



The scalar potential of the 331 model: theoretical constraints

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arXiv:2001.08550

AC, M. Ghezzi and G.M. Pruna

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Menu

Appetizer - Higgs in the SM: Unitarity constraint

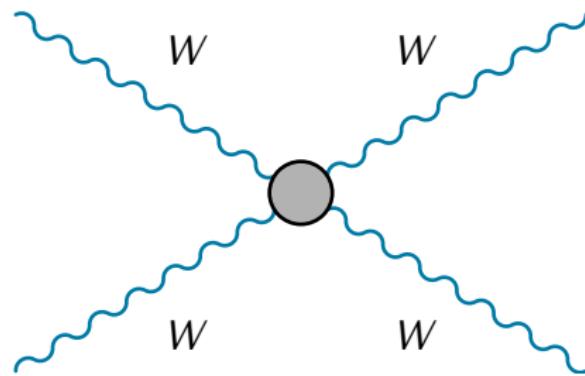
First Course - The 331 Model: General Features

Second Course - Unitarity of the 331 Model

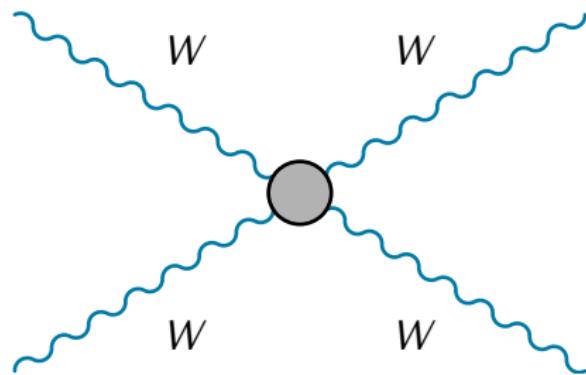
Dessert - The Scalar Potential: Boundedness from Below

Higgs in the SM: Unitarity constraint

Higgs in the SM: Unitarity constraint



Higgs in the SM: Unitarity constraint



$$A(W^+W^- \rightarrow W^+W^-) \xrightarrow{s \gg M_W^2} \frac{1}{v^2} \left[s + t - \frac{s^2}{s - M_H^2} - \frac{t^2}{t - M_H^2} \right]$$

Higgs in the SM: Unitarity constraint

$$A = 16\pi \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(\cos \theta) a_\ell$$

Higgs in the SM: Unitarity constraint

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$$\begin{aligned}\sigma &= \frac{8\pi}{s} \sum_{\ell=0}^{\infty} \sum_{\ell'=0}^{\infty} (2\ell + 1)(2\ell' + 1) a_\ell a_{\ell'} \int_{-1}^1 d\cos\theta P_\ell(\cos\theta) P_{\ell'}(\cos\theta) \\ &= \frac{16\pi}{s} \sum_{\ell=0}^{\infty} (2\ell + 1) |a_\ell|^2\end{aligned}$$

with the optical theorem

Higgs in the SM: Unitarity constraint

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with the optical theorem

$$\begin{aligned}|a_\ell|^2 &= \text{Im}(a_\ell) \Rightarrow [\text{Re}(a_\ell)]^2 + [\text{Im}(a_\ell)]^2 = \text{Im}(a_\ell) \\ &\Rightarrow [\text{Re}(a_\ell)]^2 + [\text{Im}(a_\ell) - \frac{1}{2}]^2 = \frac{1}{4}\end{aligned}$$

Higgs in the SM: Unitarity constraint

$$\begin{aligned} a_0 &= \frac{1}{16\pi s} \int_s^0 dt |A| \\ &= -\frac{M_H^2}{16\pi v^2} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log \left(1 + \frac{s}{M_H^2} \right) \right] \end{aligned}$$

$$a_0 \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}$$
$$\Downarrow$$

Higgs in the SM: Unitarity constraint

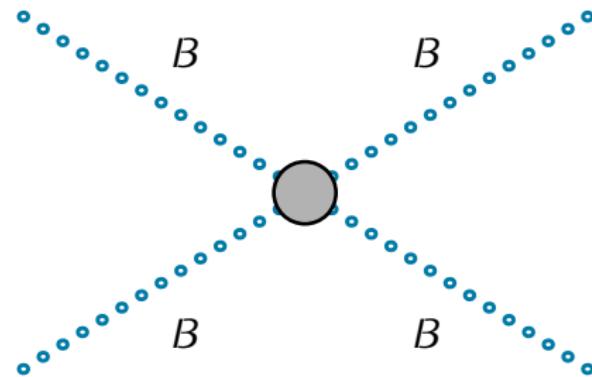
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$$a_0 \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}$$

↓

$$M_H \lesssim 870 \text{ GeV}$$

Higgs in the SM: Unitarity constraint



$$B = W, Z, H$$

$$\left(W_L^+ W_L^- , \frac{1}{\sqrt{2}} Z_L Z_L , \frac{1}{\sqrt{2}} H H , Z_L H , W_L^+ H , W_L^+ Z_L \right)$$

Higgs in the SM: Unitarity constraint

$$a_0 \propto \frac{M_H^2}{v^2} \begin{pmatrix} 1 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

⇓

$$M_H \lesssim 710 \text{ GeV}$$

Any need for BSM?

- ◊ dark matter
- ◊ neutrino masses
- ◊ EWPT
- ◊ matter-antimatter asymmetry
- ◊ hierarchy problem
- ◊ strong CP problem
- ◊ $n_{Q_f} = n_{L_f} = 3$
- ◊ hierarchy in fermion masses

...

The 331 Model: General Features

Extended Gauge Group

$$SU(3)_c \times SU(3)_L \times U(1)_X$$

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$$SU(3)_c \times SU(3)_L \times U(1)_X$$

$SU(3)$ has two diagonal generators

Extended Gauge Group

$$SU(3)_c \times SU(3)_L \times U(1)_X$$

$SU(3)$ has two diagonal generators



$$\mathbb{Q} = \mathbb{T}_3 + \beta_Q \mathbb{T}_8 + X \mathbb{I}$$

Field Content

$$Q_1 = \begin{pmatrix} u \\ d \\ D \end{pmatrix}, \quad Q_2 = \begin{pmatrix} c \\ s \\ S \end{pmatrix}, \quad Q_{1,2} \in (3, 3, X_{Q_{1,2}})$$

$$Q_3 = \begin{pmatrix} b \\ t \\ T \end{pmatrix}, \quad Q_3 \in (3, \bar{3}, X_{Q_3})$$

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$$Q_3 = \begin{pmatrix} b \\ t \\ T \end{pmatrix}, \quad Q_3 \in (3, \bar{3}, X_{Q_3})$$

$$L = \begin{pmatrix} l \\ \nu_l \\ E_l \end{pmatrix}, \quad l \in (1, \bar{3}, X_L), \quad l = e, \mu, \tau$$

Field Content

$$Q_1 = \begin{pmatrix} u \\ d \\ D \end{pmatrix}, \quad Q_2 = \begin{pmatrix} c \\ s \\ S \end{pmatrix}, \quad Q_{1,2} \in (3, 3, X_{Q_{1,2}})$$

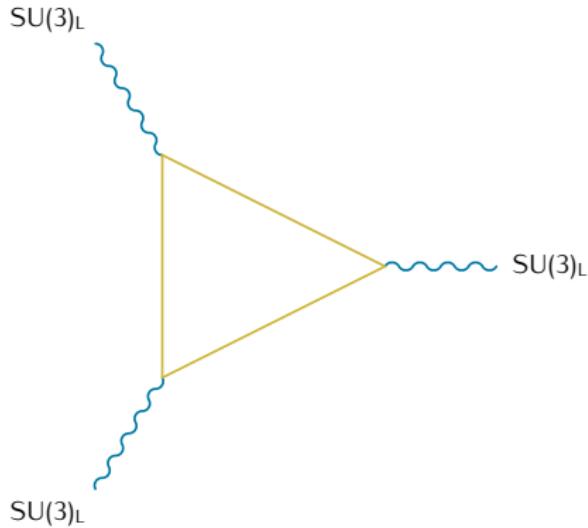
$$Q_3 = \begin{pmatrix} b \\ t \\ T \end{pmatrix}, \quad Q_3 \in (3, \bar{3}, X_{Q_3})$$

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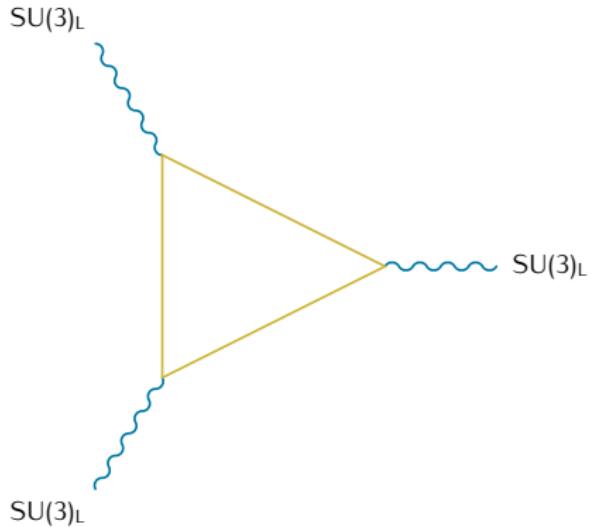
$$\chi = \begin{pmatrix} \chi^A \\ \chi^B \\ \chi^0 \end{pmatrix} \in (1, 3, X_\chi), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix} \in (1, 3, X_\rho), \quad \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^- \end{pmatrix} \in (1, 3, X_\eta)$$

$$Q^A = \frac{1}{2} + \frac{\sqrt{3}}{2} \beta_Q, \quad Q^B = -\frac{1}{2} + \frac{\sqrt{3}}{2} \beta_Q$$

Anomaly Cancellation: the $SU(3)_L$ example



Anomaly Cancellation: the $SU(3)_L$ example



$$Q_1 = +3 \times 3_c$$

$$Q_2 = +3 \times 3_c$$

$$Q_3 = -3 \times 3_c$$

$$L = -3 \times 3_f$$

$$n_{Q_f} = n_{L_f} = 3 \kappa$$

From $SU(3)_L \times U(1)_X$ to $U(1)_{em}$

$SU(3)_L \times U(1)_X$

From $SU(3)_L \times U(1)_X$ to $U(1)_{em}$

$SU(3)_L \times U(1)_X$

$$\langle \chi \rangle$$
$$\Downarrow$$

$SU(2)_L \times U(1)_Y$

From $SU(3)_L \times U(1)_X$ to $U(1)_{\text{em}}$

$SU(3)_L \times U(1)_X$

$\langle \chi \rangle$
↓

$SU(2)_L \times U(1)_Y$

$\langle \eta \rangle, \langle \rho \rangle$
↓

$U(1)_{\text{em}}$

From $SU(3)_L \times U(1)_X$ to $U(1)_{em}$

$$SU(3)_L \times U(1)_X \quad W_1, \dots, W_8, B_X$$

$$\langle \chi \rangle$$
$$\Downarrow$$

$$SU(2)_L \times U(1)_Y$$

$$\langle \eta \rangle, \langle \rho \rangle$$
$$\Downarrow$$

$$U(1)_{em}$$

From $SU(3)_L \times U(1)_X$ to $U(1)_{em}$

$SU(3)_L \times U(1)_X$

W_1, \dots, W_8, B_X

$\langle \chi \rangle$
↓

$\langle \chi \rangle$
↓

$SU(2)_L \times U(1)_Y$

$W_1, W_2, W_3, B_Y, Z', Y^{\pm A}, V^{\pm B}$

$\langle \eta \rangle, \langle \rho \rangle$
↓

$U(1)_{em}$

From $SU(3)_L \times U(1)_X$ to $U(1)_{em}$

$SU(3)_L \times U(1)_X$

W_1, \dots, W_8, B_X

$\langle \chi \rangle$
↓

$\langle \chi \rangle$
↓

$SU(2)_L \times U(1)_Y$

$W_1, W_2, W_3, B_Y, Z', Y^{\pm A}, V^{\pm B}$

$\langle \eta \rangle, \langle \rho \rangle$
↓

$\langle \eta \rangle, \langle \rho \rangle$
↓

$U(1)_{em}$

$\gamma, Z, W^\pm, Z', Y^{\pm A}, V^{\pm B}$

The Scalar Potential

The (β_Q -invariant) potential is

$$\begin{aligned}V = & m_1 \rho^* \rho + m_2 \eta^* \eta + m_3 \chi^* \chi \\& + \lambda_1 (\rho^* \rho)^2 + \lambda_2 (\eta^* \eta)^2 + \lambda_3 (\chi^* \chi)^2 \\& + \lambda_{12} \rho^* \rho \eta^* \eta + \lambda_{13} \rho^* \rho \chi^* \chi + \lambda_{23} \eta^* \eta \chi^* \chi \\& + \zeta_{12} \rho^* \eta \eta^* \rho + \zeta_{13} \rho^* \chi \chi^* \rho + \zeta_{23} \eta^* \chi \chi^* \eta \\& + \sqrt{2} f_{\rho \eta \chi} (\rho \eta \chi + h.c.)\end{aligned}$$

$$[m_i] = M^2$$

$$[f_{\rho \eta \chi}] = M^1$$

$$[\lambda_j] = [\zeta_k] = M^0$$

Massive Scalar States

$$\chi = \begin{pmatrix} \chi^A \\ \chi^B \\ \chi^0 \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^-B \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^{-A} \end{pmatrix}$$

- #3 neutral scalar, #1 neutral pseudoscalar ($G_Z, G_{Z'}$),
- #1 singly-charged (G_W),
- #1 A-charged (G_{Y^A}), #1 B-charged (G_{V^B})

Massive Scalar States

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#3 neutral scalar, #1 neutral pseudoscalar ($G_Z, G_{Z'}$),
#1 singly-charged (G_W),
#1 A-charged (G_{Y^A}), #1 B-charged (G_{V^B})

diagonalisation



$$m_{h_i} = f_{h_i}(\vec{\lambda}, \vec{\zeta}, f_{\rho \eta \chi}, v_j)$$

$$m_a = f_a(\vec{\lambda}, \vec{\zeta}, f_{\rho \eta \chi}, v_j)$$

$$m_{h^{\pm Q}} = f_{h^{\pm Q}}(\vec{\lambda}, \vec{\zeta}, f_{\rho \eta \chi}, v_j)$$

Massive Scalar States

$$\chi = \begin{pmatrix} \chi^A \\ \chi^B \\ \chi^0 \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^- \end{pmatrix}$$

- #3 neutral scalar, #1 neutral pseudoscalar ($G_Z, G_{Z'}$),
- #1 singly-charged (G_W),
- #1 A-charged (G_{Y^A}), #1 B-charged (G_{V^B})

"inverse" diagonalization
↓

$$\lambda = F_\lambda(m_{h_i}, m_a, m_{h^\pm Q}, v_j, \alpha_k)$$
$$f_{\rho\eta\chi} = F_{f_{\rho\eta\chi}}(m_{h_i}, m_a, m_{h^\pm Q}, v_j, \alpha_k)$$
$$\zeta = F_\zeta(m_{h_i}, m_a, m_{h^\pm Q}, v_j, \alpha_k)$$

Trading the Parameter: Explicit Example

$$\begin{aligned}\lambda_1 = & -\frac{m_{a_1}^2 \tan^2 \beta}{2v^2} + m_{h_1}^2 \frac{c_2^2 c_3^2 \sec^2 \beta}{2v^2} \\ & + m_{h_2}^2 \frac{\sec^2 \beta (s_1 s_2 c_3 - c_1 s_3)^2}{2v^2} \\ & + m_{h_3}^2 \frac{\sec^2 \beta (c_1 s_2 c_3 + s_1 s_3)^2}{2v^2} + O\left(\frac{m}{v_X}\right)\end{aligned}$$

$$\begin{aligned}\lambda_2 = & -\frac{m_{a_1}^2 \cot^2 \beta}{2v^2} + m_{h_1}^2 \frac{c_2^2 s_3^2 \csc^2 \beta}{2v^2} \\ & + m_{h_2}^2 \frac{\csc^2 \beta (s_1^2 s_2^2 s_3^2 + 2s_1 c_1 s_2 c_3 s_3 + c_1^2 c_3^2)}{2v^2} \\ & + m_{h_3}^2 \frac{\csc^2 \beta (c_1 s_2 (c_1 s_2 s_3^2 - 2s_1 c_3 s_3) + s_1^2 c_3^2)}{2v^2} + O\left(\frac{m}{v_X}\right)\end{aligned}$$

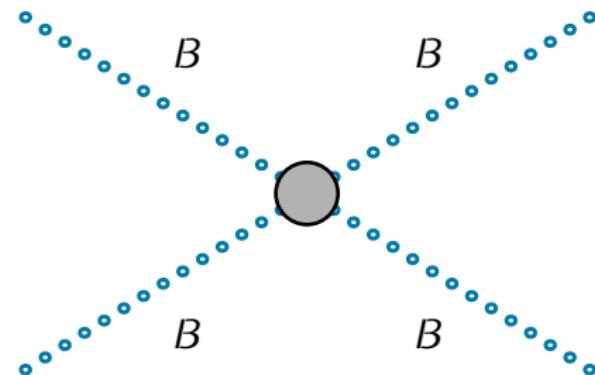
$$\begin{aligned}\lambda_{12} = & \frac{m_{a_1}^2}{v^2} + m_{h_1}^2 \frac{c_2^2 s_3 c_3 \csc \beta \sec \beta}{v^2} \\ & + m_{h_2}^2 \frac{\csc \beta \sec \beta}{4v^2} \left(4c_1 s_1 s_2 (c_3^2 - s_3^2) \right. \\ & \left. - c_3 s_3 (2s_1^2 (c_2^2 - s_2^2) + 6c_1 s_1 + 1) \right) \\ & - m_{h_3}^2 \frac{\csc \beta \sec \beta}{4v^2} \left(4c_1 s_1 s_2 (c_3^2 - s_3^2) \right. \\ & \left. + c_3 s_3 (2c_1^2 (c_2^2 - s_2^2) - 3(c_1^2 - s_1^2) + 1) \right) + O\left(\frac{m}{v_X}\right)\end{aligned}$$

$$\begin{aligned}\zeta_{12} = & \frac{2}{v^2} \left(m_{h_1^\pm}^2 - m_{a_1}^2 \right) + O\left(\frac{m}{v_X}\right) \\ \lambda_3 = \lambda_{13} = \lambda_{23} = \zeta_{13} = \zeta_{23} = & O\left(\frac{m}{v_X}\right)\end{aligned}$$

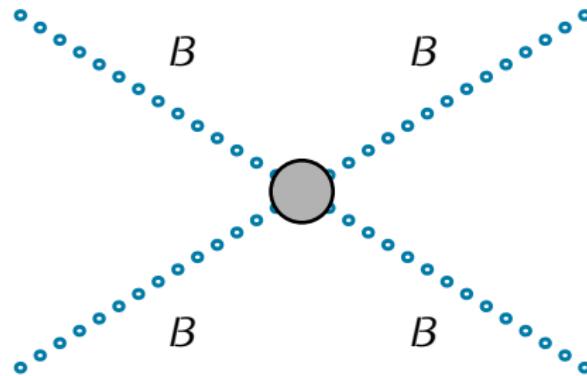
perturbativity of the couplings requires certain degeneracy among masses

Unitarity of the 331 Model

Unitarity of the 331 Models



Unitarity of the 331 Models



$$B = Z, W^\pm, Z', Y^{\pm A}, V^{\pm B}, h_i, a_1, h^\pm, h^{\pm A}, h^{\pm B}$$

$$\begin{aligned} Q = & 0, 1, 2, Q^A, Q^B, Q^A + 1, Q^B + 1, Q^A - 1, Q^B - 1, \\ & Q^A + Q^B, Q^A - Q^B, 2Q^A, 2Q^B \end{aligned}$$

Unitarity of the 331 Models

perturbative unitarity



$$|\mathbf{a}| \leq \frac{1}{2}$$

a are the eigenvalues of \mathcal{S} matrix of all $2 \rightarrow 2$ bosonic amplitudes (in the $s \rightarrow \infty$ limit)

Unitarity of the 331 Models

$$\mathbf{a} = \left\{ \frac{\lambda_i}{8\pi}, \frac{\lambda_{ij}}{16\pi}, \frac{\lambda_{ij} \pm \zeta_{ij}}{16\pi}, \frac{\lambda_{ij} + 2\zeta_{ij}}{16\pi}, \frac{\lambda_i + \lambda_j \pm \sqrt{(\lambda_i - \lambda_j)^2 + \zeta_{ij}^2}}{16\pi}, \right. \\ \left. \frac{\mathcal{P}_1^3(\lambda_m, \lambda_{mn}, \zeta_{mn})}{32\pi}, \frac{\mathcal{P}_2^3(\lambda_m, \lambda_{mn}, \zeta_{mn})}{32\pi} \right\}$$

Unitarity of the 331 Models

$$\mathbf{a} = \left\{ \frac{\lambda_i}{8\pi}, \frac{\lambda_{ij}}{16\pi}, \frac{\lambda_{ij} \pm \zeta_{ij}}{16\pi}, \frac{\lambda_{ij} + 2\zeta_{ij}}{16\pi}, \frac{\lambda_i + \lambda_j \pm \sqrt{(\lambda_i - \lambda_j)^2 + \zeta_{ij}^2}}{16\pi}, \right.$$
$$\left. \frac{\mathcal{P}_1^3(\lambda_m, \lambda_{mn}, \zeta_{mn})}{32\pi}, \frac{\mathcal{P}_2^3(\lambda_m, \lambda_{mn}, \zeta_{mn})}{32\pi} \right\}$$

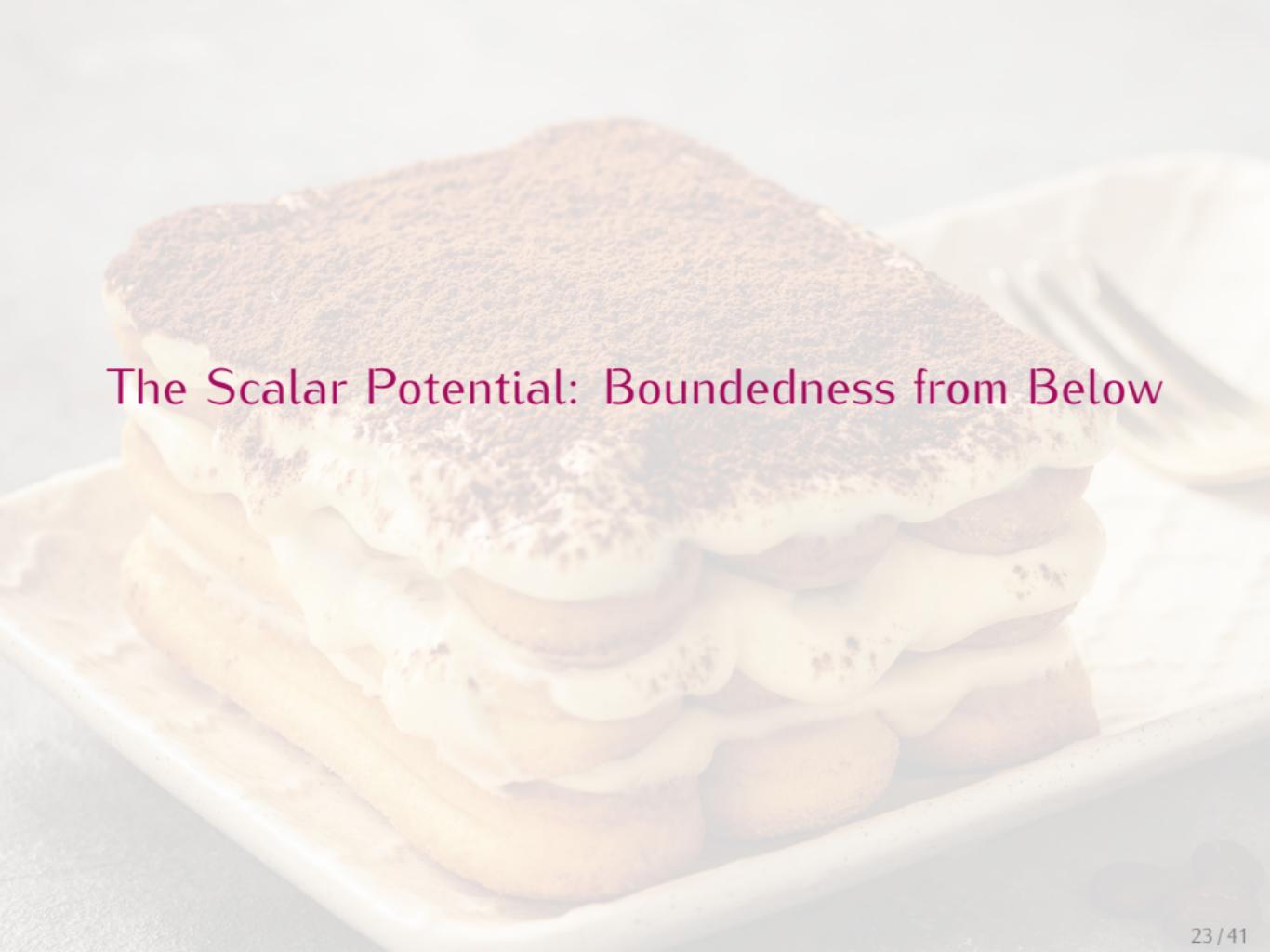
$$\sum_{i,j,k=1}^3 \left[\frac{x^3}{27} - \frac{4}{9}\lambda_i x^2 + \left(2(4\lambda_i\lambda_j - \zeta_{ij}^2)x - \frac{8}{3}(\zeta_{ij}\zeta_{ik}\zeta_{jk} - 3\lambda_i\zeta_{jk}^2 \right. \right.$$

$$\left. \left. + 4\lambda_i\lambda_j\lambda_k \right) (\varepsilon_{ijk})^2 \right],$$

$$\sum_{i,j,k=1}^3 \left[\frac{x^3}{27} - \frac{16}{9}\lambda_i x^2 + \left(2(64\lambda_i\lambda_j - (3\lambda_{ij} + \zeta_{ij})^2)x \right. \right.$$

$$\left. \left. - \frac{8}{3}(\zeta_{ik}\zeta_{jk}(9\lambda_{ij} + \zeta_{ij}) + 27\lambda_{ij}\lambda_{ik}(\lambda_{jk} + \zeta_{jk}) \right) + 4\lambda_i(64\lambda_j\lambda_k - 3(3\lambda_{jk} + \zeta_{jk})^2) \right) (\varepsilon_{ijk})^2 \right],$$

with $\lambda_{ji} = \lambda_{ij}$, $\zeta_{ji} = \zeta_{ij}$.

A stack of white cream puffs with chocolate drizzle on top, arranged in a pyramid shape.

The Scalar Potential: Boundedness from Below

BFB in the 331 Models

boundedness from below of the scalar potential



analysis of the highest powers of the fields

BFB in the 331 Models

boundedness from below of the scalar potential



analysis of the highest powers of the fields

not-so-easy in multi-Higgs models, results for n -doublets

Hadeler(1983); Klimenko(1984); Ivanov, Köpke, Mühlleitner(2018); Maniatis,
von Manteuffel, Nachtmann, Nagel(2006);
Degee, Ivanov, Kesu(2012); Kannike(2012); Maniatis, Nachtmann(2015);
Kannike(2016); Faro, Ivanov (2019);

BFB in the 331 Models

$$\Phi_i = \sqrt{r_i} e^{i \gamma_i} \begin{pmatrix} \sin a_i \cos b_i \\ e^{i \beta_i} \sin a_i \sin b_i \\ e^{i \alpha_i} \cos a_i \end{pmatrix}$$

$$V^{(4)} = V_R + \zeta'_{12} \tau_{12} + \zeta'_{13} \tau_{13} + \zeta'_{23} \tau_{23} = V_R + V_A,$$
$$V_R = \lambda_1 (\rho^* \rho)^2 + \lambda_2 (\eta^* \eta)^2 + \lambda_3 (\chi^* \chi)^2$$
$$+ \lambda'_{12} \rho^* \rho \eta^* \eta + \lambda'_{13} \rho^* \rho \chi^* \chi + \lambda'_{23} \eta^* \eta \chi^* \chi$$

$$\tau_{ij} = \left(\Phi_i^\dagger \Phi_i \right) \left(\Phi_j^\dagger \Phi_j \right) - \left(\Phi_i^\dagger \Phi_j \right) \left(\Phi_j^\dagger \Phi_i \right)$$

BFB in the 331 Models

BFB \equiv co-positivity of Q_{ij}

$$V^{(4)} = A_{ij} r_i r_j$$

BFB in the 331 Models

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co-positivity constraints of a (generic) rank-3 matrix A are

$$A_{ii} \geq 0, \quad \text{with } i = 1, 2, 3,$$

$$\mathring{A}_{ij} \equiv \sqrt{A_{ii} A_{jj}} + A_{ij} \geq 0, \quad \text{with } i, j = 1, 2, 3,$$

$$\begin{aligned} & \sqrt{A_{11} A_{22} A_{33}} + A_{12} \sqrt{A_{33}} + A_{13} \sqrt{A_{22}} + A_{23} \sqrt{A_{11}} \\ & + \sqrt{2\mathring{A}_{12}\mathring{A}_{13}\mathring{A}_{23}} \geq 0 \end{aligned}$$

BFB in the 331 Models

Relevant part of the 331 potential

$$V^{(4)} = V_R + V_A$$

$$\begin{aligned} V_R &= \lambda_1(\rho^*\rho)^2 + \lambda_2(\eta^*\eta)^2 + \lambda_3(\chi^*\chi)^2 \\ &\quad + \lambda'_{12}\rho^*\rho\eta^*\eta + \lambda'_{13}\rho^*\rho\chi^*\chi + \lambda'_{23}\eta^*\eta\chi^*\chi \\ V_A &= \zeta'_{12}\tau_{12} + \zeta'_{13}\tau_{13} + \zeta'_{23}\tau_{23} \end{aligned}$$

BFB in the 331 Models

Relevant part of the 331 potential

$$V^{(4)} = V_R + V_A$$

$$\begin{aligned} V_R &= \lambda_1(\rho^*\rho)^2 + \lambda_2(\eta^*\eta)^2 + \lambda_3(\chi^*\chi)^2 \\ &\quad + \lambda'_{12}\rho^*\rho\eta^*\eta + \lambda'_{13}\rho^*\rho\chi^*\chi + \lambda'_{23}\eta^*\eta\chi^*\chi \\ V_A &= \zeta'_{12}\tau_{12} + \zeta'_{13}\tau_{13} + \zeta'_{23}\tau_{23} \end{aligned}$$

V_R ok for extracting BFB conditions

V_A needs to be treated properly



angular minimization of V_A

BFB in the 331 Models

Simplest case: $\zeta'_{12} = \zeta'_{13} = \zeta'_{23} = 0$

$$V_R = Q_{ij} r_i r_j.$$

BFB in the 331 Models

Simplest case: $\zeta'_{12} = \zeta'_{13} = \zeta'_{23} = 0$

$$V_R = Q_{ij} r_i r_j.$$

Next-to-Simplest: at least one ζ' is zero

$$V_R + \min(V_A)_k^T = \tilde{Q}_k^{ij} r_i r_j, \quad k = 1, \dots, 4$$

$$\min(V_A)_1^T = \zeta'_{12} r_1 r_2 + \zeta'_{23} r_2 r_3,$$

$$\min(V_A)_2^T = \zeta'_{13} r_1 r_3 + \zeta'_{23} r_2 r_3,$$

$$\min(V_A)_3^T = \zeta'_{12} r_1 r_2 + \zeta'_{13} r_1 r_3,$$

$$\min(V_A)_4^T = \zeta'_{12} r_1 r_2 + \zeta'_{13} r_1 r_3 + \zeta'_{23} r_2 r_3.$$

BFB in the 331 Models

Next-to-Next-to-Simplest Case: all $\zeta' \neq 0$

$$V_R + \min(V_A)^{NT} = \hat{Q}^{ij} r_i r_j$$

AND

$$V_R + \min(V_A)_k^T = \tilde{Q}_k^{ij} r_i r_j, \quad k = 1, \dots, 4$$

where

$$\min(V_A)^{NT} = \frac{\zeta'_{12} \zeta'_{13} \zeta'_{23}}{4} \left(\frac{r_1}{\zeta'_{23}} + \frac{r_2}{\zeta'_{13}} + \frac{r_3}{\zeta'_{12}} \right)^2$$

Conclusions

- ◊ SM issues: dark matter, neutrino masses ... → BSM
- ◊ 331 model(s) explain the observed number of fermion families ($n_{Q_f} = n_{L_f} = 3\kappa$)
- ◊ 331 model is phenomenologically appealing as embrace different scenarios
- ◊ analysis of the scalar potential is β_Q -independent
(theoretical constraints: unitarity, perturbativity, BFB
NEW!!!)
- ◊ possible application for phenomenological studies



BACKUP

Minimization Conditions

$$\rho^0 = \frac{1}{\sqrt{2}}v_\rho + \frac{1}{\sqrt{2}}(\operatorname{Re} \rho^0 + i \operatorname{Im} \rho^0)$$

$$\eta^0 = \frac{1}{\sqrt{2}}v_\eta + \frac{1}{\sqrt{2}}(\operatorname{Re} \eta^0 + i \operatorname{Im} \eta^0)$$

$$\chi^0 = \frac{1}{\sqrt{2}}v_\chi + \frac{1}{\sqrt{2}}(\operatorname{Re} \chi^0 + i \operatorname{Im} \chi^0)$$

Minimization conditions ($\frac{\partial V}{\partial \Phi}|_{\Phi=0} = 0$) are

$$m_1 v_\rho + \lambda_1 v_\rho^3 + \frac{\lambda_{12}}{2} v_\rho v_\eta^2 - f_{\rho\eta\chi} v_\eta v_\chi + \frac{\lambda_{13}}{2} v_\rho v_\chi^2 = 0$$

$$m_2 v_\eta + \lambda_2 v_\eta^3 + \frac{\lambda_{12}}{2} v_\rho^2 v_\eta - f_{\rho\eta\chi} v_\rho v_\chi + \frac{\lambda_{23}}{2} v_\eta v_\chi^2 = 0$$

$$m_3 v_\chi + \lambda_3 v_\chi^3 + \frac{\lambda_{13}}{2} v_\rho^2 v_\chi - f_{\rho\eta\chi} v_\rho v_\eta + \frac{\lambda_{23}}{2} v_\eta^2 v_\chi = 0$$

Scalars

CP-even neutral scalars mix

$$h_i = \mathcal{R}_{ij}^S H_j$$

$$\vec{H} = (\text{Re } \rho^0, \text{Re } \eta^0, \text{Re } \chi^0), \vec{h} = (h_1, h_2, h_3)$$

$$f_{\rho\eta\chi} = \kappa v_\chi$$

$$\beta = \tan^{-1} v_\eta / v_\rho, v = \sqrt{v_\eta^2 + v_\rho^2}.$$

$$m_h^2 = \begin{pmatrix} \kappa \tan \beta v_\chi^2 + 2\lambda_1 v^2 \cos^2 \beta & \lambda_{12} v^2 \cos \beta \sin \beta - \kappa v_\chi^2 & v_\chi v (\lambda_{13} \cos \beta - \kappa \sin \beta) \\ \sim & \kappa \cot \beta v_\chi^2 + 2\lambda_2 v^2 \sin^2 \beta & v_\chi v (\lambda_{23} \sin \beta - \kappa \cos \beta) \\ \sim & \sim & 2\lambda_3 v_\chi^2 + \kappa v^2 \cos \beta \sin \beta \end{pmatrix}$$

Pseudocalar

CP-odd neutral scalars mix

$$a_i = \mathcal{R}_{ij}^P A_j$$

$$\vec{A} = (\text{Im } \rho^0, \text{Im } \eta^0, \text{Im } \chi^0), \vec{a} = (a_{G_Z}, a_{G_{Z'}}, a_1).$$

$$m_a^2 = \begin{pmatrix} \kappa v_\chi^2 \tan \beta & \kappa v_\chi^2 & \kappa v_\chi v \sin \beta \\ \sim & \kappa v_\chi^2 \cot \beta & \kappa v_\chi v \cos \beta \\ \sim & \sim & \kappa v^2 \cos \beta \sin \beta \end{pmatrix}$$



$$m_{a_1}^2 = \kappa(v_\chi^2 \csc \beta \sec \beta + v^2 \cos \beta \sin \beta)$$

Singly-Charged State

Singly-charged states mix

$$h_i^- = \mathcal{R}_{ij}^C H_j^-$$

$$\vec{H}^- = ((\rho^+)^*, \eta^-), \vec{h}^- = (h_{G_W}^-, h_1^-)$$

$$m_{h^\pm}^2 = \begin{pmatrix} \kappa \tan \beta v_\chi^2 + \frac{1}{2} \zeta_{12} v^2 \sin^2 \beta & \kappa v_\chi^2 + \frac{1}{2} \zeta_{12} v^2 \cos \beta \sin \beta \\ \sim & \kappa \cot \beta v_\chi^2 + \frac{1}{2} \zeta_{12} v^2 \cos^2 \beta \end{pmatrix}$$

⇓

$$m_{h_1^\pm}^2 = \frac{1}{2} \zeta_{12} v^2 + \kappa v_\chi^2 \csc \beta \sec \beta$$

A-Charged State

A -charged states mix

$$h_i^A = \mathcal{R}_{ij}^A H_j^A$$

$$\vec{H}^A = ((\eta^{-A})^*, \chi^A), \vec{h}^A = (h_{G_{V^A}}^A, h_1^A)$$

$$m_{h^{\pm A}}^2 = \begin{pmatrix} \frac{1}{2} v_\chi^2 (\zeta_{23} + 2\kappa \cot \beta) & \frac{1}{2} v_\chi v (2\kappa \cos \beta + \zeta_{23} \sin \beta) \\ \sim & \frac{1}{2} v^2 \sin \beta (2\kappa \cos \beta + \zeta_{23} \sin \beta) \end{pmatrix}$$



$$m_{h_1^{\pm A}}^2 = \frac{1}{4} (\zeta_{23} + 2\kappa \cot \beta) (2v_\chi^2 + v^2 - v^2 \cos 2\beta)$$

B-Charged State

B -charged states mix

$$h_i^B = \mathcal{R}_{ij}^B H_j^B$$

$$\vec{H}^B = ((\rho^{-B})^*, \chi^B), \vec{h}^B = (h_{G_{V^B}}^B, h_1^B)$$

$$m_{h^{\pm B}}^2 = \begin{pmatrix} \frac{1}{2} v_\chi^2 (\zeta_{13} + 2\kappa \tan \beta) & \frac{1}{2} v_\chi v (2\kappa \sin \beta + \zeta_{13} \cos \beta) \\ \sim & \frac{1}{2} v^2 \cos \beta (2\kappa \sin \beta + \zeta_{13} \cos \beta) \end{pmatrix}$$



$$m_{h_1^{\pm B}}^2 = \frac{1}{4} (\zeta_{13} + 2\kappa \tan \beta) (2v_\chi^2 + v^2 - v^2 \cos 2\beta)$$

Angular Minimization

$$\frac{\partial_{a_2} V_A}{r_2} = \sin 2a_2 r_1 \zeta'_{12} + (\sin 2a_3 \cos 2a_2 \sin b_3 + \sin 2a_2 (\cos^2 a_3 - \sin^2 a_3 \sin^2 b_3)) r_3 \zeta'_{23},$$

$$\frac{\partial_{a_3} V_A}{r_3} = \sin 2a_3 r_1 \zeta'_{13} + (\sin 2a_2 \cos 2a_3 \sin b_3 + \sin 2a_3 (\cos^2 a_2 - \sin^2 a_2 \sin^2 b_3)) r_2 \zeta'_{23},$$

$$\frac{\partial_{b_3} V_A}{r_2 r_3} = \frac{1}{2} \cos b_3 (\sin 2a_2 \sin 2a_3 - 4 \sin^2 a_2 \sin^2 a_3 \sin b_3) \zeta'_{23}.$$

