Supersymmetric theories and Graphene

Condensed matter physics from geometry

Antonio Gallerati

Politecnico di Torino - Dipartimento di Scienza Applicata e Tecnologia

Istituto Nazionale di Fisica Nucleare - Sez. TO

Outline

Outline

1 **Introduction**

- Graphene and high energy physics
- Mass gaps in graphene-like materials

Outline

POLITECNICO DI TORINO
DISAT

1. **Introduction**
   - Graphene and high energy physics
   - Mass gaps in graphene-like materials

2. **Geometrical top-down approach**
   - Holography
   - AVZ model
   - Microscopic from macroscopic

Outline

1 Introduction

- Graphene and high energy physics
- Mass gaps in graphene-like materials

2 Geometrical top-down approach

- Holography
- AVZ model
- Microscopic from macroscopic

3 Conclusions

- Obtained results and future developments

POLITECNICO DI TORINO

DISAT

Graphene: suitable framework to study what is believed to be, as close as possible, a quantum field in a curved space-time:

Graphene: suitable framework to study what is believed to be, as close as possible, a quantum field in a curved space-time:

- natural description of its electronic properties in terms of Dirac pseudoparticles;

Graphene: suitable framework to study what is believed to be, as close as possible, a quantum field in a curved space-time:

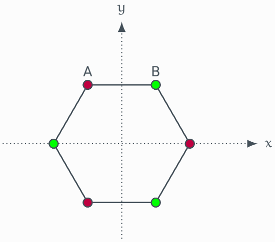
- natural description of its electronic properties in terms of Dirac pseudoparticles;
- possibility to study quasi-relativistic particle behaviour at sub-light speed regime;

Graphene: suitable framework to study what is believed to be, as close as possible, a quantum field in a curved space-time:

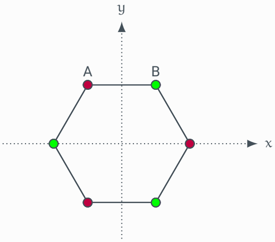
- natural description of its electronic properties in terms of Dirac pseudoparticles;
- possibility to study quasi-relativistic particle behaviour at sub-light speed regime;
- curved space configurations: possibility of new direct observation of quantum behaviour in the curved background of a solid state system.

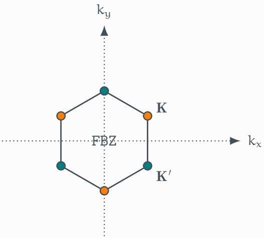
Graphene honeycomb lattice

Graphene honeycomb lattice

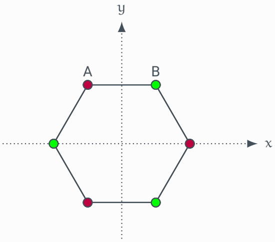


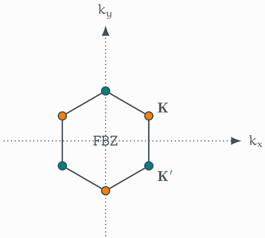
Graphene honeycomb lattice





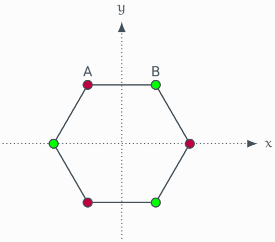
Graphene honeycomb lattice

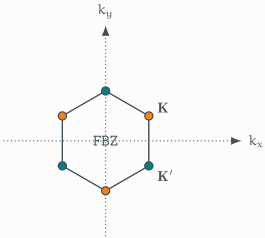




1. Pristine graphene :

Graphene honeycomb lattice

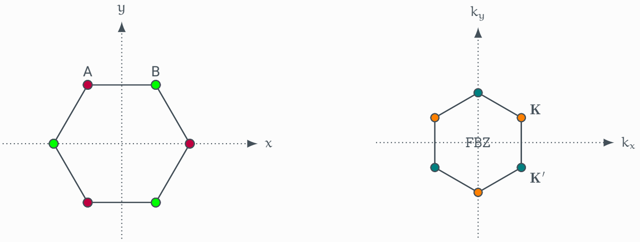




1. Pristine graphene : single-state per site honeycomb lattice, far apart ions

$$H_1 = t_1 \sum_{\langle i,j \rangle} c_i^\dagger \, c_j \; ;$$

Graphene honeycomb lattice

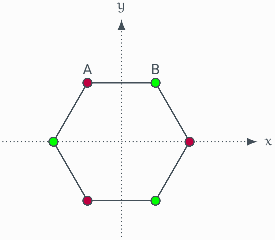


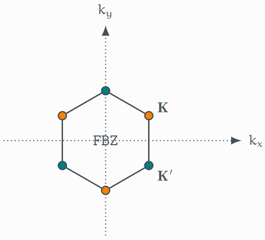
1. Pristine graphene : single-state per site honeycomb lattice, far apart ions

$$H_1 = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j \;;$$

- electrons can tunnel to their first neighbor atoms;

Graphene honeycomb lattice





1. Pristine graphene : single-state per site honeycomb lattice, far apart ions

$$H_1 = t_1 \sum_{\langle i,j \rangle} c_i^\dagger c_j \; ;$$

- electrons can tunnel to their first neighbor atoms;
- robust massless formulation, protected, to a certain extent, by combination of parity and time-reversal symmetry.

2. Haldane model:

2. Haldane model: breaking model symmetries with second-neighbor hopping terms plus on-site staggered potential $\pm M$

$$H = H_1 + H_2 = H_1 + t_2 \sum_{\langle i,j \rangle_2} e^{i\,\varphi\,\alpha_{ij}}\, c_i^\dagger c_j + \epsilon_i\, M \sum_i c_i^\dagger c_i\,, \qquad \epsilon_i = \pm 1;$$

POLITECNICO DI TORINO

DISAT

2. Haldane model: breaking model symmetries with second-neighbor hopping terms plus on-site staggered potential $\pm M$

$$H = H_1 + H_2 = H_1 + t_2 \sum_{\langle i,j \rangle_2} e^{i\,\varphi\,\alpha_{ij}}\, c_i^\dagger c_j + \epsilon_i\, M \sum_i c_i^\dagger c_i\,, \qquad \epsilon_i = \pm 1;$$

- second-neighbor hopping terms with unimodular phase factor, the phase sign depending on the ‘chirality’ of the electron path;

2. Haldane model: breaking model symmetries with second-neighbor hopping terms plus on-site staggered potential $\pm M$

$$H = H_1 + H_2 = H_1 + t_2 \sum_{\langle i,j \rangle_2} e^{i\,\varphi\,\alpha_{ij}}\, c_i^\dagger c_j + \epsilon_i\, M \sum_i c_i^\dagger c_i\,, \qquad \epsilon_i = \pm 1;$$

- second-neighbor hopping terms with unimodular phase factor, the phase sign depending on the ‘chirality’ of the electron path;
- parity breaking terms that spoil sublattices equivalence;

2. Haldane model: breaking model symmetries with second-neighbor hopping terms plus on-site staggered potential $\pm M$

$$H = H_1 + H_2 = H_1 + t_2 \sum_{\langle i,j \rangle_2} e^{i\,\varphi\,\alpha_{ij}}\, c_i^\dagger c_j + \epsilon_i\, M \sum_i c_i^\dagger c_i\,, \qquad \epsilon_i = \pm 1\,;$$

- second-neighbor hopping terms with unimodular phase factor, the phase sign depending on the ‘chirality’ of the electron path;
- parity breaking terms that spoil sublattices equivalence;
- fermion masses in the two inequivalent valleys:

$$m_{\mathbf{K}} = M - 3\sqrt{3}\, t_2 \sin\varphi\,, \qquad m_{\mathbf{K'}} = M + 3\sqrt{3}\, t_2 \sin\varphi\,.$$

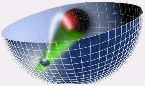
POLITECNICO DI TORINO
DISAT

Holographic correspondence

POLITECNICO DI TORINO DISAT

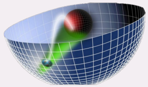
Holographic correspondence

Infer properties of a strongly coupled D dimensional quantum models defined at the boundary from a classical $\mathrm{D}+1$ dimensional (AdS) gravity theory in the bulk;



Holographic correspondence

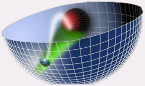
Infer properties of a strongly coupled D dimensional quantum models defined at the boundary from a classical $\mathrm{D}+1$ dimensional (AdS) gravity theory in the bulk;



- at low energy there is a one-to-one correspondence between quantum operators in the boundary and fields of the bulk gravity theory;

Holographic correspondence

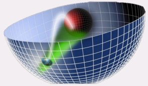
Infer properties of a strongly coupled D dimensional quantum models defined at the boundary from a classical $\mathrm{D}+1$ dimensional (AdS) gravity theory in the bulk;



- at low energy there is a one-to-one correspondence between quantum operators in the boundary and fields of the bulk gravity theory;
- boundary conditions for gravity fields in $\mathrm{D}+1$ dimensional act as sources for operators of the D dimensional QCFT.

Holographic correspondence

Infer properties of a strongly coupled D dimensional quantum models defined at the boundary from a classical $\mathrm{D}+1$ dimensional (AdS) gravity theory in the bulk;

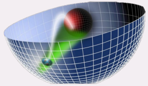


- at low energy there is a one-to-one correspondence between quantum operators in the boundary and fields of the bulk gravity theory;
- boundary conditions for gravity fields in $\mathrm{D}+1$ dimensional act as sources for operators of the D dimensional QCFT.

1. **Shot** : write down a $\mathrm{D}=4$ (super)gravity model whose $\mathrm{D}=3$ boundary features an effective theory for a spin-1/2 fermion defined on a curved geometry;

Holographic correspondence

Infer properties of a strongly coupled D dimensional quantum models defined at the boundary from a classical $\mathrm{D}+1$ dimensional (AdS) gravity theory in the bulk;



- at low energy there is a one-to-one correspondence between quantum operators in the boundary and fields of the bulk gravity theory;
- boundary conditions for gravity fields in $\mathrm{D}+1$ dimensional act as sources for operators of the D dimensional QCFT.

1. **Shot** : write down a $\mathrm{D}=4$ (super)gravity model whose $\mathrm{D}=3$ boundary features an effective theory for a spin-1/2 fermion defined on a curved geometry;
2. check if the generated $\mathrm{D}=3$ spin-1/2 fermion can be identified with Dirac electronic charge carriers in graphene.

POLITECNICO DI TORINO

DISAT

The procedure in a nutshell [JHEP 01 (2020) 084] :

POLITECNICO DI TORINO

DISAT

The procedure in a nutshell [JHEP 01 (2020) 084] :

- start from a maximally supersymmetric AdS_4 vacuum of an $\mathcal{N}$–extended supergravity and consider fluctuations on this background;

The procedure in a nutshell [JHEP 01 (2020) 084] :

- start from a maximally supersymmetric AdS_4 vacuum of an $\mathcal{N}$−extended supergravity and consider fluctuations on this background;

- write a covariant form for the Maurer-Cartan equations at the UV-boundary located at radial infinity;

POLITECNICO DI TORINO DISAT

The procedure in a nutshell [JHEP 01 (2020) 084] :

- start from a maximally supersymmetric AdS_4 vacuum of an $\mathcal{N}-$extended supergravity and consider fluctuations on this background;
- write a covariant form for the Maurer-Cartan equations at the UV-boundary located at radial infinity;
- consider the boundary limit, defining the correct form for the boundary vielbein, spinors and connections;

The procedure in a nutshell [JHEP 01 (2020) 084] :

- start from a maximally supersymmetric AdS_4 vacuum of an $\mathcal{N}$−extended supergravity and consider fluctuations on this background;
- write a covariant form for the Maurer-Cartan equations at the UV-boundary located at radial infinity;
- consider the boundary limit, defining the correct form for the boundary vielbein, spinors and connections;
- the resulting $\mathrm{D} = 3$ world volume describes a generalized AVZ model featuring a local NYW symmetry.

POLITECNICO DI TORINO

DISAT

AVZ model

POLITECNICO DI TORINO
DISAT

AVZ model

- Effective theory in 2+1 dimensions for a spin-1/2 fermion χ;

Geometrical top-down approach

AVZ model

- Effective theory in 2+1 dimensions for a spin-1/2 fermion χ;
- generically defined on a curved geometry and minimally coupled to the background gravity;

AVZ model

- Effective theory in 2+1 dimensions for a spin-1/2 fermion χ;
- generically defined on a curved geometry and minimally coupled to the background gravity;
- the system exhibits an unconventional form of supersymmetry based on a suitable graded Lie algebra;

POLITECNICO DI TORINO
DISAT

AVZ model

- Effective theory in 2+1 dimensions for a spin-1/2 fermion χ;
- generically defined on a curved geometry and minimally coupled to the background gravity;
- the system exhibits an unconventional form of supersymmetry based on a suitable graded Lie algebra;
- Dirac fermion fields of the theory may describe the wave functions of the π-electrons in graphene at the Dirac points **K**, **K'**.

Geometrical top-down approach

AVZ model

- Effective theory in 2+1 dimensions for a spin-1/2 fermion χ;
- generically defined on a curved geometry and minimally coupled to the background gravity;
- the system exhibits an unconventional form of supersymmetry based on a suitable graded Lie algebra;
- Dirac fermion fields of the theory may describe the wave functions of the π-electrons in graphene at the Dirac points **K**, **K'**.

☑ The AVZ model can be (holographically) realized as the boundary theory of a $\mathrm{D} = 4$ supergravity for an AdS_4 spacetime;

Geometrical top-down approach

AVZ model

- Effective theory in 2+1 dimensions for a spin-1/2 fermion χ;
- generically defined on a curved geometry and minimally coupled to the background gravity;
- the system exhibits an unconventional form of supersymmetry based on a suitable graded Lie algebra;
- Dirac fermion fields of the theory may describe the wave functions of the π-electrons in graphene at the Dirac points **K**, **K'**.

☑ The AVZ model can be (holographically) realized as the boundary theory of a $D = 4$ supergravity for an AdS_4 spacetime;

☑ an explicit curved space Dirac equation for the charge carriers is obtained;

POLITECNICO DI TORINO
DISAT

AVZ model

- Effective theory in 2+1 dimensions for a spin-1/2 fermion χ;
- generically defined on a curved geometry and minimally coupled to the background gravity;
- the system exhibits an unconventional form of supersymmetry based on a suitable graded Lie algebra;
- Dirac fermion fields of the theory may describe the wave functions of the π-electrons in graphene at the Dirac points **K**, **K'**.

☑ The AVZ model can be (holographically) realized as the boundary theory of a $\mathrm{D} = 4$ supergravity for an AdS_4 spacetime;

☑ an explicit curved space Dirac equation for the charge carriers is obtained;

☑ fermion masses depend on the geometrical parameters (torsion) of the considered spacetime.

POLITECNICO DI TORINO DISAT

Unconventional model :

Unconventional model : peculiar ansatz for the fermionic gauge fields

$$\Psi^A = i\,\gamma_i\, e^i\, \chi^A\,,$$

expressed in terms of the vielbein and spin 1/2 fields χ.

The AVZ model

Unconventional model : peculiar ansatz for the fermionic gauge fields

$$\Psi^A = i\gamma_i e^i \chi^A ,$$

expressed in terms of the vielbein and spin 1/2 fields χ .

- The structure of the algebra is given in terms of a connection $\mathbb{A} = \vartheta^i \otimes E_i$ satisfying Maurer-Cartan equations $d\mathbb{A} + \mathbb{A} \wedge \mathbb{A} = 0$;

The AVZ model

Unconventional model: peculiar ansatz for the fermionic gauge fields

$$\Psi^A = i\gamma_i \, e^i \, \chi^A \,,$$

expressed in terms of the vielbein and spin 1/2 fields χ.

- The structure of the algebra is given in terms of a connection $\mathbb{A} = \vartheta^i \otimes E_i$ satisfying Maurer-Cartan equations $d\mathbb{A} + \mathbb{A} \wedge \mathbb{A} = 0$;
- the construction leads to the description of a propagating charged fermion satisfying a Dirac equation;

The AVZ model

Unconventional model : peculiar ansatz for the fermionic gauge fields

$$\Psi^A = i\,\gamma_i\, e^i\,\chi^A\,,$$

expressed in terms of the vielbein and spin 1/2 fields χ .

- The structure of the algebra is given in terms of a connection $\mathbb{A} = \vartheta^i \otimes \mathrm{E}_i$ satisfying Maurer-Cartan equations $\mathrm{d}\mathbb{A} + \mathbb{A} \wedge \mathbb{A} = 0$;
- the construction leads to the description of a propagating charged fermion satisfying a Dirac equation;
- the model can describe graphene electronic charge carriers in the vicinity of Dirac points, in a lattice with non-vanishing curvature and torsion.

We started from an $\mathcal{N}$-extended supergravity model featuring an AdS_4 vacuum with $OSp(\mathcal{N}|4)$ symmetry and then related it to an $OSp(p,2)_+ \times OSp(q,2)_-$ theory at the boundary, with $p + q = \mathcal{N}$.

We started from an $\mathcal{N}$-extended supergravity model featuring an AdS_4 vacuum with $OSp(\mathcal{N}|4)$ symmetry and then related it to an $OSp(p,2)_+ \times OSp(q,2)_-$ theory at the boundary, with $p+q=\mathcal{N}$.

Let us now restrict to the $p=q$ case :

We started from an $\mathcal{N}$-extended supergravity model featuring an AdS_4 vacuum with $\mathrm{OSp}(\mathcal{N}|4)$ symmetry and then related it to an $\mathrm{OSp}(p,2)_+ \times \mathrm{OSp}(q,2)_-$ theory at the boundary, with $p + q = \mathcal{N}$.

Let us now restrict to the $p = q$ case :

- parity is a discrete symmetry of the whole theory;

We started from an $\mathcal{N}$-extended supergravity model featuring an AdS_4 vacuum with $\mathrm{OSp}(\mathcal{N}|4)$ symmetry and then related it to an $\mathrm{OSp}(p,2)_+ \times \mathrm{OSp}(q,2)_-$ theory at the boundary, with $p + q = \mathcal{N}$.

Let us now restrict to the $p = q$ case :

- parity is a discrete symmetry of the whole theory;
- the AdS_4 boundary is reached in the limit $r \to \infty$, where the leading term of the $D = 4$ vielbein is defined in terms of the $D = 3$ vielbein

$$E^{i}_{\pm} = \pm \frac{1}{2} \left(\frac{r}{\ell} \right)^{\pm 1} E^{i} + \dots$$

We started from an $\mathcal{N}$-extended supergravity model featuring an AdS_4 vacuum with $OSp(\mathcal{N}|4)$ symmetry and then related it to an $OSp(p,2)_+ \times OSp(q,2)_-$ theory at the boundary, with $p + q = \mathcal{N}$.

Let us now restrict to the $p = q$ case :

- parity is a discrete symmetry of the whole theory;

- the AdS_4 boundary is reached in the limit $r \to \infty$, where the leading term of the $D = 4$ vielbein is defined in terms of the $D = 3$ vielbein

$$E^i_\pm = \pm\frac{1}{2}\left(\frac{r}{\ell}\right)^{\pm 1} E^i + \dots \qquad \text{with} \quad E^i = f\, e^i ;$$

POLITECNICO DI TORINO DISAT

We started from an $\mathcal{N}$-extended supergravity model featuring an AdS_4 vacuum with $OSp(\mathcal{N}|4)$ symmetry and then related it to an $OSp(p,2)_+ \times OSp(q,2)_-$ theory at the boundary, with $p + q = \mathcal{N}$.

Let us now restrict to the $p = q$ case :

- parity is a discrete symmetry of the whole theory;
- the AdS_4 boundary is reached in the limit $r \to \infty$, where the leading term of the $D = 4$ vielbein is defined in terms of the $D = 3$ vielbein

$$E^i_\pm = \pm\frac{1}{2}\left(\frac{r}{\ell}\right)^{\pm 1} E^i + \dots \qquad \text{with} \quad E^i = f\, e^i \, ;$$

- the $+$ and $-$ sectors of the theory can be naturally interpreted as related to the graphene Dirac points **K, K’** ;

- the *torsion* can be explicitly written as

$$T^i_{\pm} = \mathcal{D}_\Omega e^i = \beta\, e^i + \tau_{\pm}\, \varepsilon^{ijk} e_j \wedge e_k \,,$$

- the *torsion* can be explicitly written as

$$T^i_{\pm} = \mathcal{D}_\Omega e^i = \beta\, e^i + \tau_{\pm}\, \varepsilon^{ijk} e_j \wedge e_k \,,$$

where $\tau_{\pm} = \tau \mp 2\,\frac{f}{\ell}$ is written in terms of geometric quantities;

- the *torsion* can be explicitly written as

$$T^i_{\pm} = \mathcal{D}_{\Omega} e^i = \beta\, e^i + \tau_{\pm}\, \varepsilon^{ijk} e_j \wedge e_k\,,$$

where $\tau_{\pm} = \tau \mp 2\,\frac{f}{\ell}$ is written in terms of geometric quantities;

- the *Dirac equation* has the form

$$\not{\mathcal{D}}\, \chi_{\pm} = -\frac{3}{2}\, i\, \tau_{\pm}\, \chi_{\pm}$$

- the *torsion* can be explicitly written as

$$T^i_\pm = \mathcal{D}_\Omega e^i = \beta\, e^i + \tau_\pm\, \varepsilon^{ijk} e_j \wedge e_k\,,$$

where $\tau_\pm = \tau \mp 2\,\frac{f}{\ell}$ is written in terms of geometric quantities;

- the *Dirac equation* has the form

$$\not{\mathcal{D}}\, \chi_\pm = -\frac{3}{2}\, i\, \tau_\pm\, \chi_\pm \quad \Rightarrow \quad m_\pm = \frac{3}{2}\, \tau_\pm$$

- the *torsion* can be explicitly written as

$$T^i_{\pm} = \mathcal{D}_{\Omega} e^i = \beta\, e^i + \tau_{\pm}\, \varepsilon^{ijk} e_j \wedge e_k\,,$$

where $\tau_{\pm} = \tau \mp 2\,\frac{f}{\ell}$ is written in terms of geometric quantities;

- the *Dirac equation* has the form

$$\not{\mathcal{D}}\,\chi_{\pm} = -\frac{3}{2}\, i\, \tau_{\pm}\, \chi_{\pm} \quad \Rightarrow \quad m_{\pm} = \frac{3}{2}\,\tau_{\pm} = \frac{3}{2}\,\tau \mp 3\,\frac{f}{\ell}\,;$$

- the *torsion* can be explicitly written as

$$T^i_{\pm} = \mathcal{D}_{\Omega} e^i = \beta\, e^i + \tau_{\pm}\, \varepsilon^{ijk} e_j \wedge e_k \,,$$

where $\tau_{\pm} = \tau \mp 2\,\frac{f}{\ell}$ is written in terms of geometric quantities;

- the *Dirac equation* has the form

$$\not{\mathcal{D}}\,\chi_{\pm} = -\frac{3}{2}\, i\, \tau_{\pm}\, \chi_{\pm} \quad \Rightarrow \quad m_{\pm} = \frac{3}{2}\,\tau_{\pm} = \frac{3}{2}\,\tau \mp 3\,\frac{f}{\ell}\,;$$

- the choice for the boundary spin connection can be labeled by $\lambda \in \mathbb{R}$ that, in turns, yields for the τ parameter

$$\tau = \frac{\lambda}{\ell}\,(f - 1)$$

POLITECNICO DI TORINO DISAT

- the *torsion* can be explicitly written as

$$T^i_{\pm} = \mathcal{D}_{\Omega} e^i = \beta\, e^i + \tau_{\pm}\, \varepsilon^{ijk} e_j \wedge e_k \,,$$

where $\tau_{\pm} = \tau \mp 2\,\frac{f}{\ell}$ is written in terms of geometric quantities;

- the *Dirac equation* has the form

$$\not{D}\chi_{\pm} = -\frac{3}{2}\, i\, \tau_{\pm}\, \chi_{\pm} \quad \Rightarrow \quad m_{\pm} = \frac{3}{2}\tau_{\pm} = \frac{3}{2}\tau \mp 3\,\frac{f}{\ell}\,;$$

- the choice for the boundary spin connection can be labeled by $\lambda \in \mathbb{R}$ that, in turns, yields for the τ parameter

$$\tau = \frac{\lambda}{\ell}(f-1) \quad \Rightarrow \quad \tau_{\pm} = \frac{1}{\ell}\big(\lambda(f-1) \mp f\big)\,.$$

- the *torsion* can be explicitly written as

$$T^i_{\pm} = \mathcal{D}_{\Omega} e^i = \beta\, e^i + \tau_{\pm}\, \varepsilon^{ijk} e_j \wedge e_k\,,$$

where $\tau_{\pm} = \tau \mp 2\,\frac{f}{\ell}$ is written in terms of geometric quantities;

- the *Dirac equation* has the form

$$\not{\mathcal{D}}\,\chi_{\pm} = -\frac{3}{2}\, i\, \tau_{\pm}\, \chi_{\pm} \quad \Rightarrow \quad m_{\pm} = \frac{3}{2}\,\tau_{\pm} = \frac{3}{2}\,\tau \mp 3\,\frac{f}{\ell}\,;$$

- the choice for the boundary spin connection can be labeled by $\lambda \in \mathbb{R}$ that, in turns, yields for the τ parameter

$$\tau = \frac{\lambda}{\ell}\,(f-1) \quad \Rightarrow \quad \tau_{\pm} = \frac{1}{\ell}\,(\lambda(f-1) \mp f)\,.$$

[JHEP 01 (2020) 084]

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, U^{-1}\chi$, one finds:

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, U^{-1} \chi$, one finds:

- fermion masses explicitly depend on the geometry (torsion) of the three-dimensional spacetime $m_{\pm} = \frac{3}{2} \tau_{\pm}$;

POLITECNICO DI TORINO DISAT

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, U^{-1}\chi$, one finds:

- fermion masses explicitly depend on the geometry (torsion) of the three-dimensional spacetime $m_{\pm} = \frac{3}{2}\tau_{\pm}$;

- well-established top-down approach, in that the effective $D = 3$ theory derived at the boundary originates from a well defined effective supergravity in the bulk;

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, U^{-1}\chi$, one finds:

- fermion masses explicitly depend on the geometry (torsion) of the three-dimensional spacetime $m_\pm = \frac{3}{2}\tau_\pm$;
- well-established top-down approach, in that the effective $D = 3$ theory derived at the boundary originates from a well defined effective supergravity in the bulk;
- the fermions of the theory may describe the physics of graphene charge carriers at the Dirac points **K**, **K'**; in particular, it is possible to generate Semenoff and Haldane-type effective masses:

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, U^{-1}\chi$, one finds:

- fermion masses explicitly depend on the geometry (torsion) of the three-dimensional spacetime $m_\pm = \frac{3}{2}\tau_\pm$;
- well-established top-down approach, in that the effective $D = 3$ theory derived at the boundary originates from a well defined effective supergravity in the bulk;
- the fermions of the theory may describe the physics of graphene charge carriers at the Dirac points **K**, **K'**; in particular, it is possible to generate Semenoff and Haldane-type effective masses:

$$m_\pm = \frac{3}{2}\tau_\pm = \frac{3}{2}\tau \mp 3\,\frac{f}{\ell}$$

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, U^{-1}\chi$, one finds:

- fermion masses explicitly depend on the geometry (torsion) of the three-dimensional spacetime $m_{\pm} = \frac{3}{2}\tau_{\pm}$;
- well-established top-down approach, in that the effective $D = 3$ theory derived at the boundary originates from a well defined effective supergravity in the bulk;
- the fermions of the theory may describe the physics of graphene charge carriers at the Dirac points **K**, **K'**; in particular, it is possible to generate Semenoff and Haldane-type effective masses:

$$m_{\pm} = \frac{3}{2}\tau_{\pm} = \frac{3}{2}\tau \mp 3\,\frac{f}{\ell} \quad \Leftrightarrow \quad m_{\mathbf{K},\mathbf{K}'} = M \mp 3\sqrt{3}\, t_2 \sin\varphi$$

POLITECNICO DI TORINO
DISAT

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, U^{-1}\chi$, one finds:

- fermion masses explicitly depend on the geometry (torsion) of the three-dimensional spacetime $m_\pm = \frac{3}{2}\tau_\pm$;
- well-established top-down approach, in that the effective $D = 3$ theory derived at the boundary originates from a well defined effective supergravity in the bulk;
- the fermions of the theory may describe the physics of graphene charge carriers at the Dirac points **K**, **K'**; in particular, it is possible to generate Semenoff and Haldane-type effective masses:

$$m_\pm = \frac{3}{2}\tau_\pm = \frac{3}{2}\tau \mp 3\,\frac{f}{\ell} \quad \Leftrightarrow \quad m_{\mathbf{K},\mathbf{K'}} = M \mp 3\sqrt{3}\,t_2 \sin\varphi$$

$$\Longrightarrow$$

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, U^{-1}\chi$, one finds:

- fermion masses explicitly depend on the geometry (torsion) of the three-dimensional spacetime $m_\pm = \frac{3}{2}\tau_\pm$;
- well-established top-down approach, in that the effective $D = 3$ theory derived at the boundary originates from a well defined effective supergravity in the bulk;
- the fermions of the theory may describe the physics of graphene charge carriers at the Dirac points **K**, **K'**; in particular, it is possible to generate Semenoff and Haldane-type effective masses:

$$m_\pm = \frac{3}{2}\tau_\pm = \frac{3}{2}\tau \mp 3\,\frac{f}{\ell} \quad \Leftrightarrow \quad m_{\mathbf{K},\mathbf{K}'} = M \mp 3\sqrt{3}\, t_2 \sin\varphi$$

$$\Longrightarrow \qquad M \leftrightarrow \frac{3}{2}\tau\,,$$

If we describe the single electron wave function of graphene in terms of a two-component Dirac spinor, $\psi = \sqrt{\frac{2}{\ell}}\, \mathrm{U}^{-1}\chi$, one finds:

- fermion masses explicitly depend on the geometry (torsion) of the three-dimensional spacetime $m_{\pm} = \frac{3}{2}\tau_{\pm}$;
- well-established top-down approach, in that the effective $D = 3$ theory derived at the boundary originates from a well defined effective supergravity in the bulk;
- the fermions of the theory may describe the physics of graphene charge carriers at the Dirac points **K**, **K'**; in particular, it is possible to generate Semenoff and Haldane-type effective masses:

$$m_{\pm} = \frac{3}{2}\tau_{\pm} = \frac{3}{2}\tau \mp 3\,\frac{f}{\ell} \quad \Leftrightarrow \quad m_{\mathbf{K},\mathbf{K}'} = M \mp 3\sqrt{3}\, t_2 \sin\varphi$$

$$\Longrightarrow \qquad M \leftrightarrow \frac{3}{2}\tau\,, \qquad \sqrt{3}\, t_2 \sin\varphi \leftrightarrow \frac{f}{\ell}\,.$$

Conclusions

Conclusions

1. We have seen how the extension of unconventional supersymmetry to the superalgebra can be instrumental in describing the electronic properties of graphene-like systems in the **K** and **K’** valleys and thus physical situations in which the symmetry between them is broken.

Conclusions

1. We have seen how the extension of unconventional supersymmetry to the superalgebra can be instrumental in describing the electronic properties of graphene-like systems in the **K** and **K'** valleys and thus physical situations in which the symmetry between them is broken.

2. Top down approach: the effective $\mathrm{D} = 3$ model that we derive at the boundary of AdS_4 originates from a well defined supergravity theory defined in the bulk.

Conclusions

POLITECNICO DI TORINO
DISAT

1. We have seen how the extension of unconventional supersymmetry to the superalgebra can be instrumental in describing the electronic properties of graphene-like systems in the **K** and **K'** valleys and thus physical situations in which the symmetry between them is broken.

2. Top down approach: the effective $\mathrm{D} = 3$ model that we derive at the boundary of AdS_4 originates from a well defined supergravity theory defined in the bulk.

3. Identification between Haldane parameters and geometrical quantities suggests a relation between the ratio f/ℓ and the Berry phase parameters.

Conclusions

POLITECNICO DI TORINO
DISAT

1. We have seen how the extension of unconventional supersymmetry to the superalgebra can be instrumental in describing the electronic properties of graphene-like systems in the **K** and **K'** valleys and thus physical situations in which the symmetry between them is broken.

2. Top down approach: the effective $\mathrm{D} = 3$ model that we derive at the boundary of AdS_4 originates from a well defined supergravity theory defined in the bulk.

3. Identification between Haldane parameters and geometrical quantities suggests a relation between the ratio $\mathtt{f}/\ell$ and the Berry phase parameters.

4. The embedding of the effective description in an $\mathcal{N}$-extended four-dimensional supergravity sets the stage of a holographic analysis which will be pursued in a future work.

Thank you for listening!