Scalar-Vector EFTs from Soft Limits

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Introduction and Motivation

1986: Parke and Taylor calculated 6-point tree-level gluon amplitudes and the result was remarkably simple

$$\mathcal{A}_n(1^-2^-3^+\dots n^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\dots\langle n1\rangle}$$

Motivation:

- better way to calculate amplitudes than the traditional Lagrangian approach
- the mapping Lagrangian $\rightarrow$ Amplitude is many-to-one
- new variables may uncover previously hidden structures
- reformulation of QFT without Lagrangians

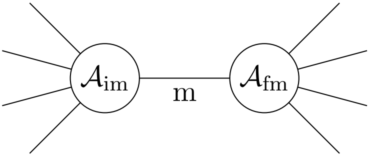
Part I

Bootstrap method

Amplitude properties

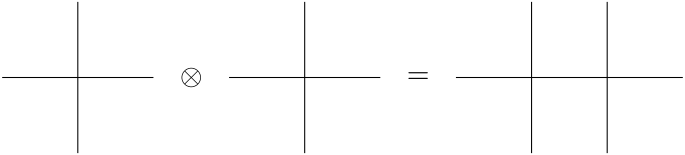
- Lorentz invariance
- Locality – tree amplitudes are rational functions, denominators come from propagators
- Unitarity – in this case, the 1-particle unitarity says the amplitude factorizes

$$\mathrm{Im}\ \mathcal{A}_{\mathrm{fi}} = \pi\delta(P_{\mathrm{f}}^2 - m_{\mathrm{m}})\mathcal{A}_{\mathrm{fm}}\mathcal{A}_{\mathrm{im}}^*$$

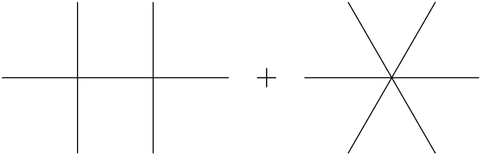


- Gauge invariance, SUSY, soft behavior (soft theorems), etc. (soft behavior: $\mathcal{A} \sim \mathcal{O}(t^\sigma)$ for $p \sim \mathcal{O}(t)$ in the limit $t \to 0$)

Basic idea



$\mathcal{A} =$



Power counting

For a tree amplitude composed of vertices V_i, it holds

$$d - 2 = \sum_i (d_i - 2)$$
$$n - 2 = \sum_i (n_i - 2),$$

where d_i and n_i are the mass dimension and number of legs of V_i respectively.

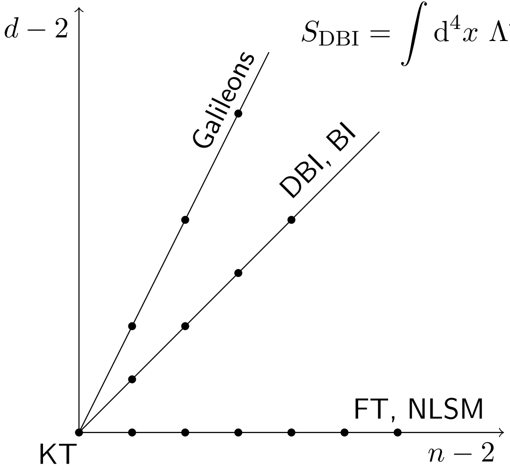
Thus, it makes sense to define the power-counting parameter ϱ as the following

$$\varrho \equiv \frac{d-2}{n-2}.$$

Two tree amplitudes with the same parameter ϱ produce an amplitude with the same ϱ when they merge. Amplitudes with the same ϱ "talk" to each other.

Power counting

$$S_{\rm DBI} = \int \mathrm{d}^4x\ \Lambda^4(1 - \sqrt{1 - \partial\varphi \cdot \partial\varphi/\Lambda^4})$$



Contact amplitudes

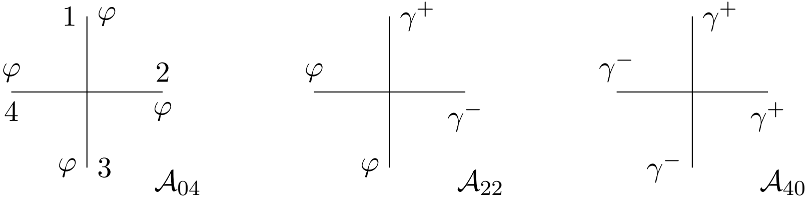
Contact amplitudes are the most basic. How to construct them? We have the following rules:

- The amplitude should be a polynomial in spinor brackets of external momenta
- The power counting sets the amplitude mass dimension (via the power-counting parameter ϱ)
- The amplitude should also have the correct little group scaling, i.e. $\mathcal{A} \to z^{2h_i}\mathcal{A}$ when the i-th particle with helicity h_i is scaled with z being the scaling parameter

  (the square and angle spinors scale as $|i] \to z|i]$, $|i\rangle \to z^{-1}|i\rangle$ with respect to the little group)

Is there a unique theory with Galileon-like power counting ($\varrho = 2$) containing Special Galileon coupled to photon? [H. Elvang, M. Hadjiantonis, C. R. T. Jones, S. Paranjape, 1806.06079]

Assuming helicity conservation, there are three 4-point "seed" amplitudes

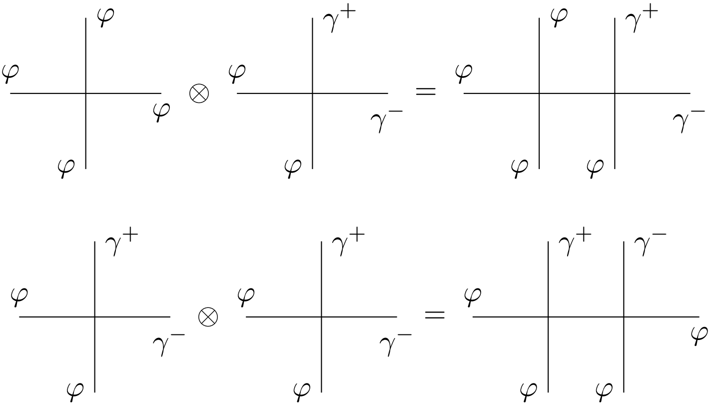


$$\mathcal{A}_{04} = c_{04} \sum\nolimits_{\text{Bose}} \langle 12 \rangle^3 [12]^3 + \text{p.c.}$$
$$\mathcal{A}_{22} = c_{22} \sum\nolimits_{\text{Bose}} \langle 13 \rangle \langle 23 \rangle^2 [13]^3 + \text{p.c.}$$
$$\mathcal{A}_{40} = c_{40} \sum\nolimits_{\text{Bose}} \langle 12 \rangle \langle 34 \rangle^2 [12]^3 + \text{p.c.},$$

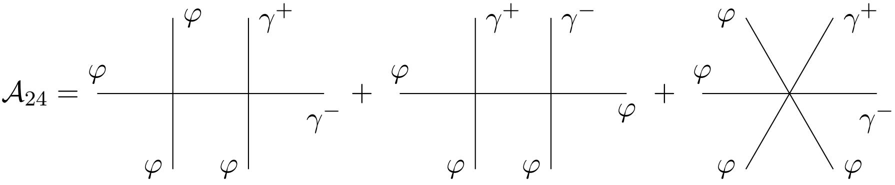
where p.c. stands for the parity conjugated term.

Higher amplitudes? (6-point)

How to get the amplitude $\mathcal{A}_{24}$?



Higher amplitudes? (6-point)

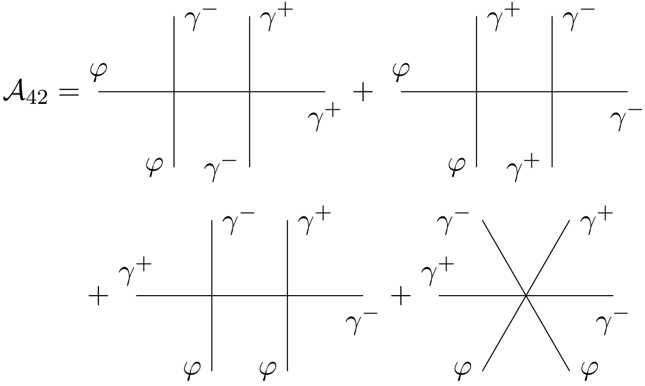


29 terms

- For the case of $\mathcal{A}_{24}$, demanding $\mathcal{O}(t^3)$ soft behavior for any Galileon leg yields a system of equations for all the constants with a unique solution.
- All the 6-point constants (29 in total) and one of the 4-point constants are fixed. The other 4-point constant represents the overall normalization of the amplitude.

Higher amplitudes? (6-point)

What about the amplitude $\mathcal{A}_{42}$?



42 terms

- For the case of $\mathcal{A}_{42}$, demanding $\mathcal{O}(t^3)$ soft behavior for any Galileon leg is not enough, some 6-point constants remain unfixed.

Higher amplitudes? (6-point)

- The case of $\mathcal{A}_{42}$ can be saved by demanding a special soft limit, where we make all the photons with the same helicity soft simultaneously (so-called multichiral limit). [C. Cheung, K. Kampf, J. Novotný, C.-H. Shen, J. Trnka, C. Wen, 1801.01496]

- By demanding both $\mathcal{O}(t^3)$ soft behavior for any Galileon leg and $\mathcal{O}(t)$ soft behavior for all the "$+$" photons (or all the "$-$" photons), all the 6-point constants (42 in total) and one of the 4-point constants are fixed.

- What about $\mathcal{A}_{60}$? This purely photon amplitude can be again fixed by a multichiral limit but $\mathcal{O}(t)$ is not strong enough. We need $\mathcal{O}(t^2)$.

- And $\mathcal{A}_{06}$? This is a purely Galileon amplitude. [C. Cheung, K. Kampf, J. Novotný, J. Trnka, 1412.4095 and C. Cheung, K. Kampf, J. Novotný, C.-H. Shen, J. Trnka, 1509.03309]

Even higher amplitudes? (8-point)

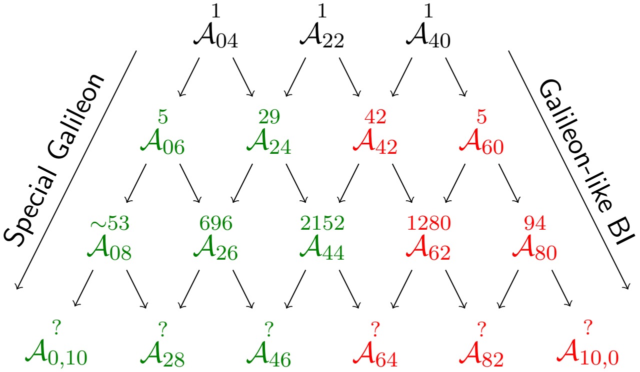
- The amplitude $\mathcal{A}_{26}$ is fixed completely using only the Galileon soft limit. It contains 696 8-point constants.

- The amplitude $\mathcal{A}_{44}$ contains 2152 8-point constants and the Galileon soft limit should be enough to fix it. Currently, we are in the process of verifying our calculations.

- For the case of the amplitude $\mathcal{A}_{62}$, the Galileon soft limit is not enough to fix it. Another additional limit possibly might help. We do not know currently. It contains 1280 8-point constants.

- Finally, $\mathcal{A}_{80}$ is the purely photon case. There are 94 8-point constants, and so far, we also have no idea how to fix this one.

Tree of Photon-Galileon tree amplitudes



Zero residue at infinity demands a naive condition (analogy from BCFW)

$$n_\gamma < n_\varphi + 2.$$

Green amplitudes should be fixed only using the Galileon soft limit.
Amplitudes $\mathcal{A}_{42}$ and $\mathcal{A}_{60}$ can be fixed using additional photon soft limits.

Part II

Lagrangian method

Galileons

- decoupling limit of Horndeski theory (a scalar-tensor gravity theory)
- the EOM are second order despite the action contains higher order of derivatives ($\Rightarrow$ no Ostrogradsky ghosts)
- exceptional EFTs

Lagrangian of a general Galileon (in D dimensions)

$$\mathcal{L} = \sum_{n=1}^{D+1} d_n \varphi \left(\varepsilon^{\mu_1 \ldots \mu_D} \varepsilon^{\nu_1 \ldots \nu_D} \prod_{i=1}^{n-1} \partial_{\mu_i} \partial_{\nu_i} \varphi \prod_{j=n}^{D} \eta_{\mu_j \nu_j} \right),$$

to get the Special Galileon, we set

$$d_{2k} = -\frac{(-1)^{k+D-1}}{D!\,2k} \binom{D}{2k-1} \alpha^{2(1-k)}, \qquad d_{2k+1} = 0.$$

Special Galileon

- Galileon symmetry

$$\delta\varphi = a + b^\mu x_\mu$$

- Special Galileon hidden symmetry [K. Hinterbichler, A. Joyce, 1501.07600]

$$\delta\varphi = -\frac{1}{2}G^{\mu\nu}\left(\alpha^2 x_\mu x_\nu + \partial_\mu\varphi\partial_\nu\varphi\right),$$

  where $G^{\mu\nu}$ is symmetric and traceless.

- The former symmetry ensures $\mathcal{O}(t^2)$ soft behavior, the latter symmetry even $\mathcal{O}(t^3)$.

- These symmetry properties carry over to 1-loop amplitudes. [FP, J. Novotný, 1909.06214]

Special Galileon – geometry

- fluctuations of a D-dimensional brane in a $2D$-dimensional flat space
- metric on the brane

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{\alpha^2}\partial_\mu\partial\varphi \cdot \partial\partial_\nu\varphi$$

- the external curvature tensor

$$\mathcal{K}_{\mu\nu\varrho} = -\frac{1}{\alpha}\partial_\mu\partial_\nu\partial_\varrho\varphi$$

- Gauss equation implies

$$R_{\alpha\beta\mu\nu} = g^{\varrho\sigma}(\mathcal{K}_{\varrho\mu\alpha}\mathcal{K}_{\sigma\nu\beta} - \mathcal{K}_{\varrho\mu\beta}\mathcal{K}_{\sigma\nu\alpha})$$

Special Galileon – geometry

- invariant measures

$$\mathrm{d}^D Z \equiv (-1)^{D-1} \det\left(\eta + \frac{\mathrm{i}}{\alpha}\partial\partial\varphi\right) \mathrm{d}^D x$$

$$\mathrm{d}^D \bar{Z} \equiv (-1)^{D-1} \det\left(\eta - \frac{\mathrm{i}}{\alpha}\partial\partial\varphi\right) \mathrm{d}^D x$$

- σ invariant

$$\sigma \equiv \frac{\alpha}{2\mathrm{i}} \ln \frac{\det\left(\eta + \frac{\mathrm{i}}{\alpha}\partial\partial\varphi\right)}{\det\left(\eta - \frac{\mathrm{i}}{\alpha}\partial\partial\varphi\right)}$$

Any Lagrangian constructed using the presented building blocks is automatically invariant under the Special Galileon hidden symmetry. If we want to add photons to the story, the symmetry can be implemented using

$$\delta A_\mu = -G^{\alpha\beta}\partial_\alpha\varphi\partial_\beta A_\mu - G^{\alpha\beta}A_\alpha\partial_\beta\varphi\partial_\mu\varphi.$$

Lagrangians

The minimal Lagrangian (2-photon) [J. Bonifacio, K. Hinterbichler, L. A. Johnson, A. Joyce, R. A. Rosen, 1911.04490]

$$\mathcal{L}_{\text{min.}} = -\frac{1}{4}\sqrt{|g|}V_{FF}\left(\frac{\sigma}{\alpha}\right)F_{\mu\alpha}F_{\nu\beta}g^{\mu\nu}g^{\alpha\beta}$$

Note: For $V'_{FF}(0) \neq 0$, this Lagrangian can actually produce helicity-violating amplitudes.

Examples of non-minimal Lagrangians (4-photon)

$$\mathcal{L}_1 = \sqrt{|g|}V_{F^4\mathcal{K}^2}\left(\frac{\sigma}{\alpha}\right)F^4\mathcal{K}^2 \qquad \text{(12 contractions)}$$

$$\mathcal{L}_2 = \sqrt{|g|}V_{(DF)^2F^2}\left(\frac{\sigma}{\alpha}\right)(DF)^2F^2$$

$$\mathcal{L}_3 = \sqrt{|g|}V_{F^4(D\mathcal{K})}\left(\frac{\sigma}{\alpha}\right)F^4(D\mathcal{K})$$

$$\mathcal{L}_4 = \sqrt{|g|}V_{F^3(DF)\mathcal{K}}\left(\frac{\sigma}{\alpha}\right)F^3(DF)\mathcal{K}$$

Lagrangians

For example, what about $\mathcal{A}_{42}$?

- Minimal Lagrangian $\mathcal{L}_{\mathrm{min.}}$ itself can not produce the correct amplitude $\mathcal{A}_{42}$ that actually possesses the multichiral limit.
- Nevertheless, the limit can be restored by adding $\mathcal{L}_1$ to the mix.
- Adding $\mathcal{L}_2$ does not change anything for this amplitude, it only generates structures already present (and thus only shifts constant in front of these structures).
- The remaining two Lagrangians $\mathcal{L}_3$ and $\mathcal{L}_4$ do not contribute into this amplitude at all.

Summary (Part I)

- We have two approaches: the bootstrap (bottom-up) construction and the Lagrangian approach.

- Because the number of contact amplitudes grows very harshly (already hundreds or thousands at 8-point), we cannot go too far with the bootstrap method. On the other hand, the bootstrap method has no redundancies typical for Lagrangian approaches.

- From the bootstrap method, it seems that a subtheory with amplitudes that obey some multichiral limits exists, but so far we have not been able to find the corresponding Lagrangian. The bootstrap method only gives a hint, not a proof.

Summary (Part II)

- At this point, we have an infinite amount of theories with $\mathcal{O}(t^3)$ Special Galileon soft behavior, parametrized by possible structures built from the aforementioned building blocks and the corresponding potentials $V_{\Box}(\sigma/\alpha)$.

- Our task is to try using the constraints obtained from the bootstrap method to reduce the set of possible Lagrangians.

- For example, the multichiral limit reduces this set a lot, but we have not found a unique theory yet (if it exists).

- It would be nice to find some additional properties (symmetries) that could help to restrict the set of possible Lagrangians even more.

Thank you.