Università degli Studi
di Milano

INFN Istituto Nazionale di Fisica Nucleare

Impact of *pdfs* and α_s uncertainties on Higgs production via gluon fusion

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Freiburg, April 12th 2010

based on:
F. Demartin, S. Forte, E. Mariani, J. Rojo, AV, arXiv:1004.0962

Motivations and content of the talk

Gluon-fusion Higgs production cross section depends on

- hard matrix element
- parton densities
- strong coupling

- Compare the recipes by CTEQ6.6, MSTW2008, NNPDF1.2 (preliminary with NNPDF2.0) to assess the uncertainties due to pdfs and to alphas (procedure and results)

- Study correlation between pdfs and alphas both for central values and for uncertainty bands

- Disentangle the contribution to the discrepancy (of central values and of uncertainty bands) given by the pdfs from the one given by alphas

- Disentangle the contribution to the uncertainty from the pdfs and from alphas

- Find a common agreement on the recipes and procedures to be adopted to obtain a reliable prediction of central values and total uncertainty bands

  2 proposal from the PDF4LHC working group will be shown

The hadronic cross section

$$\sigma(h_1 h_2 \to H + X) = \sum_{a,b} \int_0^1 dx_1 dx_2 \; f_{a,h_1}(x_1, \mu_F^2) f_{b,h_2}(x_2, \mu_F^2) \times \int_0^1 dz \; \delta\left(z - \frac{\tau_H}{x_1 x_2}\right) \hat{\sigma}_{ab}(z)$$

We concentrate on the gluon fusion production process

All the results in this talk at NLO-QCD (code GGSCA by Aglietti, Bonciani, Degrassi, AV)

we want to study the pdf+alphas interplay

➔ we need parton sets extracted with different alphas reference values

This process represents a "worst case" scenario to study the pdf+alphas interplay

In fact

$$\sigma_{tot} = \alpha_s^2 \sigma_0 + \alpha_s^3 \sigma_1 + \ldots \qquad \sigma_0 \sim \alpha_s \sigma_1$$

and the sensitivity to a variation of alphas is roughly approximated by

$$\boxed{\frac{\Delta \sigma_{\alpha_s}}{\sigma} \sim 2.5 \; \frac{\delta \alpha_s}{\alpha_s}}$$

Which central value and 1-σ error for $\alpha_s(m_Z)$?

World average (PDG) $\alpha_s(m_Z) = 0.1176(20)$

World average (Bethke) $\alpha_s(m_Z) = 0.1184(7)$
(new value on the PDG web update)

$\alpha_s(m_Z) =$

CTEQ6.6	0.118	} PDG values
NNPDF1.2	0.119	} PDG values
MSTW2008nlo	0.12018	MSTW determination

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As a compromise for the 1-σ error we take

$\delta^{(90)}_{\alpha_s}$ = 0.002 as a 90% C.L.

$\delta^{(68)}_{\alpha_s}$ = 0.002/1.64885 = 0.0012 as a 68% C.L.

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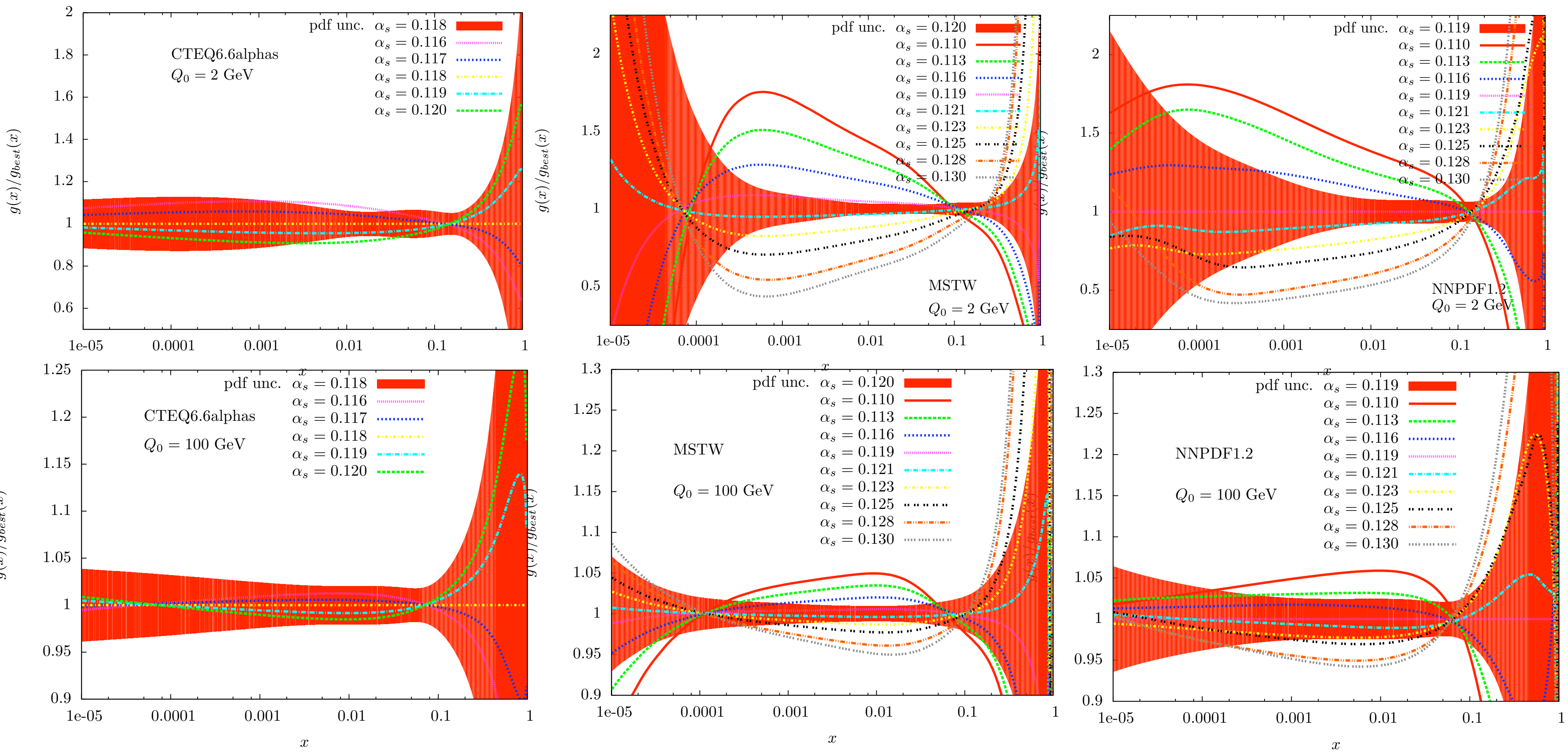
It is crucial to use the same variation of alphas when computing the uncertainty bands

Comparison of gluon densities extracted with different values of α_s



Red bands: gluon *pdf* uncertainty, normalized the corresponding central value

Colored lines: gluon central values, extracted with different α_s , normalized to the best central value

gluon density and α_s at small-*x* are anticorrelated

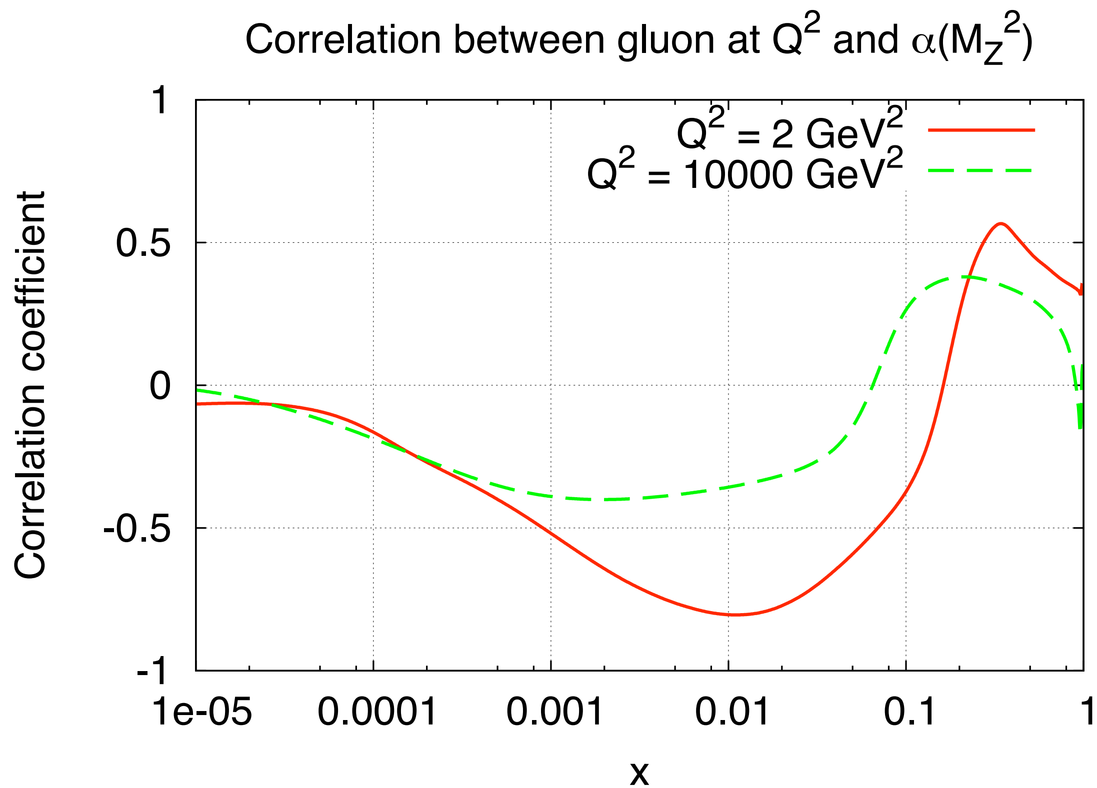
Correlation of gluon density and α_s

$$\rho\left[\alpha_s\left(M_Z^2\right), g\left(x,Q^2\right)\right] = \frac{\left\langle \alpha_s\left(M_Z^2\right) g\left(x,Q^2\right)\right\rangle_{\rm rep} - \left\langle \alpha_s\left(M_Z^2\right)\right\rangle_{\rm rep}\left\langle g\left(x,Q^2\right)\right\rangle_{\rm rep}}{\sigma_{\alpha_s\left(M_Z^2\right)}\sigma_{g(x,Q^2)}}$$

Obtained with NNPDF1.2

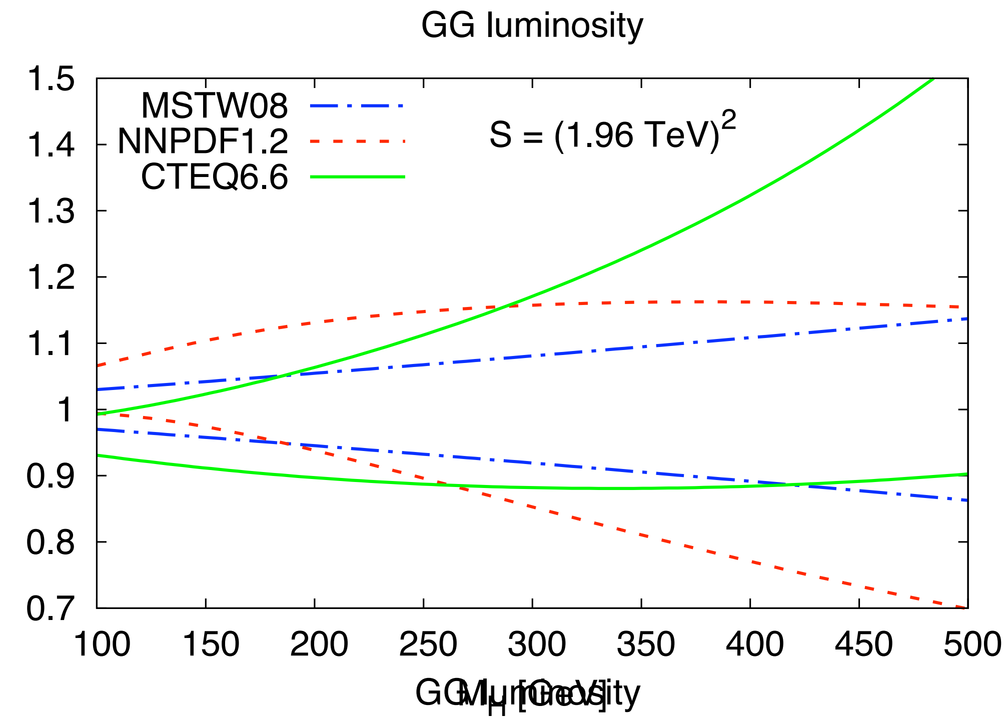
Gaussian distribution assumed for α_s

Both the various central gluons and their uncertainties enter in $\rho\left[\alpha_s\left(M_Z^2\right), g\left(x,Q^2\right)\right]$



The evolution de-correlates the gluon from the strong coupling

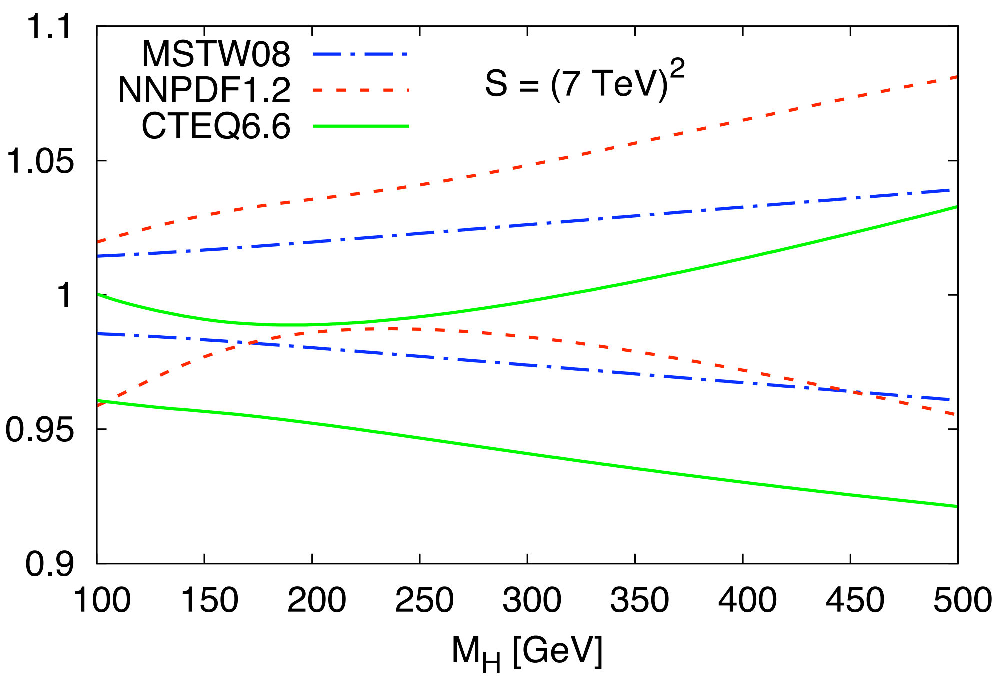
Comparison of gluon-gluon luminosity (normalized to MSTW2008)

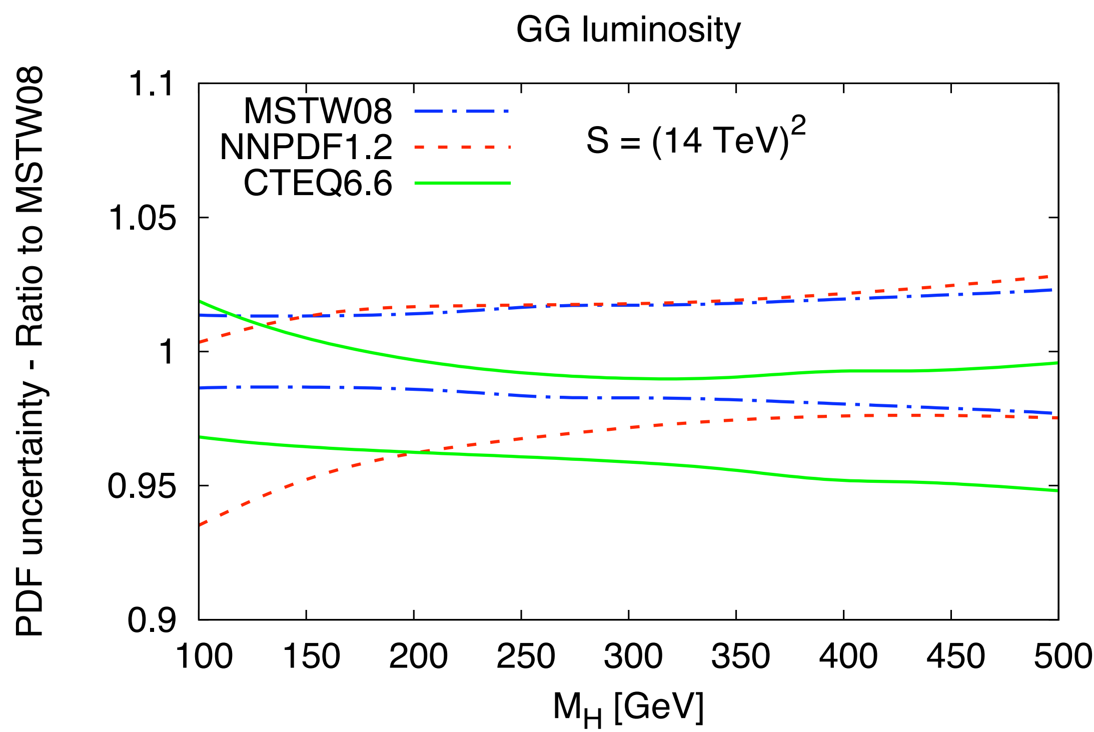


the gluon-gluon luminosity
with 68% C.L. *pdf* error bands

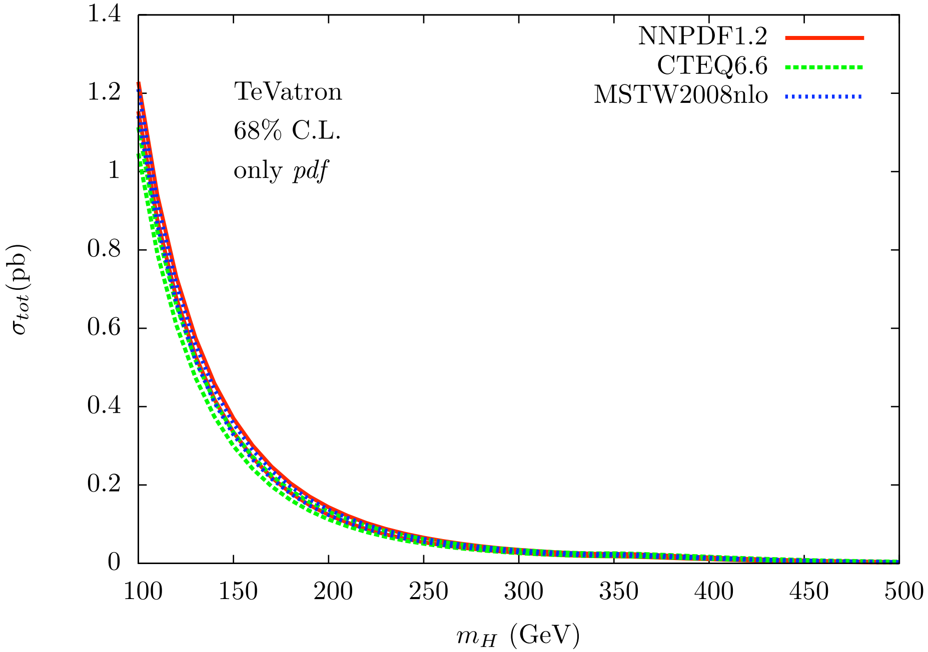
the cross section is proportional to this luminosity

the gg-lumi bands have been computed
with the preferred alphas(MZ)
and DO overlap



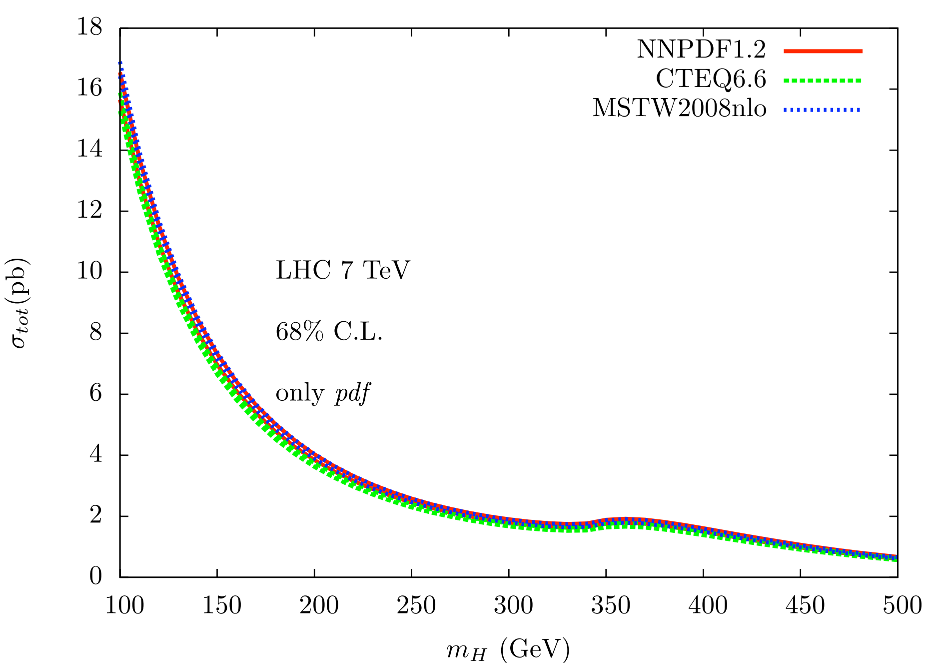


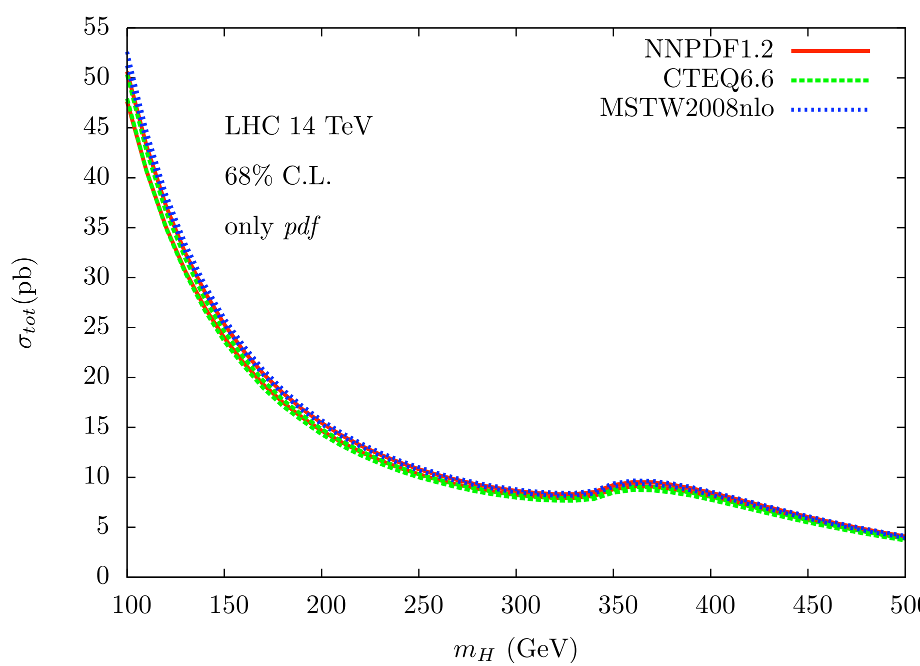
Uncertainties due only to the *pdfs* 68% C.L.



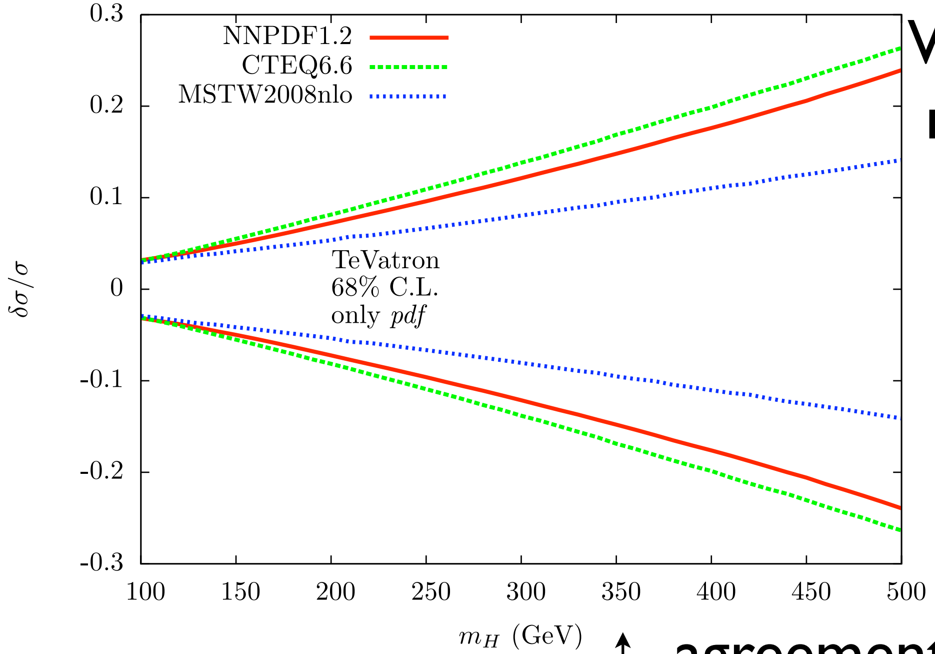
pdf sets used with LHAPDF interface

cteq66.LHgrid
MSTW2008nlo68cl.LHgrid
NNPDF12_100.LHgrid





Uncertainties due only to the *pdfs* 68% C.L.

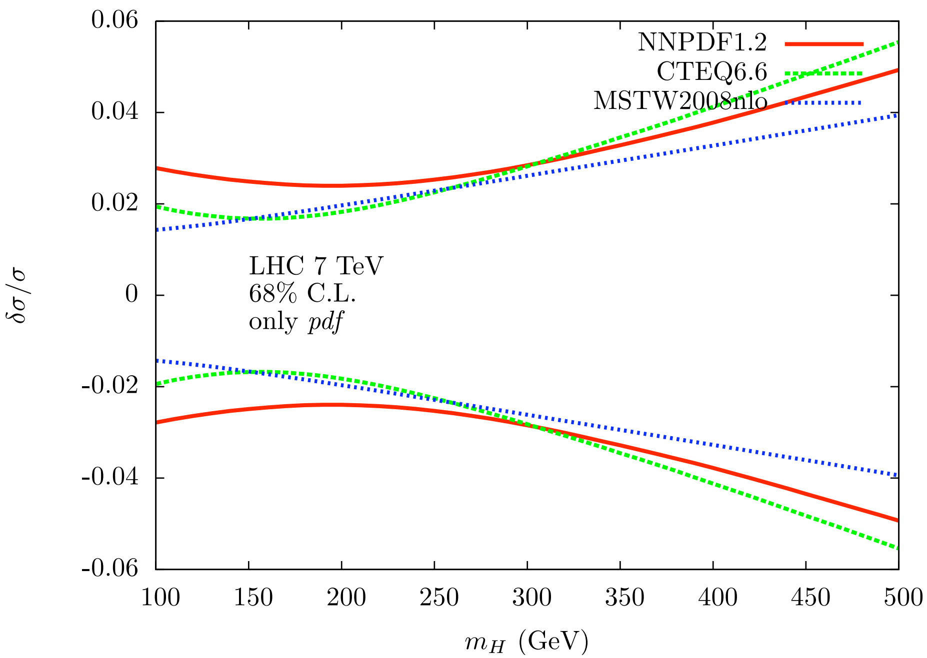


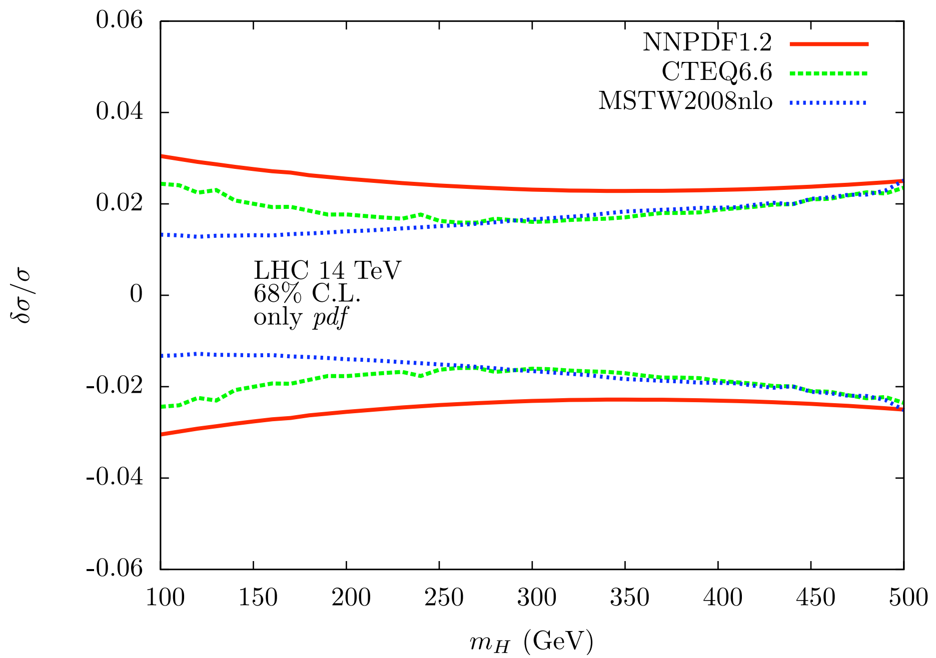
Width of the uncertainty bands
normalized to the respective central value

We used:
MSTW2008nlo68cl.LHgrid
NNPDF12_100.LHgrid
cteq6.6.LHgrid with rescaling of error
band from 90% to 68% C.L.

agreement of CTEQ and MSTW for medium-large mh
deviation at LHC for small mh

plot scales are different

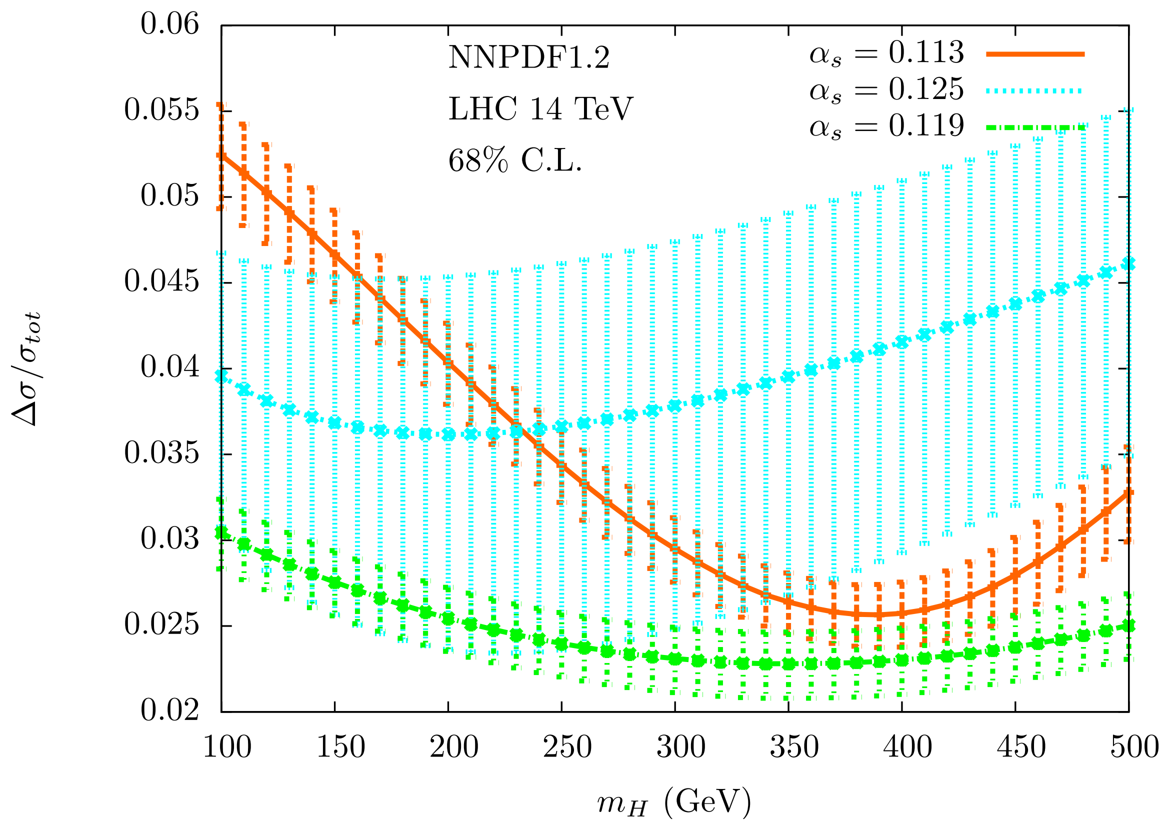




Uncertainty of the *pdf* uncertainty

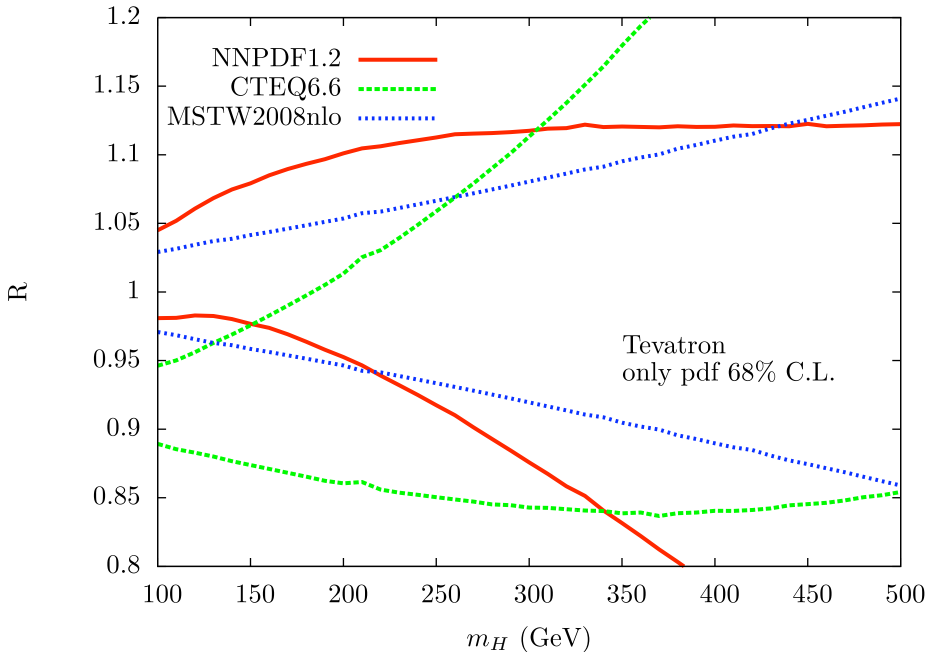
The width of the *pdf* uncertainty band is an observable whose variance can be computed

$$\sigma^2[\sigma^2] = \frac{1}{N_{\rm rep}}\left[m_4[q] - \frac{N_{\rm rep}-3}{N_{\rm rep}-1}\left(\bar{\sigma}^2\right)^2\right]$$



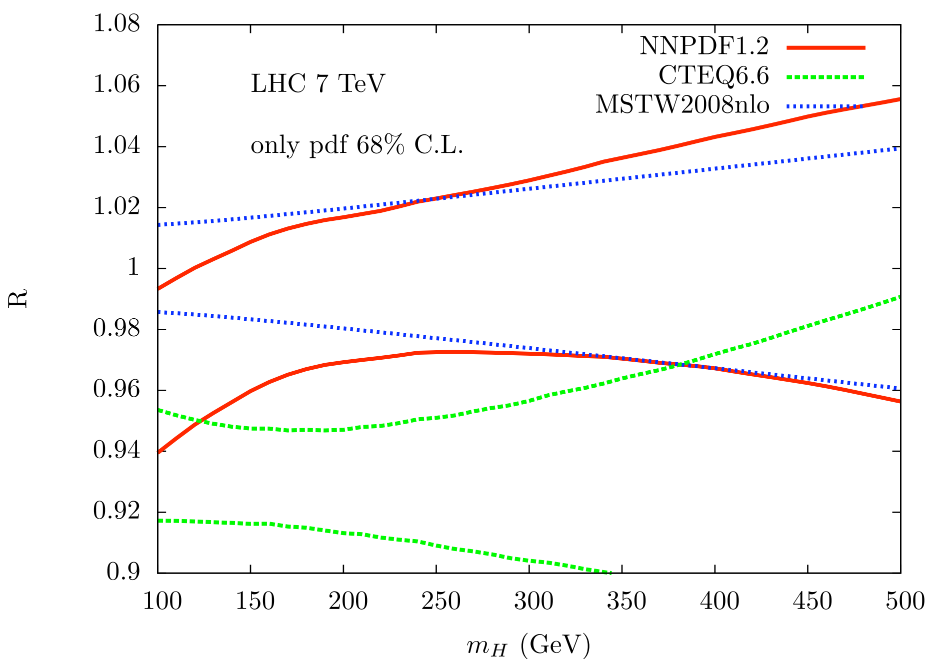
The compatibility of *pdf* uncertainty bands corresponding to different alphas can be checked comparing the overlap of $\sqrt{N_{\rm rep}}\ \sigma[\sigma^2]$ where σ is plotted in the figure

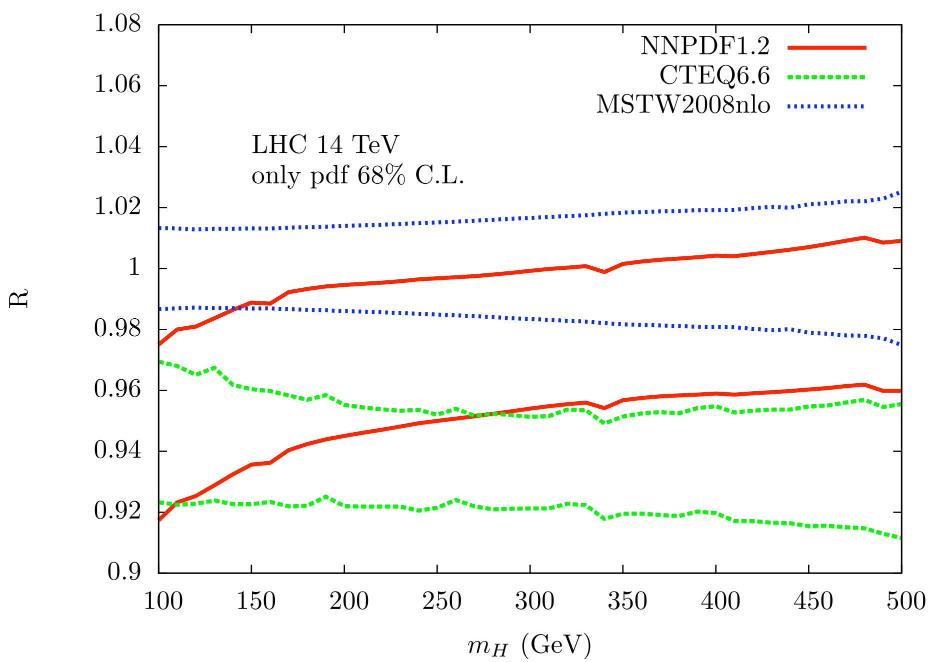
Comparison only-*pdf* bands (normalized to MSTW2008) 68% C.L.



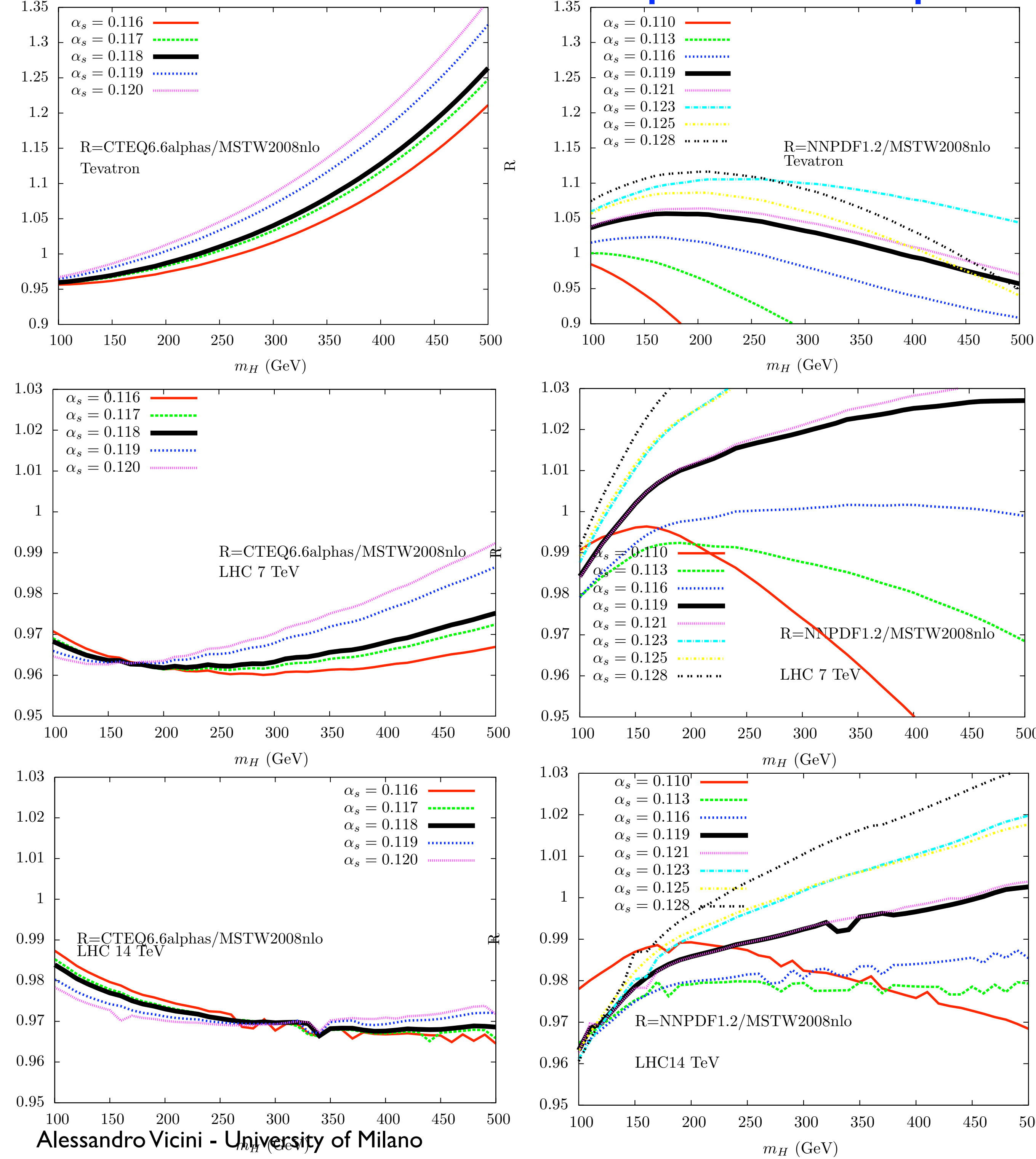
Central values by CTEQ6.6 and MSTW2008 differ between
6% (LHC 14 TeV) and 9% (Tevatron)
Uncertainty bands do not overlap

$\alpha_s(m_Z) =$	0.118	CTEQ6.6
	0.119	NNPDF1.2
	0.12018	MSTW2008nlo





Cross sections with same alphas in the pdfs and in the partonic xsec



Cross sections computed with
a given alphas in the partonic xsec
and with
pdf sets extracted using the very
same alphas

In this comparison
discrepancies are only due to
intrinsic differences in the pdf sets:
different data sets,
parametrizations,...

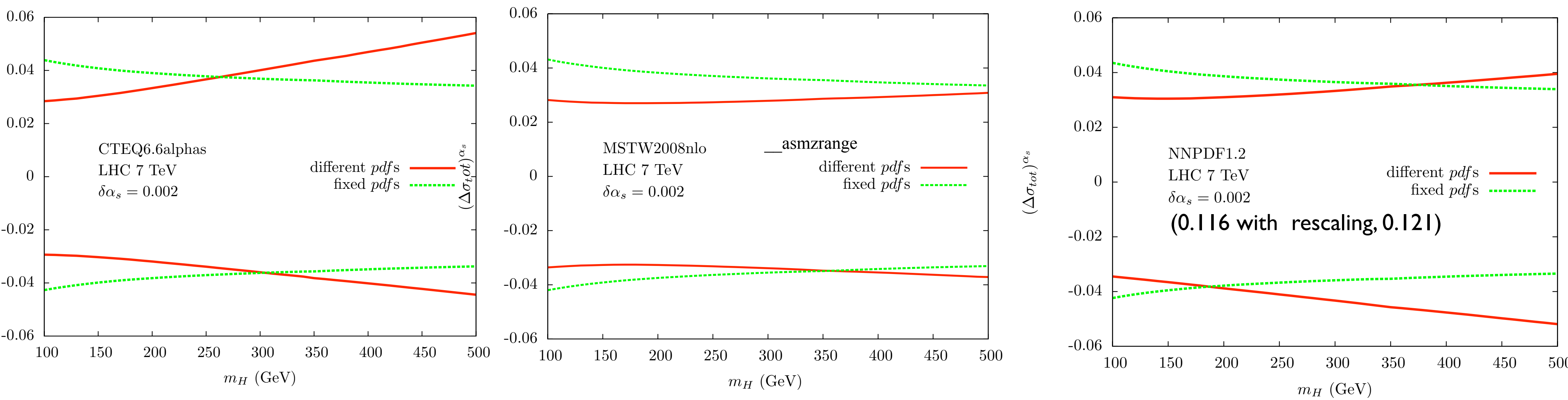
Discrepancies

- do not exceed the 4% level
- are of the same size or smaller than the *pdf* uncertainty bands

it is precisely the order of magnitude
we expect for these discrepancies

Evaluation of the uncertainty associated to α_s

$$\Delta\sigma^{\pm}_{\alpha_s} = \frac{\sigma(\alpha_s^0 \pm \delta\alpha_s)}{\sigma(\alpha_s^0)} - 1 \qquad \delta\alpha_s^{(90)} = 0.002$$



(red line) variation of α_s in the partonic xsec AND in the *pdfs*

- The spread is measured with respect to the central value of the best set

(green line) *pdfs* fixed to their central value, α_s changed only in the partonic xsec

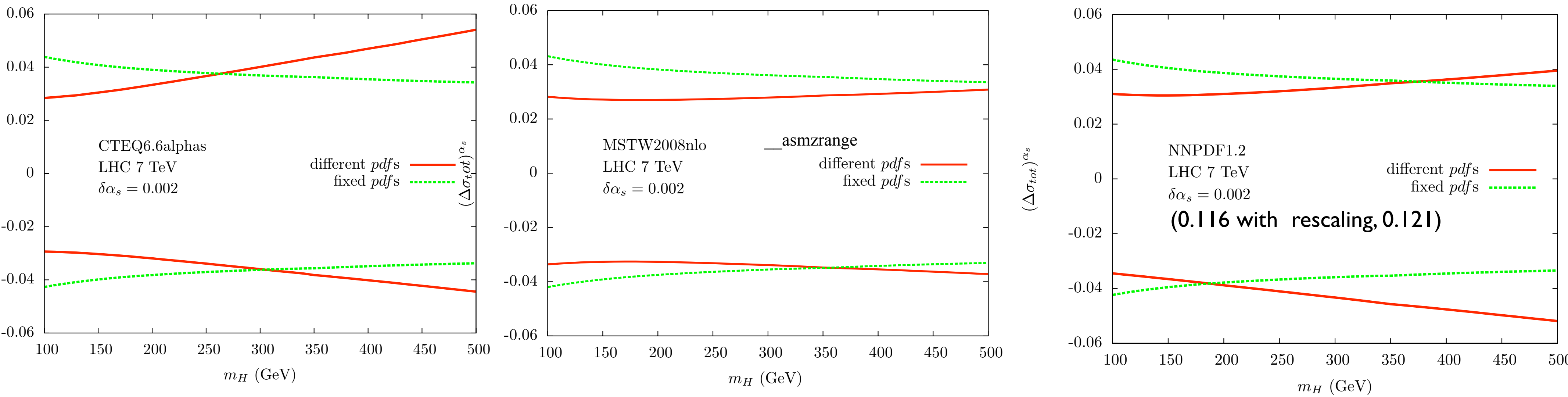
- A change of α_s only in the partonic xsec, keeping the *pdfs* fixed to their central value, (green lines) overestimates the effect by at most 35%

$$\frac{\Delta\sigma_{\alpha_s}}{\sigma} \sim 2.5\, \frac{\delta\alpha_s}{\alpha_s}$$

Evaluation of the uncertainty associated to α_s

$$\Delta\sigma^{\pm}_{\alpha_s} = \frac{\sigma(\alpha_s^0 \pm \delta\alpha_s)}{\sigma(\alpha_s^0)} - 1$$

$$\delta\alpha_s^{(90)} = 0.002$$



variation of α_s in the partonic xsec AND in the *pdfs*

- The spread is measured with respect to the central value of the best set

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$$\frac{\Delta\sigma_{\alpha_s}}{\sigma} \sim 2.5\,\frac{\delta\alpha_s}{\alpha_s}$$

Combination of the *pdfs* and of the α_s uncertainties

There are 3 possible recipes:

Sum the two uncertainties in quadrature
computing the α_s uncertainty with a

1) variation of α_s in the partonic xsec, AND in the *pdfs*

2) variation of α_s in the partonic xsec, keeping fixed the *pdfs*

3) Combine the two uncertainties
taking into account their full correlation (with some recipe)

Combination of *pdfs* and α_s uncertainties : full correlation

- MSTW2008 recipe described in **Eur.Phys.J.C64:653-680,2009. arXiv:0905.3531**
  a variation of α_s by $\delta_{\alpha_s}^{MSTW} = 0.12018^{+0.0012}_{-0.0015}\ (68\%)\ ^{+0.0032}_{-0.0039}\ (90\%)$
  is used in the different *pdf* sets

- NNPDF1.2 recommends to treat α_s as a gaussian variable
  and to combine accordingly the replicas extracted with different α_s
  in a Montecarlo way

- CTEQ6.6 remarks the weak correlation between *pdfs* and α_s
  and recommends to sum them in quadrature

  The α_s uncertainty is obtained keeping pdf fixed and change α_s in the partonic cross section

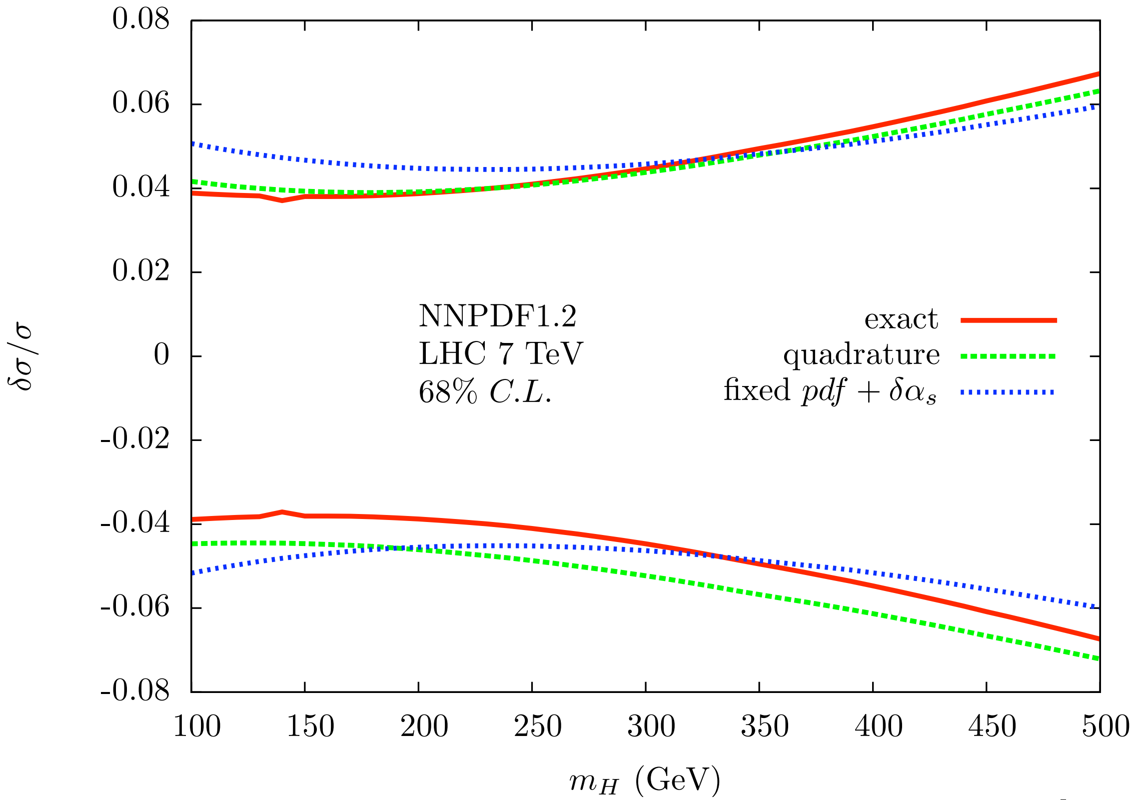
Combination α_s + *pdfs*: NNPDF1.2

1) full correlation

sum the two uncertainties in quadrature with

2) variation of α_s in the partonic xsec AND in the *pdfs*

3) variation of α_s in the partonic xsec, keeping fixed the *pdfs*



68% C.L.

$$\delta_{\alpha_s}^{(68)} = 0.0012$$

using a rescaling in
to obtain a 68% spread

NNPDF12_113_100.LHgrid
NNPDF12_116_100.LHgrid
NNPDF12_119_100.LHgrid
NNPDF12_121_100.LHgrid
NNPDF12_123_100.LHgrid
NNPDF12_125_100.LHgrid
NNPDF12_128_100.LHgrid

The sum in quadrature is a quite good approximation of the full correlation recipe

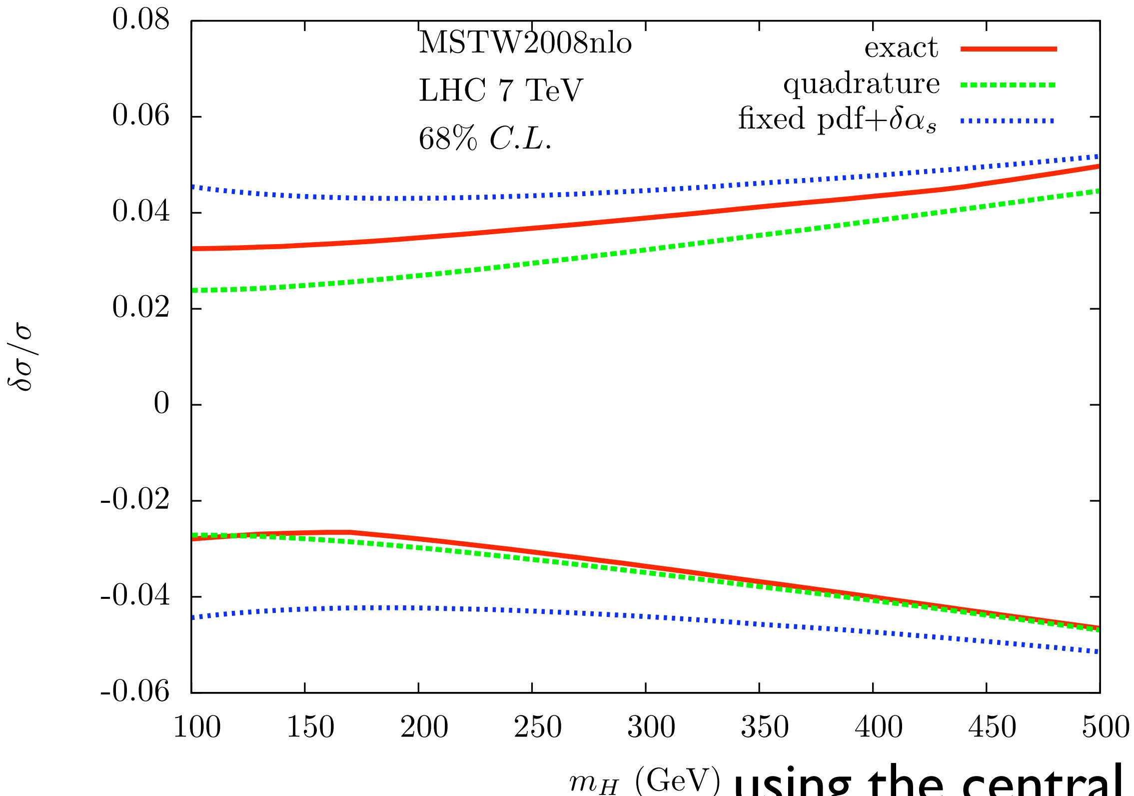
Combination α_s + *pdfs*: MSTW2008nlo

1) full correlation

sum the two uncertainties in quadrature with

2) variation of α_s in the partonic xsec AND in the *pdfs*

3) variation of α_s in the partonic xsec, keeping fixed the *pdfs*



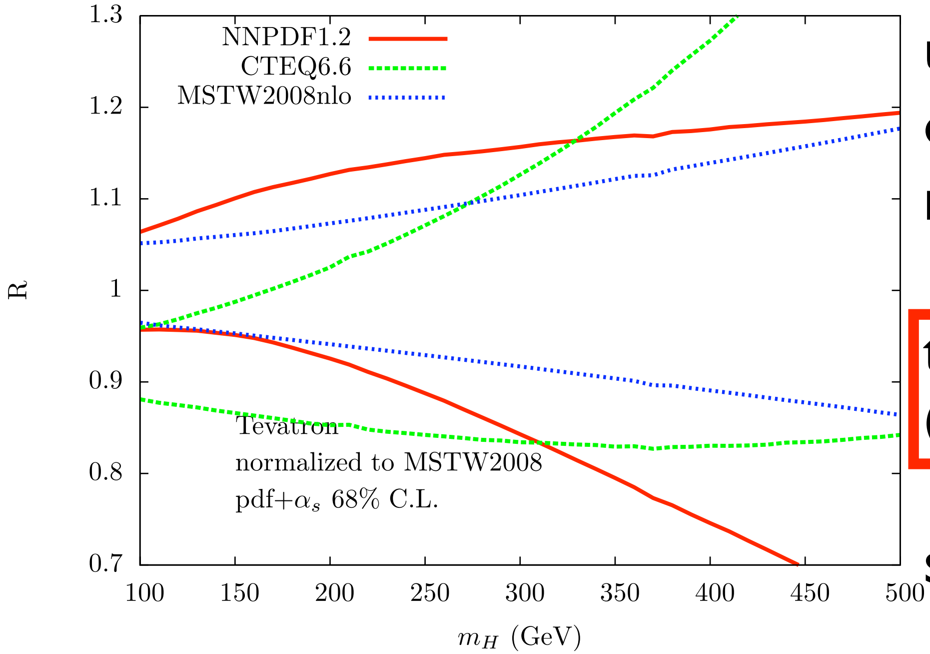
68% C.L.

$$\delta^{(68)}_{\alpha_s} = {}^{+0.0012}_{-0.0015}$$

using the central values of MSTW2008nlo68cl_asmz+68cl.LHgrid MSTW2008nlo68cl_asmz-68cl.LHgrid in the green curve

The sum in quadrature is a quite good approximation of the full correlation recipe

Combined *pdf*+alphas uncertainties at 68% C.L.

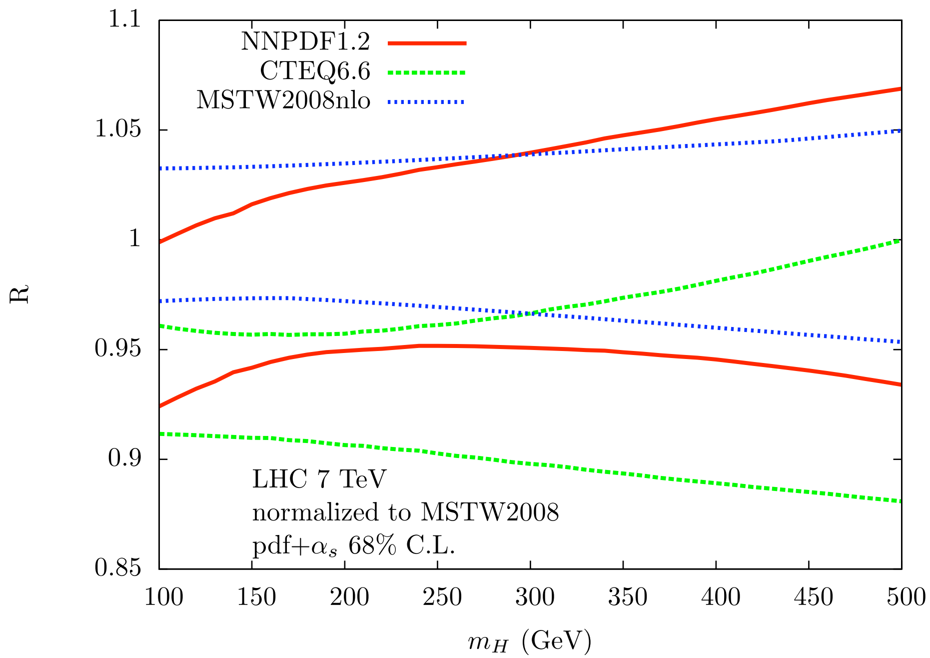


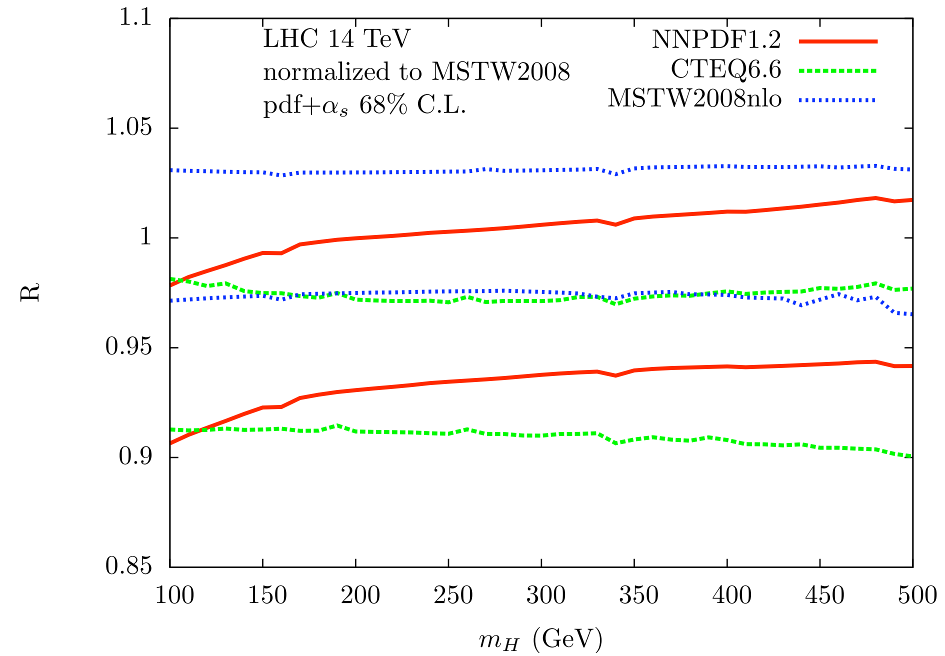
uncertainty bands
obtained with the best recipes of the 3 groups
normalized to the MSTW2008 central value

the pdf+alphas 1-σ band now almost overlap
(the cross section central values did not change)

still with $\alpha_s(m_Z) =$

0.118	CTEQ6.6
0.119	NNPDF1.2
0.12018	MSTW2008nlo





Estimates of the total uncertainty

- the pdf and alphas uncertainties are weakly correlated

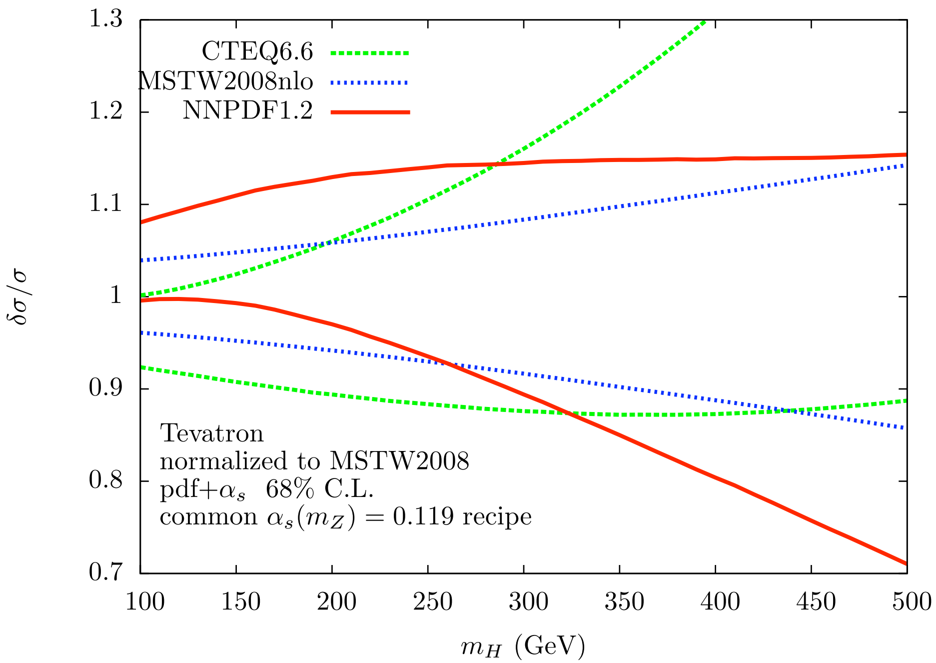
- the gluon luminosities of the three collaborations overlap
  with their own preferred value of alphas(MZ) and *a fortiori* with common alphas=0.119

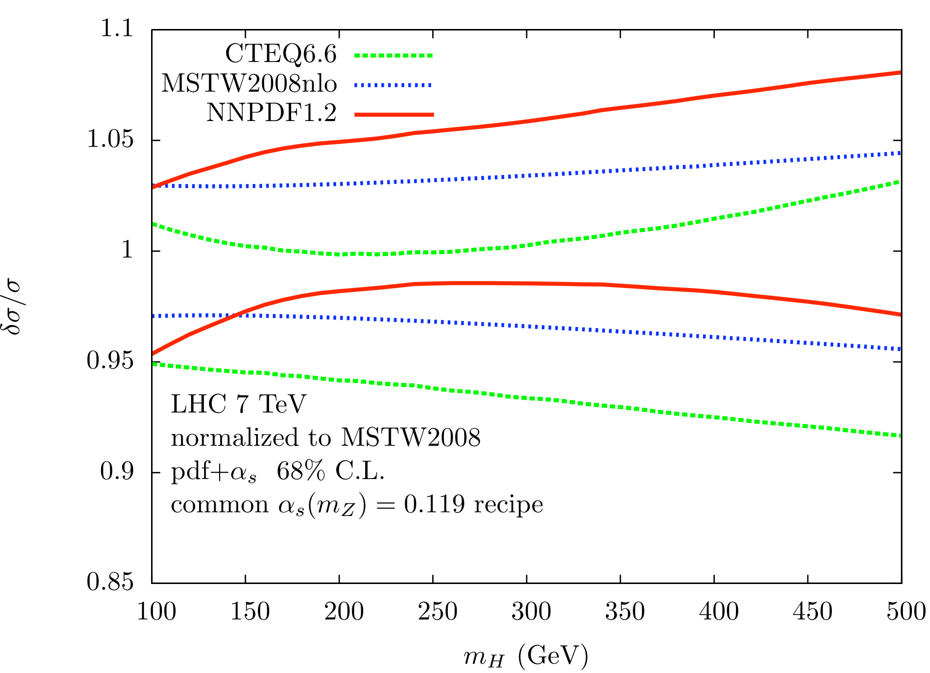
2 possible ways to estimate the total uncertainty

➔ make a common choice of alphas=0.119 for the three groups
and use the combination of (pdf+alphas) uncertainties

➔ take the envelope of the only-pdf uncertainty bands
with the cross sections evaluated with different alphas

Common alphas=0.119 recipe



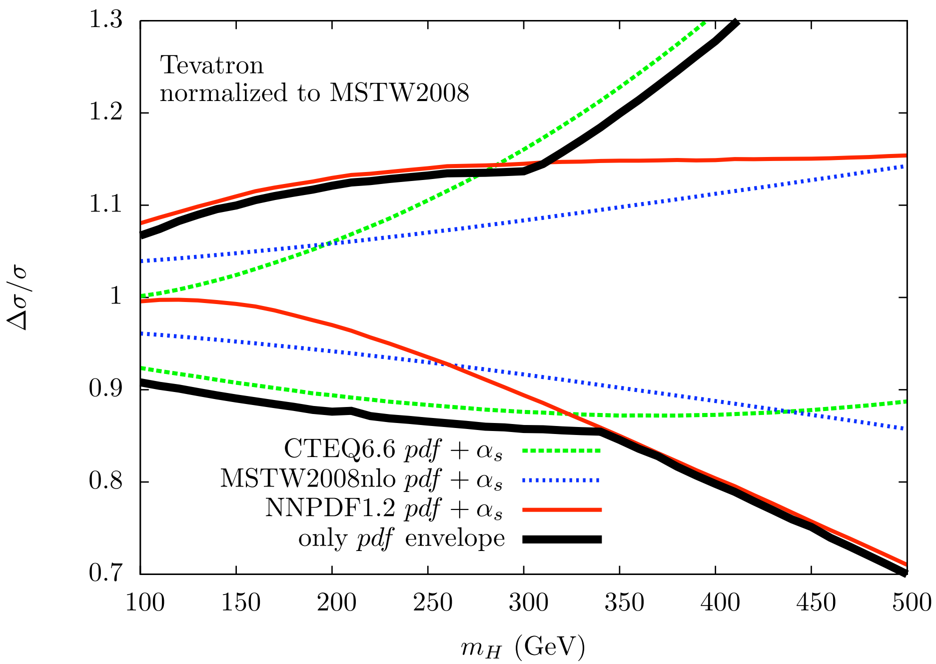


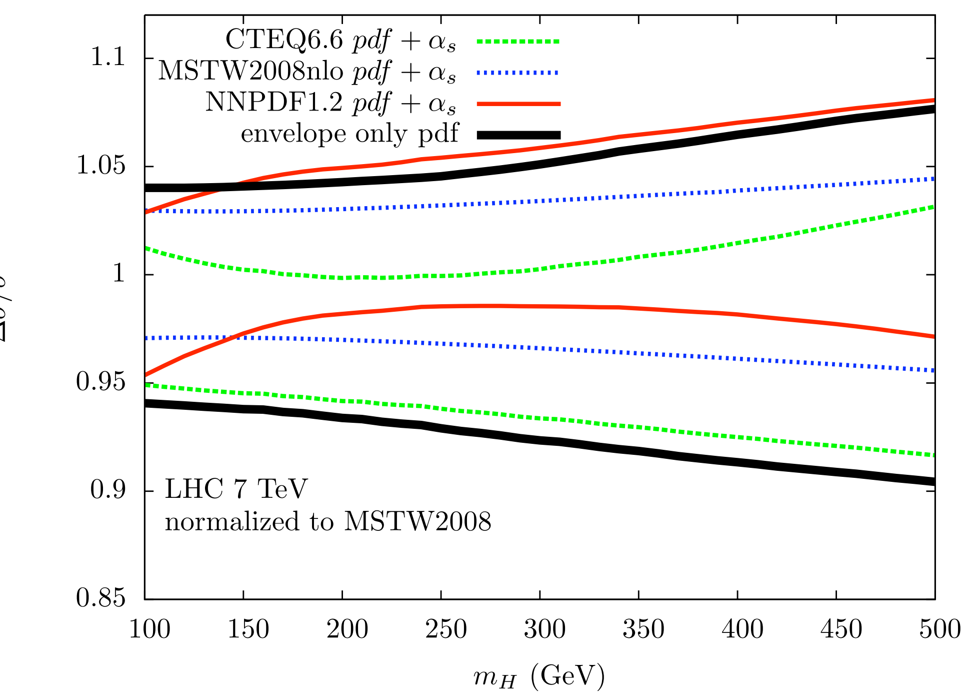
- Evaluate the partonic cross section and the pdf uncertainty bands with alphas=0.119 using the best *pdf* sets by the 3 groups

- Keep the *pdf* sets fixed, and vary alphas by ±0.0012; from the difference of the central values derive the 68% C.L. alphas uncertainty

- Sum in quadrature 68% C.L. *pdf* and alphas uncertainties

Comparison "common alphas=0.119" *vs* "envelope" recipes





The envelope is computed taking the (min/max) predictions

- with the preferred alphas of each group
- including only the 68% C.L. *pdf* uncertainty

The different alphas values used provide an estimate of the alphas uncertainty

The envelope (black) is normalized to MSTW2008nlo used with alphas=0.119

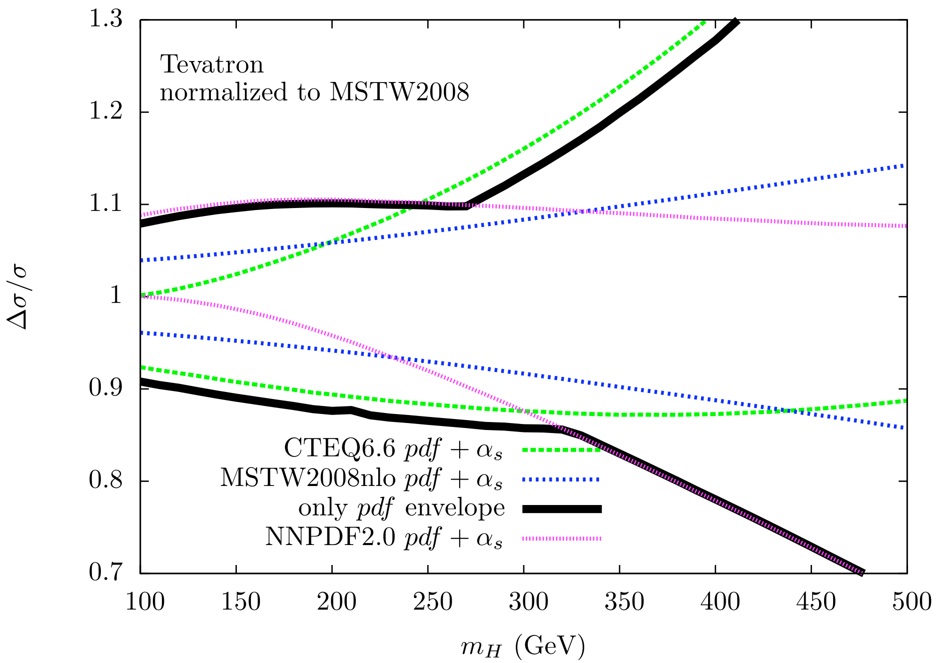
The two recipes to estimate the total uncertainty are in good agreement

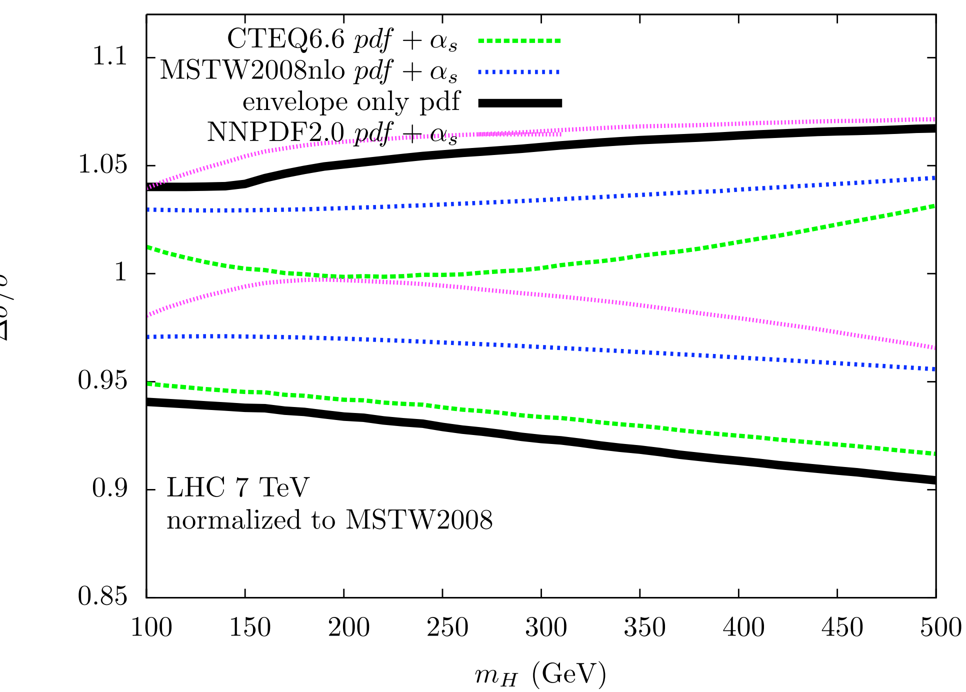
A faithful envelope is obtained using ALL the three pdf sets

An estimate of the "true" central value seems to be achievable using MSTW2008nlo with alphas=0.119

Conservative estimate of the total uncertainty: 10% at the Tevatron, 5% at the LHC

Further *pdf* improvement: NNPDF2.0





The new parton set NNPDF2.0 has:

- reduced uncertainty bands
- slightly higher (LHC7 TeV) central values

- The comparison of common alphas *vs* envelope recipes still shows a good agreement

- The size of the total uncertainty remains similar to the one obtained with NNPDF1.2

- A substantial reduction of the pdf uncertainty requires a dedicated effort

Conclusions

- the pdf central values by CTEQ66, MSTW2008 and NNPDF12
  are consistent within their uncertainties,   as shown by the luminosity plots

- the different estimates of the size of the pdf uncertainty band are consistent
  (even for small mh as soon as the uncertainty on the uncertainty is taken into account)

- the estimate of the alphas uncertainty yields very similar results
  for the three collaborations (weak correlation of alphas and gluon density;
  the partonic xsec plays the major role)

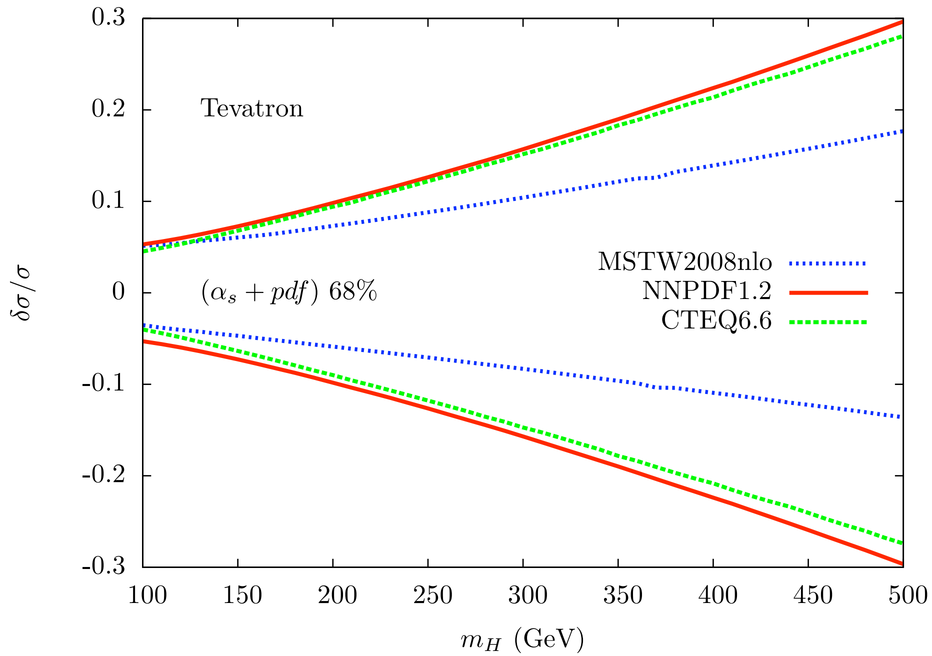
- pdf and alphas uncertainties are weakly correlated:
  their sum in quadrature is a good approximation of exact recipes

- the common alphas recipe provides a solid estimate of the size of the total uncertainty
  (once an agreement on delta alphas has been found)
  the envelope of the only-pdf results with different alphas brings to results
  very similar to the common alphas approach

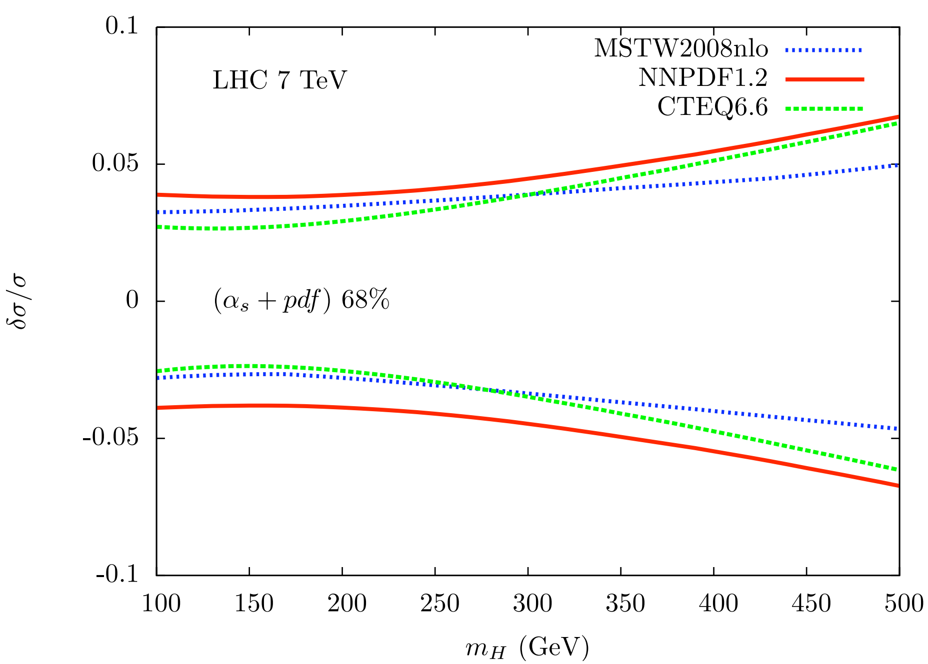
- Conservative estimate of the total uncertainty for light Higgs:
  10% at the Tevatron,   5% at the LHC (a factor 2 larger than individual sets estimates)

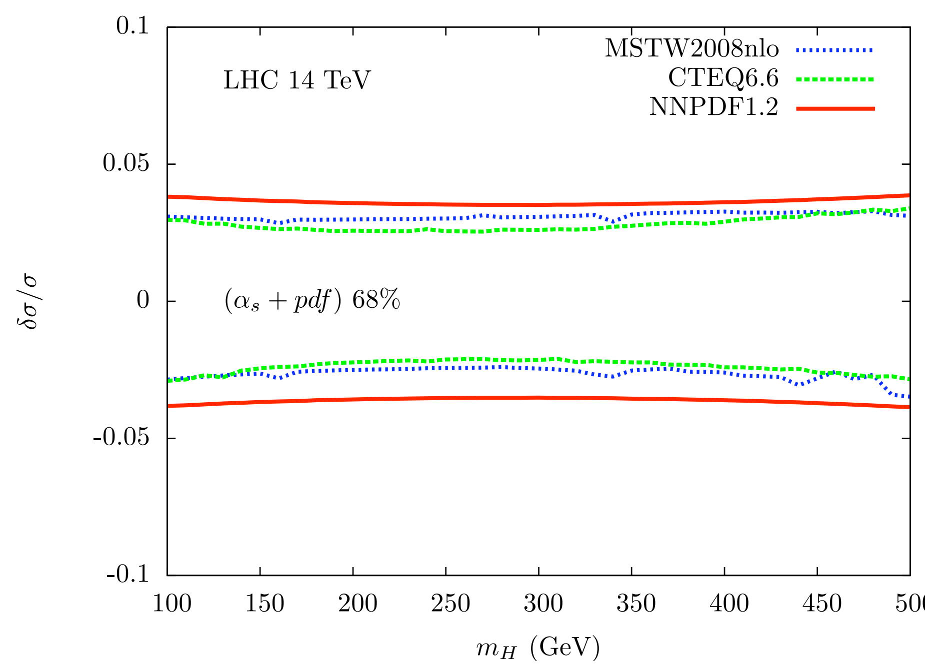
Back-up slides

Combined *pdf*+alphas uncertainties at 68% C.L.

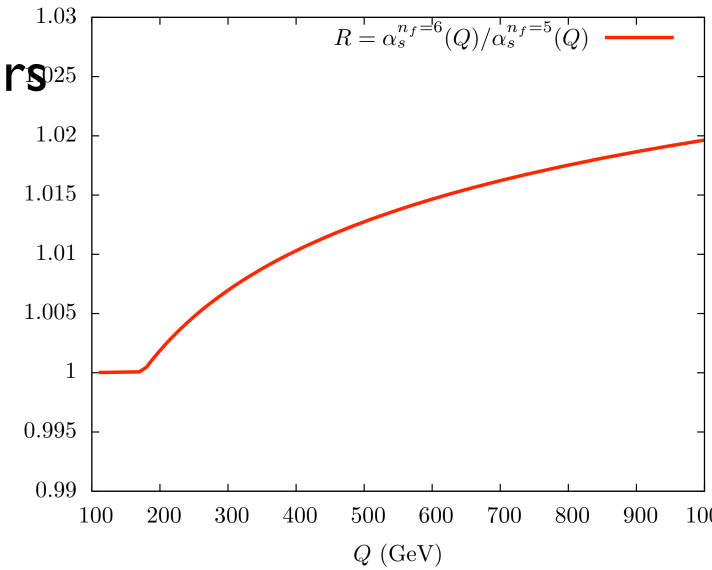


relative uncertainty
normalized to the central value
of each group





The running of α_s



The running of α_s depends on the number of active flavors

The routines for α_s, provided by CTEQ and by MSTW, implemented in LHAPDF, use $Q > m_b \quad n_f = 5$

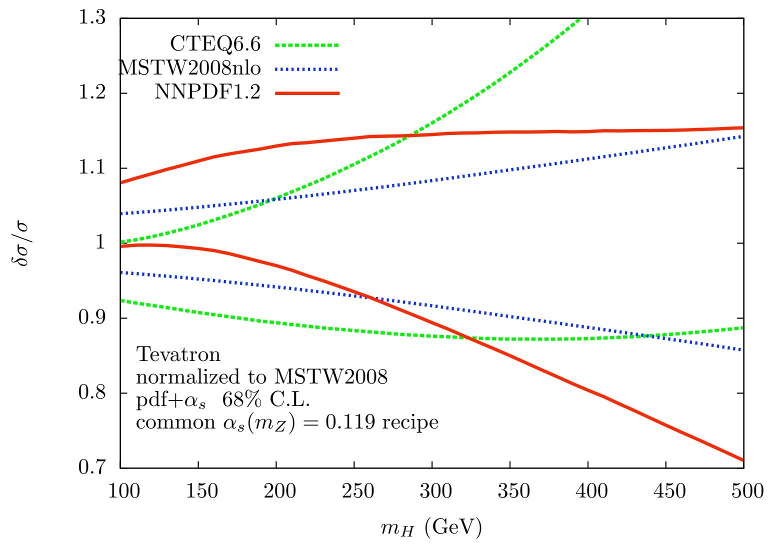
The routines for α_s, provided by NNPDF, implemented in LHAPDF, use the variable nf

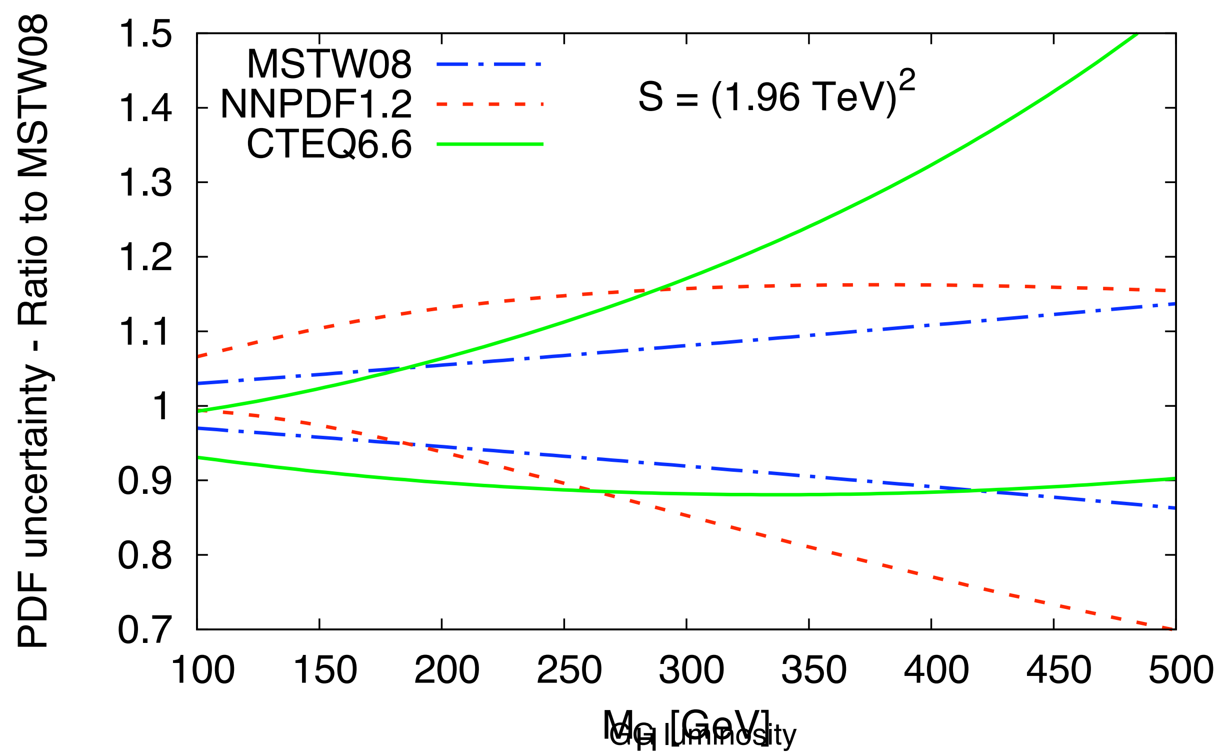
$$m_b < Q < m_t \qquad n_f = 5$$
$$Q > m_t \qquad n_f = 6$$

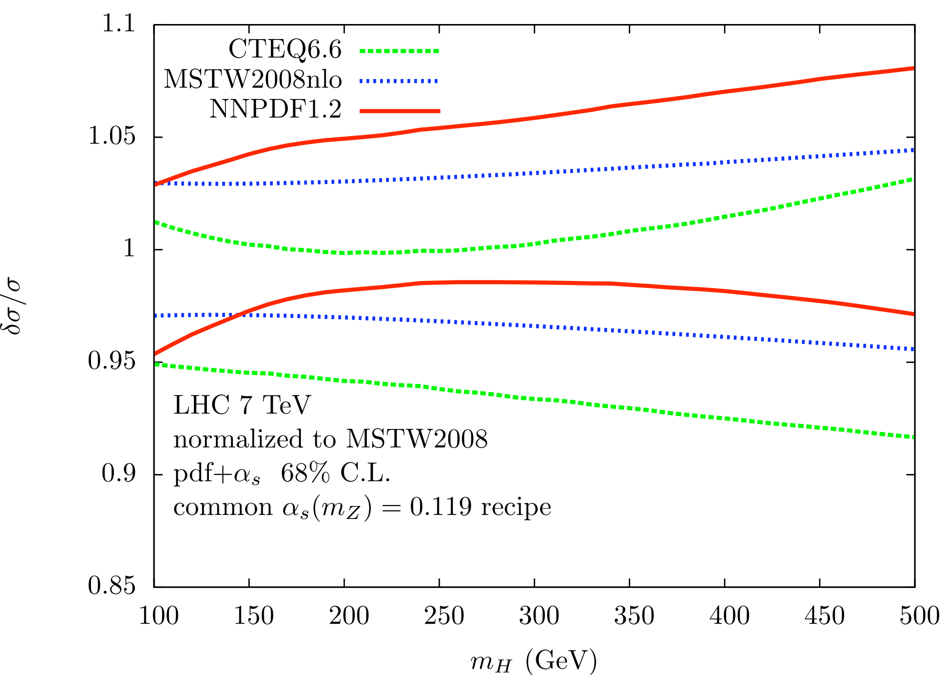
In a code like HIGLU (or like GGSCA) where the top mass is renormalized on-shell the variable number of active flavours has to be adopted
The cross sections in the two cases might differ at the percent level !

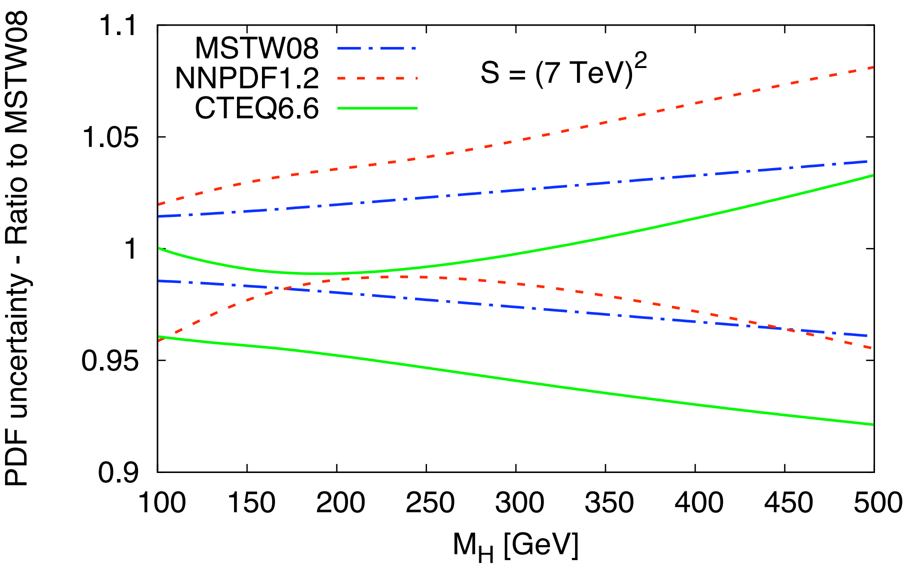
In the rest of the talk all the cross-sections evaluated with nf=6 above the top mass
There will be a missing cancellation of α_s running effects between partonic xsec and *pdf* evolution, when using CTEQ or MSTW *pdfs*

Cross sections "common alphas" *vs* gg-luminosities

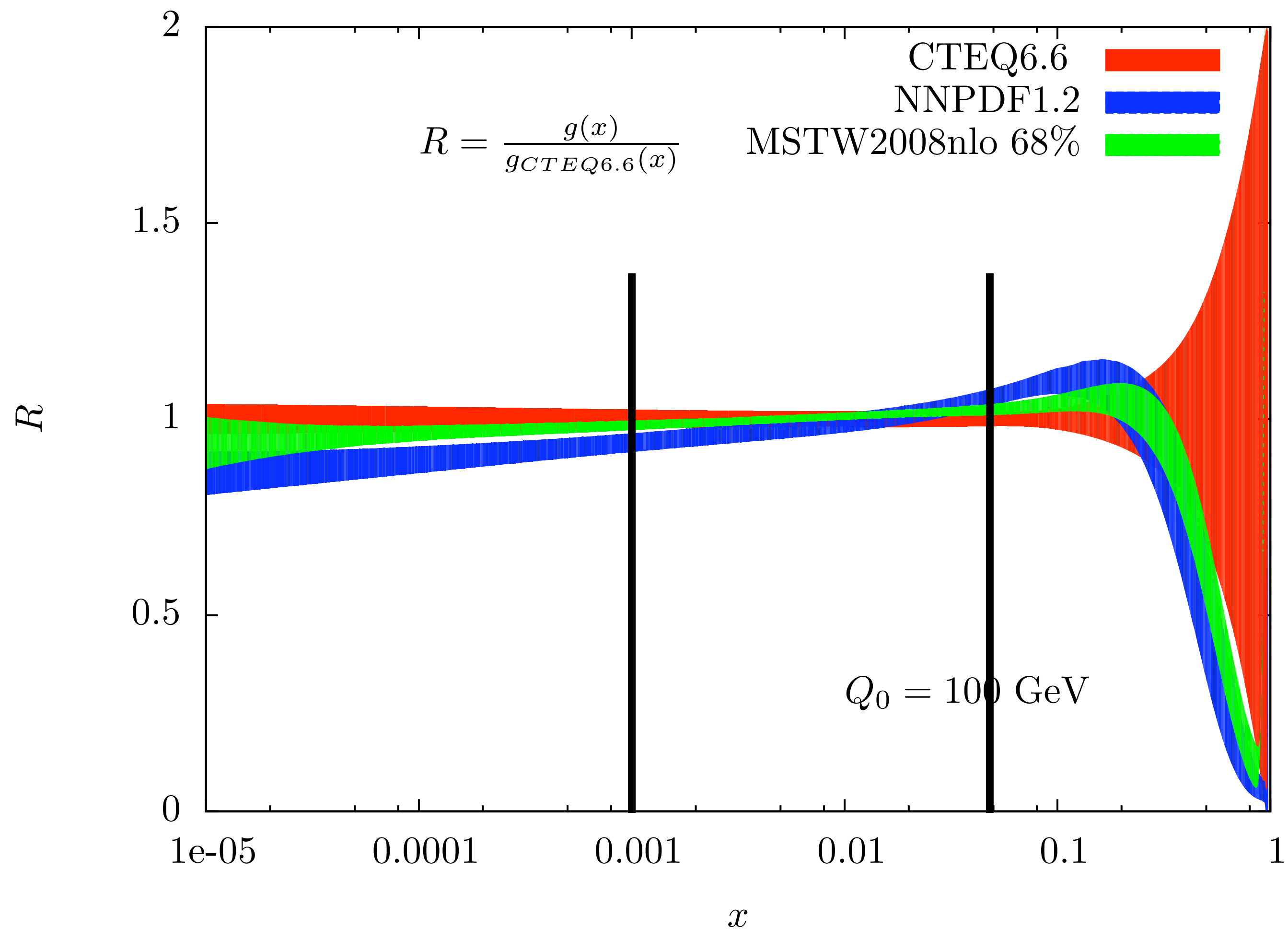








Comparison of gluon densities: best fits



At LO-QCD, the central production of a 120 GeV Higgs (|y|<2) corresponds to:

Tevatron		$0.008 < x < 0.45$
LHC	7 TeV	$0.002 < x < 0.12$
LHC	14 TeV	$0.001 < x < 0.06$

Combination of the *pdfs* and of the α_s uncertainties: MSTW2008

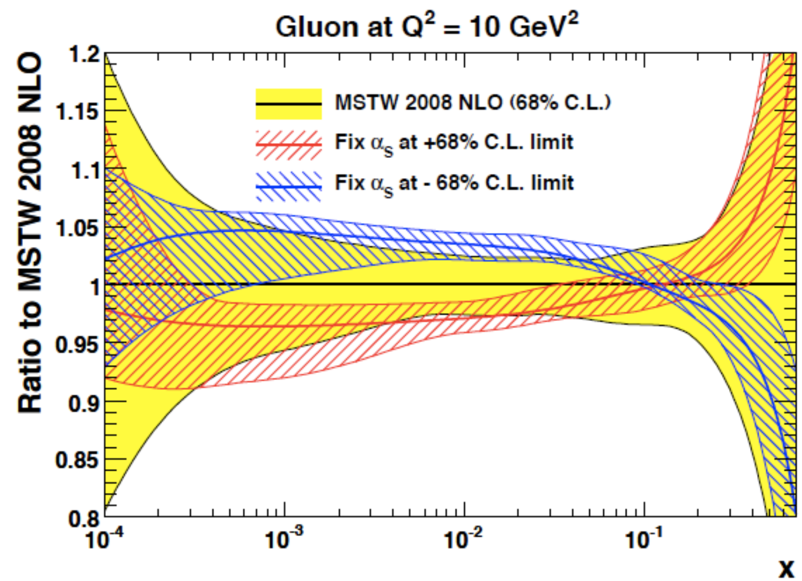
For MSTW, the uncertainty on the *pdfs* and on alpha_s are correlated

$$\alpha_s \in [\alpha_s^0 - 1\sigma,\ \alpha_s^0 + 1\sigma] \qquad \alpha_s^0 \equiv \alpha_s(m_Z) = 0.1202^{+0.0012}_{-0.0015}$$

For each of the 5 values: $\alpha_s^0 - 1\sigma,\ \alpha_s^0 - 0.5\sigma,\ \alpha_s^0,\ \alpha_s^0 + 0.5\sigma,\ \alpha_s^0 + 1\sigma$

there are 40 pdf sets

MSTW2008nlo68cl_asmz+68cl.LHgrid
MSTW2008nlo68cl_asmz+68clhalf.LHgrid
MSTW2008nlo68cl.LHgrid
MSTW2008nlo68cl_asmz-68clhalf.LHgrid
MSTW2008nlo68cl_asmz-68cl.LHgrid



Some *pdfs* spreads are much smaller that the central-value spread

Combination of the *pdfs* and of the α_S uncertainties: MSTW2008

For each of the 5 values compute the *pdf* spread (not necessarily symmetric)

$$(\Delta F^{\alpha_S}_{\rm PDF})_+ = \sqrt{\sum_{k=1}^{n} \left\{\max\left[\, F^{\alpha_S}(S_k^+) - F^{\alpha_S}(S_0),\ F^{\alpha_S}(S_k^-) - F^{\alpha_S}(S_0),\ 0\right]\right\}^2},$$

$$(\Delta F^{\alpha_S}_{\rm PDF})_- = \sqrt{\sum_{k=1}^{n} \left\{\max\left[\, F^{\alpha_S}(S_0) - F^{\alpha_S}(S_k^+),\ F^{\alpha_S}(S_0) - F^{\alpha_S}(S_k^-),\ 0\right]\right\}^2},$$

The (pdf+alpha_s) spread is obtained as follows

$$(\Delta F_{\mathrm{PDF}+\alpha_S})_+ = \max_{\alpha_S}\left(\{F^{\alpha_S}(S_0) + (\Delta F^{\alpha_S}_{\rm PDF})_+\}\right) - F^{\alpha_S^0}(S_0),$$

$$(\Delta F_{\mathrm{PDF}+\alpha_S})_- = F^{\alpha_S^0}(S_0) - \min_{\alpha_S}\left(\{F^{\alpha_S}(S_0) - (\Delta F^{\alpha_S}_{\rm PDF})_-\}\right),$$

Combination of the *pdfs* and of the α_s uncertainties: NNPDF1.2

$$\sigma_{\mathcal{F}} = \left(\frac{1}{N_{set}-1} \sum_{j=1}^{N_\alpha} \sum_{k_j=1}^{N_{rep}^{\alpha_s^{(j)}}} \left(\mathcal{F}[\{q^{(k_j,j)}\}] - \mathcal{F}[\{q^{(0)}\}] \right)^2 \right)^{1/2}$$

N_α number of distinct values of α_s used

$$N_{rep}^{\alpha_s^{(j)}} \propto \exp\left(-\frac{\left(\alpha_s^{(j)} - \alpha_s^{(0)}\right)^2}{2\left(\delta_{\alpha_s}^{(68)}\right)^2} \right)$$

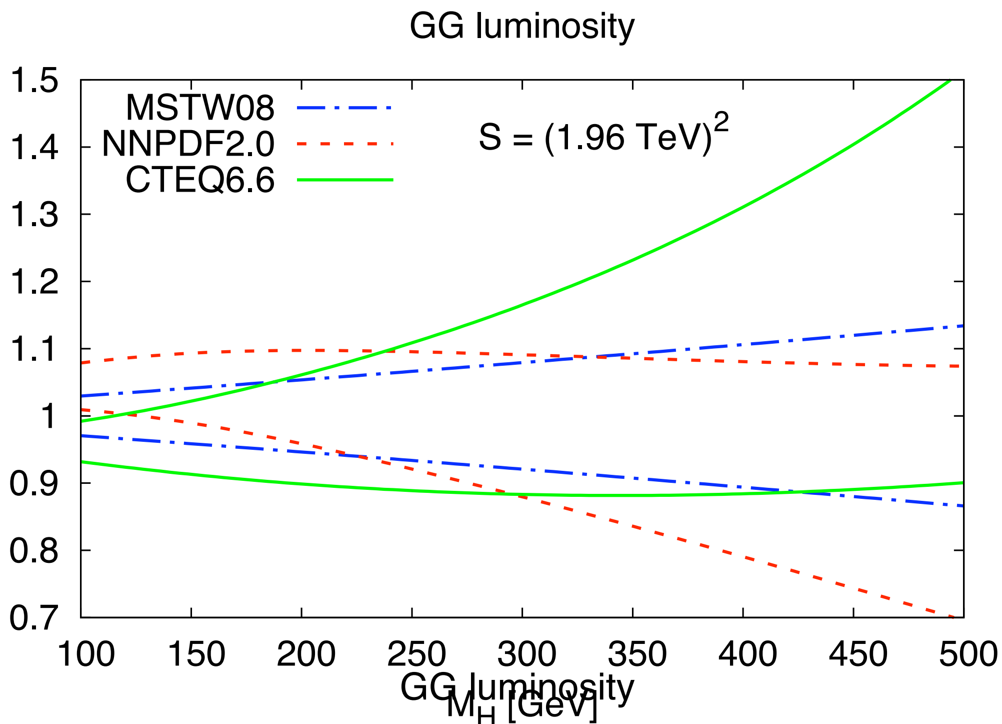
number of replicas used extracted with $\alpha_s^{(j)}$

$$N_{set} = \sum_j N_{rep}^{\alpha_s^{(j)}}$$

total number of replicas

$$\delta_{\alpha_s}^{(68)} = 0.0012$$

Gluon-gluon luminosity (normalized to MSTW2008) with NNPDF2.0



the gluon-gluon luminosity
with 68% C.L. *pdf* error bands

the gg-lumi bands have been computed
with the preferred alphas(MZ)
and DO overlap