Electroweak precision pseudo-observables at the e^+e^- Z-resonance region

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NATIONAL SCIENCE CENTRE
POLAND

Outline

- Based on - literature

1. Electrowek pseudo-observables (EWPOs) - formalism, importance
2. This talk
   - Discussed
   - Not discussed
3. Motivation - precision measurements at future colliders
4. Complete 2-loop results
5. Theoretical errors: Needs for EWPOs beyond NNLO
6. 3-loop calculations: Needs for new methods and tools
7. Summary and Outlook
8. Backup slides

"Report 1", https://e-publishing.cern.ch/index.php/CYRM/issue/view/89

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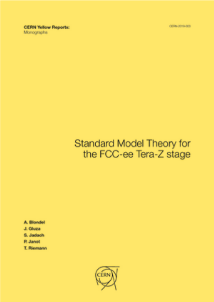


Report on the Mini Workshop Precision EW and QCD Calculations for the FCC Studies: Methods and Tools, 12–13 January 2018, CERN, Geneva.

DOI: https://doi.org/10.23731/CYRM-2019-003

Other important works for this talk

- "Report 2", A. Blondel et al., "Theory for the FCC-ee : Report on the 11th FCC-ee Workshop Theory and Experiments",
  https://e-publishing.cern.ch/index.php/CYRM/issue/view/110
- "Report 3", Input to the European Strategy Particle Physics 2018-2020
  A. Blondel et al., "Theory Requirements and Possibilities for the FCC-ee and other Future High Energy and Precision Frontier Lepton Colliders",
  https://inspirehep.net/literature/1712839
- "Report 4"
  A. Freitas et al., "Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee",
  https://arxiv.org/abs/1906.05379
- "Report 5", Input to the European Strategy Particle Physics 2018-2020
  A. Blondel et al., "FCC-ee: Your Questions Answered",
  https://inspirehep.net/literature/1738661

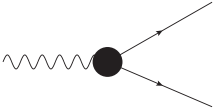
Published results on EWPOs in the SM @NNLO

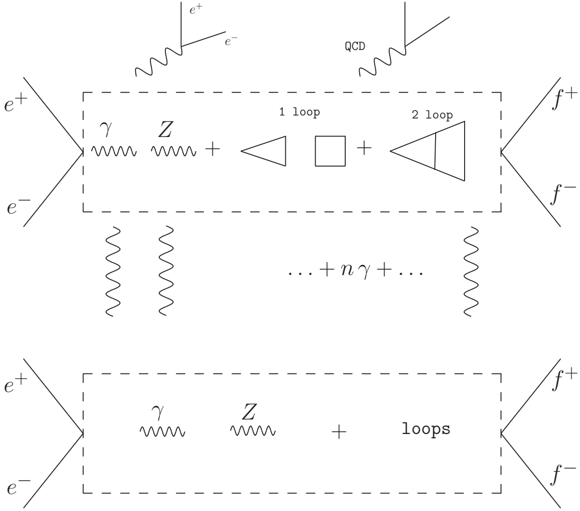
Complete corrections $\Delta r, \sin^2\theta^l_{\text{eff}}$:	Freitas, Hollik, Walter, Weiglein: '00
	Awramik,Czakon: '02,Onishchenko,Veretin: '02
	Awramik,Czakon,Freitas,Weiglein: '04
	Awramik,Czakon,Freitas: '06
	Hollik,Meier,Uccirati: '05,'07
	Degrassi,Gambino,Giardino: '14
Fermionic corrections $\sin^2\theta^b_{\text{eff}}, a_f, v_f$:	Awramik,Czakon,Freitas,Kniehl: '09
	Czarnecki,Kühn: '96
	Harlander,Seidensticker,Steinhauser: '98
	Freitas: '13,'14 Freitas: '13,'14
Bosonic corrections: $\sin^2\theta^{\text{b}}_{\text{eff}}$:	Dubovyk, Freitas, JG, Riemann, Usovitsch PLB'16
Bosonic corrections: $\Gamma_Z, R_l, \ldots$:	Dubovyk, Freitas, JG, Riemann, Usovitsch '18,'19

$\longrightarrow$ Chen, Freitas, "Leading fermionic three-loop corrections to electroweak precision observables", https://arxiv.org/abs/2002.05845

Rough scheme for extracting the $Zf\bar{f}$ vertex and EW corrections [Unfolding]

How to extract FF?





EWPOs

$$
\begin{aligned}
\sigma^0_{\rm had} &= \sigma[e^+e^- \to \text{hadrons}]_{s=M_Z^2}, \\
\Gamma_Z &= \sum_f \Gamma[Z \to f\bar{f}], \\
R_\ell &= \frac{\Gamma[Z \to \text{hadrons}]}{\Gamma[Z \to \ell^+\ell^-]}, \quad \ell = e, \mu, \tau, \\
R_q &= \frac{\Gamma[Z \to q\bar{q}]}{\Gamma[Z \to \text{hadrons}]}, \quad q = u, d, s, c, b.
\end{aligned}
$$

The remaining EWPOs are cross section asymmetries, measured at the Z pole, e.g., forward-backward asymmetry

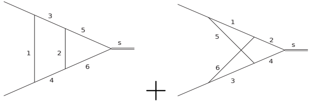
$$
A^f_{\rm FB} = \frac{\sigma_f\left[\theta < \frac{\pi}{2}\right] - \sigma_f\left[\theta > \frac{\pi}{2}\right]}{\sigma_f\left[\theta < \frac{\pi}{2}\right] + \sigma_f\left[\theta > \frac{\pi}{2}\right]},
$$

where θ is the scattering angle between the incoming e^- and the outgoing f.

EWPOs and Form Factors

fermionic,bosonic

$$V_\mu^{Zb\bar{b}} = \gamma_\mu[v_b(s) + a_b(s)\gamma_5] = \cdots +$$



$$+ \cdots$$

planar,non−planar

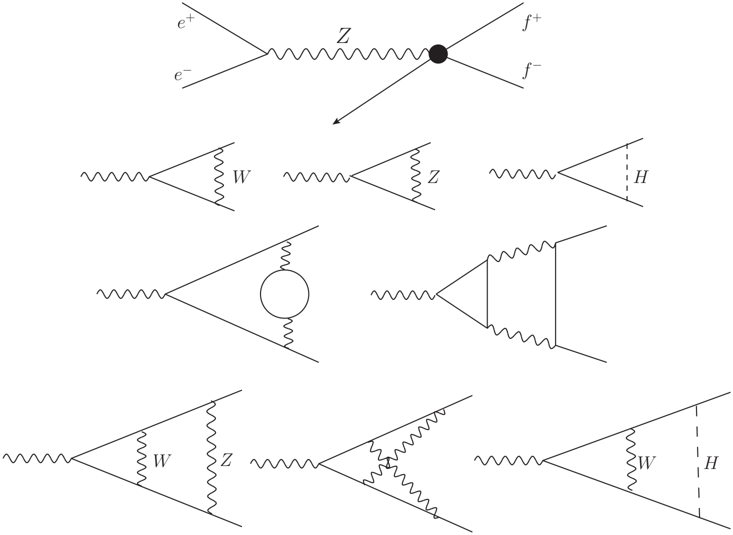
Note approximate factorization of weak couplings

$$A_{FB} = \frac{\left[\int_0^1 dcos\theta - \int_{-1}^0 dcos\theta\right] \frac{d\sigma}{dcos\theta}}{\sigma_T} \sim \overbrace{\frac{2a_e v_e}{a_e^2 + v_e^2}}^{A_e} \overbrace{\frac{2a_f v_f}{a_f^2 + v_f^2}}^{A_f} + \text{corrections}$$

$$A_f = \frac{2\Re e\frac{v_f}{a_f}}{1 + \left(\Re e\frac{v_f}{a_f}\right)^2} = \frac{1 - 4|Q_f|\sin^2\theta_{\rm eff}^{\rm f}}{1 - 4|Q_f|\sin^2\theta_{\rm eff}^{\rm f} + 8(Q_f\sin^2\theta_{\rm eff}^{\rm f})^2}, \quad \sin^2\theta_{\rm eff}^{\rm f} = F\left(\Re e\frac{v_f}{a_f}\right)$$

Rough scheme for extracting the $Zf\bar{f}$ vertex and EW corrections [loop corrections]

This talk.



General remarks on usefulness of EWPOs

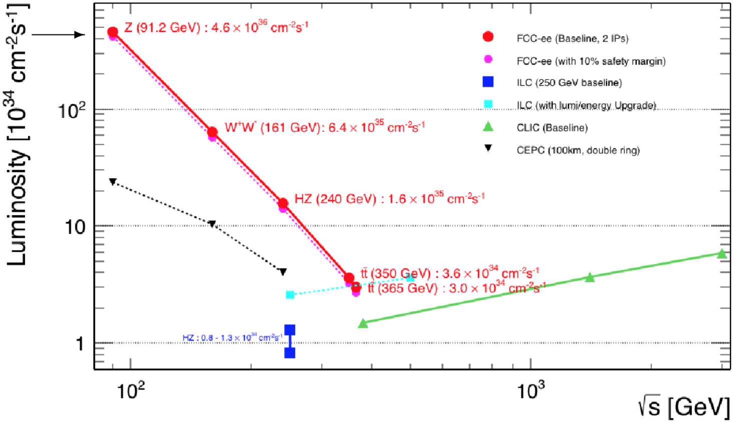
1. EWPOs encapsulate experimental data after extraction of well known and controllable QED and QCD effects, in a model-independent manner.
2. They provide a convenient bridge between real data and the predictions of the SM (or SM plus New Physics).
3. Contrary to raw experimental data (like differential crosssections), EWPOs are well suited for archiving and long term exploitation.
4. In particular archived EWPOscan be exploited over long periods of time for comparisons with steadily improving theoretical calculationsof the SM predictions, and for validations of the New Physics models beyond the SM.
5. They are also useful for comparison and combination of results from different experiments.

Not discussed in this talk

I will focus on 2- and 3-loop Z-boson vertex corrections, I will not talk about:

1. EWPOs and LHC/HL-LHC
   $\longrightarrow$ M. Chiesa, F. Piccinini and A. Vicini, "Direct determination of $\sin^2\theta^{\ell}_{eff}$ at hadron colliders", Phys. Rev. D **100** (2019) no.7, 071302 (scheme with G_μ, $\sin^2\theta^{\ell}_{eff}$, M_Z), see also E.Richter-Wąs:
   https://indico.cern.ch/event/801961/contributions/3361500/attachments/1823702/2984160/ERW_Durham_EWprecision__4April_v1.pdf
2. EWPOs and BSM
   See Sven Heinemeyer Snowmass 07.2020 talk
   "Electroweak Precision Observables and BSM Physics":
   https://indico.fnal.gov/event/43577/contributions/191539/attachments/131503/161060/sven.pdf
   $\longrightarrow$ How to fix beyond SM effects? EFT vs concrete models
3. Extraction of non-factorizable corrections, QED-resummations, MC generators, ...
   $\longrightarrow$ Staszek Jadach in "Report 1", https://doi.org/10.1140/epjc/s10052-019-7255-9

Motivation - precision of future colliders



See ICHEP poster, A. Blondel, "FCC-ee: Polarization and energy calibration"
https://indico.cern.ch/event/938611/contributions/3943373/attachments/2076416/3488354/Po
EPOL-Alain-v0.pdf

Z,W,H,t electroweak factories

Table: Run plan for FCC-ee in its baseline configuration with two experiments. The WW event numbers are given for the entirety of the FCC-ee r unning at and above the WW threshold.

Phase	Run duration (years)	Center-of-mass Energies (GeV)	Integrated Luminosity (ab^{-1})	Event Statistics
FCC-ee-Z	4	88-95	150	$3 \cdot 10^{12}$ **visible Z decays**
FCC-ee-W	2	158-162	12	10^8 WW events
FCC-ee-H	3	240	5	10^6 ZH events
FCC-ee-tt	5	345-365	1.5	10^6 $t\bar{t}$ even ts

Table from "Report 1" and FCC-ee CDR https://doi.org/10.1140/epjc/s10052-019-6904-3.

Precision. FCC-ee is the most demanding HEP project for theoretical SM calculations.

Expected precision in 2040

J. Gluza

❑ Conclusion of the 2018 Workshop

"We anticipate that, at the beginning of the FCC-ee campaign of precision measurements, the theory will be precise enough not to limit their physics interpretation. This statement is however conditional to sufficiently strong support by the physics community and the funding agencies, including strong training programmes".

- **Numerical evaluation with three-loops calculations:** arXiv:1901.02648

	$\delta\Gamma_Z$ [MeV]	δR_l $[10^{-4}]$	δR_b $[10^{-5}]$	$\delta \sin^{2,l}_{eff}\theta$ $[10^{-6}]$
Present EWPO theoretical uncertainties				
EXP-2018	2.3	250	66	160
TH-2018	0.4	60	10	45
EWPO theoretical uncertainties when FCC-ee will start				
EXP-FCC-ee	~~0.1~~ 0.025	10	$2 \div 6$	~~6~~ 3
TH-FCC-ee	0.07	7	3	7

- **500 person-years needed over 20 years – Recognized as strategic priority.**

How to get such precision?

Let's assume we unfolded EWPOs properly
And adjusted all corrections, taking into account non-factorizable effects.

Then we can calculate comfortable SM corrections Z-boson vertex corrections, order by order.

Complete NNLO results

- The standard model prediction for the effective weak mixing angle can be wri tten as

$$\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa_{\mathrm{b}})$$

ions.

- The bosonic electroweak two-loop corrections amount to

$$\boxed{\Delta\kappa_{\mathrm{b}}^{(\alpha^2,bos)} = -0.9855 \times 10^{-4}}$$

DFGRU, Phys.Lett. B762 (2016) 184

Collection of radiative corrections: Full stabilization at 10^{-4}!

Order	Value $[10^{-4}]$	Order	Value $[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
α^2_{ferm}	3.866	$\alpha_t\alpha_s^2$	-7.074
α^2_{bos}	**-0.9855**	$\alpha_t\alpha_s^3$	-1.196

$\pm 0.001 \xrightarrow{!}$

Table: *Comparison of different orders of radiative corrections to* $\Delta\kappa_{\mathrm{b}}$.

Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s *and* $\Delta\alpha$

- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]

Partial higher-order corrections

$\mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^2)$	Avdeev: 1994,Chetyrkin: 1995
$\mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^3)$	Schroder: 2005,Chetyrkin: 2006,Boughezal: 2006
$\mathcal{O}(\alpha^2\alpha_{\mathrm{t}})$ and $\mathcal{O}(\alpha_{\mathrm{t}}^3)$	vanderBij: 2000,Faisst: 2003

The 2-loops EWPOs results for $\mathcal{O}(\alpha_{\rm bos}^2)$, https://doi.org/10.1016/j.physletb.2018.06.037

	$\Gamma_{\rm Z}$ [GeV]	$\sigma^0_{\rm had}$ [nb]
Born	2.53601	41.6171
$+\ \mathcal{O}(\alpha)$	2.49770	41.4687
$+\ \mathcal{O}(\alpha\alpha_{\rm s})$	2.49649	41.4758
$+\ \mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2,\ \alpha_{\rm t}\alpha_{\rm s}^3,\ \alpha_{\rm t}^2\alpha_{\rm s},\ \alpha_{\rm t}^3)$	2.49560	41.4770
$+\ \mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	2.49441	41.4883
$+\ \mathcal{O}(\alpha_{\rm bos}^2)$	**[+0.34 MeV]=**2.49475	**[+1.3 pb]=**41.4896

Results for $\Gamma_{\rm Z}$ and $\sigma^0_{\rm had}$, with $M_{\rm W}$ calculated from G_μ using the same order of perturbation theory as indicated in each line.

The 2-loops EWPOs results for $\mathcal{O}(\alpha^2_{\mathrm{bos}})$, https://doi.org/10.1016/j.physletb.2018.06.037

	R_ℓ	R_c	R_b
Born	21.0272	0.17306	0.21733
$+\ \mathcal{O}(\alpha)$	20.8031	0.17230	0.21558
$+\ \mathcal{O}(\alpha\alpha_{\mathrm{s}})$	20.7963	0.17222	0.21593
$+\ \mathcal{O}(\alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^2,\ \alpha_{\mathrm{t}}\alpha_{\mathrm{s}}^3,\ \alpha_{\mathrm{t}}^2\alpha_{\mathrm{s}},\ \alpha_{\mathrm{t}}^3)$	20.7943	0.17222	0.21593
$+\ \mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	20.7512	0.17223	0.21580
$+\ \mathcal{O}(\alpha^2_{\mathrm{bos}})$	20.7516	0.17222	0.21585

Results for the ratios R_ℓ, R_c and R_b, with M_{W} calculat ed from G_μ to the same order as indicated in each line.

The 2-loops EWPOs results for $\mathcal{O}(\alpha^2_{\rm bos})$, https://doi.org/10.1016/j.physletb.2018.06.037

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
Born	81.142	160.096	371.141	292.445	369.56	2420.2
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_{\rm s})$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha^2_{\rm bos})$	**0.017**	**0.019**	**0.058**	**0.057**	**0.167**	**0.505**
$\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2,\ \alpha_{\rm t}\alpha_{\rm s}^3,\ \alpha_{\rm t}^2\alpha_{\rm s},\ \alpha_{\rm t}^3)$	0.038	0.059	0.191	0.170	**0.190**	1.20

1. 2016, estimation, bosonic NNLO $\sim 0 \pm 0.1$ MeV
   2018, exact result: 0.505 MeV

* Fixed values of M_W

Currently most precise prediction for $\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}}$

$$\begin{aligned}\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}} = & s_0 + d_1\,L_H + d_2\,L_H^2 + d_3\,\Delta_\alpha + d_4\,\Delta_t + d_5\,\Delta_t^2 \\ & + d_6\,\Delta_t\,L_H + d_7\,\Delta_{\alpha_{\mathrm{s}}} + d_8\,\Delta_t\,\Delta_{\alpha_{\mathrm{s}}} + d_9\,\Delta_Z\end{aligned} \qquad (1)$$

$$\begin{aligned} L_H &= \log\left(\frac{M_{\mathrm{H}}}{125.7\mathrm{GeV}}\right), \quad \Delta_t = \left(\frac{m_{\mathrm{t}}}{173.2\mathrm{GeV}}\right)^2 - 1, \quad \Delta_Z = \frac{M_{\mathrm{Z}}}{91.1876\mathrm{GeV}} - 1, \\ \Delta_\alpha &= \frac{\Delta\alpha}{0.0059} - 1, \quad \Delta_{\alpha_{\mathrm{s}}} = \frac{\alpha_{\mathrm{s}}}{0.1184} - 1. \end{aligned} \qquad (2)$$

$$\begin{aligned} s_0 &= 0.232704, \quad d_1 = 4.723\times10^{-4}, \quad d_2 = 1.97\times10^{-4}, \quad d_3 = 2.07\times10^{-2}, \\ & \quad d_4 = -9.733\times10^{-4}, \quad d_5 = 3.93\times10^{-4}, \quad d_6 = -1.38\times10^{-4}, \\ & \quad d_7 = 2.42\times10^{-4}, \quad d_8 = -8.10\times10^{-4}, \quad d_9 = -0.664. \end{aligned} \qquad (3)$$

- M_{W} is calculated from the Fermi constant G_μ [Awramik, et al., 2004]
- The deviations to the full calculation amount to average (maximal) 2×10^{-7} (1.3×10^{-6}), in the input parameter ranges.

DFGRU, Phys.Lett. B762 (2016) 184

Decreasing theoretical errors

Complicated subject, theoretical, parametric errors.
See:

1. "Report 1"
2. "Report 4"

Errors, a simple observation:

1. Lack of knowledge about HO corrections is a real pain, estimates even in the perturbative regime can differ substantially from concrete results.
2. Estimations for each next piece of HO take into account **AMOUNT** of the correction
3. Real calculation gives a **CONCRETE** number, with an error which is at least 2 digits.

These points are essential when we are at the level of accuracy which approaches experimental precision.

E.g.: Intrinsic theory error estimation for Γ_Z, 1804.10236 [1604.00406]

1. Geometric series

$$\delta_1 : \mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.20 \text{ MeV } [0.26 \text{ MeV}]$$

$$\delta_2 : \mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.21 \text{ MeV } [0.3 \text{ MeV}]$$

$$\delta_3 : \mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\delta_4 : \mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\delta_5 : \mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1} \text{ MeV } \underline{[\text{Now we know it!}]}$$

Total: $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim \mathbf{0.4}$ **MeV** [0.5 MeV]

Accuracy of calculations

For 2-loops we maintained 4 digits for EWPOs.

A calculation of the radiative corrections $\delta_1 \div \delta_4$ and $\delta'_1 \div \delta'_3$ with a 10% accuracy (corresponding to two significant digits) should suffice to meet future experimental demands.

Minimal precision of 3-loop EW calculations, an example.

1. Calculating N^3LO with 10% accuracy (two digits), we can replace intrinsic error estimation $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} \delta_i^2} \sim 0.4$ MeV by $\delta\Gamma_Z = \sqrt{\sum_{i=1}^{5} (\delta_i/10)^2} \sim 0.04$ MeV.

1. The requirement of FCC-ee$^{exper.\ error}(\Gamma_Z) \sim 0.1$ MeV can be met and the condition

$$\delta[\mathrm{FCCee}^{\mathrm{theor.}}(\Gamma_{\mathrm{Z}})] \ \sim 0.04 \ \mathrm{MeV} < \delta[\mathrm{FCCee}^{exper.}(\Gamma_Z)] \ \sim 0.1 \ \mathrm{MeV}$$

will be fullfilled.

More EWPOs, taken from ESPPU "Report 3"

	$\delta\Gamma_Z$ [MeV]	δR_l $[10^{-4}]$	δR_b $[10^{-5}]$	$\delta \sin^{2,l}_{eff}\theta$ $[10^{-6}]$
Present EWPO theoretical uncertainties				
EXP-2018	2.3	250	66	160
TH-2018	0.4	60	10	45
EWPO theoretical uncertainties when FCC-ee will start				
EXP-FCC-ee	0.1	10	$2 \div 6$	6
TH-FCC-ee	0.07	7	3	7

Table: *Comparison for selected precision observables of present experimental measurements (EXP-2018), current theory errors (TH-2018), FCC-ee precision goals at the end of the Tera-Z run (EXP-FCC-ee) and rough estimates of the theory errors assuming that electroweak 3-loop corrections and the dominant 4-loop EW-QCD corrections* $\mathcal{O}(\alpha\alpha_s^2), \mathcal{O}(N_f\alpha^2\alpha_s), \mathcal{O}(N_f^2\alpha^3)$ *are available at the start of FCC-ee (TH-FCC-ee). Based on discussion in 1809.01830.*

For more details, see Executive Summary and Chapter 2 in "Report 1".

A: Proper preparation of future colliders for the Z-resonance physics studies.

$$A \rightleftharpoons B$$

B: The progress in developing methods and tools is a MUST to crackdown the $\geq N^3LO$ EWPOs SM corrections.

Direct numerical approach

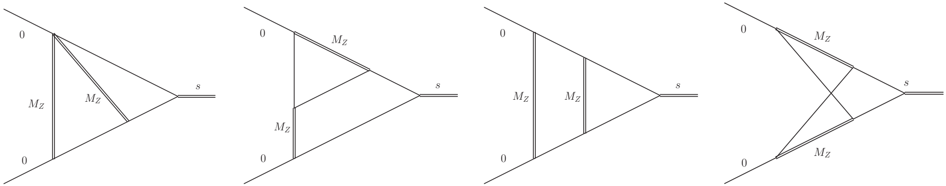
- Sector decomposition (SD)
  - FIESTA 3 [2014], *FIESTA 4 [2016] [A.V.Smirnov, V.A.Smirnov]
  - SecDec 3 [2015], *pySecDec [2017] [S. Borowka, G. Heinrich, et. al.]
- The Mellin-Barnes (MB) method:
  - PlanarityTest [I.Dubovyk, K.Bielas, 2013]
  - AMBRE 2 [J.Gluza, et. al., 2011], AMBRE 3 [I.Dubovyk, et. al., 2015]
  - MB [M.Czakon, 2006], MBresolve [A.V.Smirnov, V.A.Smirnov, 2009]
  - MBnumerics [J.Usovitsch, I.Dubovyk, T.Riemann, 2015] – Minkowskian kinematics
  - QMB [I.Dubovyk, JG, T.Riemann, 2019] – Minkowskian kinematics
- Computation:
  - Comparison for Eucledian kinematics
  - SD in Minkowskian, defining bottlenecks
  - Improving with MB and comparison
  - Comparison to analytical results for some single-scale integrals [Fleischer, Kotikov, Veretin 99, Aglietti, Bonciani 03,04, Aglietti, Bonciani, Grassi, Remiddi 08]
  - 10^{-8} accuracy achieved for most of Feynman integrals and at least 10^{-6} for the few worst integrals with one of the methods

Mellin-Barnes and Sector Decomposition methodsare very much complementary!

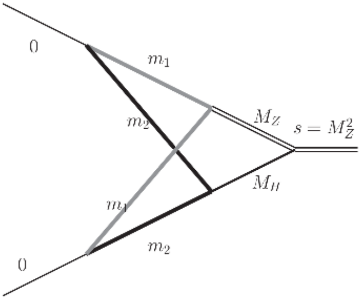
- MB works well for hard threshold, on-shell cases, not many internal masses (more IR); SD more useful for integrals with many internal masses

10^{-8} accuracy achieved for **any** self-energy and vertex Feynman integral with one of the methods - in **Minkowskian region**.

Available for several years!



2-loops $\longrightarrow$ 3-loops



$$m_1 = M_t, m_2 = M_W$$

The integrals contain up to three dimensionless parameters

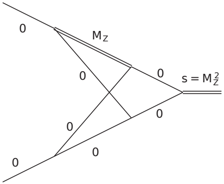
$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

Towards 3-loop results ("Report 1")

$Z \to b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	$14 \to^{(\mathbf{A})} 7 \to^{(\mathbf{B})}$ **5**	$211 \to^{(\mathbf{A})} 84 \to^{(\mathbf{B})}$ **50**
Number of diagrams	15	$2383 \to^{(\mathbf{A},\mathbf{B})}$ **1114**	$490387 \to^{(\mathbf{A},\mathbf{B})}$ **120187**
Fermionic loops	0	150	**17580**
Bosonic loops	15	**964**	102607
Planar diagrams	1T/15D	4T/981D	35T/84059D
Non-planar diagrams	0	1T/133D	15T/36128D

Table: Some statistical overview for $Z \to b\bar{b}$ multiloop studies. At 3 loops there are in total almost half a million of diagrams present. After basic refinements (A) and (B) about 10^5 genuine 3-loop vertex diagrams remain. In (A) tadpoles and products of lower loops are excluded, in (B) symmetries of topologies are taken into account.

Substantial progress for critical cases (here pySecDec)



Euclidean results (constant part):

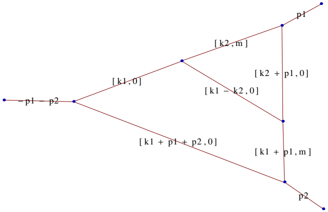
$$\begin{array}{ll} \text{Analytical}: & -0.4966198306057021 \\ \text{MB(Vegas)}: & -0.4969417442183914 \\ \text{MB(Cuhre)}: & -0.4966198313219404 \\ \text{FIESTA}: & -0.4966184488196595 \\ \text{SecDec}: & -0.4966192150541896 \end{array}$$

Minkowskian results (constant part):

$$\begin{array}{ll} \text{Analytical}: & -0.778599608979684 - 4.123512593396311 \cdot i \\ \text{MBnumerics}: & -0.778599608324769 - 4.123512600516016 \cdot i \\ \text{MB(Vegas)}: & \text{big error} \\ \text{MB(Cuhre)}: & \text{NaN} \\ \text{FIESTA}: & \text{big error} \\ \text{SecDec}: & \text{big error } [\mathbf{2016}], -0.77 - \mathrm{i} \cdot 4.1\ [\mathbf{2017}], -0.778 - \mathrm{i} \cdot 4.123\ [\mathbf{2019}] \end{array}$$

Substantial progress for critical cases (new: QMC). https://www.actaphys.uj.edu.pl/R/50/11/1993/pdf

With QMC, we can approach MB integrals with dim > 5 (!)



$$I = \frac{1}{(2\pi i)^3}\frac{1}{s^2}\int_{-i\infty}^{i\infty} dz_1 \int_{-i\infty}^{i\infty} dz_2 \int_{-i\infty}^{i\infty} dz_3 \left(\frac{m^2}{-s}\right)^{z_1} \frac{\Gamma(-1-z_1)\ldots\Gamma(-z_1-z_2+z_3)}{\Gamma(-z_1)\Gamma(1-z_2)\Gamma(1-z_1+z_3)}.$$

Numerical results for I with $s = m^2 = 1$. AB - analytical solution.

AB	$-1.\mathbf{199526183135}$	$+5.\mathbf{567365907880}i$	
MB	$-1.\mathbf{1995261831}68$	$+5.\mathbf{5673659079}04i$	Cuhre, 10^7, 10^{-8}
MB	$-1.\mathbf{20}4597845834$	$+5.\mathbf{567}518701898i$	Vegas, 10^7, 10^{-3}
MB	$-1.\mathbf{1995}16455248$	$+5.\mathbf{5673}76681167i$	QMC, 10^6, 10^{-5}
MB	$-1.\mathbf{199527}580305$	$+5.\mathbf{567367}345229i$	QMC, 10^7, 10^{-6}

Summary and Outlook

Challenges for Z-pole:

1. 3-loop EW and mixed EW-QCD corrections for $Z \to 2f$ vertices
2. Leading 4-loop effects
3. QED interference effects, non-factorizable corrections
4. Adjusting MC generators at NNLO and beyond: LEP basic programs for MC and fits must be reorganized (KKMC, DIZET, ZFITTER, ...)
5. **New independent software is very welcome!** E.g.:
    1. Factorization to infinite order of multi-photon soft- virtual- QED contributions, and re-summations in MC
    2. Disentangling QED and EW corrections beyond one-loop with soft-photon fact/resum
    3. Implamentation of higher loop effects properly in Laurent series around Z-peak

Backup slides

STANDARD MODEL

E.g. effective weak mixing angle

The weak mixing angle $s_W^2 \equiv \sin^2\theta_W$ has three potential different meanings or functions in the model-building:

(i) It describes the ratio of the two gauge couplings,

$$g'/g = c_W/s_W, \tag{4}$$

usually in the $\overline{\text{MS}}$ scheme.

(ii) It describes the ratio of two gauge boson (on-shell) masses,

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \tag{5}$$

(iii) It describes the ratio of the vector and axial-vector couplings of an (on-shell) Z boson to fermions,

$$\frac{v_f}{a_f} = 1 - 4|Q_f|s_W^2. \tag{6}$$

This definition is called the effective weak mixing angle, denoted as $\sin^2\theta_W^{f,\text{eff}}$.

EW corrections will become more important also at HL-LHC

HL-LHC will be tangible to EW physics, see this month "LHC EW Precision sub-group workshop", `https://indico.cern.ch/event/801961/`.
or

- Standard Model Physics at the HL-LHC and HE-LHC
  HL-LHC and HE-LHC Working Group, e-Print: arXiv:1902.04070

and ESPPU contribution `https://indico.cern.ch/event/765096`, e.g.:

100. Precision calculations for high-energy collider processes (Charalampos Anastasiou, Stefan Dittmaier, Thomas Gehrmann, Nigel Glover, Massimiliano Grazzini, Michelangelo Mangano, Stefano Pozzorini, Gavin Salam, Giulia Zanderighi)

Z-resonance: QED and EW

1. Z-resonance and $\gamma, Z', \ldots \longrightarrow$ Laurent series,

$$\mathcal{M} = \frac{R}{s - s_0} + \sum_{n=0}^{\infty} (s - s_0)^n \; B^{(n)}, \quad s_0 = \bar{M}_Z^2 + i \bar{M}_Z \bar{\Gamma}_Z.$$

2. We want to extract EW Z-vertex couplings and definitions like $\sin^2 \theta^f_{\text{eff}}$, but in reality, we deal with complicated process

$$e^+ e^- \to f^+ f^- \quad + \text{invisible} \; (n \; \gamma + \mathrm{e}^+ \mathrm{e}^- \text{pairs} + \cdots)$$

$$\sigma^{e^+ e^- \to f^+ f^- + \cdots}(s) = \int dx \; \overbrace{f(x)} \; \underbrace{\sigma^{e^+ e^- \to f^+ f^-}(s')} \; \delta(x - s'/s)$$

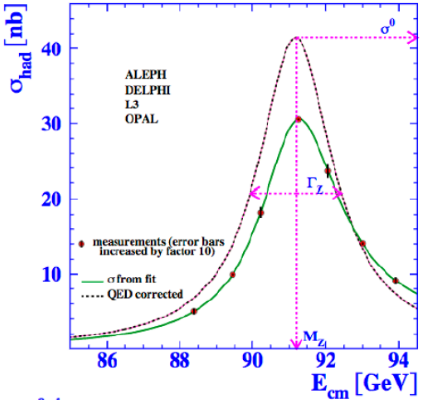
$\longrightarrow$ form factors, QED separation/deconvolution, non-factorizations, ...

To determine the structure function/flux function kernels and hard scattering ansatz for data preparation or for unfolding is one of the challenges of FCC-ee-Z physics.

QED unfolding

Altogether $17 \cdot 10^6$ Z-boson decays at LEP

- Cross section : Z mass and width



- ~30% QED corrections (ISR)

QED perturbative leading and subleading corrections, 1903.09895

$(a) \quad 0.5\%$

1				
αL	α			
α^2L^2	α^2L	α^2		
α^3L^3	α^3L^2	α^3L	α^3	
α^4L^4	α^4L^3	α^4L^2	α^4L^1	α^4
$\vdots$	$\vdots$	$\vdots$		

$(b) \quad 0.02\%$

1				
αL	α			
α^2L^2	α^2L	α^2		
α^3L^3	α^3L^2	α^3L	α^3	
α^4L^4	α^4L^3	α^4L^2	α^4L^1	α^4
$\vdots$	$\vdots$	$\vdots$		

$(c) \quad 0.001\%$

1				
αL	α			
α^2L^2	α^2L	α^2		
α^3L^3	α^3L^2	α^3L	α^3	
α^4L^4	α^4L^3	α^4L^2	α^4L^1	α^4
α^5L^5	$\vdots$	$\vdots$	$\vdots$	

ISR ($e^{\pm}$) and FSR ($\mu^{\pm}$) at the Z peak

$$\alpha \equiv \alpha_{QED}$$

$$L \equiv L_f = \ln(s/m_f^2), \;\; f = e, \mu$$

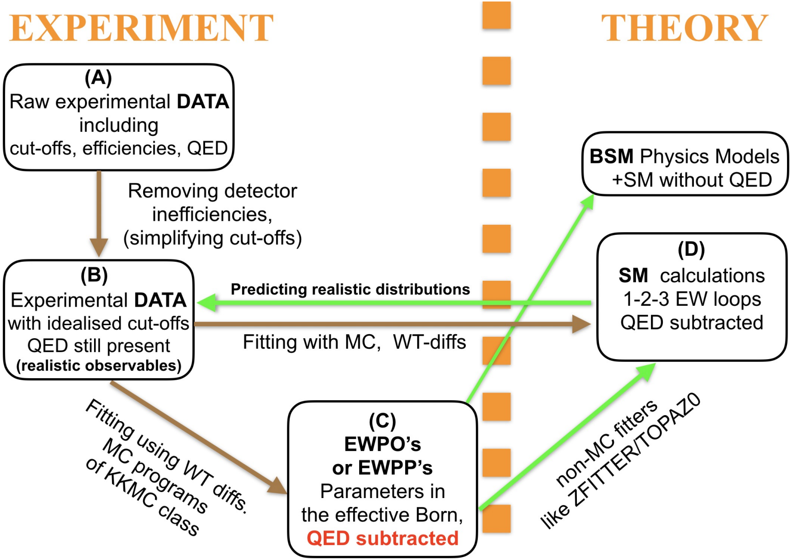
QED, LEP/FCC-ee, 1903.09895

Observable	Source LEP	Err.{QED} LEP	Stat[Syst] FCC-ee	$\frac{\text{LEP}}{\text{FCC-ee}}$	main development to be done
M_Z [MeV]	Z linesh.	$2.1\{0.3\}$	$0.005[0.1]$	$3\times3^{\star}$	light fermion pairs
Γ_Z [MeV]	Z linesh.	$2.1\{0.2\}$	$0.008[0.1]$	$2\times3^{\star}$	fermion pairs
$R_l^Z \times 10^3$	$\sigma(M_Z)$	$25\{12\}$	$0.06[1.0]$	$12\times3^{\star\star}$	better FSR
$\sigma^0_{\rm had}$ [pb]	$\sigma^0_{\rm had}$	$37\{25\}$	$0.1[4.0]$	$6\times3^{\star}$	better lumi MC
$N_\nu \times 10^3$	$\sigma(M_Z)$	$8\{6\}$	$0.005[1.0]$	$6\times3^{\star}$	CEEX in lumi MC
$N_\nu \times 10^3$	$Z\gamma$	$150\{60\}$	$0.8[<1]$	$60\times3^{\star\star}$	$\mathcal{O}(\alpha^2)$ for $Z\gamma$
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$	$53\{28\}$	$0.3[0.5]$	$55\times3^{\star\star}$	h.o. and EWPOs
$\sin^2\theta_W^{eff} \times 10^5$	$\langle\mathcal{P}_\tau\rangle, A_{\rm FB}^{pol,\tau}$	$41\{12\}$	$0.6[<0.6]$	$20\times3^{\star\star}$	better τ decay MC
M_W [MeV]	mass rec.	$33\{6\}$	$0.5[0.3]$	$12\times3^{\star\star\star}$	QED at threshold
$A_{FB,\mu}^{M_Z\pm3.5\text{GeV}} \times 10^5$	$\frac{d\sigma}{d\cos\theta}$	$2000\{100\}$	$1.0[0.3]$	$100\times3^{\star\star\star}$	improved IFI

Rating from * to *** marks whether the needed improvement is relatively straightforward, difficult or very difficult to achieve.

Scheme of construction and the use of EWPO/EWPP at FCC-ee

Report 1 and 1903.0985



EWPOs - refers to $|M|^2$; EWPPs - refers to M

Beyond Born level, one can write

$$\begin{aligned}
\mathcal{M}_{\gamma}^{(0)}(e^- e^+ \to f^- f^+) &= \frac{4\pi i \alpha_{em}(s)}{s} Q_e Q_f \, \gamma_\alpha \otimes \gamma^\alpha, \\
\mathcal{M}_{Z}^{(0)}(e^- e^+ \to f^- f^+) &= 4ie^2 \frac{\chi_Z(s)}{s} \big[M_{vv}^{ef} \, \gamma_\alpha \otimes \gamma^\alpha - M_{av}^{ef} \, \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \\
&\quad - M_{va}^{ef} \, \gamma_\alpha \times \gamma^\alpha \gamma_5 + M_{aa}^{ef} \, \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \gamma_5 \big].
\end{aligned}$$

In the <u>pole scheme</u>, where $\bar{M}_Z$ is defined as the real part of the pole of the S matrix, one has

$$\chi_Z(s) = \frac{G_F M_Z^2}{\sqrt{2}\, 8\pi \alpha_{em}} \, K_Z(s) \simeq \frac{1}{1 + i\frac{\bar{\Gamma}_Z}{\bar{M}_Z}} \frac{s}{s - \bar{M}_Z^2 + i\bar{M}_Z \bar{\Gamma}_Z} \simeq \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z(s)},$$

$$\Gamma_Z(s) = \frac{s}{M_Z^2} \, \Gamma_Z$$

EWPOs - refers to $|M|^2$; EWPPs - refers to M

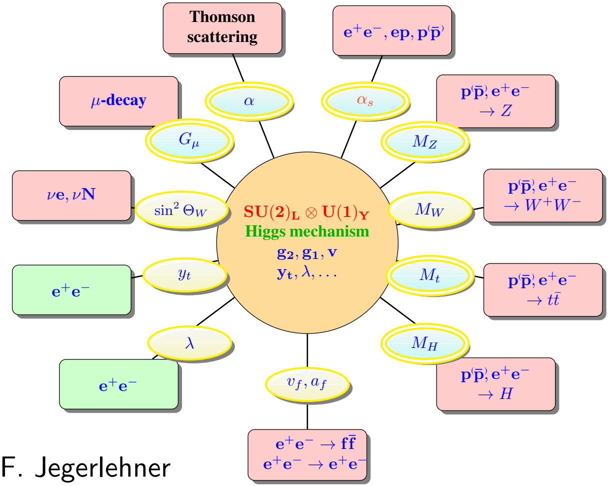
Definitions are related:

$$\bar{M}_Z \approx M_Z - \frac{1}{2}\frac{\Gamma_Z^2}{M_Z} \approx M_Z - 34\ \mathrm{MeV},$$
$$\bar{\Gamma}_Z \approx \Gamma_Z - \frac{1}{2}\frac{\Gamma_Z^3}{M_Z^2} \approx \Gamma_Z - 0.9\ \mathrm{MeV}.$$

- Known from LEP. One of examples why changing frameworks/assumptions/simplifications of calculations matter (!).
- However, at FCC-ee $\delta\Gamma_Z \sim 0.1$ MeV. Non-facotrization effects must be added properly beyond 1-loop.
- Is it necessary for FCC-ee accuracy to implement MC with radiative corrections calculated at the amplitudes level?
- At this precision it is important which parameters are taken as input parameters in schemes.

Input and renormalization schemes

1. Input and calculated/measured parameters (how many independent parameters?)



α_{QED} in "Report 2", F. Jegerlehner

Input, theoretical and parametric errors

EWPO	Exp. direct error	Exp. param. error	Main source	Theory uncert.
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$	0.07
R_b $[10^{-5}]$	6	1	$\delta\alpha_s$	3
R_ℓ $[10^{-3}]$	1	1.3	$\delta\alpha_s$	0.7
$\sin^2\theta^\ell_{eff}$ $[10^{-5}]$	0.5	1	$\delta(\Delta\alpha)$	0.7
M_W [MeV]	0.3	0.6	$\delta(\Delta\alpha)$	0.3

Estimated experimental precision for the direct measurement of several important EWPOs at FCC-ee (column two) and experimental parametric error (column three), with the main source shown in the forth column. Important input parameter errors are $\delta(\Delta\alpha) = 3 \cdot 10^{-5}$, $\delta\alpha_s = 0.00015$ see FCC CDR, vol. 2. Last column shows anticipated theory uncertainties at start of FCC-ee.

Input and renormalization schemes

- E.g. the bosonic 2-loop corrections shift the value of Γ_Z by 0.51 MeV when using M_W as input and 0.34 MeV when using G_μ as input. $\delta\Gamma_{FCC-ee} = 0.1$ MeV (Dubovyk et al, `http://arxiv.org/pdf/1804.10236.pdf`)
- In general, there are many different approaches. Which measured parameters to choose as an independent input parameters? E.g. recently Piccinini et al, Durham talk

  `https://indico.cern.ch/event/801961/contributions/3361495/attachments/1823019/2982558/piccinini.pdf`

  are proposing to take for LHC $(\alpha/G_\mu, \sin^2\theta^f_{\text{eff}}, M_Z)$

  $\sin^2\theta^f_{\text{eff}}$ fixed at measured leptonic $\sin^2\theta^f_{\text{eff}}$ requiring v_l/a_l does not get radiative corrections. Procedure independent of QED corrections (both couplings get the same QED corrections and we have a ratio).

Table 3.1: Measurement of selected electroweak quantities at the FCC-ee, compared with the present precisions.

Observable	present value ± error	FCC-ee Stat.	FCC-ee Syst.	Comment and dominant exp. error
m_Z (keV/c^2)	91186700 ± 2200	5	100	From Z line shape scan Beam energy calibration
Γ_Z (keV)	2495200 ± 2300	8	100	From Z line shape scan Beam energy calibration
R_ℓ^Z ($\times 10^3$)	20767 ± 25	0.06	0.2-1	ratio of hadrons to leptons acceptance for leptons
$\alpha_s(m_Z)$ ($\times 10^4$)	1196 ± 30	0.1	0.4-1.6	from R_ℓ^Z above [29]
R_b ($\times 10^6$)	216290 ± 660	0.3	<60	ratio of $b\bar{b}$ to hadrons stat. extrapol. from SLD [30]
σ^0_{had} ($\times 10^3$) (nb)	41541 ± 37	0.1	4	peak hadronic cross-section luminosity measurement
N_ν($\times 10^3$)	2991 ± 7	0.005	1	Z peak cross sections Luminosity measurement
$\sin^2\theta_W^{eff}$($\times 10^6$)	231480 ± 160	3	2 - 5	from $A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{QED}(m_Z)$($\times 10^3$)	128952 ± 14	4	small	from $A_{FB}^{\mu\mu}$ off peak [20]
$A_{FB}^b, 0$ ($\times 10^4$)	992 ± 16	0.02	1-3	b-quark asymmetry at Z pole from jet charge
$A_{FB}^{pol,\tau}$ ($\times 10^4$)	1498 ± 49	0.15	<2	τ polarisation and charge asymmetry τ decay physics

Flavour physics numbers for FCC-ee

Table 7.1: Expected production yields of heavy-flavoured particles at Belle II (50 ab^{-1}) and FCC-ee.

Particle production (10^9)	B^0 / $\overline{B}^0$	B^+ / B^-	B_s^0 / $\overline{B}_s^0$	Λ_b / $\overline{\Lambda}_b$	$c\bar{c}$	$\tau^+\tau^-$
Belle II	27.5	27.5	n/a	n/a	65	45
FCC-*ee*	1000	1000	250	250	550	170

Table 7.2: Comparison of orders of magnitude for expected reconstructed yields of a selection of electroweak penguin and pure dileptonic decay modes in Belle II, LHCb upgrade and FCC-ee experiments. Standard model branching fractions are assumed. The yields for the electroweak penguin decay $\bar{B}^0 \rightarrow K^{*0}(892)e^+e^-$ are given in the low q^2 region.

Decay mode	$B^0 \rightarrow K^*(892)e^+e^-$	$B^0 \rightarrow K^*(892)\tau^+\tau^-$	$B_s(B^0) \rightarrow \mu^+\mu^-$
Belle II	$\sim$ 2 000	$\sim$ 10	n/a (5)
LHCb Run I	150	-	$\sim$ 15 (–)
LHCb Upgrade	$\sim$ 5000	-	$\sim$ 500 (50)
FCC-ee	$\sim$ 200000	$\sim$ 1000	$\sim$1000 (100)

EW SM theory at loops, an example ($\Delta_{ef} \neq 0$)

$$\left\{\begin{array}{l} \Gamma_Z, \Gamma_{partial} \\ \\ A^{eff.,Born}_{FB,peak}, A^{eff.,Born}_{LR,peak} \\ \\ R_b, R_\ell, \dots \end{array}\right. \longrightarrow \left\{\begin{array}{l} v^{eff}_{\ell,\nu,u,d,b} \\ \\ a^{eff}_{\ell,\nu,u,d,b} \\ \\ \sin^2\theta^{\mathrm{b}}_{\mathrm{eff}}, \sin^2\theta^{lept}_{\mathrm{eff}} \end{array}\right.$$

e.g. : improvements needed for subtle corrections $\Delta_{1,2}$ (e.g. boxes, **2L-boxes**)

$$\begin{aligned} A^{eff.,Born}_{FB,peak} &= \frac{2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\right] 2\Re e\left[\frac{v_f a_f^*}{|a_f|^2}\right]}{\left(1+\frac{|v_e|^2}{|a_e|^2}\right)\left(1+\frac{|v_f|^2}{|a_f|^2}\right)} + \Delta_1 - \Delta_2 \simeq \frac{3}{4} A_e A_f, \\ \Delta_1 &= 2\Re e\left[\Delta_{ef}\right], \; \Delta_2 = |\Delta_{ef}|^2 + 2\Re e\left[\frac{v_e a_e^*}{|a_e|^2}\frac{v_f a_f^*}{|a_f|^2}\Delta^*_{ef}\right], \\ \Delta_{ef} &= 16|Q_e Q_f| s_W^4 (\kappa_{ef} - \kappa_e \kappa_f) \end{aligned}$$

LEP uncertainities, A. Freitas: 1604.00406

	Experiment	Theory error	Main source
M_{W}	80.385 ± 0.015 GeV	4 MeV	α^3, $\alpha^2\alpha_{\mathrm{s}}$
Γ_{Z}	2495.2 ± 2.3 MeV	0.5 MeV	α_{bos}^2, α^3, $\alpha^2\alpha_{\mathrm{s}}$, $\alpha\alpha_{\mathrm{s}}^2$
σ_{had}^0	41540 ± 37 pb	6 pb	α_{bos}^2, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$R_b \equiv \Gamma_{\mathrm{Z}}^b/\Gamma_{\mathrm{Z}}^{\mathrm{had}}$	0.21629 ± 0.00066	0.00015	α_{bos}^2, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$\sin^2\theta_{\mathrm{eff}}^{\ell}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_{\mathrm{s}}$

Comparisons, A. Freitas: 1604.00406

	Measurement error			Theory error	
	ILC	CEPC	FCC-ee	Current	Future†
M_W [MeV]	3–4	3	1	4	1-1.5
Γ_Z [MeV]	0.8	0.5	**0.1**	**0.5**	**0.2**
R_b $[10^{-5}]$	14	17	6	15	7
$\sin^2\theta^{\ell}_{\rm eff}$ $[10^{-5}]$	1	2.3	**0.6**	4.5	**1.5**

Table: Projected experimental and theoretical uncertainties for some electroweak precision pseudo-observables.

† Based on estimations for: $\mathcal{O}(\alpha^2_{bos})$,$\mathcal{O}(\alpha\alpha^2_s)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^3)$

Updates for error estimations

- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_{t}, m_b, α_{s}, $\Delta\alpha_{\text{had}}$, ...)

see, Ayres Freitas: 1604.00406

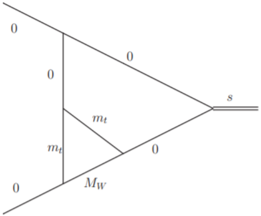
Summary: estimations for higher order EW and QCD corrections

δ_1 :	δ_2 :	δ_3 :	δ_4 :	δ_5 :	$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^2\alpha_s)$	$\mathcal{O}(\alpha\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$	$\mathcal{O}(\alpha_{bos}^2)$	$=\sqrt{\sum_{i=1}^5 \delta_i^2}$
TH1 (estimated error limits from geometric series of perturbation)					
0.26	0.3	0.23	0.035	0.1	0.5
TH1-new (estimated error limits from geometric series of perturbation)					
0.2	0.21	0.23	0.035	$< 10^{-4}$	**0.4**

δ_1' :	δ_2' :	δ_3' :	δ_4 :		$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(N_f^{\leq 1}\alpha^3)$	$\mathcal{O}(\alpha^3\alpha_s)$	$\mathcal{O}(\alpha^2\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$		$\sqrt{\delta_1'^2 + \delta_2'^2 + \delta_2'^3 + \delta_4^2}$
TH2 (extrapolation through prefactor scaling)					
0.04	0.1	0.1	0.035	10^{-4}	**0.15**

"Report 2"

soft7 ϵ^0:[MB - 3 dim] [SD - 5 dim], ϵ^{-1}:[MB - 2 dim] [SD - 4 dim], ϵ^{-2}:[MB - 1 dim] [SD - 3 dim]



MB	$0.060266486557699\boldsymbol{9}\,\epsilon^{-2}$	
SD - 90 Mio	$0.0602664865\boldsymbol{5}\,\epsilon^{-2}$	
MB	$(-0.0315124890\boldsymbol{0}3$	$+0.18933275\boldsymbol{1}42i)\,\epsilon^{-1}$
SD - 90 Mio	$(-0.03151248\boldsymbol{1}6$	$+0.18933271\boldsymbol{6}96i)\,\epsilon^{-1}$
MB 1	$(-0.228231867\boldsymbol{5}11$	$-0.0882479456\boldsymbol{9}1i) + \mathcal{O}(\epsilon)$
MB 2	$(-0.22823186755\boldsymbol{5}1$	$-0.0882479457\boldsymbol{3}9i) + \mathcal{O}(\epsilon)$
SD - 90 Mio	$(-0.228226\boldsymbol{5}3$	$-0.088245\boldsymbol{9}6i) + \mathcal{O}(\epsilon)$
SD - 15 Mio	$(-0.22816\boldsymbol{6}2$	$-0.0882\boldsymbol{0}9i) + \mathcal{O}(\epsilon)$

SM precision parameters determination: $\alpha(M_Z^2)$

1. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics: besides top Yukawa y_t and Higgs self-coupling λ

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions**

50% non-perturbative $\quad \sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$
\begin{array}{lcccl}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 0.9 \div 1.6 & \times & 10^{-4} \quad (\text{present : lost } 10^5 \text{ in precision!}) \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 5.3 & \times & 10^{-5} \quad (\mathtt{FCC-ee/ILC\ requirement})
\end{array}
$$

LEP/SLD: $\sin^2\Theta_{\mathrm{eff}} = (1 - v_l/a_l)/4 = 0.23148 \pm \boxed{0.00017}$

$\delta\Delta\alpha(M_Z) = 0.00020 \quad \Rightarrow \quad \delta\sin^2\Theta_{\mathrm{eff}} = \boxed{0.00007}$ **;** $\delta M_W/M_W \sim 4.3 \times 10^{-5}$

affects most precision tests and new physics searches!!!

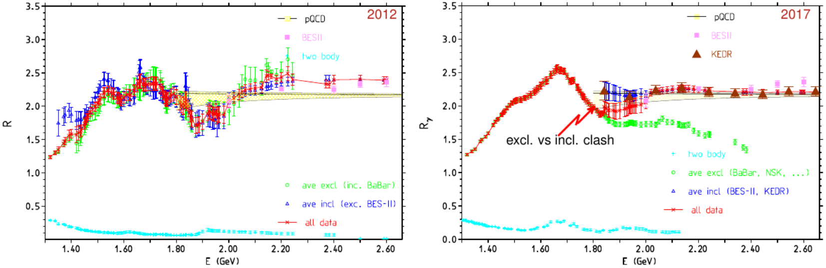
$$\frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}\,, \quad \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}\,, \quad \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3}$$

For pQCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ **Lattice-QCD**

SM precision parameters determination: $\alpha(M_Z^2)$

❐ Still an issue in HVP

❐ region 1.2 to 2 GeV data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty



● illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

Three approaches should be further explored for better error estimate

Note: theory-driven standard analyses ($R(s)$ integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

precision in α:	present	direct	1.7×10^{-4}
		Adler	1.2×10^{-4}
	future	Adler QCD 0.2%	5.4×10^{-5}
		Adler QCD 0.1%	3.9×10^{-5}
	future	via $A_{\rm FB}^{\mu\mu}$ off Z	3×10^{-5}

- Adler function method is competitive with Patrick Janot's direct near Z pole determination via forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$

$$A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3\,a^2}{4\,v^2}\frac{\mathcal{I}}{\mathcal{Z} + \mathcal{G}}$$

where

$\gamma - Z$ interference term	$\mathcal{I} \propto \alpha(s)\, G_\mu$
Z alone	$\mathcal{Z} \propto G_\mu^2$
γ only	$\mathcal{G} \propto \alpha^2(s)$
v vector Z coupling	also depends on $\alpha(s \sim M_Z^2)$ and $\sin^2\Theta_f(s \sim M_Z^2)$
a axial Z coupling	sensitive to ρ-parameter (strong M_t dependence)

❏ using v, a as measured at Z-peak

$e^+e^- \to \mu^+\mu^-$ and $\alpha^2(s)$

$\sigma_{\mu\mu}$:

1. the photon-exchange term, $\mathcal{G}$, proportional to $\alpha^2(s)$;
2. the Z-exchange term, $\mathcal{Z}$, proportional to G_F^2 (where G_F is the Fermi constant);
3. the Z-photon interference term, $\mathcal{I}$, proportional to $\alpha(s) \times G_F$

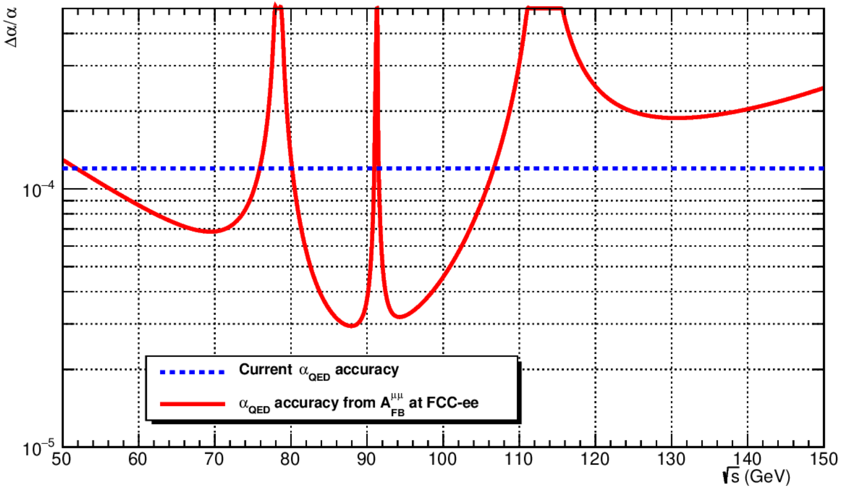
The muon forward-backward asymmetry, $A_{\mathrm{FB}}^{\mu\mu}$, is maximally dependent on the interference term

$$A_{\mathrm{FB}}^{\mu\mu} = A_{\mathrm{FB},0}^{\mu\mu} + \frac{3}{4}\frac{2}{2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}},$$

varies with $\alpha_{\mathrm{QED}}(s)$ as follows:

$$\Delta A_{\mathrm{FB}}^{\mu\mu} = \left(A_{\mathrm{FB}}^{\mu\mu} - A_{\mathrm{FB},0}^{\mu\mu}\right) \times \frac{\mathcal{Z}-\mathcal{G}}{\mathcal{Z}+\mathcal{G}} \times \frac{\Delta\alpha}{\alpha}.$$

$e^+e^- \to \mu^+\mu^-$ and $\alpha^2(s)$



The best accuracy is obtained for one year of running either just below or just above the Z pole, at 87.9 and 94.3 GeV, respectively.