Axial vector transition form factors in holographic QCD and their contribution to the anomalous magnetic moment of the muon

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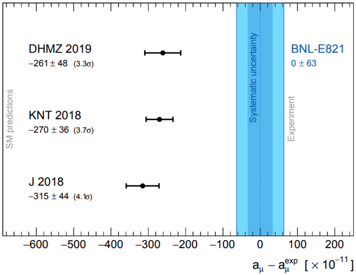
The muon $g-2$ discrepancy

Since ~ 20 years,
3—4 σ discrepancy between theorical and experimental results on the anomalous magnetic moment of the muon:



Current world average of $a_\mu = (g_\mu - 2)/2$ (dominated by BNL's E821 experiment):

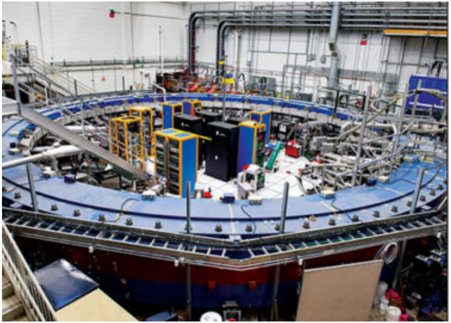
$$a_\mu^{\text{exp.}} = (11659209.1 \pm 6.3) \times 10^{-10}$$

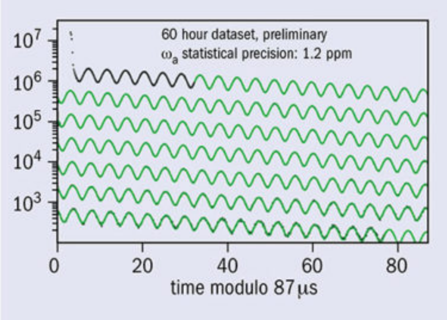
Standard model prediction [Keshavarzi et al. PRD101 (2020)] <u>3.8σ</u> below that:

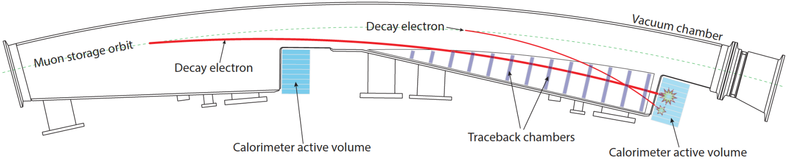
$$a_\mu^{\text{theory}} = (11659181.1 \pm 3.8) \times 10^{-10} = a_\mu^{\text{exp.}} - (28 \pm 7.4) \times 10^{-10}$$

Fermilab E989 experiment

Since 2018: new round of experimental results from Fermilab E989 experiment
first results to be released some time this year (hopefully)!

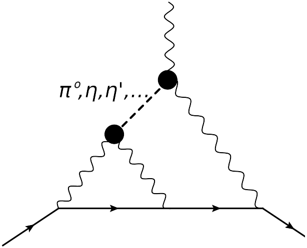






HLBL contribution to muon $g-2$

Theoretical uncertainty ($\sim 4\times 10^{-10}$) is dominated by **hadronic vacuum polarization** and **hadronic light-by-light scattering** contributions:



e.g. the dominant pion exchange contribution:

method/model	$a_\mu^{\pi^0}\times 10^{10}$
LMD+V [Nyffeler 2016]	7.2 ± 1.2
dispersive [Hoferichter et al.]	6.3 ± 0.3
lattice (Mainz, 2016)	6.5 ± 0.8
lattice (Mainz, 2019)	6.0 ± 0.4
Danilkin et al. (DRV,2019)	5.6 ± 0.2

but virtually all hadronic models miss **Melnikov-Vainshtein short-distance constraint** from nonrenormalization theorem for axial anomaly

Melnikov and Vainshtein [PRD70(2004)]:

estimated effect in (controversial) MV model: $\Delta a_\mu^{\rm PS,MV}=2.35\times 10^{-10}$

with current input data even $\Delta a_\mu^{\rm PS,MV}=3.8\times 10^{-10}$

HLBL contribution to muon $g-2$ & holographic QCD

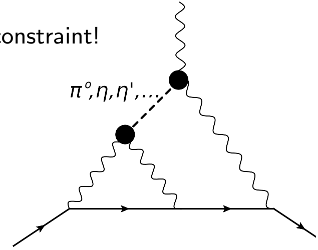
This talk:

- hQCD results (bottom-up and top-down) for
  single and double virtual (pion) transition form factor $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

  [Grigoryan, Radyushkin, PRD76,77,78 (2007-8)]
  [Cappiello, Cata, D'Ambrosio, PRD83 (2011)]
  [J. Leutgeb, J. Mager, AR, PRD100 (2019)]

  - comparison with recent low-energy data (BESIII)
  - hQCD prediction for $a_\mu^{\pi^0,\eta,\eta'}$

- **Axial vector meson contributions** [J. Leutgeb, AR, PRD101 (2020)]
  - comparison with $f_1 \to \gamma\gamma^*$ data from L3
  - crucial role in saturation of Melnikov-Vainshtein constraint!
  - hQCD prediction for $a_\mu^{a_1,f_1,f_1',\ldots}$



$+$ permutations of $\mu\gamma$-vertices

Anomalous TFF from holographic QCD

In various bottom-up hQCD models and also the top-down Sakai-Sugimoto model, (axial) vector mesons and pions are described by YM fields $\mathcal{F}^{L,R}_{MN} = \mathcal{F}^{V}_{MN} \mp \mathcal{F}^{A}_{MN}$ and 5-dimensional action

$$S_{\rm YM}^{U(N_f)\times U(N_f)} \propto \; \mathsf{tr} \int d^4x \int_0^{z_0} dz\, e^{-\Phi(z)} \sqrt{-g}\, g^{PR} g^{QS} \left(\mathcal{F}^{(L)}_{PQ} \mathcal{F}^{(L)}_{RS} + \mathcal{F}^{(R)}_{PQ} \mathcal{F}^{(R)}_{RS} \right),$$

where $P,Q,R,S = 0,\ldots,3,z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

conformal boundary at $z = 0$,

either sharp cut-off of AdS$_5$ at z_0 (HW) or with nontrivial dilaton $z_0 = \infty$ (SW)

(SS: not asymptotically AdS$_5$, finite z_0, corresponding to point where D8 branes join)

Chiral symmetry breaking either from extra bifundamental scalar field (HW1), or through different boundary conditions for vector/axial-vector fields at z_0 (Hirn-Sanz (HW2), SS)

Anomalies from
Wess-Zumino-Witten term: (by hand in bottom-up models, from D8 branes in SS model)

$$S^L_{\rm CS} - S^R_{\rm CS}, \quad S_{\rm CS} = \frac{N_c}{24\pi^2} \int \mathsf{tr} \left(\mathcal{B}\mathcal{F}^2 - \frac{i}{2}\mathcal{B}^3\mathcal{F} - \frac{1}{10}\mathcal{B}^5 \right).$$

Pion TFF from bottom-up and top-down holographic QCD

Electromagnetic (background) fields included through nonnormalizable modes of $\mathcal{B}^V$

$$\mathcal{B}^V_\mu\Big|_{boundary} = e\,\mathrm{diag}(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3})A^{e.m.}_\mu$$

E.g.: HW models have normalizable vector modes $\psi_n(z) \propto z J_1(M_n z)$ w/ $\psi_n(0) = \psi_n'(z_0) = 0$;
lowest mode $M_1 = m_\rho = 775$ MeV $\to z_0 = 3.103\,\mathrm{GeV}^{-1}$.
Vector bulk-to-boundary propagator $\mathcal{J}$ through $M_n^2 \to -Q^2$ and $\mathcal{J}(Q,0) = 1, \partial_z \mathcal{J}(Q, z_0) = 0$

$$\to \mathcal{J}^{\mathrm{HW}}(Q,z) = Qz\left[K_1(Qz) + \frac{K_0(Qz_0)}{I_0(Qz_0)} I_1(Qz)\right].$$

SW model with $\Phi(z) = \kappa^2 z^2$, $\kappa = m_\rho/2$: $\mathcal{J}^{\mathrm{SW}}(Q,\kappa,z) = \Gamma(1 + \frac{Q^2}{4\kappa^2})U(\frac{Q^2}{4\kappa^2}, 0, (\kappa z)^2)$

(no closed form for $\mathcal{J}$ in SS model)

Pion TFF from CS action:

$$F(Q_1^2, Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\int_0^{z_0} \mathcal{J}(Q_1, z)\mathcal{J}(Q_2, z)\partial_z\alpha(z)dz,$$

where $\partial_z\alpha(z)$ is the holographic pion wave function

HW1 model: needs modification by IR boundary term [Grigoryan, Radyushkin, PRD77 (2008)]

Pion TFF from bottom-up and top-down holographic QCD

Short distance behavior:

• Amazingly, bottom-up models with asymptotic AdS_5 geometry reproduce asymptotic momentum dependence of pQCD (Brodsky-Lepage):

$$F^{\mathrm{HW1}}(Q_1^2,Q_2^2) \to \frac{2f_\pi}{Q_1^2+Q_2^2}\sqrt{1-w^2}\int_0^\infty d\xi\,\xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$

$$= \frac{2f_\pi}{Q_1^2+Q_2^2}f(w), \quad f(w)=\frac{1}{w^2}-\frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}, \quad w=\frac{Q_1^2-Q_2^2}{Q_1^2+Q_2^2}$$

HW1: Extra parameter for quark condensate achieves complete fit including prefactor
HW2 (Hirn-Sanz): Prefactor already fixed by m_ρ fit $\to \approx 62\%$ of LO pQCD result
SW: $\kappa = m_\rho/2 \to \approx 89\%$ of LO pQCD result

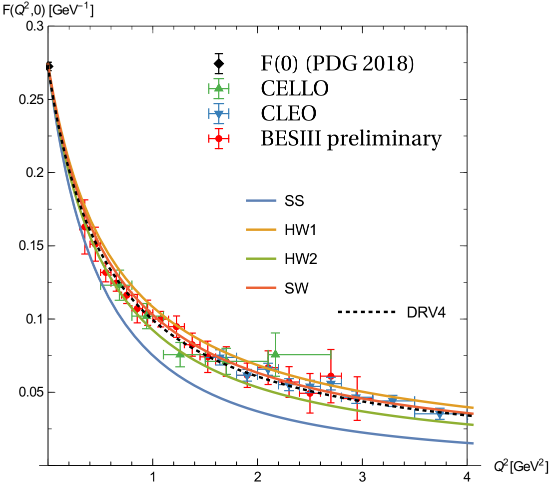
• SS-model only meaningful in low-energy limit:

nevertheless w-dependence roughly similar, but faster fall-off at large Q_i:

$$F^{\mathrm{SS}}(Q_1^2,Q_2^2) \to \frac{16}{36\pi^3 f_\pi}\left(\frac{2M_{\mathrm{KK}}^2}{Q_1^2+Q_2^2}\right)^{\frac{3}{2}}\frac{2+5\sqrt{1-w^2}}{(\sqrt{1-w}+\sqrt{1+w})^5}.$$

Holographic pion TFF and experimental data

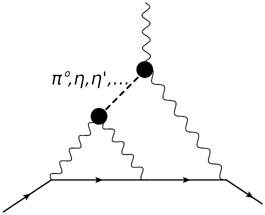
Comparison with single-virtual TFF from CELLO, CLEO, and BESIII (preliminary):



Hardly any experimental information on double-virtual case (only for η)!

Holographic pion TFF and $a_\mu^{\pi^0}$ predictions

In contrast to older phenomenological ansätze (e.g. LMD+V),
Brodsky-Lepage constraints for double virtual TFF satisfied by bottom-up hQCD models,
also good agreement with recent dispersive approach and lattice results for pion TFF



method/model	$a_\mu^{\pi^0} \times 10^{10}$
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dispersive [Hoferichter et al.]	6.3 ± 0.3
lattice (Mainz, 2016)	6.5 ± 0.8
lattice (Mainz, 2019)	6.0 ± 0.4
Danilkin et al. (DRV,2019)	5.6 ± 0.2
SS	4.84
HW1	6.10
HW2	5.69
SW	5.92
hQCD (HW,SW)	5.9 ± 0.2

Previous estimates of $a_\mu^{\pi^0}$ in bottom-up models [Cappiello et al. 2011]
10% higher due to approximation by interpolators

Axial vector mesons $\leftrightarrow \gamma^*\gamma^*$

Holographic models all involve (infinite tower of) axial vector mesons in addition to pseudoscalar Goldstone bosons

lightest axials:

	SS	HW1	HW2	SW1	exp.
m_{a_1,f_1}[MeV]	1186.5	1375.5	1234.8	1674.1	1230(40) ; 1281.9(0.5)

$a \to \gamma^*\gamma^*$ interactions fixed by CS action (axial anomaly):

$$\mathcal{L}^{\rm CS}_{\mathcal{A}\gamma\gamma} = -i\frac{N_c}{12\pi^2}\text{tr}\,\epsilon^{\mu\nu\rho\sigma}\int_{-\infty}^{\infty} dZ\,\left(a_\mu \mathcal{V}'_\nu \partial_\rho \mathcal{V}_\sigma + \mathcal{V}_\mu a'_\nu \partial_\rho \mathcal{V}_\sigma + \mathcal{V}_\mu \mathcal{V}'_\nu \partial_\rho a_\sigma\right)$$

($'$ denotes derivative of holographic wave function)

Landau-Yang theorem (1948): axial vector mesons cannot decay into two real photons realized by $\mathcal{V}'_\mu = 0$ for $Q^2 = 0$ (part.int. in 2nd term OK in SS, HW2)

Axial vector meson TFF

$$\mathcal{M} \propto \epsilon^{\mu}_{(1)}\epsilon^{\nu}_{(2)}\epsilon^{*\rho}_{\mathcal{A}}\epsilon_{\mu\nu\rho\sigma}\left[q^{\sigma}_{(2)}Q_1^2 A(Q_1^2,Q_2^2) - q^{\sigma}_{(1)}Q_2^2 A(Q_2^2,Q_1^2)\right]$$

In contrast to most phenomenological models of TFF for axials,
the holographic result is asymmetric in double-virtual case!

$$A(Q_1^2,Q_2^2) = \frac{2}{Q_1^2}\int_0^{z_0} dz \left[\frac{d}{dz}\mathcal{J}(Q_1,z)\right]\mathcal{J}(Q_2,z)\psi^A(z) \Big/ \left[g_5^{-2}\int_0^{z_0}\frac{dz}{z}(\psi^A)^2\right]^{1/2}$$

Experimental data for (single-virtual) TFF of $f_1(1285)$ and $f_1'(1420)$ from L3 experiment:

$$|A(0,0)|^{\mathrm{exp.}} \simeq 15(2)\mathrm{GeV}^{-2} \qquad \text{at } \theta_{f_1} = 24^\circ \text{ away from ideal}$$

dipole fit $\frac{A(Q_1^2,0)}{A(0,0)} = \frac{1}{(1+Q_1^2/\Lambda_D^2)^2} \rightarrow \Lambda_D = 1040 \pm 78$ MeV for $f_1(1285)$

Holographic results:

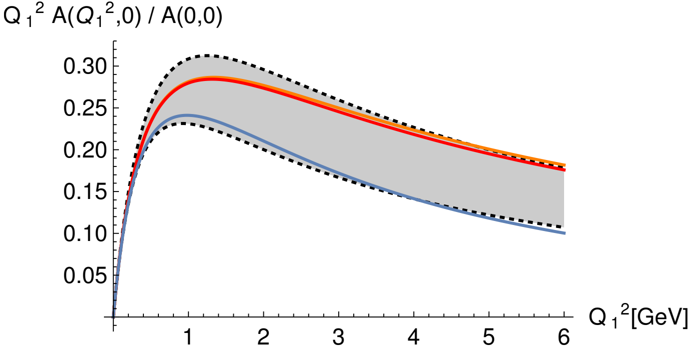
	SS	HW1	HW2
$\lvert A(0,0)\rvert$ [GeV^{-2}]	15.9	21.0	16.6

with complicated dependence on virtualities in agreement with SDC from pQCD!

[Hoferichter & Stoffer, 2004.06127]

Single-virtual axial vector meson TFF

Experiment (gray band) vs. holographic results of SS, HW1, and HW2 models:

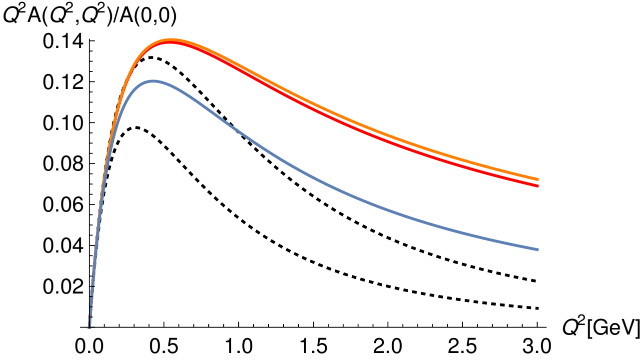


Double-virtual axial vector meson TFF

Holographic results of SS, HW1, and HW2 models

vs. symmetric dipole model $\frac{A^{\rm PV}(Q_1^2,Q_2^2)}{A(0,0)} = \frac{1}{(1+Q_1^2/\Lambda_D^2)^2(1+Q_2^2/\Lambda_D^2)^2}$ (dashed lines)

used by Pauk & Vanderhaeghen [1401.0832] in their calculation of $a_\mu^{f_1,f_1'}$

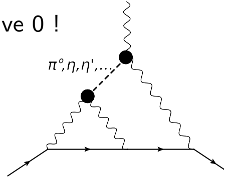


Melnikov-Vainshtein short-distance constraint

Melnikov and Vainshtein [hep-ph/0312226, PRD70(2004)]:
nonrenormalization theorem for axial anomaly implies
short-distance constraint for 4-photon-amplitude (in BTT basis w/ 54 structure functions):

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

virtually all model calculations of meson exchange contributions give 0 !



MV-SDC would be satisfied
if external TFF was replaced by constant on-shell value,
leading to significant (almost +40%) increase of $a_\mu^{\pi^0,\eta,\eta'}$
(as indeed proposed by MV)

Recently: Colangelo et al. [1910.11881] constructed Regge model of infinite tower of excited PS states which saturates MV-SDC with $\Delta a_\mu^{PS} = 1.3(6)\times 10^{-10} \approx 0.1 a_\mu^{\pi^0,\eta,\eta'}$
But: Excited PS states decouple in chiral large-N limit

Axial vector contribution to MV-SDC

Axial vector contribution to $\bar{\Pi}_1(Q,Q,Q_3)$
(involving only the longitudinal part of the axial vector propagator $q^\mu_{(3)} q^\nu_{(3)}/(M_n^A Q_3)^2$)
in holographic HW models:

$$\bar{\Pi}_1 = -\frac{g_5^2}{2\pi^4} \sum_{n=1}^{\infty} \int_0^{z_0} dz \left[\frac{d}{dz}\mathcal{J}(Q,z)\right] \mathcal{J}(Q,z)\psi_n^A(z) \frac{1}{(M_n^A Q_3)^2} \int_0^{z_0} dz' \left[\frac{d}{dz'}\mathcal{J}(Q_3,z')\right] \psi_n^A(z')$$

Each term in sum has $\lim_{Q_3\to\infty} \lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1^{(n)}(Q,Q,Q_3) = 0$

but infinite sum gives

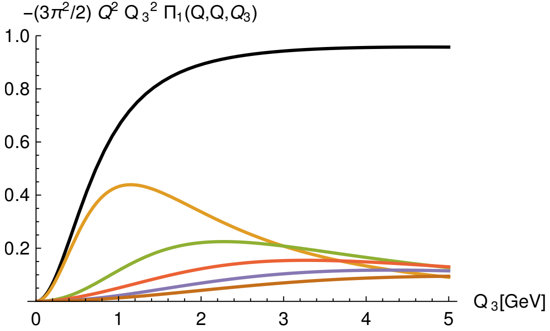
$$\lim_{Q_3\to\infty} \lim_{Q\to\infty} \bar{\Pi}_1(Q,Q,Q_3) = -\frac{g_5^2}{2\pi^4} \frac{1}{2Q_3^2 Q^2} \underbrace{\int_0^{\infty} d\xi \xi K_1(\xi) \frac{d}{d\xi}[\xi K_1(\xi)]\xi^2}_{2/3\ (!)}$$

HW1 model has $g_5^2 = 4\pi^2 \Rightarrow$: MV-SDC satisfied exactly
HW2 model: MV-SDC at 62% level, like in SDC's of pion sector
SS model: 0%, also like in pion sector

Axial vector contributions to MV-SDC

MV-SDC $\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$

HW2 model with $g_5^2 = 4\pi^2$ and large $Q = 50$GeV and increasing $Q_3 \ll Q$:

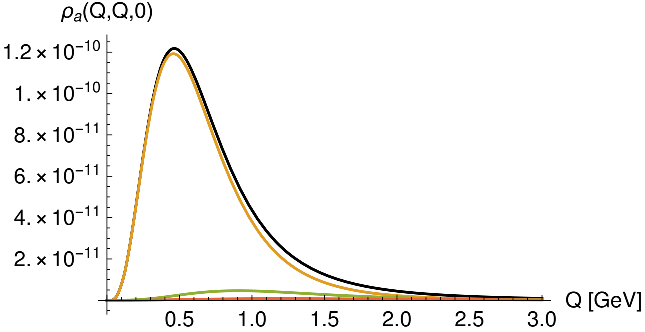


black line: infinite sum
colored lines: first 5 axial vector modes

Axial vector contributions to muon $g-2$

$a_\mu^{\rm AV} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\, \rho_a(Q_1,Q_2,\tau)$

E.g. at $\tau = 0$:



Strongly dominated by lowest axials, but nonnegligible contribution from higher modes:

	$j=1$	$j\le 2$	$j \le 3$	$j \le 4$	$j \le 5$	$a_\mu^{\rm AV}$
HW1	3.14	3.62	3.79	3.91	3.96	4.1×10^{-10}
HW2	2.30	2.62	2.74	2.79	2.82	2.9×10^{-10}
SS	1.38	1.45	1.47	1.48	1.48	1.5×10^{-10}

Pseudoscalar plus axial vector contributions to muon $g-2$

Our results [J. Leutgeb & AR, 1912.01596] combined with $a_\mu^{\rm PS}$ [1906.11795]
(z_0 s.t. $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV; f_1-f_1' mixing from L3 data)

	HW1 (100% MV-SDC)	HW2 (62% MV-SDC)
$a_\mu^{\rm PS}[\pi^0+\eta+\eta'] \times 10^{10}$	9.2 [6.13+1.67+1.42]	8.4 [5.92+1.59+1.34]
$a_\mu^{\rm AV}[L+T] \times 10^{10}$	4.1 [2.3+1.8]	2.9 [1.7+1.2]
$a_\mu^{\rm PS+AV} \times 10^{10}$	13.3	11.2

(compare with controversial MV model: longitudinal contribution estimated $\sim 3.8 \times 10^{10}$)

almost at same time:
[L. Cappiello, O. Cata, G. D'Ambrosio, D. Greynat, A. Iyer, 1912.02779]:
agreement with our HW2 results, but different parameters:
HW2$^{(1)}$: z_0 s.t. $m_\rho = 776$ MeV, $f_\pi = 93$ MeV, $f_{\eta'} = 74$ MeV
HW2$^{(2)}$: z_0 s.t. 100% UV limit (but $m_\rho = 987$ MeV !)

	HW2$^{(2)}$ (100% MV-SDC)	HW2$^{(1)}$ (62% MV-SDC)
$a_\mu^{\rm PS}[\pi^0+\eta+\eta'] \times 10^{10}$	11.2 [7.5+2.1+1.6]	8.1 [5.7+1.4+1.0]
$a_\mu^{\rm AV}[L+T] \times 10^{10}$	3.2 [1.8+1.4]	2.8 [1.4+1.4]
$a_\mu^{\rm PS+AV} \times 10^{10}$	14.4	11.0

HW1 and HW2 results bracket experimental results for pion TFF
but both overestimate experimental result for $f_1 \to \gamma\gamma^*$ from L3
(same for HW2$^{(2)}$ and HW2$^{(1)}$ results of Cappiello et al.)

Conclusions

- Holographic bottom-up results reproduce well experimental data for pseudoscalar TFF and give a_μ^{PS} in good agreement with dispersion relation approach and lattice (SS model good at low energies, but misses small but relevant UV contributions)
- Infinite tower of axial vector meson contributions responsible for MV-SDC
- Axial vector TFF reproduce experimental dipole-like results for Q^2 dependence; different form in double-virtual case, to be tested
- Axial vector meson contributions in hQCD models $a_\mu^{\mathrm{AV}} = 3\ldots4 \times 10^{-10}$
  or downscaled to match L3 data: $2.2(5) \times 10^{-10}$

  much larger than previously estimated,
  (e.g. F. Jegerlehner in his book (2017): $a_\mu^{\mathrm{AV}} = 0.76(27) \times 10^{-10}$;
  Roig & Sanchez-Puertas [1910.02881]: $a_\mu^{\mathrm{AV}} = 0.08\binom{+0.35}{-0.01} \times 10^{-10}$;
  Dorokhov et al. [1910.07815]: $a_\mu^{\mathrm{AV}} = 0.34 \times 10^{-10}$)

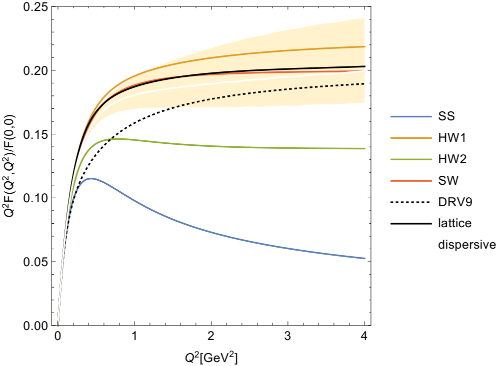
  but longitudinal part (57%) smaller than estimated by MV model ($\sim 3.8 \times 10^{-10}$)

  in the end: hQCD models suggest slight increase of total HLBL contribution
  (without removing $> 3\sigma$ discrepancy)

BACKUP SLIDES

Holographic pion TFF and recent lattice data

No experimental data yet for double-virtual pion TFF, but
• results from dispersive approach M. Hoferichter et al., 1808.04823 and
new lattice extrapolations from A. Gérardin, H. B. Meyer, and A. Nyffeler, 1903.09471:



- HW1: quickly approach LO pQCD result (but NLO negative)
- SW: (fortuitously?) close to lattice (89% of LO pQCD asymptotically)
- SS: wrong asymptotics, but below 0.3 GeV^2 closer to lattice than DRV interpolator

Holographic pion TFF and experimental data

Slope parameter:

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)/F(0,0) = 1 + \hat{\alpha}(Q_1^2 + Q_2^2) + O(Q^4)$$

Holographic predictions: (only free parameters: $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV)

Model	$\hat{\alpha}[\text{GeV}^{-2}]$
Sakai-Sugimoto	-2.043
HW1	-1.595
HW2 (Hirn-Sanz)	-1.805
SW	-1.665

(cp. with $m_\rho^{-2} = 1.665\text{GeV}^{-2}$)

Experimental data (fits):

Experiment	$\hat{\alpha}[\text{GeV}^{-2}]$
PDG (before NA62)	-1.76(22)
NA62 (Dalitz decays $\pi^0 \to \gamma e^+ e^-$)	-2.02(31)
PDG (after NA62)	-1.84(17)
DRV4 (CELLO,CLEO,BESIII)	-1.74(2)

DRV: simple monopole fits up to $Q^2 = 4$ or 9 GeV^2 with lowest $Q^2 \approx 0.3$ GeV^2