Intrinsic quantum coherence in particle oscillations

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- Neutrino flavour oscillations ($\nu_e \longleftrightarrow \nu_\mu$ etc.)

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- 2015 Nobel Prize in Physics to T. Kajita (Super-Kamiokande) and A. McDonald (Sudbury Neutrino Observatory)

  "for the discovery of **neutrino oscillations**, which shows that **neutrinos have mass**."

Outline

- Standard approach to neutrino oscillations and the theoretical challenge
- Oscillations and coherence in Quantum Mechanics
  - two-level systems
  - coherent states in quantum optics
- Intrinsically coherent oscillating particle states
- Conclusions and outlook

Standard theory of neutrino oscillations

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- Lagrangian with flavour violation (induced by Yukawa terms in SM Lagrangian) and Dirac neutrino masses:

$$\mathcal{L} = \overline{\Psi}_{\nu_e} i\partial\!\!\!/ \Psi_{\nu_e} + \overline{\Psi}_{\nu_\mu} i\partial\!\!\!/ \Psi_{\nu_\mu} - \begin{pmatrix} \overline{\Psi}_{\nu_e} & \overline{\Psi}_{\nu_\mu} \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} \end{pmatrix} \begin{pmatrix} \Psi_{\nu_e} \\ \Psi_{\nu_\mu} \end{pmatrix}.$$

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Massive neutrino states: $|\nu_{i\lambda}(\mathbf{p})\rangle = A^\dagger_{i\lambda}(\mathbf{p})|\Phi_0\rangle, \quad |\bar{\nu}_{i\lambda}(\mathbf{p})\rangle = B^\dagger_{i\lambda}(\mathbf{p})|\Phi_0\rangle$

$|\Phi_0\rangle$ – physical vacuum

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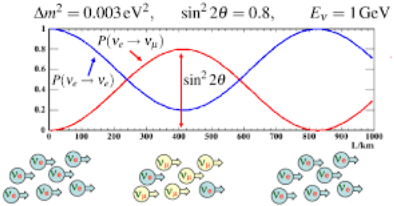
- Then oscillations can take place:

$$\mathcal{P}_{\nu_e\to\nu_\mu} = |\langle\nu_\mu(\mathbf{p})|e^{-iHt}|\nu_e(\mathbf{p})\rangle|^2 = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4E}L\right),$$

$$\Delta m^2 = m_2^2 - m_1^2, \quad \frac{m_i}{E} \ll 1.$$



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- Attempts to incorporate the oscillation phenomenon into quantum field theory:

Giunti, Kim and Lee (1992), Giunti, Kim, Lee and Lee (1993), Blasone and Vitiello (1995), Grimus and Stockinger (1996), Giunti and Bilenky (2001), Giunti (2007), Akhmedov and Kopp (2010), etc.

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Coherent flavour neutrino states cannot be derived in conventional QFT!

Oscillations of states and coherence in Quantum Mechanics

Prototypical quantum oscillations: two-level quantum systems

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Quantum mechanical system with two stationary states

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$$\mathcal{P}_{|0\rangle\to|1\rangle} = \langle 1|e^{-iH\Delta t}|0\rangle \sim \sin^2\left(\frac{\Delta E}{2}\Delta t\right)$$

Note:

- because of Stone–von Neumann theorem, all the representations of the canonical algebra for a given quantum mechanical system are equivalent, implying *unitary change of basis*:

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Can this simple quantum mechanical picture be extended straightforwardly to particle oscillations?

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The quantum mechanical interpretation of neutrino oscillation as two-level system oscillation is conceptually untenable!

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i.e. the coherent state is a superposition of an infinite number of particle states (or Fock states), all belonging to the same Fock space.

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- In QFT, the notion of coherent state appears as vacuum condensate.

How to define coherent oscillating states in quantum field theory, as superposition of finite number of particle states belonging to different Fock spaces?

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Nambu and Jona-Lasinio (1961),
see also Umezawa, Takahashi and Kamefuchi (1964)

inspired by Bardeen–Cooper–Schrieffer theory of superconductivity in Bogoliubov's formulation

Bogoliubov (1958)

The technique: Quantum Hamiltonian diagonalization

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- Flavour number-violating **Hamiltonian**

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H &= \int d^3x \Big[-\overline{\Psi}_{\nu_e} i\gamma^i \partial_i \Psi_{\nu_e} - \overline{\Psi}_{\nu_\mu} i\gamma^i \partial_i \Psi_{\nu_\mu} \Big] \\
&+ \int d^3x \Big[m_{ee} \overline{\Psi}_{\nu_e} \Psi_{\nu_e} + m_{\mu\mu} \overline{\Psi}_{\nu_\mu} \Psi_{\nu_\mu} + m_{e\mu} \left(\overline{\Psi}_{\nu_e} \Psi_{\nu_\mu} + \overline{\Psi}_{\nu_\mu} \Psi_{\nu_e} \right) \Big] = H_0 + H_{mass} .
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&+ \int d^3x \Big[m_{ee} \overline{\Psi}_{\nu_e} \Psi_{\nu_e} + m_{\mu\mu} \overline{\Psi}_{\nu_\mu} \Psi_{\nu_\mu} + m_{e\mu} \left(\overline{\Psi}_{\nu_e} \Psi_{\nu_\mu} + \overline{\Psi}_{\nu_\mu} \Psi_{\nu_e} \right) \Big] = H_0 + H_{mass} .
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$$

- Treat H_{mass} as an interaction term for massless SM flavour fields.

- Nondiagonal Hamiltonian in terms of massless (bare) particles' operators :

$$
\begin{aligned}
H &= \int d^3p \sum_\lambda \Big\{ \mathrm{p} \left(a^\dagger_{e\lambda}(\mathbf{p}) a_{e\lambda}(\mathbf{p}) + b^\dagger_{e\lambda}(\mathbf{p}) b_{e\lambda}(\mathbf{p}) + a^\dagger_{\mu\lambda}(\mathbf{p}) a_{\mu\lambda}(\mathbf{p}) + b^\dagger_{\mu\lambda}(\mathbf{p}) b_{\mu\lambda}(\mathbf{p}) \right) \\
&+ \operatorname{sgn}\lambda \Big[m_{ee} \left(a^\dagger_{e\lambda}(\mathbf{p}) b^\dagger_{e\lambda}(-\mathbf{p}) + b_{e\lambda}(\mathbf{p}) a_{e\lambda}(-\mathbf{p}) \right) + m_{\mu\mu} \Big(a^\dagger_{\mu\lambda}(\mathbf{p}) b^\dagger_{\mu\lambda}(-\mathbf{p}) + b_{\mu\lambda}(\mathbf{p}) a_{\mu\lambda}(\\
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+ \; & \mathrm{sgn}\, \lambda \Big[m_{ee} \left(a^{\dagger}_{e\lambda}(\mathbf{p}) b^{\dagger}_{e\lambda}(-\mathbf{p}) + b_{e\lambda}(\mathbf{p}) a_{e\lambda}(-\mathbf{p}) \right) + m_{\mu\mu} \Big(a^{\dagger}_{\mu\lambda}(\mathbf{p}) b^{\dagger}_{\mu\lambda}(-\mathbf{p}) + b_{\mu\lambda}(\mathbf{p}) a_{\mu\lambda}(\\
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\end{aligned}
$$

- Diagonal form:

$$
H = \int d^3 p \sum_{\lambda, i=1,2} E_{i\mathrm{p}} \left[A^{\dagger}_{i\lambda}(\mathbf{p}) A_{i\lambda}(\mathbf{p}) + B^{\dagger}_{i\lambda}(\mathbf{p}) B_{i\lambda}(\mathbf{p}) \right], \qquad E_{i\mathrm{p}} = \sqrt{\mathrm{p}^2 + m_i^2}
$$

- The eigenstates of the diagonal Hamiltonian are the physical particle states (Bogoliubov quasiparticles).

Three sets of canonical fields:

$\psi_{\nu_l}(x)$, $l = e, \mu$ massless,
$a_{l\lambda}(\mathbf{p}), b_{l\lambda}(\mathbf{p})$

$\psi_{\nu_i}(x)$, $i = 1, 2$ massless,
$a_{i\lambda}(\mathbf{p}), b_{i\lambda}(\mathbf{p})$

Two (orthogonal) vacua:

$|0\rangle$ non-physical
$a_{l\lambda}(\mathbf{p})|0\rangle = b_{l\lambda}(\mathbf{p})|0\rangle = 0$

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- Bogoliubov transformations between the "massless" and "massive" operators:

$$A_{i\lambda}(\mathbf{p}) = \alpha_{i\mathrm{p}} a_{i\lambda}(\mathbf{p}) + \beta_{i\mathrm{p}} b^{\dagger}_{i\lambda}(-\mathbf{p}), \quad i = 1, 2,$$

$$B_{i\lambda}(\mathbf{p}) = \alpha_{i\mathrm{p}} b_{i\lambda}(\mathbf{p}) - \beta_{i\mathrm{p}} a^{\dagger}_{i\lambda}(-\mathbf{p}), \quad \alpha_{i\mathrm{p}} = \sqrt{\frac{1}{2}\left(1 + \frac{\mathrm{p}}{E_{i\mathrm{p}}}\right)}, \beta_{i\mathrm{p}} = \mathrm{sgn}\,\lambda \sqrt{\frac{1}{2}\left(1 - \frac{\mathrm{p}}{E_{i\mathrm{p}}}\right)}$$

- Physical vacuum is a condensate of "Cooper-like pairs" of massless neutrino-antineutrino – coherent state!

$$|\Phi_0\rangle = \Pi_{i,\mathbf{p},\lambda} \left(\alpha_{i\mathrm{p}} - \beta_{i\mathrm{p}}\, a^{\dagger}_{i\lambda}(\mathbf{p}) b^{\dagger}_{i\lambda}(-\mathbf{p})\right) |0\rangle,$$

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such that

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- Massive neutrino states interpreted as Bogoliubov quasiparticles.

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- Oscillation amplitude is never zero!

$$\mathcal{A}_{\nu_e\to\nu_\mu} = \frac{1}{2}\sin 2\theta e^{-i\mathrm{p}t}\left[-\left(1-\frac{1}{4}\frac{m_1^2}{\mathrm{p}^2}\right)^2 e^{-i\frac{m_1^2}{2\mathrm{p}}t} + \left(1-\frac{1}{4}\frac{m_2^2}{\mathrm{p}^2}\right)^2 e^{-i\frac{m_2^2}{2\mathrm{p}}t}\right].$$

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- There is always a portion of muon neutrino in the electron neutrino and vice-versa.
- In the ultrarelativistic limit, one recover's Pontecorvo's oscillation probability:

$$P_{\nu_e\to\nu_\mu} = \sin^2 2\theta\sin^2\left(\frac{\Delta m^2}{4\mathrm{p}}t\right), \quad \Delta m^2 = m_2^2 - m_1^2.$$

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- Quantitatively significant differences for nonrelativistic neutrinos (see KATRIN and PTOLEMY experiments) and possibly for MSW effect (especially neutrinos in extreme conditions).
- First step towards elucidating

the mechanism of interaction (production and absorbtion) of oscillating particle states.

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