Cho–Maison Monopole: BPS Limit and Mass Bound

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Magnetic Monopole in Standard Model

't Hooft–Polyakov

- Based on the particular pattern of SSB
- Relevant second homotopy group is trivial

$$\pi_2\big(SU(2)\times U(1)/U(1)\big) = \{1\}$$

- $\Rightarrow$ **'t Hooft–Polyakov magnetic monopole doesn't exist in SM**

Cho–Maison: Topology

- Desired topology can be found elsewhere
- Namely in the Higgs doublet field

$$H = \frac{v}{\sqrt{2}}\rho\xi \quad \text{where} \quad \xi^\dagger\xi = 1$$

- Due to $U(1)_Y$ invariance ξ can be regarded as a $\mathbb{C}P^1$ field
- But $\pi_2(\mathbb{C}P^1) = \mathbb{Z}$
- $\Rightarrow$ **Cho–Maison magnetic monopole could, possibly, exist in SM...**

Cho–Maison: Mass

But:

- Mass of the Cho–Maison monopole is infinite:

$$M = \frac{2\pi}{g'^2}\int_0^\infty \frac{\mathrm{d}r}{r^2} + \text{finite terms} = \infty$$

- $\Rightarrow$ **Physical (i.e., finite-mass) Cho–Maison magnetic monopole cannot exist in SM!**

Regularizations of Monopole Mass

The idea:

- Perhaps some effective extension of SM might allow for a finite-mass Cho–Maison monopole
- In particular: Since the divergence is proportional to $1/g'^2$, a modification of the $U(1)_Y$ kinetic term might do the job

CKY Regularization

Cho, Kim, Yoon (1997) proposed the following model:

$$\mathcal{L}_{\mathrm{CKY}} = |D_\mu H|^2 - V(H) - \frac{1}{4g'^2}\epsilon B_{\mu\nu}^2 - \frac{1}{2g^2}\operatorname{Tr}\left[F_{\mu\nu}^2\right]$$

Notice the presence of ϵ (a function of $|H|^2/v^2$):

- If $\epsilon = 1$, we recover SM (with infinitely heavy monopole)
- If ϵ is a non-trivial function satisfying $\epsilon(0) = 0$, monopole mass is *finite*!
- Ellis, Mavromatos, You (2016) found ϵ allowing monopoles mass as low as $\sim 5.5\,\mathrm{TeV}$

$\longrightarrow$ How light can monopole be?

BPS Regularization

Inspired by CKY we propose another model, whose main virtue is that it is a BPS theory:

$$\mathcal{L}_{\mathrm{BPS}} = |D_\mu H|^2 - \frac{v^2}{4g^2|H|^2}h^2\left(\operatorname{Tr}\left[F_{\mu\nu}^2\right] - \frac{2}{|H|^4}\operatorname{Tr}\left[F_{\mu\nu}HH^\dagger\right]^2\right) - \frac{1}{4}\left(\frac{h'}{g|H|^2}\operatorname{Tr}\left[F_{\mu\nu}HH^\dagger\right] + \frac{f'}{g'}B_{\mu\nu}\right)^2$$

Model is parameterized by f' and h (again functions of $|H|^2/v^2$)

- Cannot be reduced to SM (for any f', h)
- For suitable f', h it allows for finite-mass monopoles
- Although phenomenologically not relevant *directly*, it has some *indirect* implications...

Implications of the BPS Theory $\mathcal{L}_{\mathrm{BPS}}$

Exact Monopole Solutions: Theorists' Playground

- Equations of motion ("BPS equations") are of the first order!
- $\Rightarrow$ Can be solved more easily
- *Example*: for $f' = |H|^2/v^2$ and $h = 1$ the spherically symmetric monopole solution is:

$$A_i^a = \left(1 - \exp\left\{-\frac{g}{g'}\left[\frac{\mu r}{2}\frac{2+\mu r}{1+\mu r} - \log\left(1+\mu r\right)\right]\right\}\right)\varepsilon_{iak}\frac{x_k}{r}$$

$$H = \mathrm{i}\frac{v}{\sqrt{2}\left(1+\dfrac{1}{\mu r}\right)}\begin{pmatrix}\sin(\theta/2)\,\mathrm{e}^{-\mathrm{i}\varphi} \\ -\cos(\theta/2)\end{pmatrix} \qquad \text{(where } \mu \equiv vg' \approx 86.1\,\mathrm{GeV}\text{)}$$

with mass

$$M = 4\pi v\left(\frac{1}{2g} + \frac{1}{3g'}\right) \approx 5.32\ \mathrm{TeV}$$

- Exact solutions can be found also for other f', h ...

Monopole Mass Bound: Phenomenology

By definition, BPS theory has a lower energy (Bogomolny) bound:

$$\mathcal{E} \geq \frac{1}{2}v\,\varepsilon_{ijk}\partial_i\left(\frac{1}{g}h\operatorname{Tr}\left[F_{jk}\xi\xi^\dagger\right] + \frac{1}{g'}fB_{jk}\right)$$

$\Rightarrow$ allows to derive lower bound on monopole mass:

$$\boxed{M \geq \frac{2\pi v}{g} \approx 2.37\ \mathrm{TeV}}$$

Nontrivially, this bound holds not only for the BPS theory, but also for the CKY theory:

- Regardless of the choice of f', h or ϵ, monopole mass cannot be lower
- $\Rightarrow$ mass bound is phenomenologically relevant

Reference:

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