Effective actions from string field theory

1912.05463, 2006.16270

J. Vošmera[1,2]

in collaboration with

H. Erbin, C. Maccaferri, M. Schnabl

[1]CEICO, Institute of Physics, AS CR, Prague

[2]IPNP, Charles University, Prague

ICHEP, July 30, 2020

Objectives

1. Deriving tree-level effective actions from (string) field theory

   $\rightarrow$ low-energy classical physics of superstring backgrounds

   $\rightarrow$ a useful handle on constructing certain classical solutions (marginal deformations, barely-relevant deformations)

Objectives

1. Deriving tree-level effective actions from (string) field theory

   $\rightarrow$ low-energy classical physics of superstring backgrounds

   $\rightarrow$ a useful handle on constructing certain classical solutions (marginal deformations, barely-relevant deformations)

2. Uncovering the algebraic structure of effective actions

   $\rightarrow$ strongly homotopy matrix/Lie algebras (A_∞ / L_∞)

   $\rightarrow$ inherited from the parent UV theory via homotopy transfer $\implies$ all-order results for effective physics

Objectives

1. Deriving tree-level effective actions from (string) field theory

   $\to$ low-energy classical physics of superstring backgrounds

   $\to$ a useful handle on constructing certain classical solutions (marginal deformations, barely-relevant deformations)

2. Uncovering the algebraic structure of effective actions

   $\to$ strongly homotopy matrix/Lie algebras (A_∞ / L_∞)

   $\to$ inherited from the parent UV theory via homotopy transfer $\implies$ all-order results for effective physics

3. Evaluation of effective SFT vertices

   $\to$ computing (generally off-shell) superstring amplitudes

   $\to$ simplification for superstring with global worldsheet $\mathcal{N}=2$ superconformal symmetry $\implies$ localization

Outline

String field theory

An off-shell formulation of string theory such that on-shell its Feynman diagram expansion reproduces the first-quantized string S-matrix.

String field theory

An off-shell formulation of string theory such that on-shell its Feynman diagram expansion reproduces the first-quantized string S-matrix.

What is it good for?

$\rightarrow$ consistent treatment of string perturbation theory [Sen et al.: 2015 – present]

$\rightarrow$ classical SFT solutions provide non-perturbative string backgrounds (open SFT $\implies$ D-branes) [Sen: hep-th/9911116]

$\rightarrow$ dynamics of non-perturbative phenomena (vacuum instabilities and decay) [Sen: hep-th/0203211]

String field theory

An off-shell formulation of string theory such that on-shell its Feynman diagram expansion reproduces the first-quantized string S-matrix.

What is it good for?

$\rightarrow$ consistent treatment of string perturbation theory [Sen et al.: 2015 – present]

$\rightarrow$ classical SFT solutions provide non-perturbative string backgrounds (open SFT $\implies$ D-branes) [Sen: hep-th/9911116]

$\rightarrow$ dynamics of non-perturbative phenomena (vacuum instabilities and decay) [Sen: hep-th/0203211]

Algebraic structure:

$\rightarrow$ open string: (cyclic/quantum) A_∞ algebras [Gaberdiel, Zwiebach: hep-th/9705038]

$\rightarrow$ closed string: (cyclic/quantum) L_∞ algebras [Zwiebach: hep-th/9206084]

String field theory

An off-shell formulation of string theory such that on-shell its Feynman diagram expansion reproduces the first-quantized string S-matrix.

What is it good for?

$\rightarrow$ consistent treatment of string perturbation theory [Sen et al.: 2015 – present]

$\rightarrow$ classical SFT solutions provide non-perturbative string backgrounds (open SFT $\implies$ D-branes) [Sen: hep-th/9911116]

$\rightarrow$ dynamics of non-perturbative phenomena (vacuum instabilities and decay) [Sen: hep-th/0203211]

Algebraic structure:

$\rightarrow$ open string: (cyclic/quantum) A_∞ algebras [Gaberdiel, Zwiebach: hep-th/9705038]

$\rightarrow$ closed string: (cyclic/quantum) L_∞ algebras [Zwiebach: hep-th/9206084]

Analytic structure: vertices given by 2d worldsheet matter + ghost CFT correlators integrated over the moduli space of Riemann surfaces with punctures (string insertions), boundaries (open string ends) and genus (loops)

A_∞ algebras

[Stasheff: 1963]

Vector space: $\mathcal{H}$ graded by degree $d : \mathcal{H} \longrightarrow \mathbb{Z}$.

A_∞ algebras

[Stasheff: 1963]

Vector space: $\mathcal{H}$ graded by degree $d : \mathcal{H} \longrightarrow \mathbb{Z}$.

Products: $m_k : \mathcal{H}^{\otimes k} \longrightarrow \mathcal{H}$ (degree-odd, multi-linear)

A_∞ algebras

[Stasheff: 1963]

Vector space: $\mathcal{H}$ graded by degree $d : \mathcal{H} \longrightarrow \mathbb{Z}$.

Products: $m_k : \mathcal{H}^{\otimes k} \longrightarrow \mathcal{H}$ (degree-odd, multi-linear)

A_∞ relations (for $A_1, A_2, \ldots \in \mathcal{H}$)

$$
\begin{aligned}
0 &= m_1(m_1(A_1))\,, \\
0 &= m_1(m_2(A_1, A_2)) + m_2(m_1(A_1), A_2) + \\
&\qquad\qquad + (-1)^{d(A_1)} m_2(A_1, m_1(A_2))\,, \\
0 &= m_1(m_3(A_1, A_2, A_3)) + m_2(m_2(A_1, A_2), A_3) + \\
&\qquad\qquad + (-1)^{d(A_1)} m_2(A_1, m_2(A_2, A_3)) + \\
&\quad + m_3(m_1(A_1), A_2, A_3) + (-1)^{d(A_1)} m_3(A_1, m_1(A_2), A_3) + \\
&\qquad\qquad + (-1)^{d(A_1)+d(A_2)} m_3(A_1, A_2, m_1(A_3))\,, \\
&\;\vdots
\end{aligned}
$$

or, more compactly,

$$\sum_{l=1}^{k} m_l m_{k+1-l} = 0$$

A_∞ algebras

Symplectic form: $\omega : \mathcal{H}^{\otimes 2} \longrightarrow \mathbb{C}$

$$\omega(A_1, A_2) = -(-1)^{d(A_1)d(A)_2}\omega(A_2, A_1)$$

A_∞ algebras

Symplectic form: $\omega : \mathcal{H}^{\otimes 2} \longrightarrow \mathbb{C}$

$$\omega(A_1, A_2) = -(-1)^{d(A_1)d(A)_2}\omega(A_2, A_1)$$

Cyclicity:

$$\omega(A_1, m_k(A_2, \ldots, A_{k+1})) = -(-1)^{d(A_1)}\omega(m_k(A_1, \ldots, A_k), A_{k+1})$$

A_∞ algebras

Symplectic form: $\omega : \mathcal{H}^{\otimes 2} \longrightarrow \mathbb{C}$

$$\omega(A_1, A_2) = -(-1)^{d(A_1)d(A)_2}\omega(A_2, A_1)$$

Cyclicity:

$$\omega(A_1, m_k(A_2, \ldots, A_{k+1})) = -(-1)^{d(A_1)}\omega(m_k(A_1, \ldots, A_k), A_{k+1})$$

A_∞ action: for Φ degree even

$$S(\Phi) = \frac{1}{2}\omega(\Phi, m_1(\Phi)) + \frac{1}{3}\omega(\Phi, m_2(\Phi, \Phi)) + \frac{1}{4}\omega(\Phi, m_3(\Phi, \Phi, \Phi)) + \ldots$$

$\rightarrow$ equation of motion

$$0 = \mathrm{EOM}(\Phi) \equiv m_1(\Phi) + m_2(\Phi, \Phi) + m_3(\Phi, \Phi, \Phi) + \ldots$$

$\rightarrow$ linearized gauge symmetry

$$\delta\Phi = m_1(\Lambda) + m_2(\Lambda, \Phi) + m_2(\Phi, \Lambda) + \ldots$$

L_∞ algebras

[Lada, Stasheff: hep-th/9209099]

Products: $l_k : \mathcal{H}^{\wedge k} \to \mathcal{H}$ are graded-symmetric

$$l_k(A_{\sigma(1)}, \ldots, A_{\sigma(k)}) = (-1)^\sigma l_k(A_1, \ldots, A_k)$$

L_∞ algebras

[Lada, Stasheff: hep-th/9209099]

Products: $l_k : \mathcal{H}^{\wedge k} \to \mathcal{H}$ are graded-symmetric

$$l_k(A_{\sigma(1)}, \ldots, A_{\sigma(k)}) = (-1)^\sigma l_k(A_1, \ldots, A_k)$$

A_∞ relations

$$0 = l_1(l_1(A_1))\,,$$

$$0 = l_1(l_2(A_1, A_2)) + l_2(l_1(A_1), A_2) + (-1)^{d(A_1)d(A_2)} l_2(l_1(A_2), A_1)\,,$$

$$\begin{aligned} 0 = {} & l_1(l_3(A_1, A_2, A_3)) + l_2(l_2(A_1, A_2), A_3) + \\ & + (-1)^{d(A_1)(d(A_2)+d(A_3))} l_2(l_2(A_2, A_3), A_1) + \\ & + (-1)^{d(A_3)(d(A_1)+d(A_2))} l_2(l_2(A_3, A_1), A_2) + \\ & + l_3(l_1(A_1), A_2, A_3) + (-1)^{d(A_1)} l_3(A_1, l_1(A_2), A_3) + \\ & + (-1)^{d(A_1)+d(A_2)} l_3(A_1, A_2, l_1(A_3))\,, \end{aligned}$$

$$\vdots$$

L_∞ algebras

[Lada, Stasheff: hep-th/9209099]

Products: $l_k : \mathcal{H}^{\wedge k} \to \mathcal{H}$ are graded-symmetric

$$l_k(A_{\sigma(1)}, \ldots, A_{\sigma(k)}) = (-1)^\sigma l_k(A_1, \ldots, A_k)$$

A_∞ relations

$$
\begin{aligned}
0 &= l_1(l_1(A_1))\,, \\
0 &= l_1(l_2(A_1, A_2)) + l_2(l_1(A_1), A_2) + (-1)^{d(A_1)d(A_2)} l_2(l_1(A_2), A_1)\,, \\
0 &= l_1(l_3(A_1, A_2, A_3)) + l_2(l_2(A_1, A_2), A_3) + \\
&\quad + (-1)^{d(A_1)(d(A_2)+d(A_3))} l_2(l_2(A_2, A_3), A_1) + \\
&\quad\quad + (-1)^{d(A_3)(d(A_1)+d(A_2))} l_2(l_2(A_3, A_1), A_2) + \\
&\quad + l_3(l_1(A_1), A_2, A_3) + (-1)^{d(A_1)} l_3(A_1, l_1(A_2), A_3) + \\
&\quad\quad\quad + (-1)^{d(A_1)+d(A_2)} l_3(A_1, A_2, l_1(A_3))\,, \\
&\vdots
\end{aligned}
$$

L_∞ action: for Φ degree-even

$$S(\Phi) = \frac{1}{2!}\omega(\Phi, Q\Phi) + \frac{1}{3!}\omega(\Phi, l_2(\Phi, \Phi)) + \frac{1}{4!}\omega(\Phi, l_3(\Phi, \Phi, \Phi)) + \ldots$$

L_∞ from A_∞

Symmetrization: for (cyclic) A_∞ products m_k define

$$l_k(A_1, \ldots, A_k) \equiv \sum_{\sigma \in S_k} (-1)^{\epsilon(\sigma)} m_k(A_{\sigma(1)}, \ldots, A_{\sigma(k)}),$$

that is

$$\begin{aligned} l_1(A_1) &= m_1(A_1), \\ l_2(A_1, A_2) &= m_2(A_1, A_2) + (-1)^{d(A_1)d(A_2)} m_2(A_2, A_1), \\ &\vdots \end{aligned}$$

→ l_k satisfy an L_∞ algebra

→ A_∞ action defined using m_k becomes an L_∞ action written in terms of l_k
$\implies$ all A_∞ field theories are automatically L_∞

General field theories

One can express a general classical field theory in an L_∞ form
[Hohm, Zwiebach: 1701.08824]

$$\mathcal{H} = \ldots \oplus \mathcal{H}_{-1} \oplus \mathcal{H}_0 \oplus \mathcal{H}_{+1} \oplus \ldots$$

$\mathcal{H}_{+1}$ ... gauge parameters (can be empty)

$\mathcal{H}_0$ ... dynamical fields

$\mathcal{H}_{-1}$ ... field equations

General field theories

One can express a general classical field theory in an L_∞ form
[Hohm, Zwiebach: 1701.08824]

$$\mathcal{H} = \ldots \oplus \mathcal{H}_{-1} \oplus \mathcal{H}_0 \oplus \mathcal{H}_{+1} \oplus \ldots$$

$\mathcal{H}_{+1}$	$\ldots$	gauge parameters (can be empty)
$\mathcal{H}_0$	$\ldots$	dynamical fields
$\mathcal{H}_{-1}$	$\ldots$	field equations

Examples:

→ massless scalar ϕ: L_∞ structure $l_1 = \Box$, l_2, l_3, $\ldots$ (Taylor exp. of $V(\phi)$)

→ Chern-Simons theory: A_2 structure $m_1 = d$, $m_2 = \wedge$

→ Yang-Mills, Einstein-Hilbert, $\ldots$

→ Witten's cubic open SFT: A_2 structure $m_1 = Q$, $m_2 = *$

Outline

Projectors and propagators

[Kajiura: hep-th/0112228; Berkovits, Schnabl: hep-th/0307019]

BPZ-even projector P (denote $\bar{P} \equiv 1 - P$)

$$\omega(A_1, PA_2) = \omega(PA_1, A_2)$$

Projectors and propagators

[Kajiura: hep-th/0112228; Berkovits, Schnabl: hep-th/0307019]

BPZ-even projector P (denote $\bar{P} \equiv 1 - P$)

$$\omega(A_1, PA_2) = \omega(PA_1, A_2)$$

Splitting the string field:

$$\Phi = \phi + R$$

$$\begin{aligned} \phi &\equiv P\Phi \quad \ldots \quad \text{modes which we want to keep} \\ R &\equiv \bar{P}\Phi \quad \ldots \quad \text{modes we are integrating out} \end{aligned}$$

Projectors and propagators

[Kajiura: hep-th/0112228; Berkovits, Schnabl: hep-th/0307019]

BPZ-even projector P (denote $\bar{P} \equiv 1 - P$)

$$\omega(A_1, PA_2) = \omega(PA_1, A_2)$$

Splitting the string field:

$$\Phi = \phi + R$$

$$\begin{aligned} \phi &\equiv P\Phi \quad \ldots \quad \text{modes which we want to keep} \\ R &\equiv \bar{P}\Phi \quad \ldots \quad \text{modes we are integrating out} \end{aligned}$$

Propagator $h : \mathcal{H} \longrightarrow \mathcal{H}$ for R (with gauge-fixing condition $hR = 0$) satisfies the (abstract) Hodge-Kodaira decomposition

$$hm_1 + m_1 h = 1 - P$$

as well as (consistency) annihilation conditions

$$h^2 = hP = Ph = 0$$

Classical integration-out

Recursion relation: solving the R-equation of motion

$$\mathsf{EOM}_R(\psi, R) \equiv \bar{P}\,\mathsf{EOM}(\psi + R)$$

for ϕ, we obtain

$$R(\psi) = -h\big[m_2(\Psi,\Psi) + m_3(\Psi,\Psi,\Psi) + \dots\big]\big|_{\Psi=\psi+R(\psi)}$$

so that denoting $\mathcal{G}(A) = -hm_2(A,A) - hm_3(A,A)$, we can iterate to find

$$\begin{aligned}\Psi(\psi) &\equiv \psi + R(\psi)\\ &= \psi + \mathcal{G}(\psi + \mathcal{G}(\psi + \mathcal{G}(\psi + \dots)))\\ &= \psi - hm_2(\psi,\psi) - hm_3(\psi,\psi,\psi) + \\ &\qquad + hm_2(hm_2(\psi,\psi),\psi) + hm_2(\psi, hm_2(\psi,\psi)) + \dots\end{aligned}$$

Classical integration-out

Recursion relation: solving the R-equation of motion

$$\mathsf{EOM}_R(\psi, R) \equiv \bar{P}\,\mathsf{EOM}(\psi + R)$$

for ϕ, we obtain

$$R(\psi) = -h\big[m_2(\Psi,\Psi) + m_3(\Psi,\Psi,\Psi) + \ldots\big]\big|_{\Psi=\psi+R(\psi)}$$

so that denoting $\mathcal{G}(A) = -hm_2(A,A) - hm_3(A,A)$, we can iterate to find

$$\begin{aligned}\Psi(\psi) &\equiv \psi + R(\psi)\\ &= \psi + \mathcal{G}(\psi + \mathcal{G}(\psi + \mathcal{G}(\psi + \ldots)))\\ &= \psi - hm_2(\psi,\psi) - hm_3(\psi,\psi,\psi) +\\ &\qquad + hm_2(hm_2(\psi,\psi),\psi) + hm_2(\psi, hm_2(\psi,\psi)) + \ldots\end{aligned}$$

Gauge-constraint trivialization: (denoting $\mathsf{EOM}_\psi(\psi, R) \equiv P\mathsf{EOM}(\psi + R)$)

$$\mathsf{EOM}_\psi(\psi^*, R(\psi^*)) = 0 \quad \Longrightarrow \quad \mathsf{EOM}(\psi^*, \Psi(\psi^*)) = 0$$

$\rightarrow$ no additional independent constraints apart from the e.o.m. for ψ

[Erbin, Maccaferri, Schnabl, JV: 2006.16270]

Effective dynamics

Effective equation of motion:

$$\mathrm{EOM}_\psi(\psi, R(\psi)) = \tilde{m}_1(\psi) + \tilde{m}_2(\psi,\psi) + \tilde{m}_3(\psi,\psi,\psi) + \dots$$

where we introduce effective A_∞ products [Kajiura: hep-th/0112228]

$$\begin{aligned}
\tilde{m}_1(A_1) &= Pm_1(A_1)\,, \\
\tilde{m}_2(A_1,A_2) &= Pm_2(A_1,A_2)\,, \\
\tilde{m}_3(A_1,A_2,A_3) &= Pm_3(A_1,A_2,A_3) + \\
&\quad - Pm_2(hm_2(A_1,A_2),A_3) - Pm_2(A_1,hm_2(A_2,A_3))\,, \\
&\ \vdots
\end{aligned}$$

→ tree-level Feynman diagram expansion of effective vertices

Effective dynamics

Effective equation of motion:

$$\mathsf{EOM}_\psi(\psi, R(\psi)) = \tilde{m}_1(\psi) + \tilde{m}_2(\psi,\psi) + \tilde{m}_3(\psi,\psi,\psi) + \ldots$$

where we introduce effective A_∞ products [Kajiura: hep-th/0112228]

$$\begin{aligned}
\tilde{m}_1(A_1) &= Pm_1(A_1)\,,\\
\tilde{m}_2(A_1,A_2) &= Pm_2(A_1,A_2)\,,\\
\tilde{m}_3(A_1,A_2,A_3) &= Pm_3(A_1,A_2,A_3)+\\
&\quad - Pm_2(hm_2(A_1,A_2),A_3) - Pm_2(A_1,hm_2(A_2,A_3))\,,\\
&\vdots
\end{aligned}$$

→ tree-level Feynman diagram expansion of effective vertices

Cyclicity: effective products $\tilde{m}_k$ cyclic whenever [Kajiura: math/0306332]

$$\omega(A_1, hA_2) = (-1)^{d(A_1)}\omega(hA_1, A_2)$$

Effective dynamics

Effective equation of motion:

$$\text{EOM}_\psi(\psi, R(\psi)) = \tilde{m}_1(\psi) + \tilde{m}_2(\psi,\psi) + \tilde{m}_3(\psi,\psi,\psi) + \ldots$$

where we introduce effective A_∞ products [Kajiura: hep-th/0112228]

$$\begin{aligned}
\tilde{m}_1(A_1) &= Pm_1(A_1)\,, \\
\tilde{m}_2(A_1, A_2) &= Pm_2(A_1, A_2)\,, \\
\tilde{m}_3(A_1, A_2, A_3) &= Pm_3(A_1, A_2, A_3) + \\
&\quad - Pm_2(hm_2(A_1, A_2), A_3) - Pm_2(A_1, hm_2(A_2, A_3))\,, \\
&\ \vdots
\end{aligned}$$

→ tree-level Feynman diagram expansion of effective vertices

Cyclicity: effective products $\tilde{m}_k$ cyclic whenever [Kajiura: math/0306332]

$$\omega(A_1, hA_2) = (-1)^{d(A_1)}\omega(hA_1, A_2)$$

Effective action:

$$\tilde{S}(\psi) \equiv S(\Psi(\psi)) = \frac{1}{2}\omega(\psi, \tilde{m}_1(\psi)) + \frac{1}{3}\omega(\psi, \tilde{m}_2(\psi,\psi)) + \frac{1}{4}\omega(\psi, \tilde{m}_3(\psi,\psi,\psi)) + \ldots$$

Strong deformation retract

Chain complex:

$$(V, d_V) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (W, d_W)$$

quasi-isomorphisms π, ι such that $d_W \pi = \pi d_V$, $\iota d_W = d_V \iota$

Strong deformation retract

Chain complex:

$$(V, d_V) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (W, d_W)$$

quasi-isomorphisms π, ι such that $d_W \pi = \pi d_V$, $\iota d_W = d_V \iota$

Hodge-Kodaira decomposition: contracting homotopy $\eta : V \longrightarrow V$ such that

$$\iota\pi - 1_V = d_V \eta + \eta d_V$$

Strong deformation retract

Chain complex:

$$(V, d_V) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (W, d_W)$$

quasi-isomorphisms π, ι such that $d_W \pi = \pi d_V$, $\iota d_W = d_V \iota$

Hodge-Kodaira decomposition: contracting homotopy $\eta : V \longrightarrow V$ such that

$$\iota\pi - 1_V = d_V \eta + \eta d_V$$

Retract & annihilation conditions:

$$\pi\iota = 1_W \,, \qquad \pi\eta = \eta^2 = \eta\iota = 0$$

Strong deformation retract

Chain complex:

$$(V, d_V) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (W, d_W)$$

quasi-isomorphisms π, ι such that $d_W \pi = \pi d_V$, $\iota d_W = d_V \iota$

Hodge-Kodaira decomposition: contracting homotopy $\eta : V \longrightarrow V$ such that

$$\iota\pi - 1_V = d_V \eta + \eta d_V$$

Retract & annihilation conditions:

$$\pi\iota = 1_W \, , \qquad \pi\eta = \eta^2 = \eta\iota = 0$$

$\Longrightarrow$ SDR:

$$\circlearrowright \eta \, (V, d_V) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (W, d_W)$$

Strong deformation retract

Chain complex:

$$(V, d_V) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (W, d_W)$$

quasi-isomorphisms π, ι such that $d_W \pi = \pi d_V$, $\iota d_W = d_V \iota$

Hodge-Kodaira decomposition: contracting homotopy $\eta : V \longrightarrow V$ such that

$$\iota\pi - 1_V = d_V \eta + \eta d_V$$

Retract & annihilation conditions:

$$\pi\iota = 1_W \,, \qquad \pi\eta = \eta^2 = \eta\iota = 0$$

$\Longrightarrow$ SDR:

$$\circlearrowright \eta \, (V, d_V) \underset{\iota}{\overset{\pi}{\rightleftarrows}} (W, d_W)$$

Example: decomposing $P = \Pi I$, we have

$$\circlearrowright (-h) \, (\mathcal{H}, m_1) \underset{I}{\overset{\Pi}{\rightleftarrows}} (P\mathcal{H}, \Pi m_1 I) \,,$$

Homological perturbation lemma

[see e.g. Crainic: `math/0403266`]

Perturbation: $\delta_V : V \to V$ of d_V such that

$$(d_W)^2 \equiv (d_V + \delta_V)^2 = 0\,.$$

Homological perturbation lemma

[see e.g. Crainic: math/0403266]

Perturbation: $\delta_V : V \to V$ of d_V such that

$$(d_W)^2 \equiv (d_V + \delta_V)^2 = 0\,.$$

New SDR:

$$\circlearrowright \tilde{\eta}\,(V, \tilde{d}_V) \underset{\tilde{\iota}}{\overset{\tilde{\pi}}{\rightleftarrows}} (W, \tilde{d}_W)\,.$$

with perturbed data

$$\begin{aligned}
\delta_W &= \pi \delta_V \frac{1}{1_V - \eta \delta_V} \iota\,, \\
\tilde{\iota} &= \iota + \eta \delta_V \frac{1}{1_V - \eta \delta_V} \iota\,, \\
\tilde{\pi} &= \pi + \pi \delta_V \frac{1}{1_V - \eta \delta_V} \eta\,, \\
\tilde{\eta} &= \eta + \eta \delta_V \frac{1}{1_V - \eta \delta_V} \eta\,,
\end{aligned}$$

Homological perturbation lemma

[see e.g. Crainic: math/0403266]

Perturbation: $\delta_V : V \to V$ of d_V such that

$$(d_W)^2 \equiv (d_V + \delta_V)^2 = 0\,.$$

New SDR:

$$\circlearrowright \tilde{\eta}\,(V, \tilde{d}_V) \underset{\tilde{\iota}}{\overset{\tilde{\pi}}{\rightleftarrows}} (W, \tilde{d}_W)\,.$$

with perturbed data

$$\begin{aligned}
\delta_W &= \pi \delta_V \frac{1}{1_V - \eta\delta_V}\iota\,,\\
\tilde{\iota} &= \iota + \eta\delta_V \frac{1}{1_V - \eta\delta_V}\iota\,,\\
\tilde{\pi} &= \pi + \pi\delta_V \frac{1}{1_V - \eta\delta_V}\eta\,,\\
\tilde{\eta} &= \eta + \eta\delta_V \frac{1}{1_V - \eta\delta_V}\eta\,,
\end{aligned}$$

Starting with $d_V = m_1$, we would like to view δ_V as adding higher products (interactions) "$m_2 + m_3 + \ldots$"

Tensor coalgebra language

Tensor product space: [Gaberdiel, Zwiebach: hep-th/9705038]

$$T\mathcal{H} = \mathcal{H}^{\otimes 0} \oplus \mathcal{H}^{\otimes 1} \oplus \mathcal{H}^{\otimes 2} \oplus \dots$$

with deconcatenation coproduct $\Delta \implies$ tensor coalgebra $(T\mathcal{H}, \Delta)$

Tensor coalgebra language

Tensor product space: [Gaberdiel, Zwiebach: hep-th/9705038]

$$T\mathcal{H} = \mathcal{H}^{\otimes 0} \oplus \mathcal{H}^{\otimes 1} \oplus \mathcal{H}^{\otimes 2} \oplus \dots$$

with deconcatenation coproduct $\Delta \implies$ tensor coalgebra $(T\mathcal{H}, \Delta)$

Uplift of various products on $\mathcal{H}$ to maps $T\mathcal{H} \longrightarrow T\mathcal{H}$ (coderivations, cohomomorphisms, ...)

$$m_k,\, h,\, P,\, \Pi,\, I, \quad \longrightarrow \quad \mathbf{m}_k,\, \mathbf{h},\, \mathbf{P},\, \mathbf{\Pi},\, \mathbf{I}$$

Tensor coalgebra language

Tensor product space: [Gaberdiel, Zwiebach: hep-th/9705038]

$$T\mathcal{H} = \mathcal{H}^{\otimes 0} \oplus \mathcal{H}^{\otimes 1} \oplus \mathcal{H}^{\otimes 2} \oplus \ldots$$

with deconcatenation coproduct $\Delta \implies$ tensor coalgebra $(T\mathcal{H}, \Delta)$

Uplift of various products on $\mathcal{H}$ to maps $T\mathcal{H} \longrightarrow T\mathcal{H}$ (coderivations, cohomomorphisms, …)

$$m_k,\, h,\, P,\, \Pi,\, I, \quad \longrightarrow \quad \mathbf{m}_k,\, \mathbf{h},\, \mathbf{P},\, \mathbf{\Pi},\, \mathbf{I}$$

Examples:

$\rightarrow$ $\mathbf{m}_k$ acts on $\mathcal{H}^{\otimes N}$ for $N < k$ trivially, and, for $N \geqslant k$ we have

$$\mathbf{m}_k = \sum_{n=0}^{N-k} \left[(1_{\mathcal{H}})^{\otimes N-k-n} \otimes m_k \otimes (1_{\mathcal{H}})^{\otimes n}\right]$$

$\rightarrow$ $\mathbf{h}$ acts on $\mathcal{H}^{\otimes N}$ for $N \geqslant 1$ as

$$\mathbf{h} = \sum_{l=0}^{N-1} \left[(1_{\mathcal{H}})^{\otimes l} \otimes h \otimes P^{\otimes (N-1-l)}\right] \quad \Longrightarrow \quad \begin{array}{c} \mathbf{m}_1\mathbf{h} + \mathbf{h}\mathbf{m}_1 = \mathbf{1}_{T\mathcal{H}} - \mathbf{P} \\ \mathbf{h}\mathbf{I} = \mathbf{h}^2 = \mathbf{P}\mathbf{h} = 0 \end{array}$$

Tensor coalgebra language

Tensor product space: [Gaberdiel, Zwiebach: hep-th/9705038]

$$T\mathcal{H} = \mathcal{H}^{\otimes 0} \oplus \mathcal{H}^{\otimes 1} \oplus \mathcal{H}^{\otimes 2} \oplus \dots$$

with deconcatenation coproduct $\Delta \implies$ tensor coalgebra $(T\mathcal{H}, \Delta)$

Uplift of various products on $\mathcal{H}$ to maps $T\mathcal{H} \longrightarrow T\mathcal{H}$ (coderivations, cohomomorphisms, ...)

$$m_k,\, h,\, P,\, \Pi,\, I, \quad \longrightarrow \quad \mathbf{m}_k,\, \mathbf{h},\, \mathbf{P},\, \mathbf{\Pi},\, \mathbf{I}$$

Examples:

$\rightarrow$ $\mathbf{m}_k$ acts on $\mathcal{H}^{\otimes N}$ for $N < k$ trivially, and, for $N \geqslant k$ we have

$$\mathbf{m}_k = \sum_{n=0}^{N-k} \left[(1_{\mathcal{H}})^{\otimes N-k-n} \otimes m_k \otimes (1_{\mathcal{H}})^{\otimes n}\right]$$

$\rightarrow$ $\mathbf{h}$ acts on $\mathcal{H}^{\otimes N}$ for $N \geqslant 1$ as

$$\mathbf{h} = \sum_{l=0}^{N-1} \left[(1_{\mathcal{H}})^{\otimes l} \otimes h \otimes P^{\otimes (N-1-l)}\right] \quad \Longrightarrow \quad \begin{array}{c} \mathbf{m}_1\mathbf{h} + \mathbf{h}\mathbf{m}_1 = \mathbf{1}_{T\mathcal{H}} - \mathbf{P} \\ \mathbf{h}\mathbf{I} = \mathbf{h}^2 = \mathbf{P}\mathbf{h} = 0 \end{array}$$

A_∞ relations: introducing the total coderivation $\mathbf{m} = \mathbf{m}_1 + \mathbf{m}_2 + \dots$ we have (cf. classical BV master equation)

$$[\mathbf{m}, \mathbf{m}]^2 = 0$$

Effective physics as homotopy transfer

<u>Free SDR:</u>

$$\circlearrowright (-\mathbf{h})\,(T\mathcal{H}, \mathbf{m}_1) \underset{\mathbf{I}}{\overset{\mathbf{\Pi}}{\rightleftarrows}} (TP\mathcal{H}, \mathbf{\Pi}\mathbf{m}_1\mathbf{I})$$

Effective physics as homotopy transfer

Free SDR:

$$\circlearrowright(-\mathbf{h})\,(T\mathcal{H},\mathbf{m}_1)\underset{\mathbf{I}}{\overset{\mathbf{\Pi}}{\rightleftarrows}}(TP\mathcal{H},\mathbf{\Pi}\mathbf{m}_1\mathbf{I})$$

Perturbation: deform $\mathbf{m}_1$ by adding interactions $\mathbf{m}_1 \to \mathbf{m} \equiv \mathbf{m}_1 + \delta\mathbf{m}$ where

$$\delta\mathbf{m} = \mathbf{m}_2 + \mathbf{m}_3 + \dots$$

Effective physics as homotopy transfer

Free SDR:

$$\circlearrowright (-\mathbf{h})\,(T\mathcal{H},\mathbf{m}_1) \underset{\mathbf{I}}{\overset{\mathbf{\Pi}}{\rightleftarrows}} (TP\mathcal{H},\mathbf{\Pi}\mathbf{m}_1\mathbf{I})$$

Perturbation: deform $\mathbf{m}_1$ by adding interactions $\mathbf{m}_1 \to \mathbf{m} \equiv \mathbf{m}_1 + \delta\mathbf{m}$ where

$$\delta\mathbf{m} = \mathbf{m}_2 + \mathbf{m}_3 + \ldots$$

Interacting SDR:

$$\circlearrowright (-\tilde{\mathbf{h}})\,(T\mathcal{H},\mathbf{m}) \underset{\tilde{\mathbf{I}}}{\overset{\tilde{\mathbf{\Pi}}}{\rightleftarrows}} (TP\mathcal{H},\tilde{\mathbf{m}})$$

where in particular

$$\tilde{\mathbf{m}} = \mathbf{\Pi}\mathbf{m}\frac{1}{\mathbf{1}_{T\mathcal{H}} + \mathbf{h}\delta\mathbf{m}}\mathbf{I} = \tilde{\mathbf{m}}_1 + \tilde{\mathbf{m}}_2 + \tilde{\mathbf{m}}_3 + \ldots$$

$\to$ homological perturbation lemma automatically takes care of the tree-level Feynman diagram expansion!

[Costello, Gwilliam: 2009; Konopka: 1507.08250; Erbin, Maccaferri, Schnabl, JV: 2006.16270; Arvanitakis, Hohm, Hull, Lekeu: 2007.07942]

Horizontal composition

Sequential integrating-out:

→ first integrate out DOFs given by $\mathbf{P}^{(1)}$ using propagator $\mathbf{h}^{(1)}$

→ then integrate out DOFs given by $\mathbf{P}^{(2)}$ (such that $[\mathbf{P}^{(1)}, \mathbf{P}^{(2)}] = 0$) using propagator $\mathbf{h}^{(2)}$

Horizontal composition

Sequential integrating-out:

$\rightarrow$ first integrate out DOFs given by $\mathbf{P}^{(1)}$ using propagator $\mathbf{h}^{(1)}$

$\rightarrow$ then integrate out DOFs given by $\mathbf{P}^{(2)}$ (such that $[\mathbf{P}^{(1)}, \mathbf{P}^{(2)}] = 0$) using propagator $\mathbf{h}^{(2)}$

or, equivalently,

Horizontal composition

Sequential integrating-out:

$\rightarrow$ first integrate out DOFs given by $\mathbf{P}^{(1)}$ using propagator $\mathbf{h}^{(1)}$

$\rightarrow$ then integrate out DOFs given by $\mathbf{P}^{(2)}$ (such that $[\mathbf{P}^{(1)}, \mathbf{P}^{(2)}] = 0$) using propagator $\mathbf{h}^{(2)}$

or, equivalently,

Composite integrating-out: integrate out DOFs given by $\mathbf{P}^{(2)}\mathbf{P}^{(1)}$ using the composite propagator [Erbin, Maccaferri, Schnabl, JV: 2006.16270]

$$\mathbf{h}^{(12)} \equiv \mathbf{h}^{(1)} + \mathbf{I}^{(1)}\mathbf{h}^{(2)}\mathbf{\Pi}^{(1)}$$

Horizontal composition

Sequential integrating-out:

$\rightarrow$ first integrate out DOFs given by $\mathbf{P}^{(1)}$ using propagator $\mathbf{h}^{(1)}$

$\rightarrow$ then integrate out DOFs given by $\mathbf{P}^{(2)}$ (such that $[\mathbf{P}^{(1)}, \mathbf{P}^{(2)}] = 0$) using propagator $\mathbf{h}^{(2)}$

or, equivalently,

Composite integrating-out: integrate out DOFs given by $\mathbf{P}^{(2)}\mathbf{P}^{(1)}$ using the composite propagator [Erbin, Maccaferri, Schnabl, JV: 2006.16270]

$$\mathbf{h}^{(12)} \equiv \mathbf{h}^{(1)} + \mathbf{I}^{(1)}\mathbf{h}^{(2)}\mathbf{\Pi}^{(1)}$$

Example: open SFT on a stack of Dp-branes ($\Longrightarrow$ non-Abelian YM + $\mathcal{O}(\alpha')$)

1. Integrate out massive modes using Siegel gauge propagator $h^{(1)} = (b_0/L_0)\bar{P}_0$
2. Integrate out the (auxiliary) Nakanishi-Lautrup field using algebraic propagator $h^{(2)} = (1/2)b_1 b_{-1} c_0 P_0$ to obtain gauge-invariant effective action for gluons only

$\rightarrow$ composite propagator (see also [Sen: 2002.04043])

$$h^{(12)} = \frac{b_0}{L_0}\bar{P}_0 + \frac{1}{2}b_1 b_{-1} P_0$$

$\mathcal{N}=2$ localization

Evaluation of effective couplings:

$\rightarrow$ generally need to compute off-shell tree-level string amplitudes $\implies$ hard!!

$\rightarrow$ focus on effective couplings of massless modes at zero momentum $\implies$ moduli space for the background at hand

$\rightarrow$ further simplification for superstring backgrounds with global worldsheet $\mathcal{N}=2$ superconformal symmetry such that all marginal operators carry $U(1)$ R-charge ± 1

$\mathcal{N} = 2$ localization

Evaluation of effective couplings:

→ generally need to compute off-shell tree-level string amplitudes $\Longrightarrow$ hard!!

→ focus on effective couplings of massless modes at zero momentum
$\Longrightarrow$ moduli space for the background at hand

→ further simplification for superstring backgrounds with global worldsheet $\mathcal{N} = 2$ superconformal symmetry such that all marginal operators carry $U(1)$ R-charge ± 1

Conjecture: computation of tree-level zero-momentum effective superstring couplings for backgrounds with global $\mathcal{N} = 2$ worldsheet SCA localizes on the component of the boundary of the bosonic worldsheet moduli space given by one-propagator degeneration

Verifications:

→ quartic effective potential in open SFT $\Longrightarrow$ generalized ADHM constraints [Maccaferri, Merlano: 1801.07607, 1905.04958; JV: 1910.00538]

→ quartic effective potential in heterotic SFT $\Longrightarrow$ no need for the bosonic closed string 3-product [Erbin, Maccaferri, JV: 1912.05463]

→ certain 5-point couplings in open SFT [Maccaferri, JV: in progress]

Summary

We have reviewed the homotopy-algebraic structure of (string) field theories

Summary

We have reviewed the homotopy-algebraic structure of (string) field theories

We have shown how to classically integrate out a subset of degrees of freedom:

$\rightarrow$ by solving the field equations for the unwanted modes

$\rightarrow$ by invoking homological perturbation theory

and realized that the homotopy-algebraic structure of the parent (UV) theory is transferred onto the daughter (IR) theory

Summary

We have reviewed the homotopy-algebraic structure of (string) field theories

We have shown how to classically integrate out a subset of degrees of freedom:

$\rightarrow$ by solving the field equations for the unwanted modes

$\rightarrow$ by invoking homological perturbation theory

and realized that the homotopy-algebraic structure of the parent (UV) theory is transferred onto the daughter (IR) theory

Thank you!