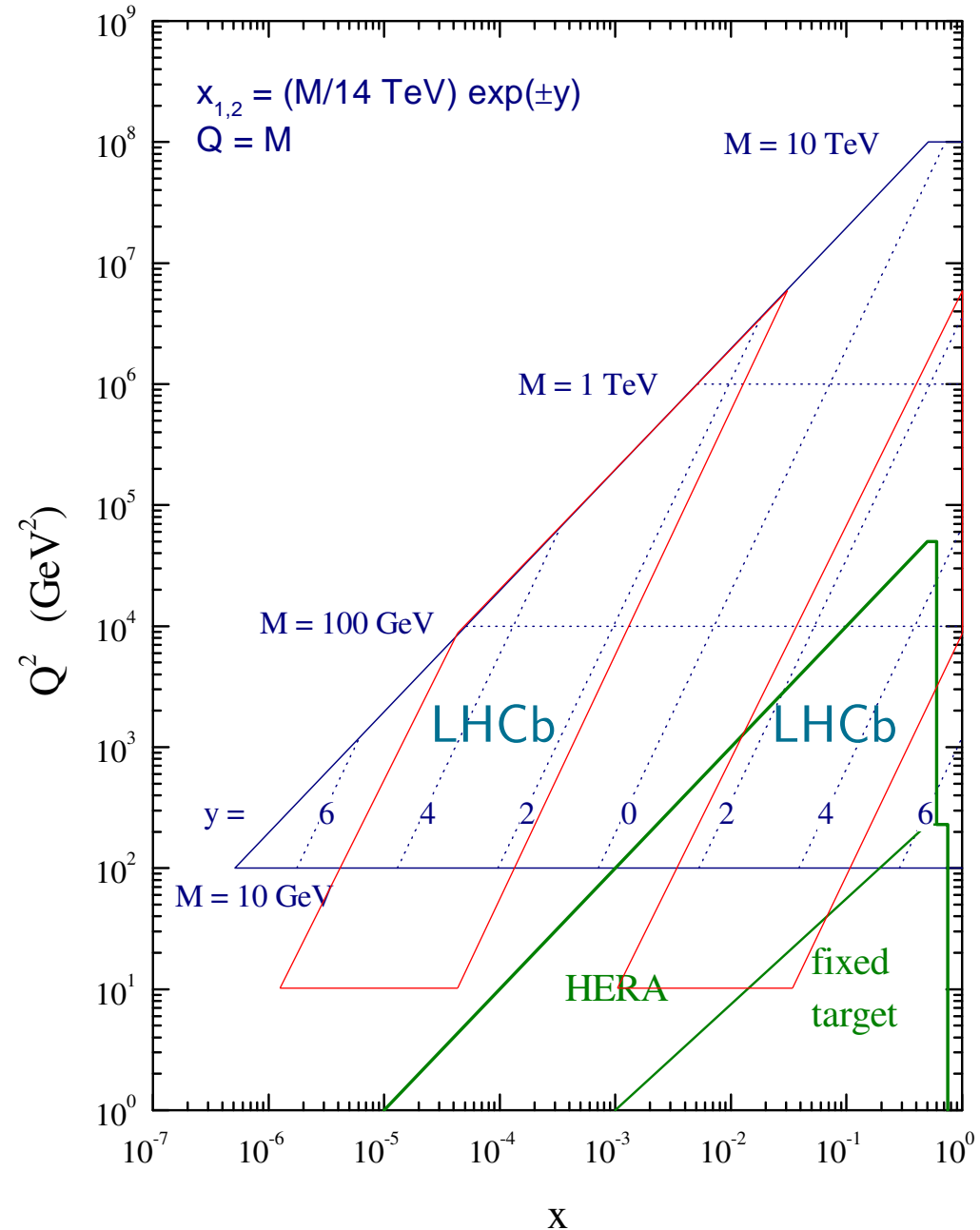
New kinematic regime.

PDFs mainly extrapolated via evolution rather than measured directly.

High scale and small- $x$  parton distributions are vital for understanding processes at the LHC.

More discrepancy at values of  $x$  away from this.

## LHC parton kinematics

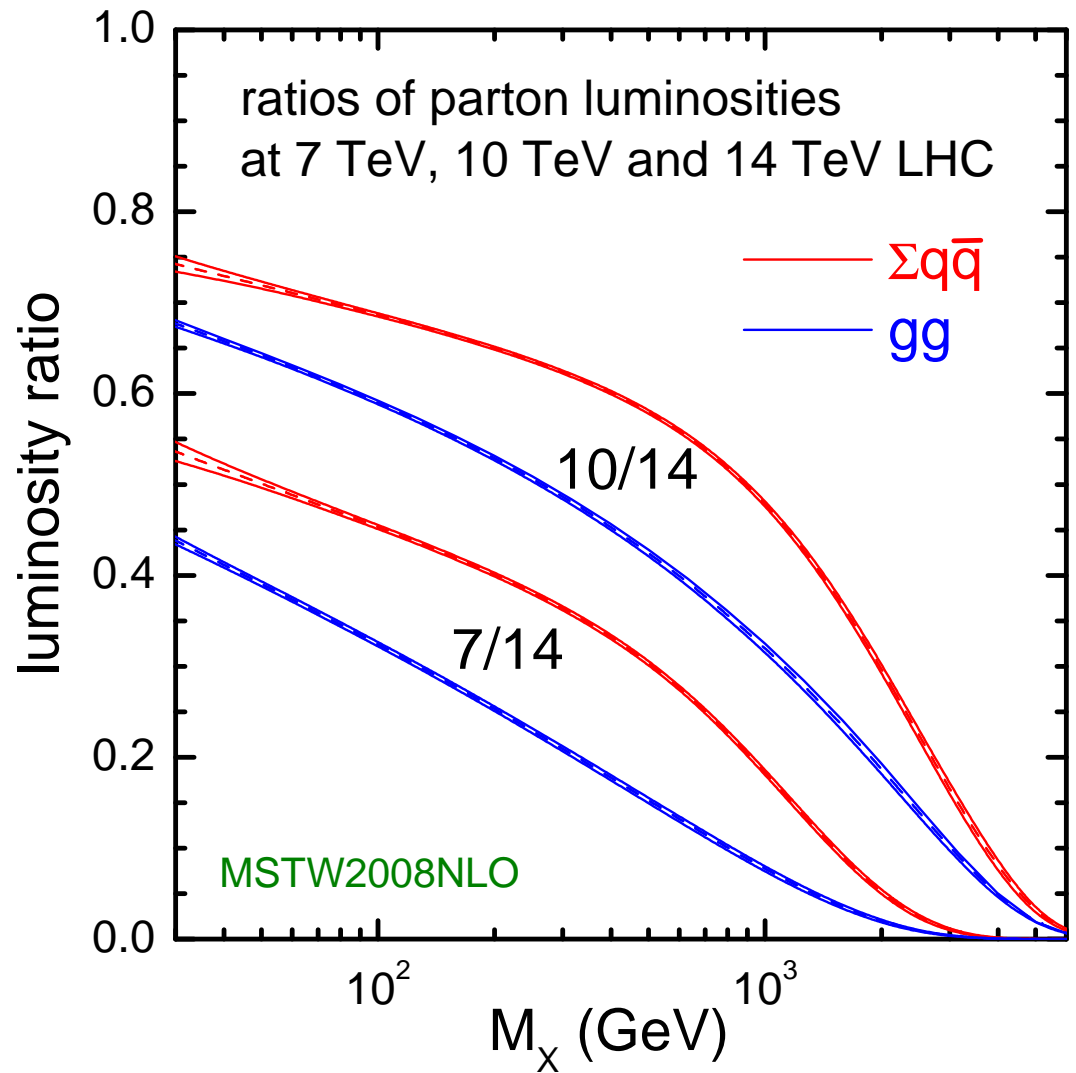


## Initial Running

Of course, will be starting the LHC running at 7 TeV rather than the full 14 TeV.

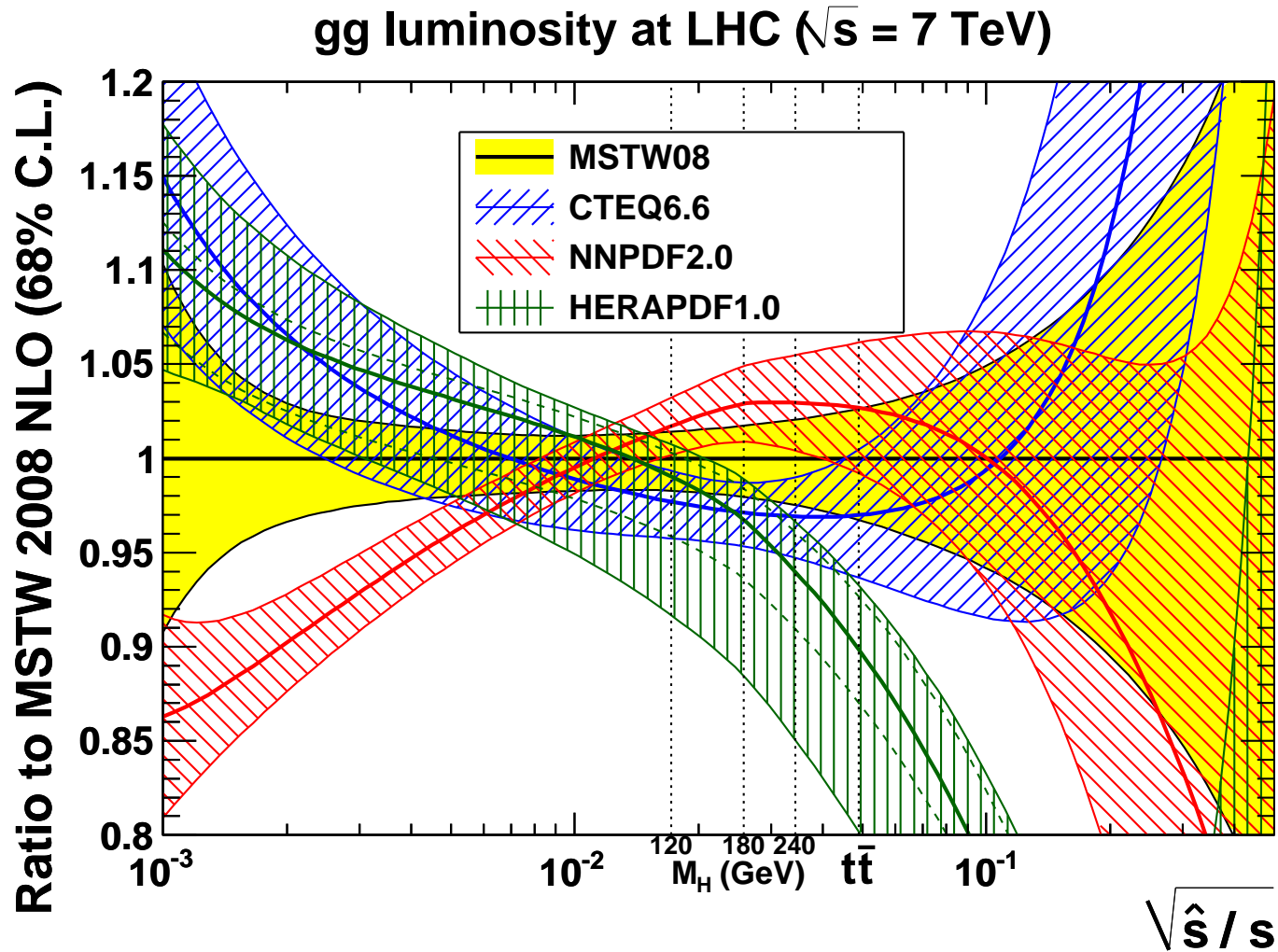
Reduces rapidity range by  $\ln 2$ .

Roughly 30 – 50% the full cross-sections for most standard model (including light Higgs) processes.





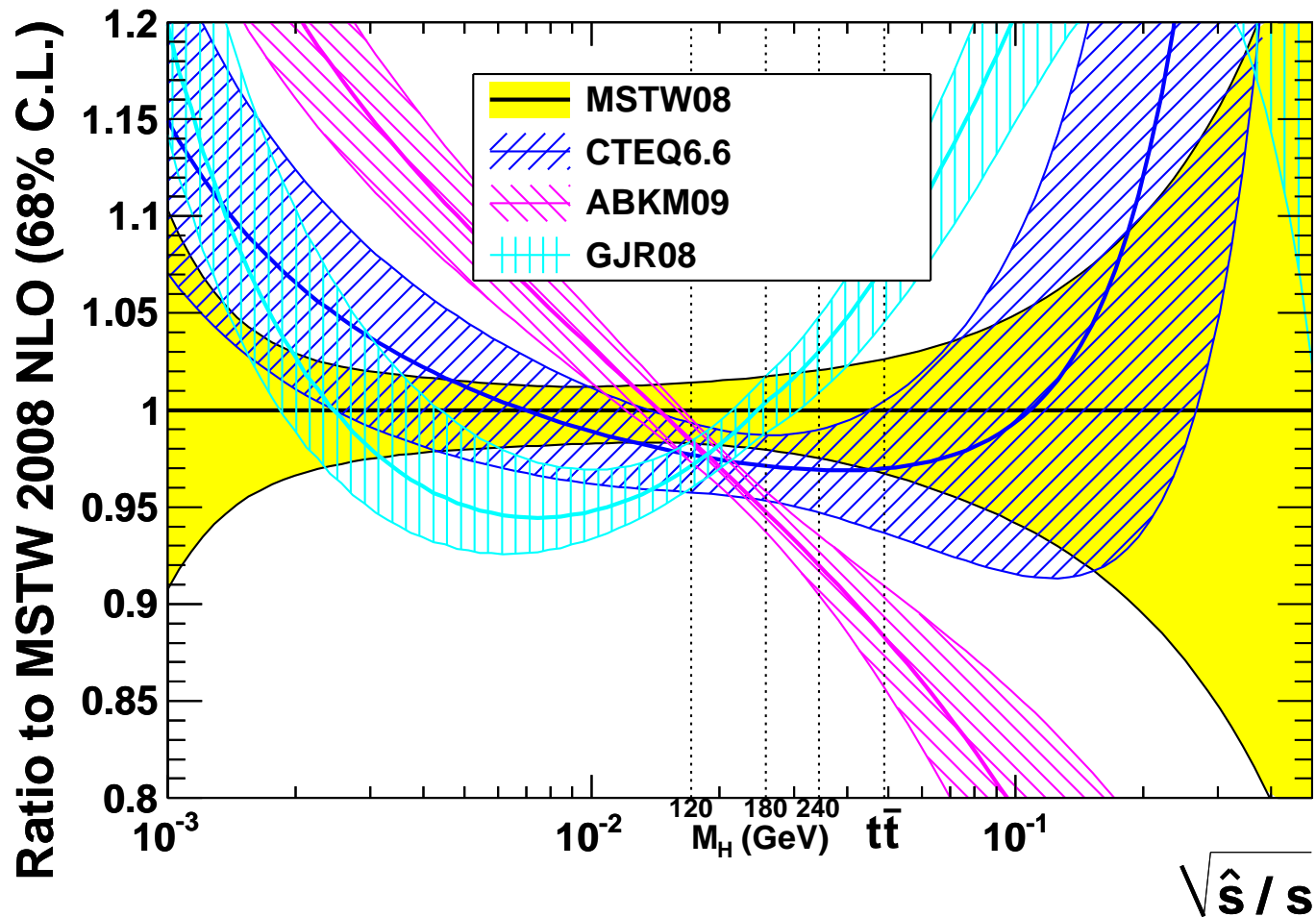
Predictions by various groups - parton luminosities – **NLO**. Plots by **G. Watt**.



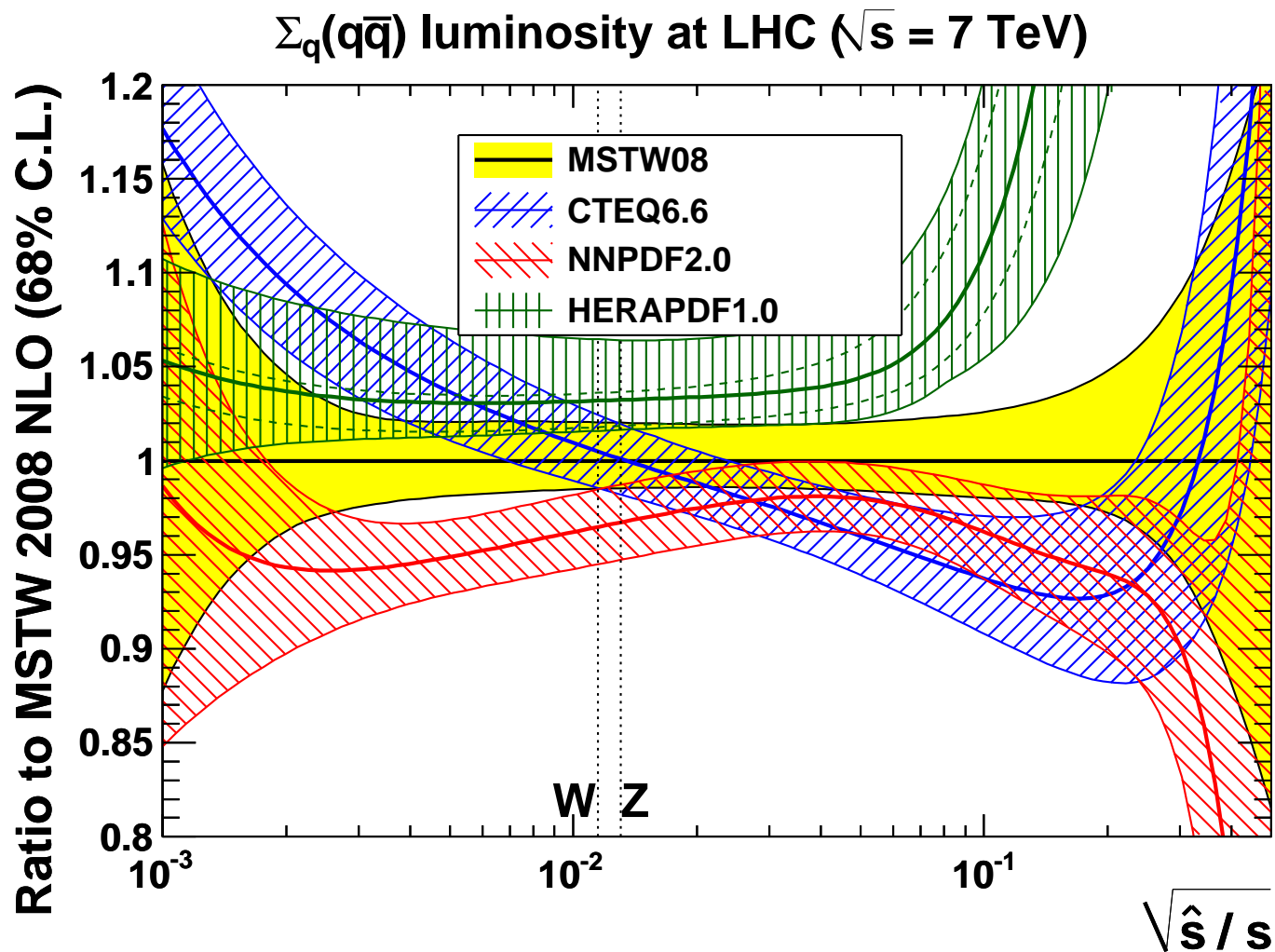
Cross-section for  $t\bar{t}$  almost identical in PDF terms to **450 GeV** Higgs.

Also  $H + t\bar{t}$  at  $\sqrt{\hat{s}}/s \sim 0.1$ .

## gg luminosity at LHC ( $\sqrt{s} = 7$ TeV)

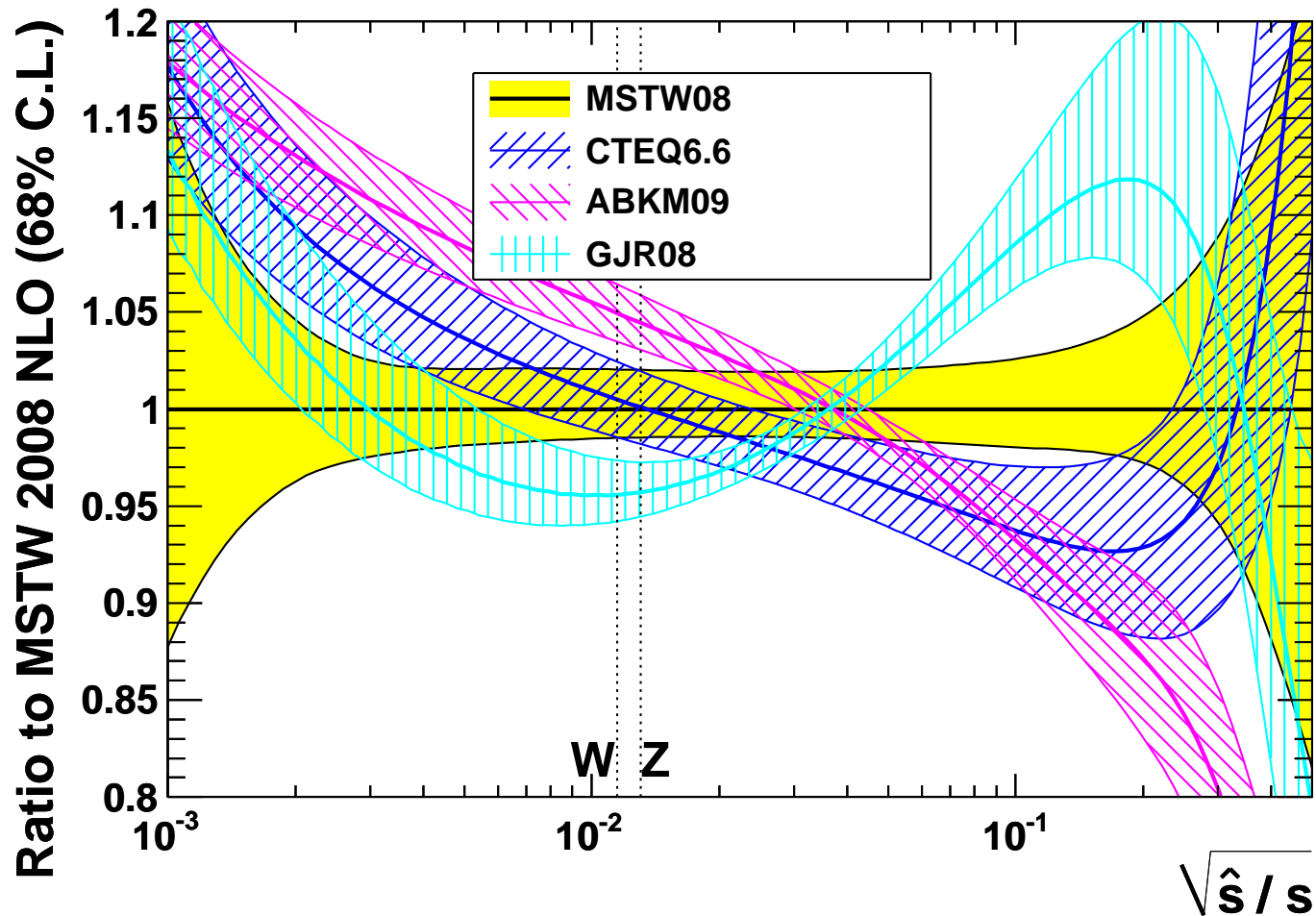


Clearly some distinct variation between groups. Much can be understood in terms of previous differences in approaches.



Many of the same general features for quark-antiquark luminosity. Some differences mainly at higher  $x$ .

# $\Sigma_q(q\bar{q})$ luminosity at LHC ( $\sqrt{s} = 7$ TeV)

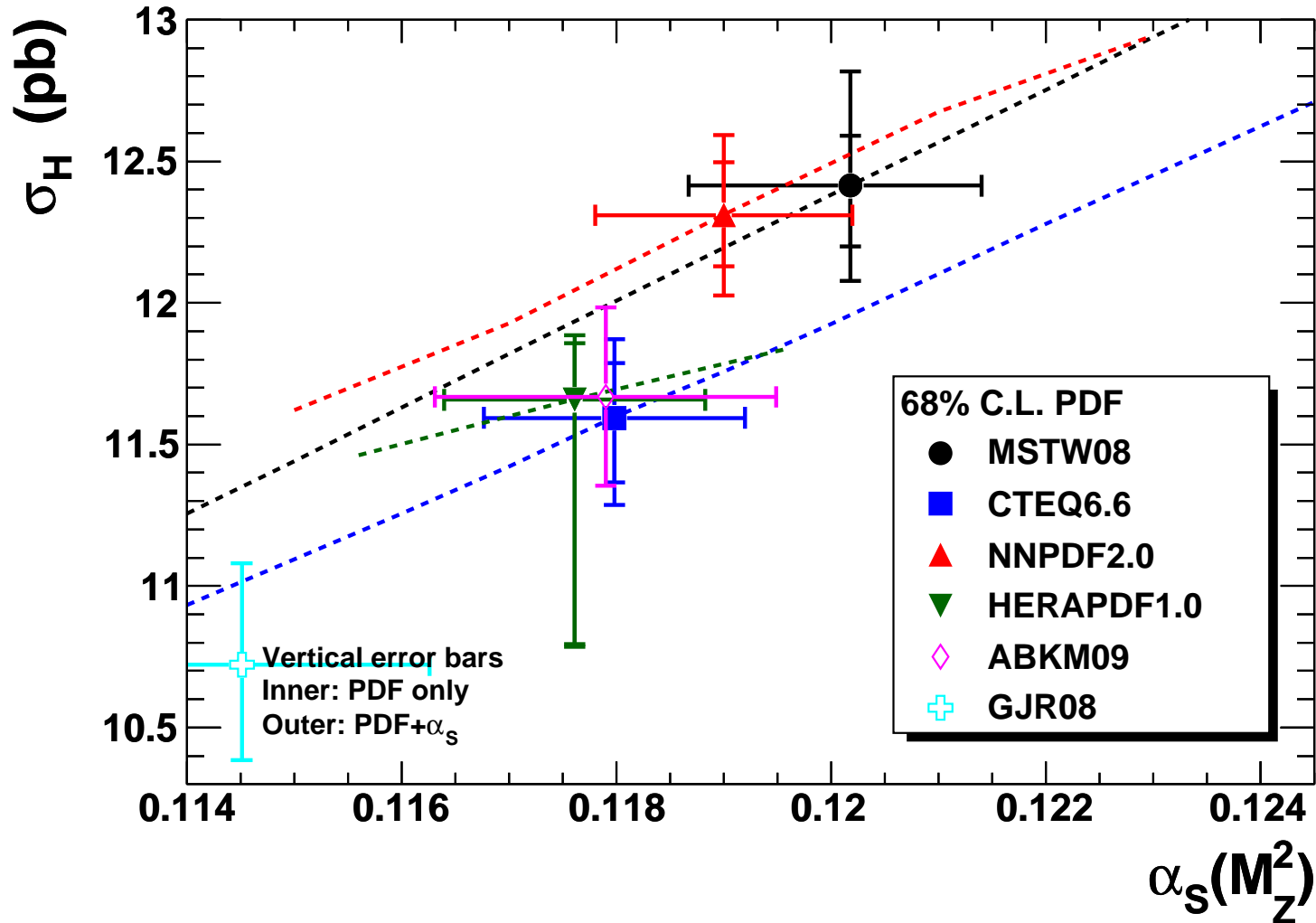


Canonical example  $W, Z$  production, but higher  $\hat{s}/s$  relevant for  $WH$  or vector boson fusion.

All plots and more at <http://projects.hepforge.org/mstwpdf/pdf4lhc>

# Variations in Cross-Section Predictions – NLO

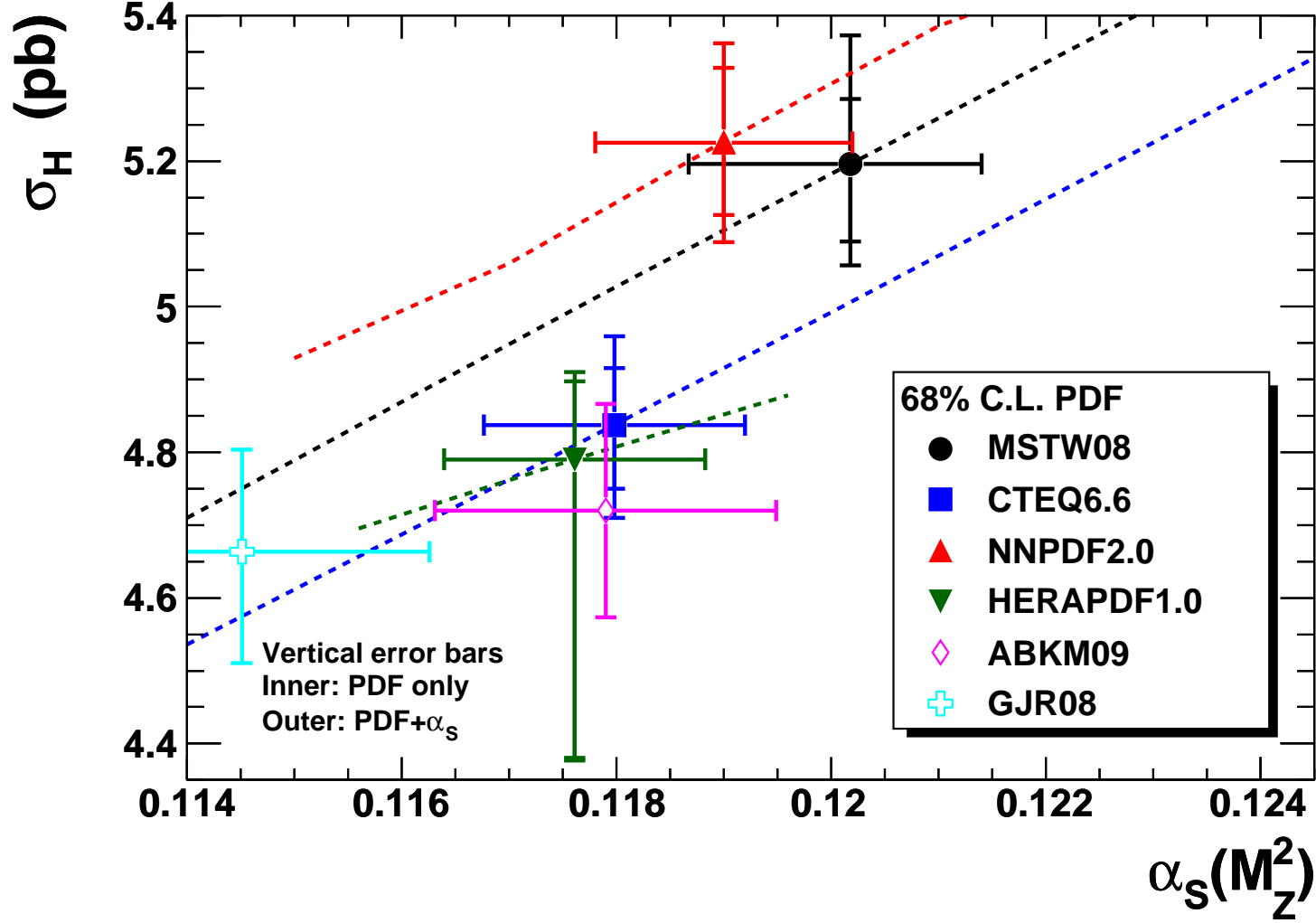
NLO  $gg \rightarrow H$  at the LHC ( $\sqrt{s} = 7$  TeV) for  $M_H = 120$  GeV



Dotted lines show how central PDF predictions vary with  $\alpha_s(M_Z^2)$ .

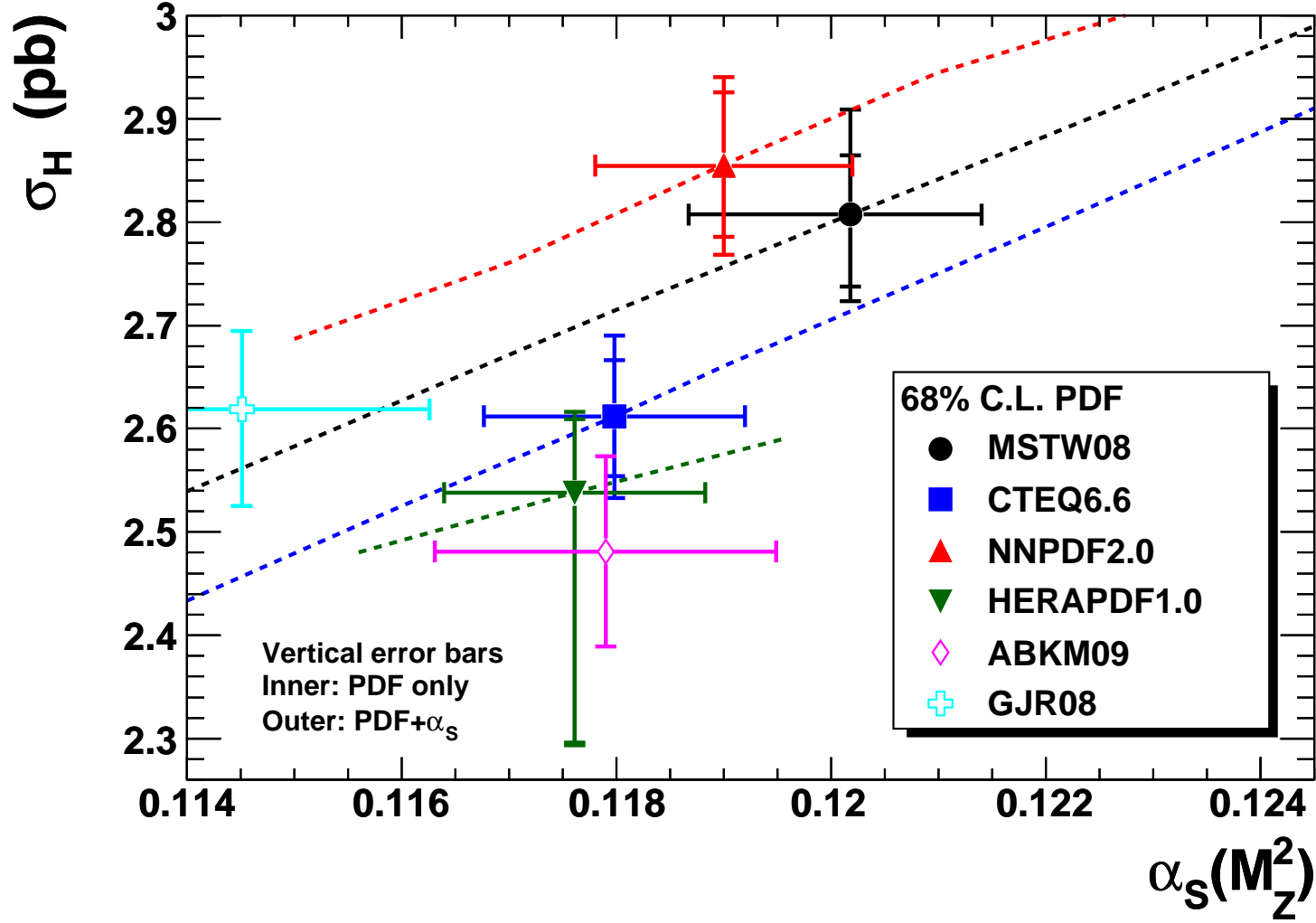
Again plots by [G Watt](#) using [PDF4LHC](#) benchmark criteria.

# NLO $gg \rightarrow H$ at the LHC ( $\sqrt{s} = 7$ TeV) for $M_H = 180$ GeV



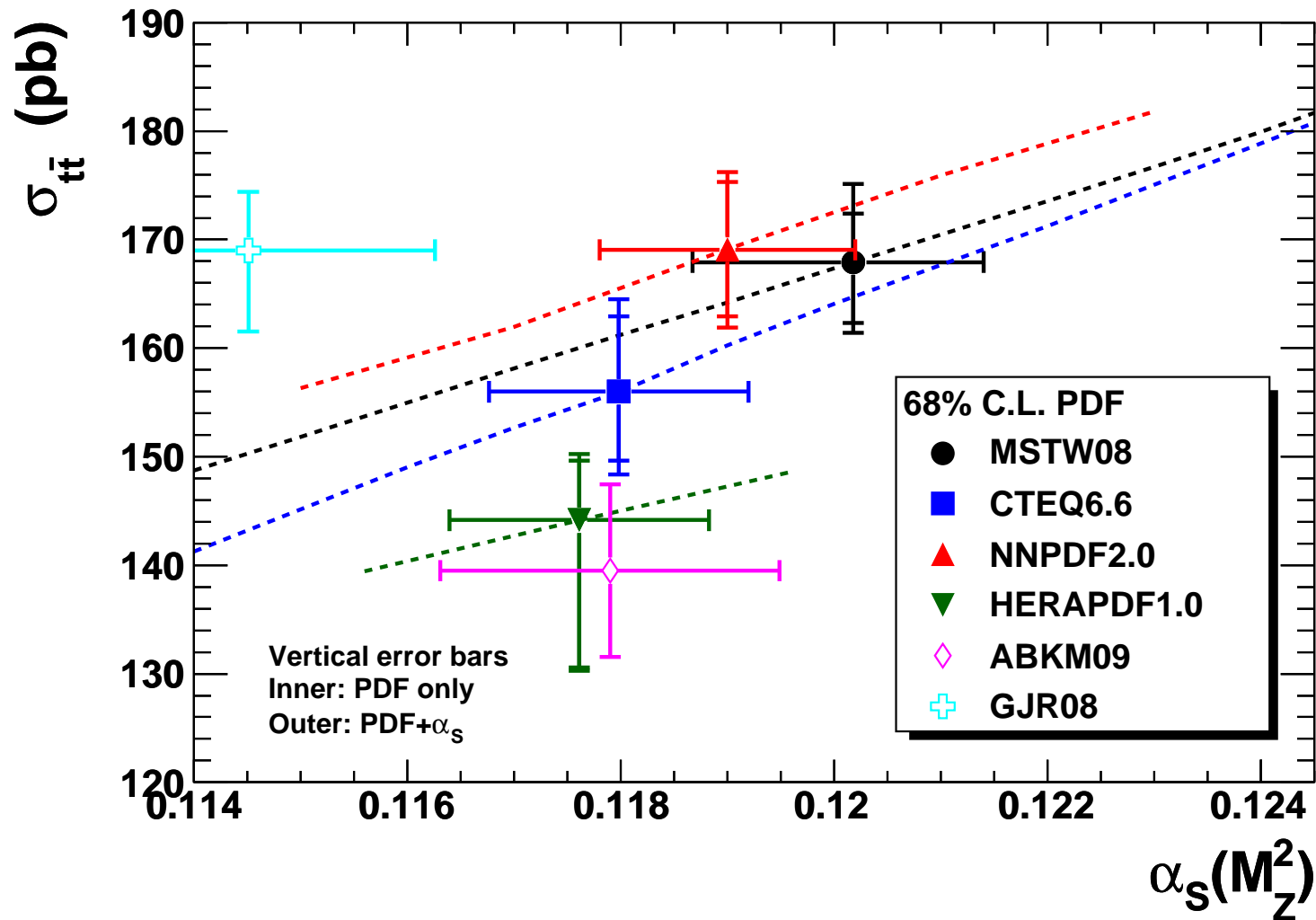
Clearly much more variation in predictions than uncertainties claimed by individual groups.

# NLO $gg \rightarrow H$ at the LHC ( $\sqrt{s} = 7$ TeV) for $M_H = 240$ GeV



Excluding GJR08 amount of difference due to  $\alpha_s(M_Z^2)$  variations 3 – 4%.

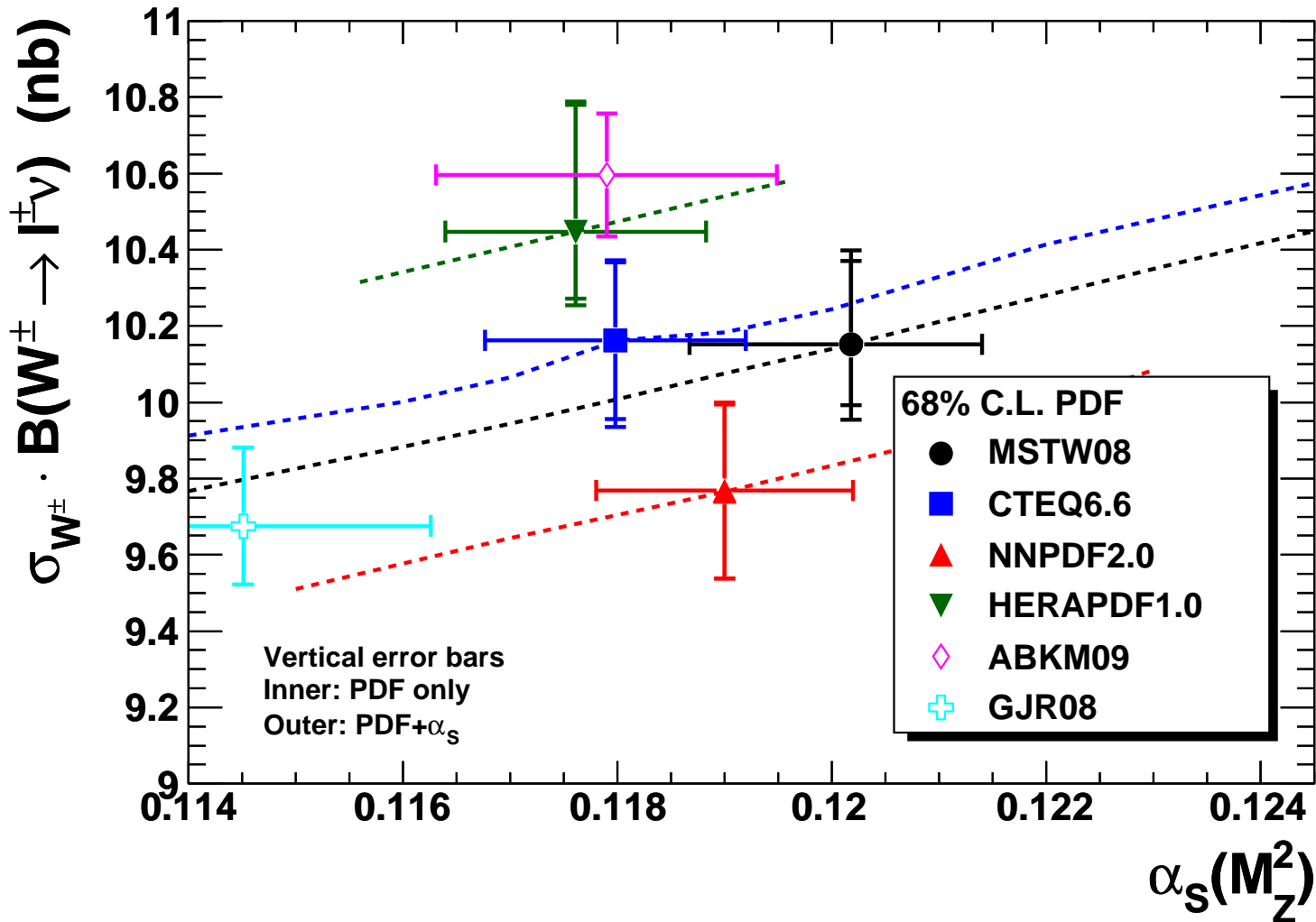
# NLO $t\bar{t}$ cross sections at the LHC ( $\sqrt{s} = 7$ TeV)



CTEQ6.6 now heading back towards MSTW08 and NNPDF2.0.

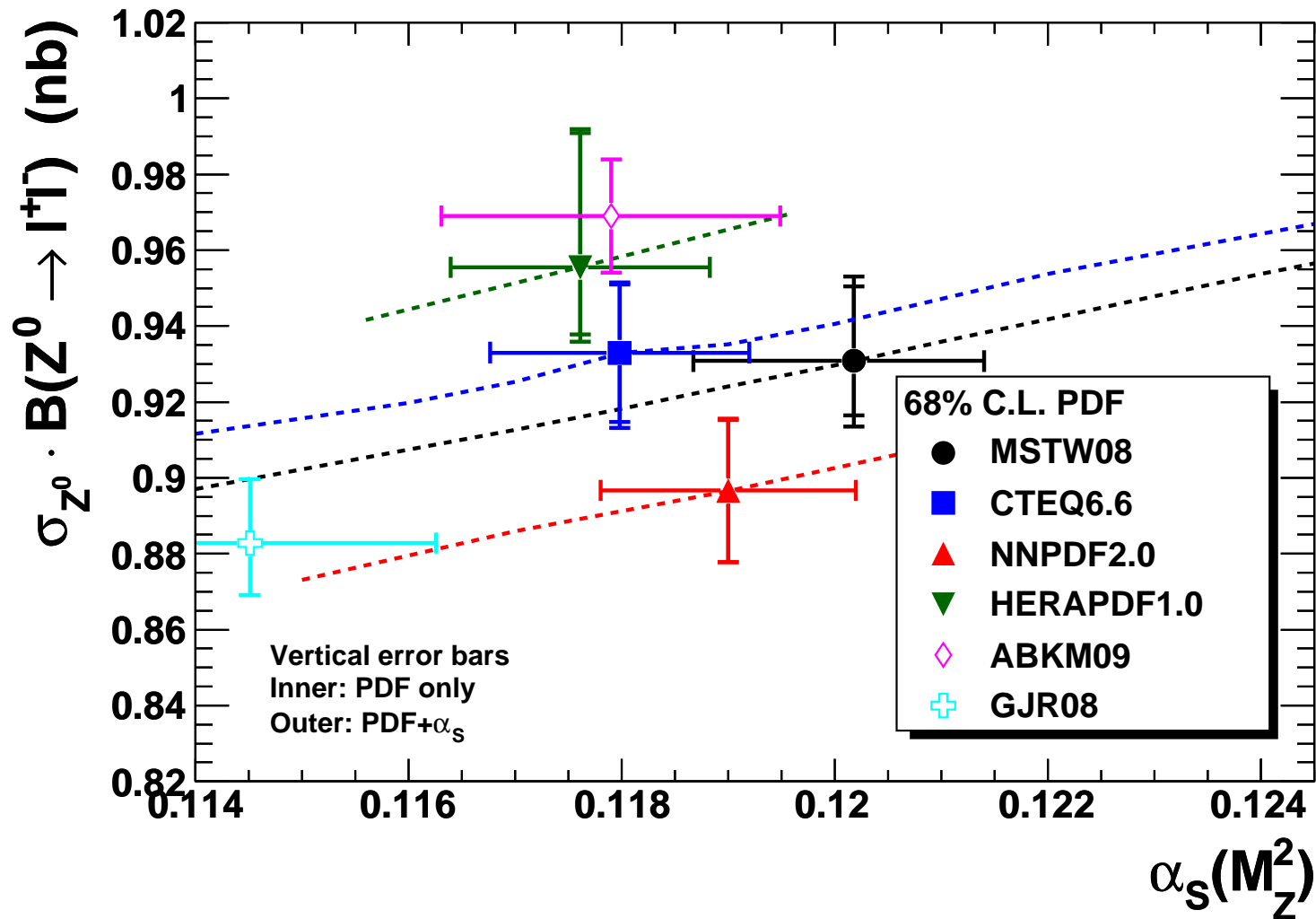


# NLO $W^\pm \rightarrow l^\pm \nu$ at the LHC ( $\sqrt{s} = 7$ TeV)



$W^+ + W^-$  cross-section.  $\alpha_s(M_Z^2)$  dependence now more due to PDF variation with  $\alpha_s(M_Z^2)$ .

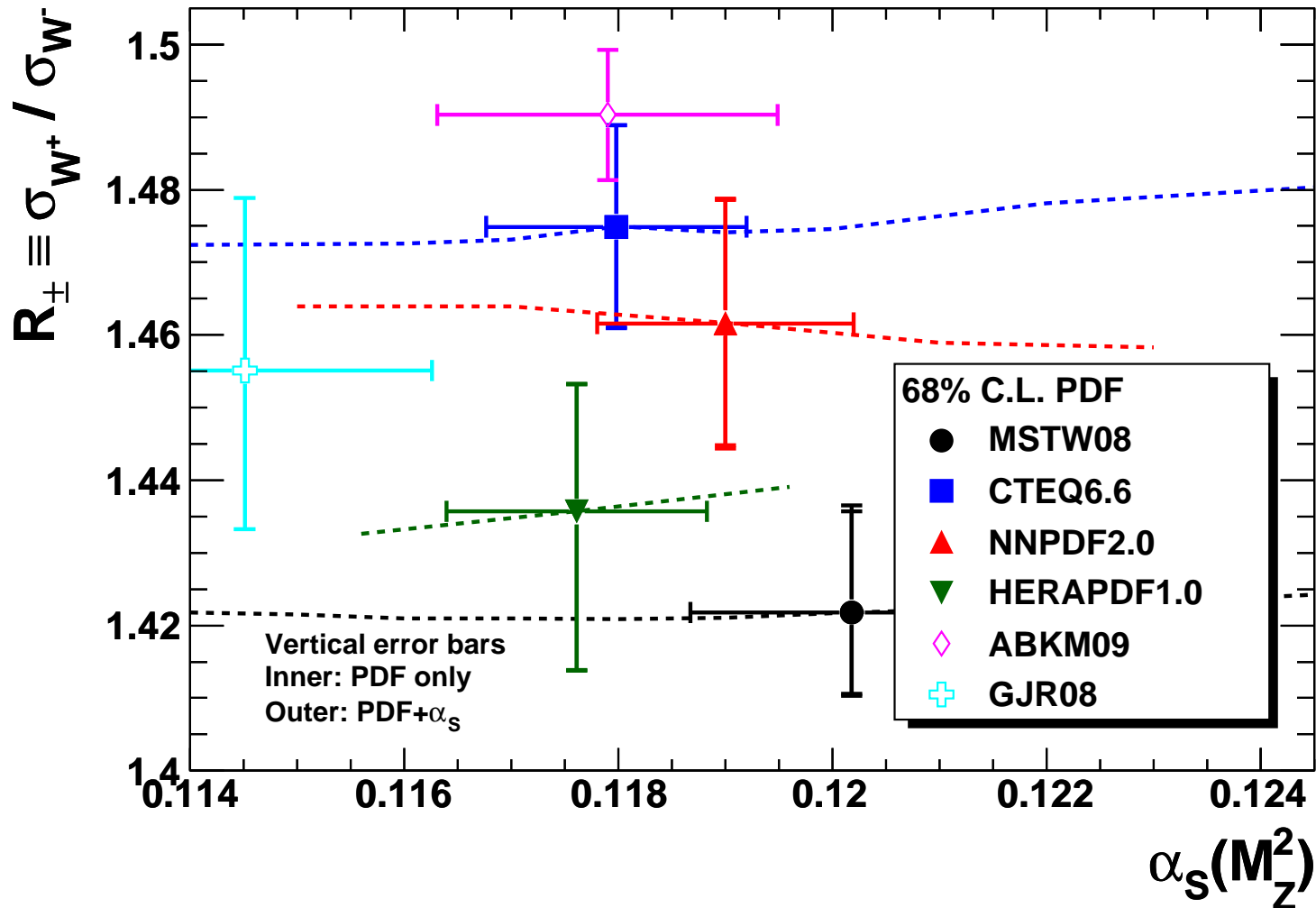
# NLO $Z^0 \rightarrow l^+l^-$ at the LHC ( $\sqrt{s} = 7$ TeV)



Again variations somewhat bigger than individual uncertainties.

Roughly similar variation for  $\hat{s}$  up to a few times higher.

## NLO $W^+/W^-$ ratio at the LHC ( $\sqrt{s} = 7$ TeV)

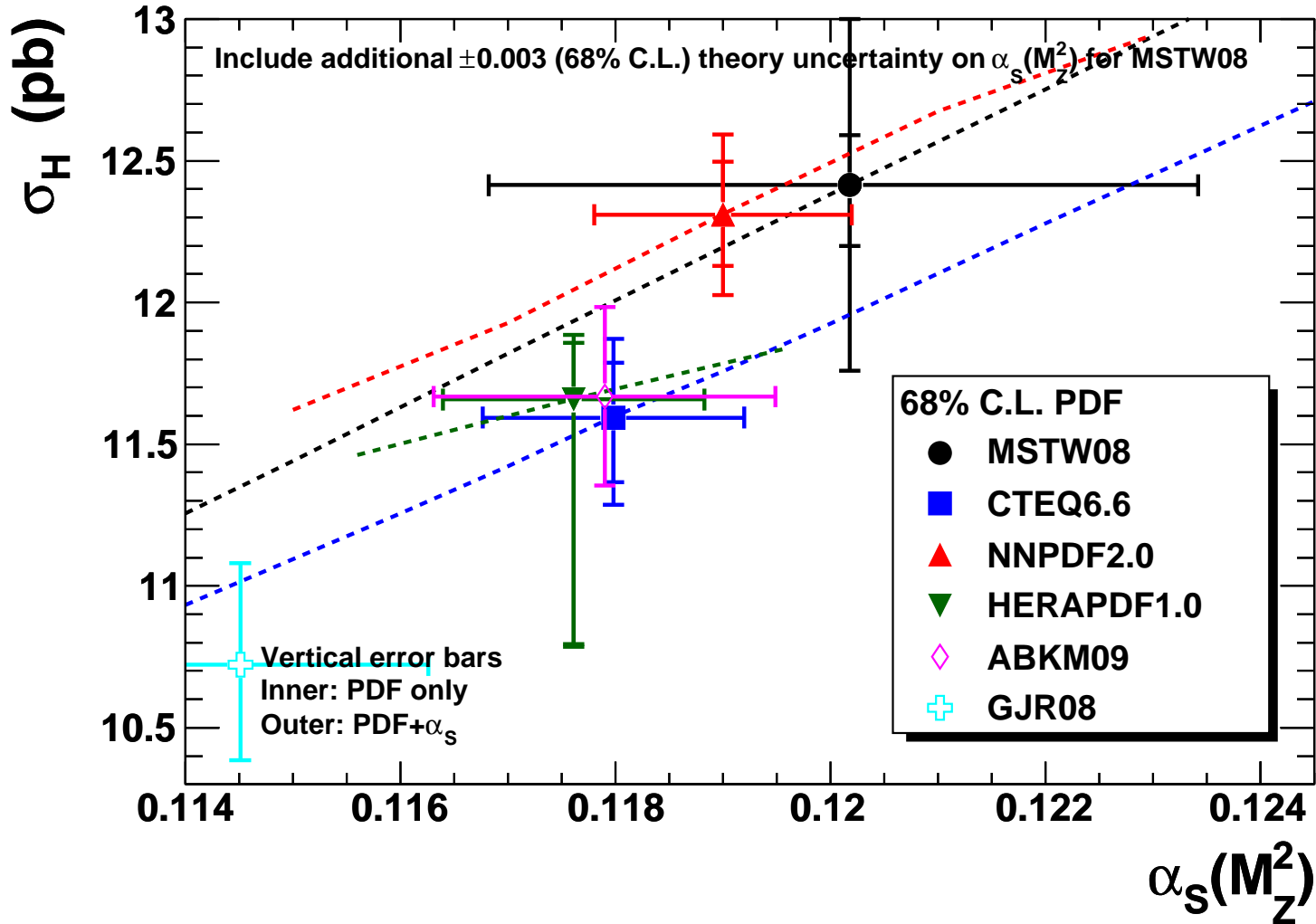


Quite a variation in ratio. Shows variations in flavour and quark-antiquark decompositions.

All plots and more at <http://projects.hepforge.org/mstwpdf/pdf4lhc>

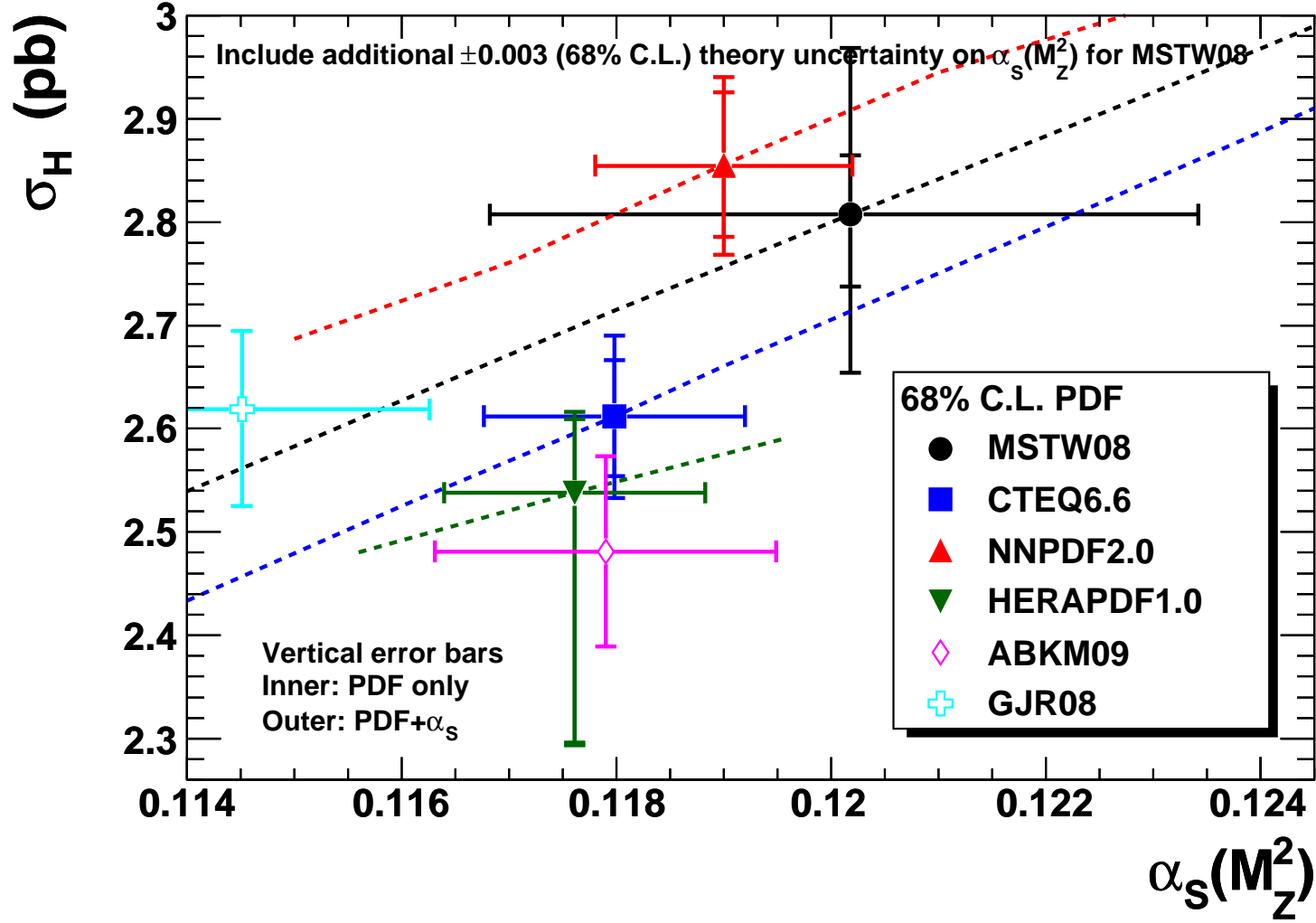
# Variations in Cross-Section Predictions – extra uncertainty at NLO

NLO  $gg \rightarrow H$  at the LHC ( $\sqrt{s} = 7$  TeV) for  $M_H = 120$  GeV



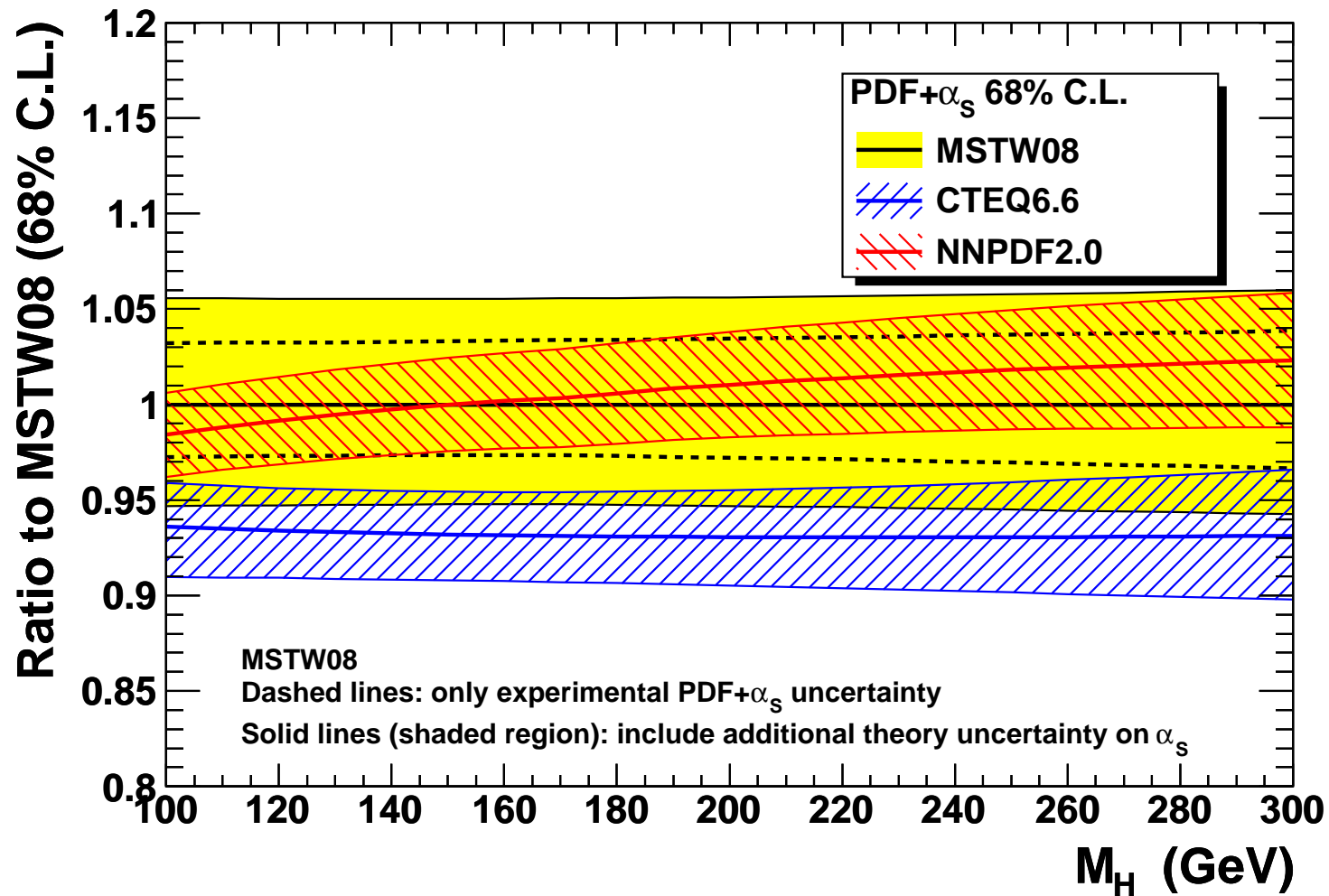
Uncertainty in prediction now includes an additional  $\alpha_s(M_Z^2) \pm 0.003$  theory uncertainty added in quadrature with original uncertainties.

# NLO $gg \rightarrow H$ at the LHC ( $\sqrt{s} = 7$ TeV) for $M_H = 240$ GeV



Even with this extra uncertainty **MSTW** doesn't completely span range of predictions, certainly at **68%** confidence level.

## NLO $gg \rightarrow H$ at the LHC ( $\sqrt{s} = 7$ TeV)



Same calculations shown as ratio for [MSTW](#), [CTEQ6.6](#) and [NNPDF2.0](#) for continuous Higgs mass.

## Sources of Uncertainty - Variation

It is vital to consider theoretical/assumption-dependent uncertainties:

- Methods of determining “best fit” and uncertainties.
- Underlying assumptions in procedure, e.g. parameterisations and data used.
- Treatment of heavy flavours.
- PDF and  $\alpha_S$  correlations.

Considered to some extent above, can explain some of the observed differences.

More details after conclusion.

## Theoretical Uncertainties

Other sources not considered even by looking at variations between groups.

- Standard higher orders (NNLO)
- QED and Weak (comparable to NNLO ?) ( $\alpha_s^3 \sim \alpha$ ). Sometime enhancements.
- Resummations, e.g. small  $x$  ( $\alpha_s^n \ln^{n-1}(1/x)$ ), or large  $x$  ( $\alpha_s^n \ln^{2n-1}(1-x)$ )
- low  $Q^2$  (higher twist), saturation

Lead to differences in current partons, and to corrections in predicted cross-sections.

Would be much the same for each group though.

Most obviously important NNLO, already considered to some extent.

Some more info in back-up slides on others, mainly small-resummations.



## Conclusions

One can determine the parton distributions and predict cross-sections at the LHC, and the fit quality using NLO or NNLO QCD is fairly good.

NNLO is strongly desirable if possible. Fewer PDFs and uncertainties available.

Various ways of looking at uncertainties due to errors on data. Uncertainties due to PDFs naively rather small –  $\sim 2 - 5\%$  for most LHC quantities.

Effects from input assumptions e.g. selection of data, cuts, input parameterisation, treatment of heavy flavour, choice of  $\alpha_S$ , can shift central predictions significantly.

Some shifts have well-understood origins, particularly some of the most extreme. Some are more difficult to tie down.

$\alpha_S$  and PDFs correlated. Differences in predictions reduced if common value taken – MSTW/CTEQ difference halved but still  $2\sigma$ . Not clear what common value is best to take. We argue very strongly it is order dependent. Groups also have (different) prescriptions for including  $\alpha_S$  uncertainty. Linked to “best-fit” value for some.

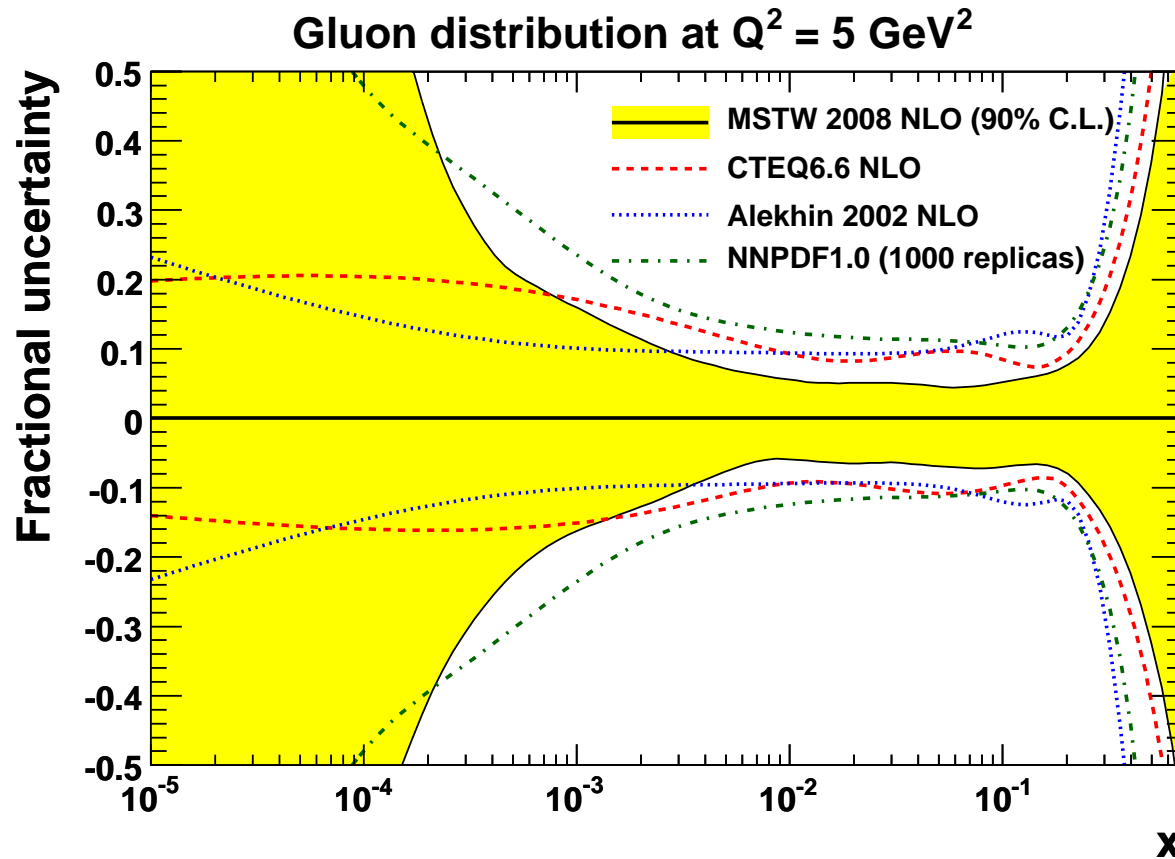
Studies suggest naive uncertainties should be about doubled to take account of the “not very well-understood” effects. For  $gg \rightarrow H$  similar to span of MSTW08, CTQ6.6 and NNPDF. Very conservative approach indeed – look at span of all sets.

Errors from higher orders/resummation and other theoretical sources potentially significant – back to NNLO again. Direct measurement of  $F_L(x, Q^2)$  at HERA now testing small- $x$  resummation, for example.

Generally same systematic type of effect for all PDFs.

At LHC early measurements, e.g.  $W, Z$  and jets would be useful in testing understanding of QCD (Standard Model).

**Gluon Parameterisation - small  $x$**  – different parameterisations lead to very different uncertainty for small  $x$  gluon.



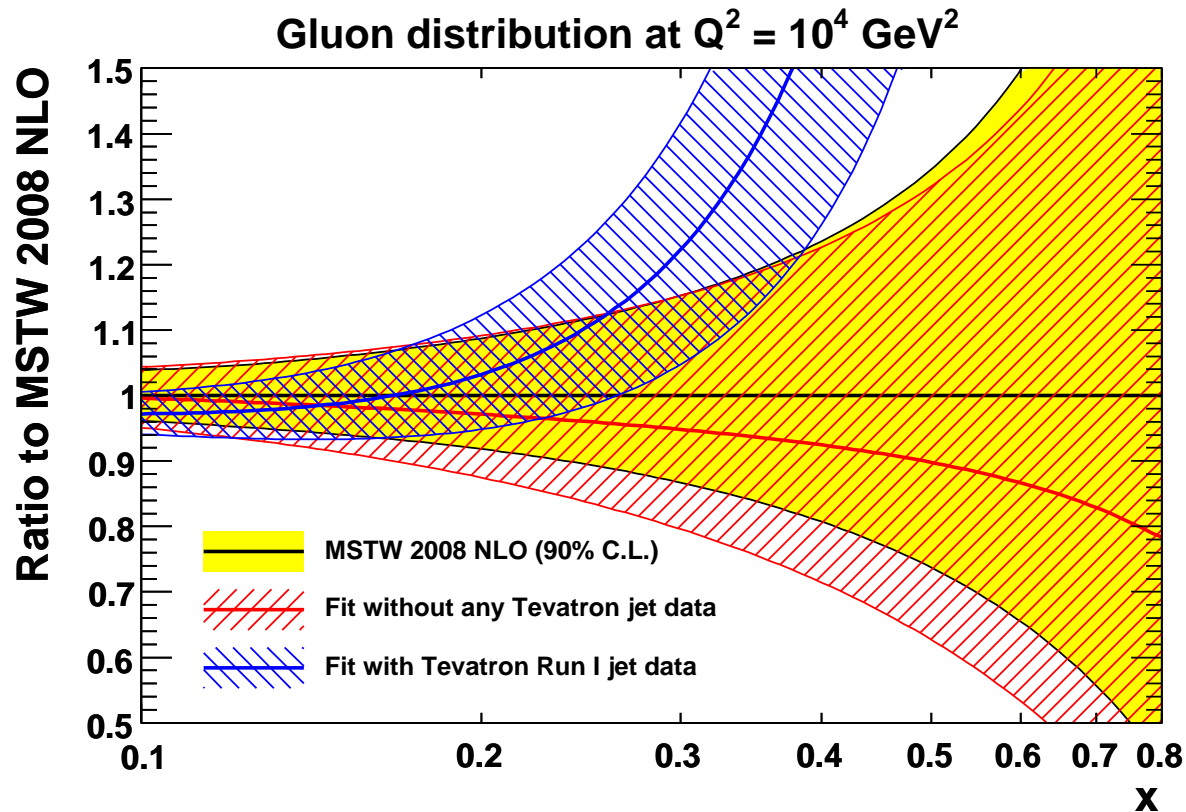
Most assume single power  $x^\lambda$  at input  $\rightarrow$  limited uncertainty. If input at low  $Q^2$   $\lambda$  positive and small- $x$  input gluon *fine-tuned* to  $\sim 0$ . Artificially small uncertainty.

If  $g(x) \propto x^{\lambda \pm \Delta\lambda}$  then  $\Delta g(x) = \Delta\lambda \ln(1/x) * g(x)$ .

**MRST/MSTW** and **NNPDF** more flexible (can be negative)  $\rightarrow$  rapid expansion of uncertainty where data runs out.

## Gluon Distribution - large $x$ .

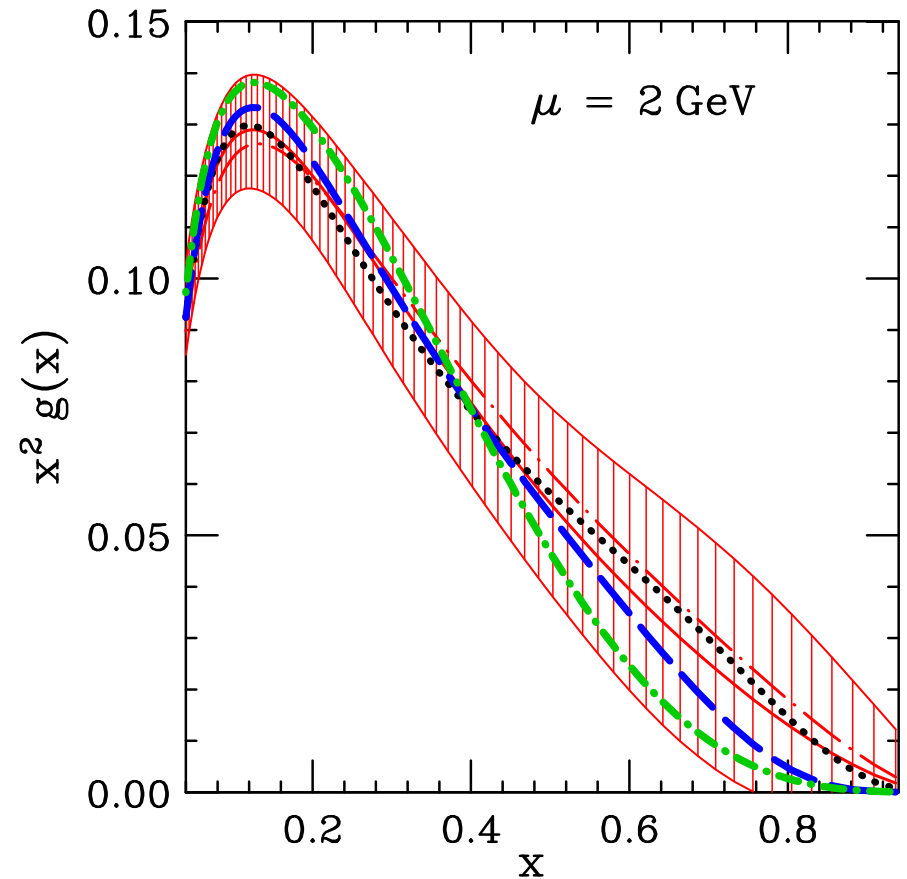
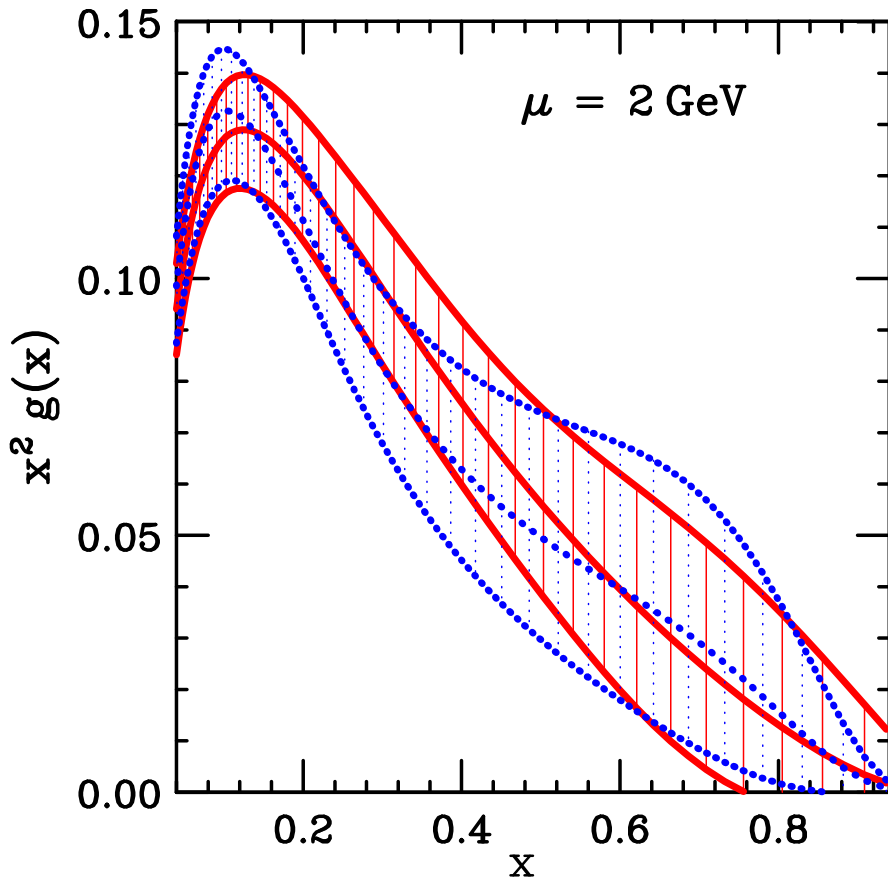
Constrained indirectly, but quite accurately, by DIS data, and directly by Tevatron high- $p_T$  jets, now Run I and Run II available. *Slightly* confusing picture.



Only fit by MSTW and CTEQ (now also NNPDF). Former found gluon much softer for Run II. Fits not very consistent between runs.

CTEQ find more compatibility between **Run I** and **Run II** fits. Fit with both sets  $\rightarrow$  little change – red CT09G, blue CTEQ6.6 (left).

Partially less strict with “consistency”, partially difference in parameterisation, partially effectively *higher weight* to jet data in global fit.

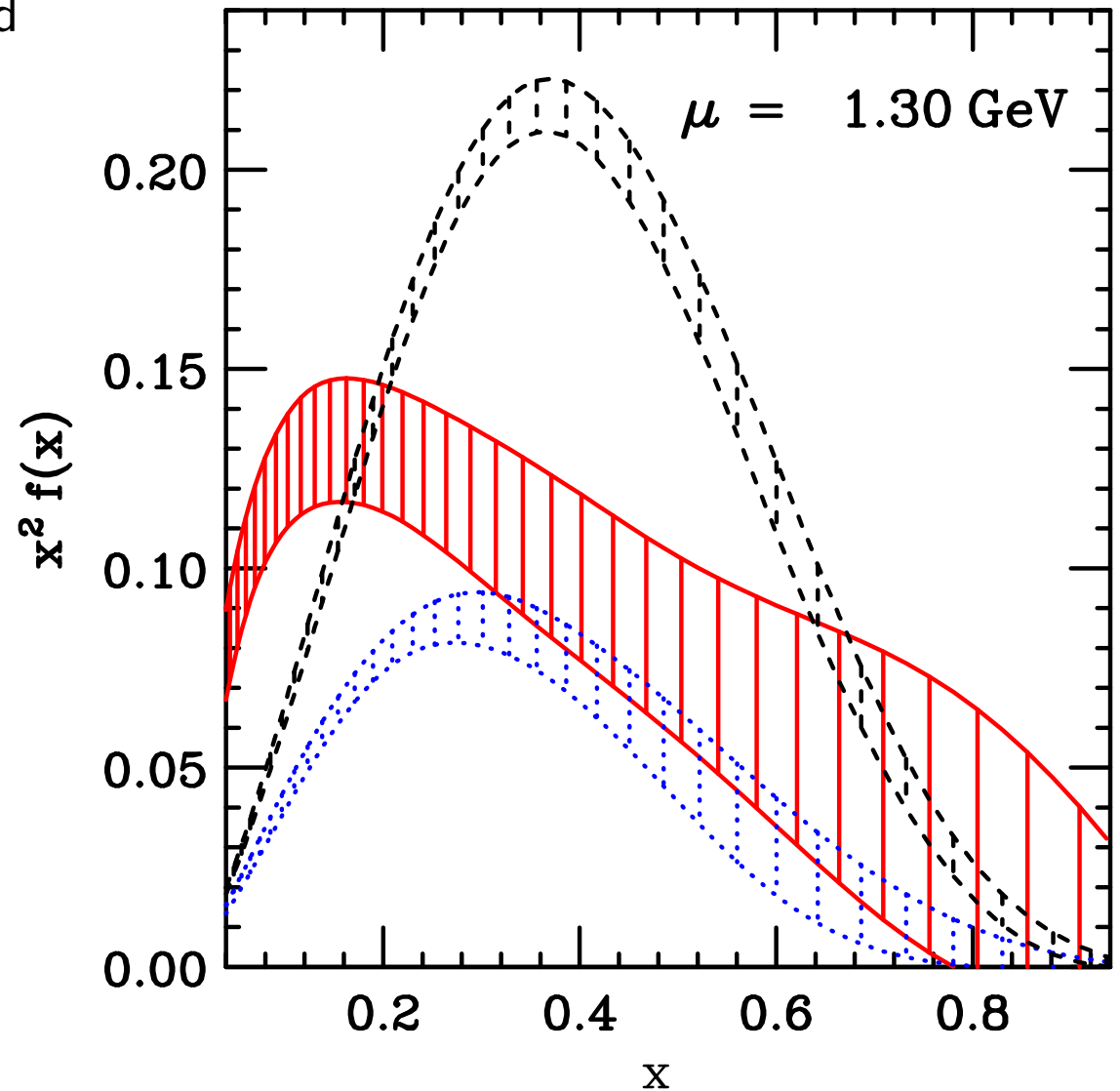


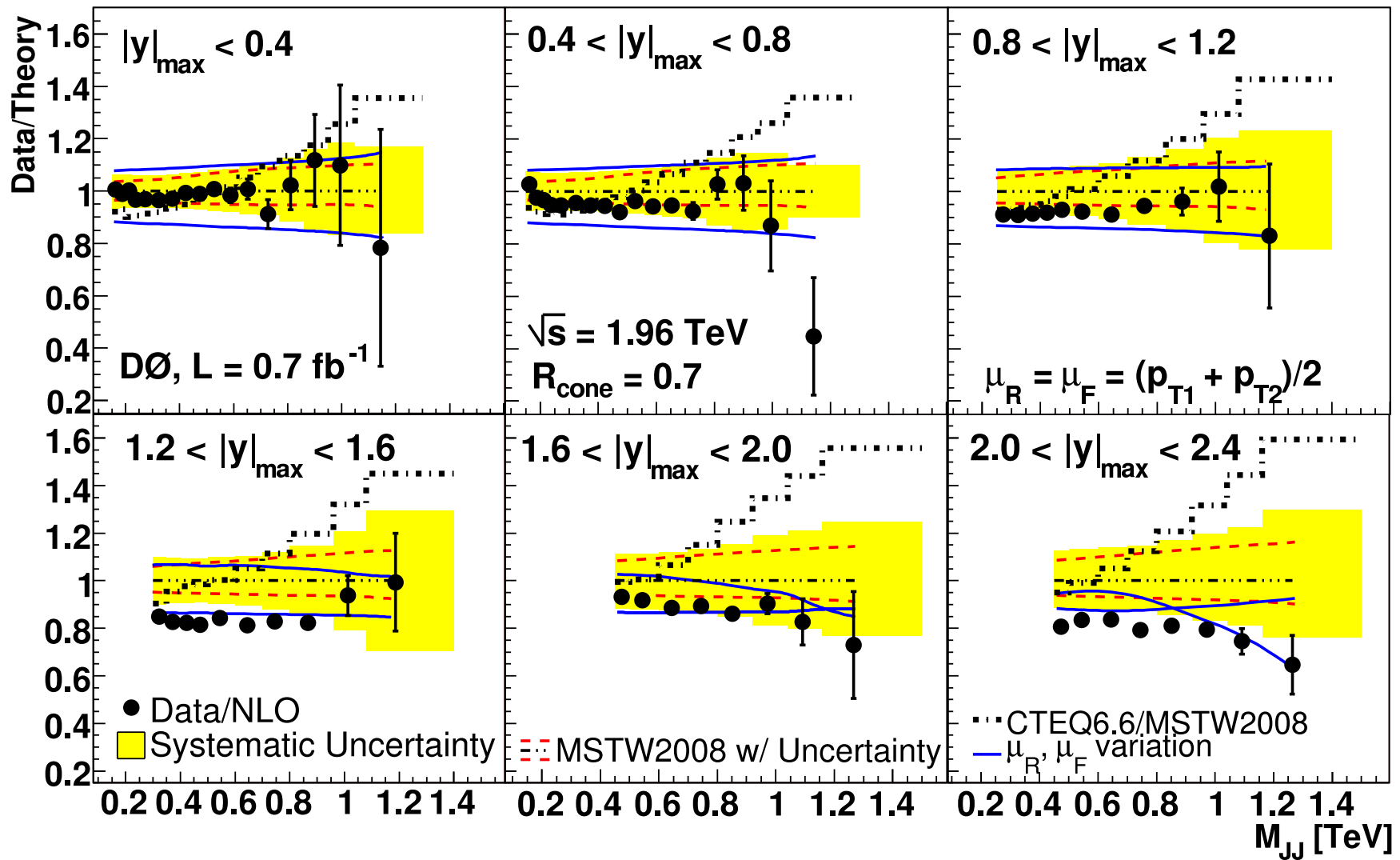
When fit to **Run II** data only and same procedure as MSTW blue (right) similar to MSTW green (right).

Generally high- $x$  PDFs parameterised so will behave like  $(1 - x)^\eta$  as  $x \rightarrow 1$ . More flexibility in CTEQ.

Very hard high- $x$  gluon distribution (more-so even than NNPDF uncertainties).

However, is gluon, which is radiated from quarks, harder than the up valence distribution for  $x \rightarrow 1$ ?





Very large high- $x$  gluon not supported by very recent  $D0$  dijet data.

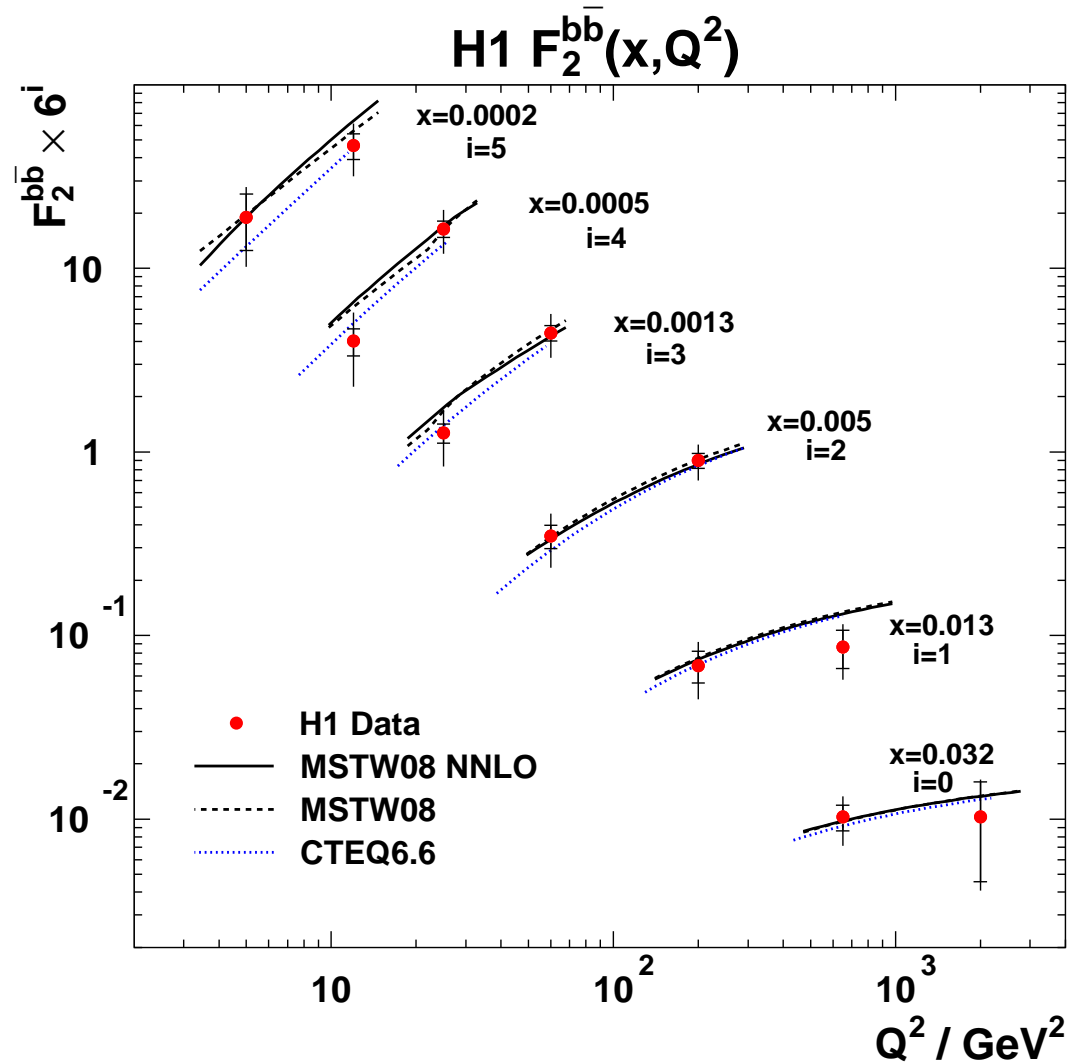
# Heavy Flavours – GM-VFNS variations.

Various definitions possible. Versions used by **MSTW** (RT) and **CTEQ** (ACOT) have converged somewhat.

Freedom in choices and consistency of kinematic limits (heavy quark pair produced in final state) introduced in **RT** scheme.

Simplest choice in heavy flavour coefficient function now commonly based on **ACOT**( $\chi$ ) prescription, i.e. scaling variable  $x$  replaced by  $\chi \equiv x(1 + 4m_H^2/Q^2)$ . (Two variations.)

Various significant differences still exist as illustrated by comparison to most recent **H1** data on bottom production.

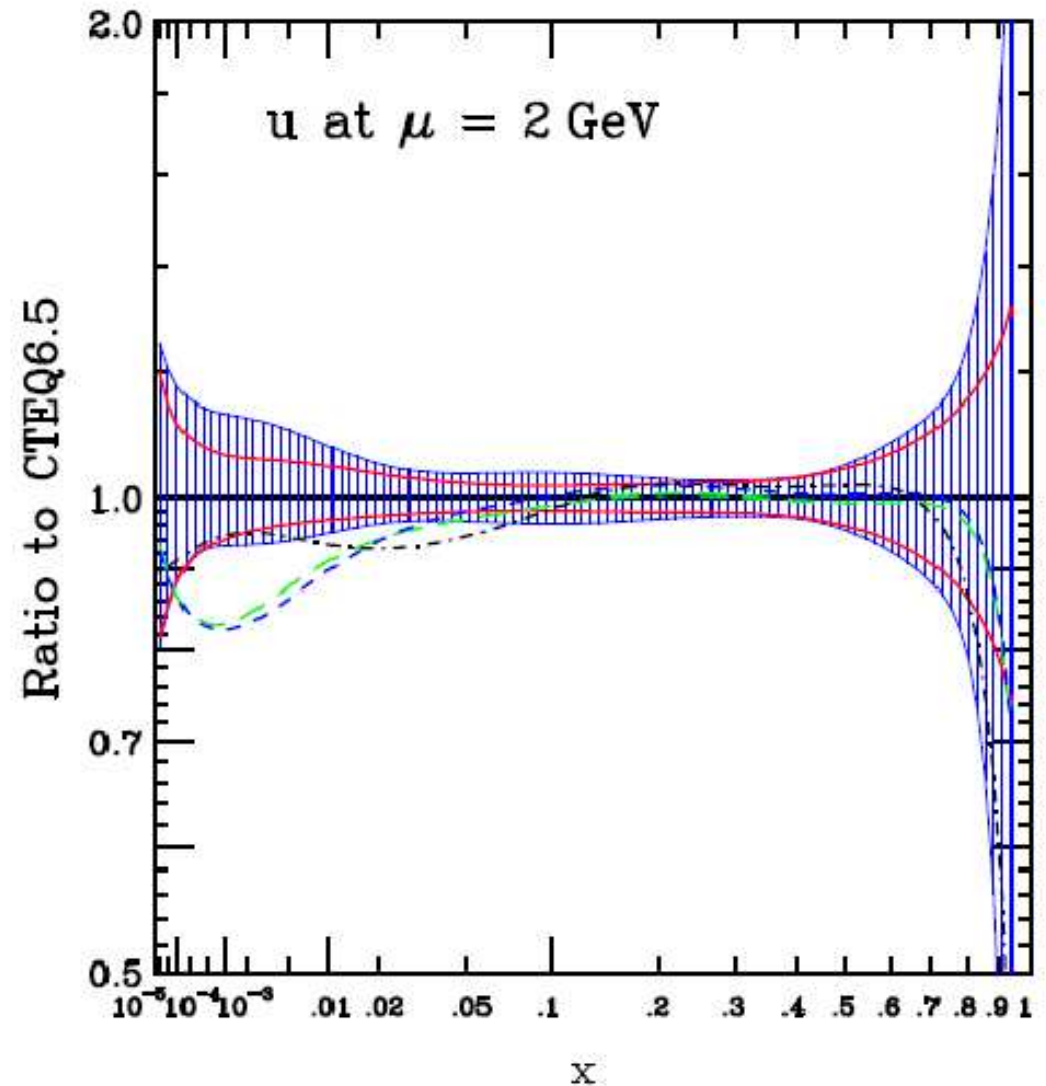




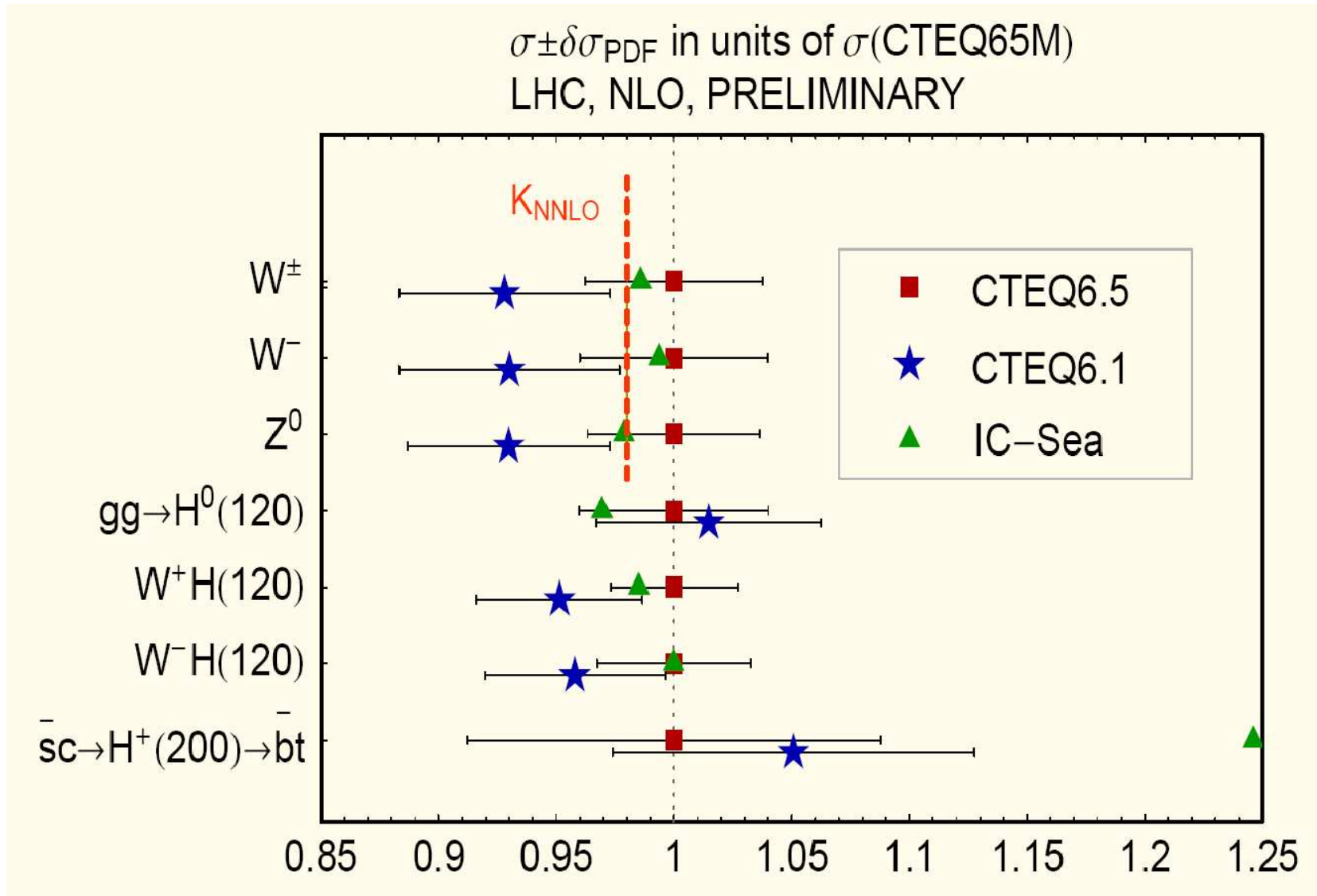
Importance of using GM-VFNS instead of massless approach illustrated by CTEQ6.5 up quark with uncertainties compared with previous versions, e.g. CTEQ6 in green.

Can be  $> 8\%$  error in PDFs. Much more than scheme uncertainty.

MRST in dash-dot line. Reasonable agreement. Already used heavy flavour treatment in default sets.



Leads to large change in predictions using CTEQ partons at LHC of 5 – 10%.



Note effects of *intrinsic charm* in final case.

The values of the predicted cross-sections at **NLO** for  $Z$  and a **120 GeV Higgs boson** at the **Tevatron** and the **LHC** (latter for **14 TeV**) as **GM-VFNS** altered.

PDF set	$B_{l+l-} \cdot \sigma_Z(\text{nb})$ TeV	$\sigma_H(\text{pb})$ TeV	$B_{l+l-} \cdot \sigma_Z(\text{nb})$ LHC	$\sigma_H(\text{pb})$ LHC
MSTW08	0.2426	0.7462	2.001	40.69
GMvar1	0.2433	0.7428	2.023	40.76
GMvar2	0.2444	0.7383	2.061	41.29
GMvar3	0.2429	0.7438	2.024	41.03
GMvar4	0.2425	0.7457	1.993	40.60
GMvar5	0.2423	0.7454	1.991	40.56
GMvar6	0.2434	0.7431	2.032	41.00
GMvarcc	0.2427	0.7451	2.001	40.65

At most **1%** variation at **Tevatron** in  $\sigma_Z$ .

Up to **+3%** and **-0.5%** variation in  $\sigma_Z$  at the **LHC**. About half as much in  $\sigma_H$  due to higher average  $x$  sampled.

Remember **8%** from **ZMVFNS** to **GMVFNS** in **CTEQ6** (**6%** for completed **NNLO GMVFNS** in **MRST06**).

The values of the predicted cross-sections at NNLO.  $\sigma_H$  calculated using Harlander, Kilgore code.

PDF set	$B_{l+l-} \cdot \sigma_Z(\text{nb})$ TeV	$\sigma_H(\text{pb})\text{TeV}$	$B_{l+l-} \cdot \sigma_Z(\text{nb})$ LHC	$\sigma_H(\text{pb})$ LHC
MSTW08	0.2507	0.9550	2.051	50.51
GMvar1	0.2509	0.9505	2.054	50.39
GMvar2	0.2514	0.9478	2.061	50.55
GMvar3	0.2516	0.9539	2.062	50.88
GMvar4	0.2507	0.9534	2.050	50.45
GMvar5	0.2509	0.9519	2.046	50.37
GMvar6	0.2509	0.9462	2.057	50.38
GMvarmod	0.2501	0.9511	2.022	50.03
GMvarmod'	0.2508	0.9482	2.052	50.57

Other than from model dependence maximum variations of order 0.5% at LHC. High- $x$  gluon leads to 1% on  $\sigma_H$  at Tevatron.

Model uncertainties can be > 1% from region at very small  $x$  and low  $Q^2$ . Can perhaps input more small- $x$  knowledge here. Effect far smaller when  $\mathcal{O}(\alpha_S^3)$  term falls with  $Q^2$ .

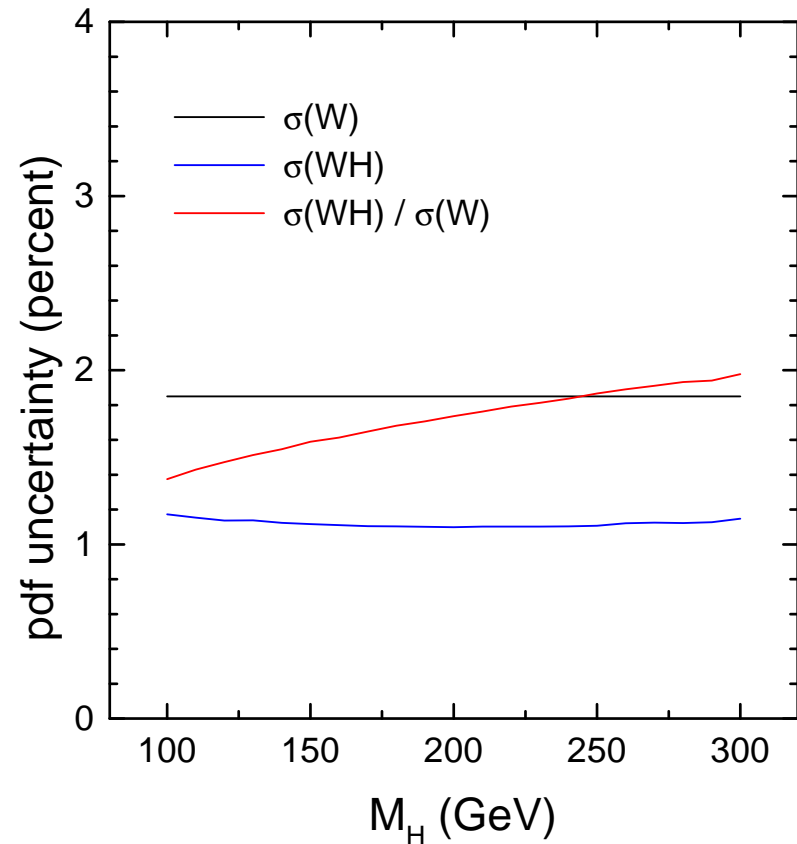
Could  $\sigma(W)$  or  $\sigma(Z)$  be used to calibrate other cross-sections, e.g.  $\sigma(WH)$ ,  $\sigma(Z')$ ?

$\sigma(WH)$  more precisely predicted because it samples quark pdfs at higher  $x$ , and scale, than  $\sigma(W)$ .

However, ratio shows no improvement in uncertainty, and can be worse.

Partons in different regions of  $x$  are often anti-correlated rather than correlated, partially due to sum rules.

pdf uncertainties on W, WH  
cross sections at LHC (MRST2001E)



No obvious advantage in using  $\sigma(tt)$  as a calibration SM cross-section, except maybe for very particular, and rather large,  $M_H$ .

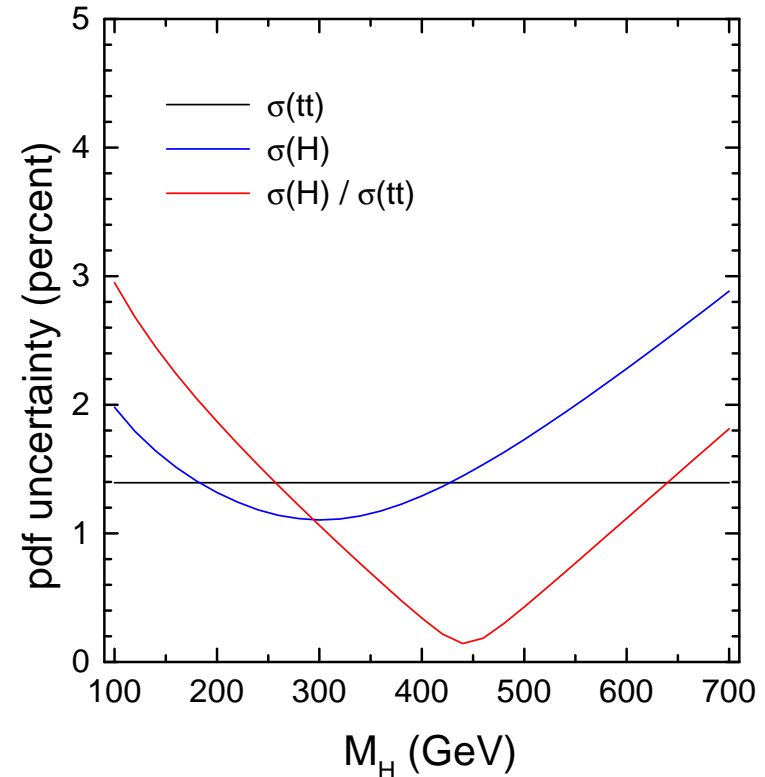
However, a light (SM or MSSM) Higgs dominantly produced via  $gg \rightarrow H$  and the cross-section has small pdf uncertainty because  $g(x)$  at small  $x$  is well constrained by HERA DIS data.

Current best (MRST) estimate, for  $M_H = 120$  GeV:  $\delta\sigma_H^{\text{NLO}}(\text{expt pdf}) = \pm 2 - 3\%$  with less sensitivity to small  $x$  than  $\sigma(W)$ .

Much smaller than the uncertainty from higher-order corrections, for example, Catani et al,

$$\delta\sigma_H^{\text{NNLL}}(\text{scale variation}) = \pm 8\%$$

pdf uncertainties on top, (gg $\rightarrow$ ) H cross sections at LHC (MRST2001E)



## Small- $x$ Theory

Reason for this instability – at each order in  $\alpha_S$  each splitting function and coefficient function obtains an extra power of  $\ln(1/x)$  (some accidental zeros in  $P_{gg}$ ), i.e.  $P_{ij}(x, \alpha_s(Q^2)), C_i^P(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x)$ .

BFKL equation for high-energy limit

$$f(k^2, x) = f_I(Q_0^2) + \int_x^1 \frac{dx'}{x'} \bar{\alpha}_S \int_0^\infty \frac{dq^2}{q^2} K(q^2, k^2) f(q^2, x),$$

where  $f(k^2, x)$  is the unintegrated gluon distribution  $g(x, Q^2) = \int_0^{Q^2} (dk^2/k^2) f(x, k^2)$ , and  $K(q^2, k^2)$  is a calculated kernel known to **NLO**.

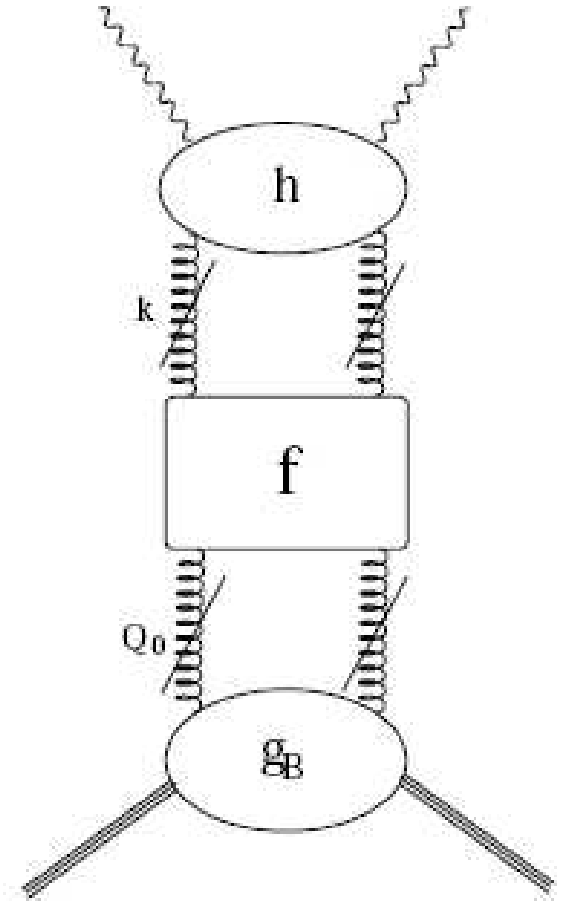
Physical structure functions obtained from

$$\sigma(Q^2, x) = \int (dk^2/k^2) h(k^2/Q^2) f(k^2, x)$$

where  $h(k^2/Q^2)$  is a calculable impact factor.

The global fits usually assume that this is unimportant in practice, and proceed regardless.

Fits work well at small  $x$ , but could improve.



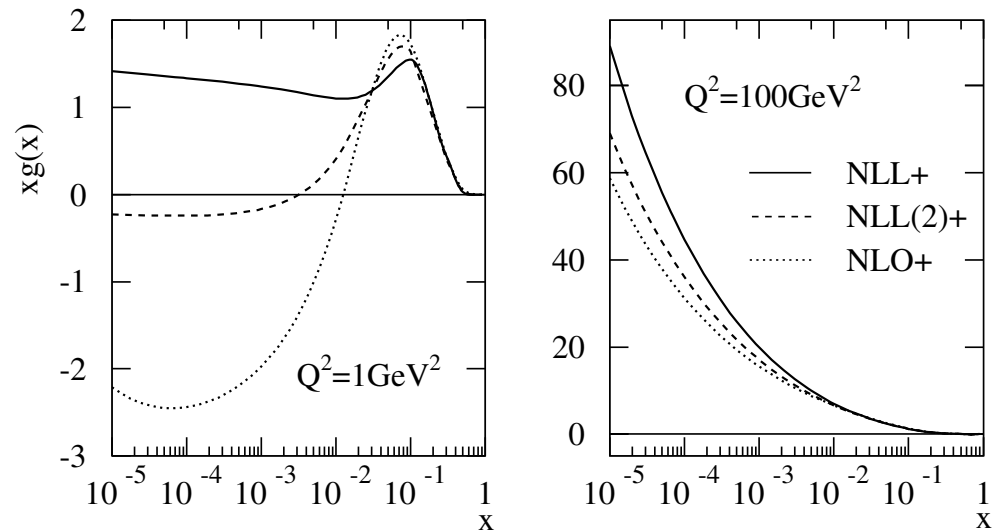
Good recent progress in incorporating  $\ln(1/x)$  resummation [Altarelli-Ball-Forte](#), [Ciafaloni-Colferai-Salam-Stasto](#) and [White-RT](#).

Include running coupling effects and variety (depending on group) of other corrections

By 2008 very similar results coming from the competing procedures, despite some differences in technique.

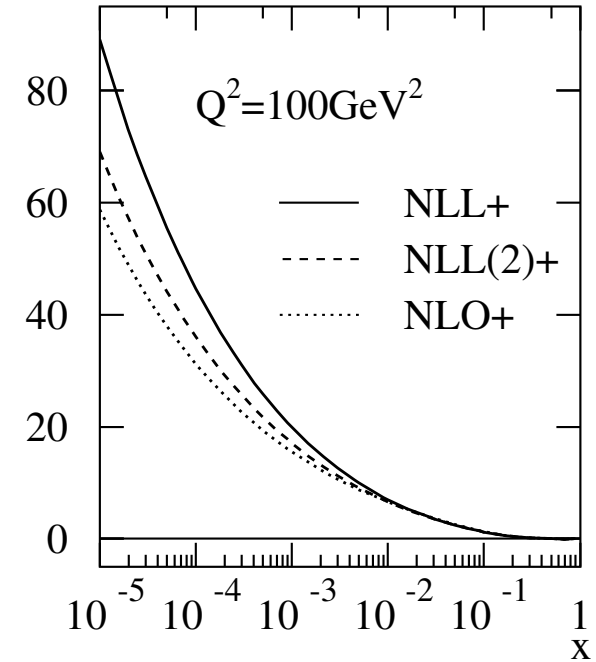
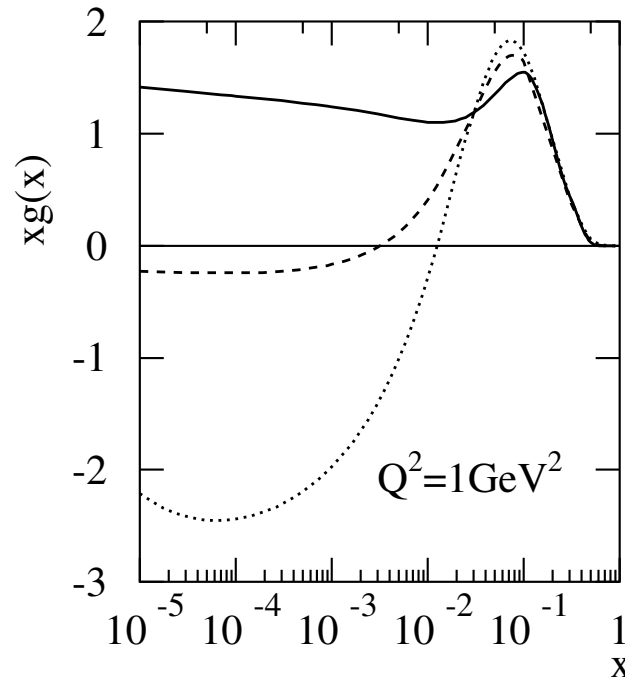
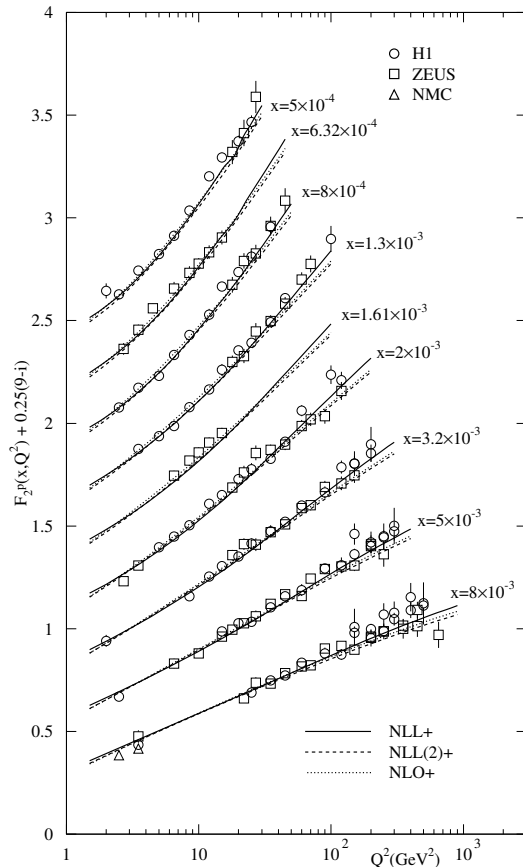
Full set of coefficient functions still to come in some cases, but splitting functions comparable.

Note, in all cases **NLO** corrections lead to dip in functions below fixed order values until slower growth (running coupling effect) at very small  $x$ .





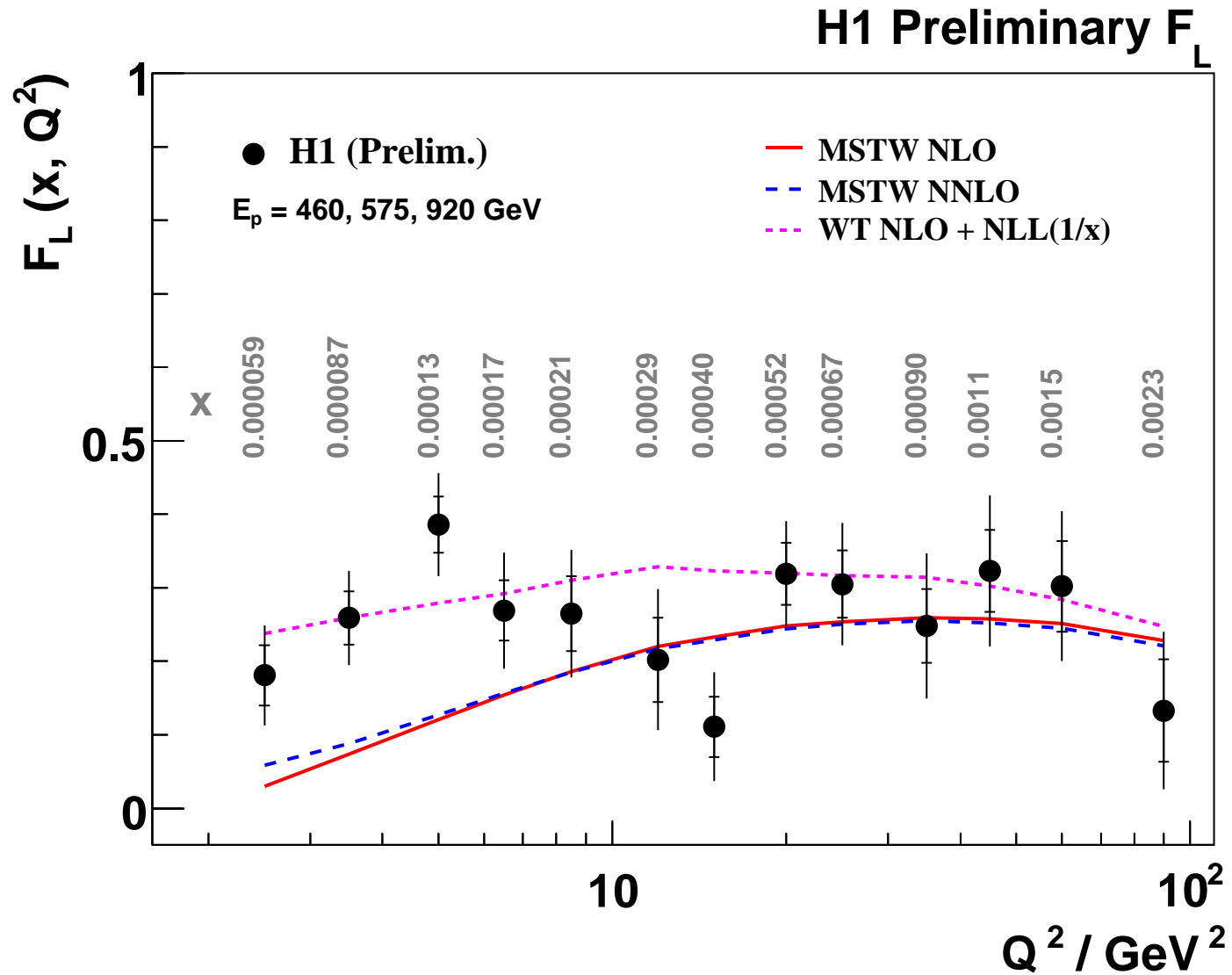
A fit to data with **NLO** plus **NLO** resummation, with heavy quarks included (**White,RT**) performed.



→ moderate improvement in fit to **HERA** data within global fit, and change in extracted gluon (more like quarks at low  $Q^2$ ).

Together with indications from **Drell Yan** resummation calculations (**Marzani, Ball**) few percent effect quite possible.

Comparison to H1 prelim data on  $F_L(x, Q^2)$  at low  $Q^2$ , only within White-RT approach, suggests resummations may be important.



Other possible (sometimes related) explanations.

## PDFs for LO Monte Carlo generators.

Often need to use generators which calculate only at LO in QCD.

LO matrix elements + LO PDFs often very inaccurate.

Using NLO PDFs suggested – sometimes better, sometimes even worse (particularly small  $x$ , important for underlying event *etc*).

Leads to introduction of new type of LO\* PDF.

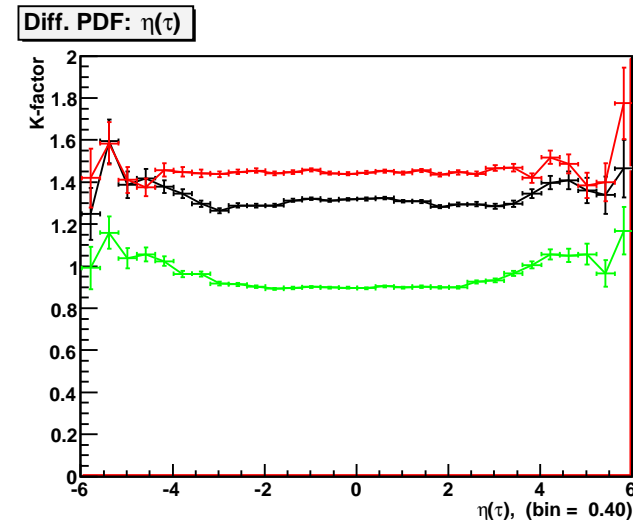
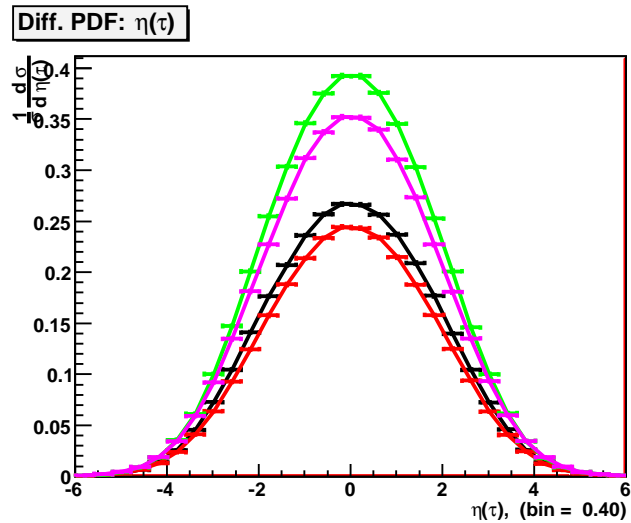
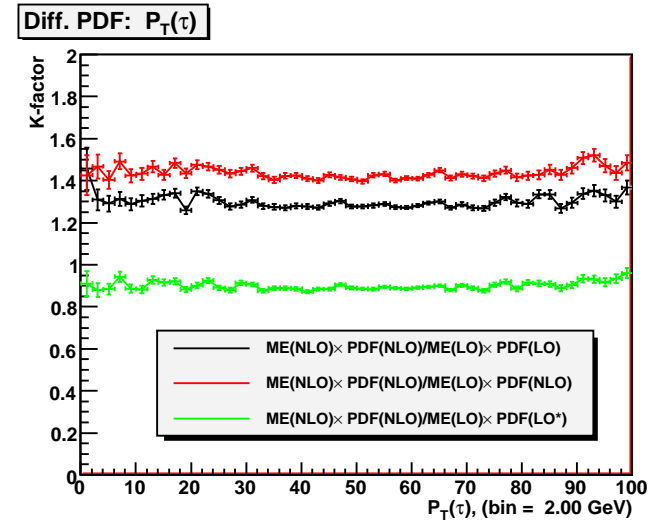
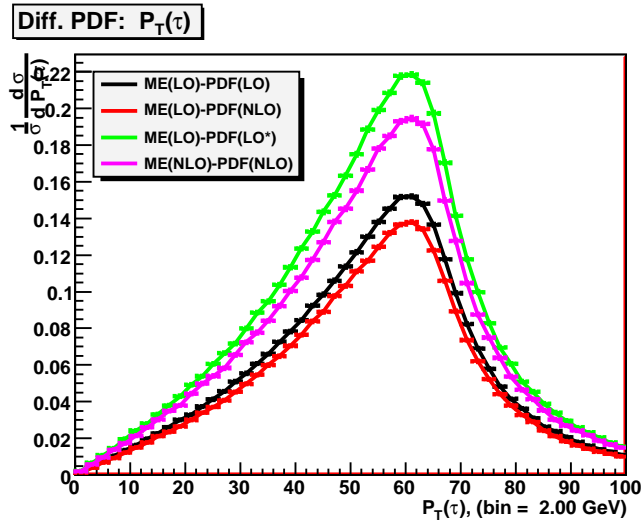
NLO corrections to cross-section usually positive  $\rightarrow$  LO PDFs bigger by allowing momentum violation in global fits, using NLO  $\alpha_S$ , fit LHC pseudo-data .....

Can also make evolution more “Monte Carlo like”, e.g. change of scale in coupling.

LO\* PDFs from MRST/MSTW followed by ones from CTEQ based on similar general principles.

Also work on fits using Monte Carlo generators directly (*Jung et al*).

Look at e.g. distributions for Higgs decaying to taus (Shertsnev, RT).



Results using LO\* partons clearly best in normalization. NLO worst and problems with shape at low scales (i.e. small  $x$ ).