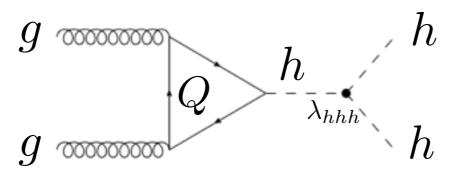
ICHEP 2020 | PRAGUE

28 July 2020 to 6 August 2020 virtual conference

HIGGS BOSON PAIR PRODUCTION

AT N3LO QCD





Work with L.-B. Chen, H.T. Li and J. Wang (1909.06808, 1912.13001)







ICHEP 2020, ONLINE (PRAGUE)
31 JULY 2020





- Undoubtably important to measure Higgs self couplings
 - Unique way to understand the Higgs potential

EW symmetry breaking



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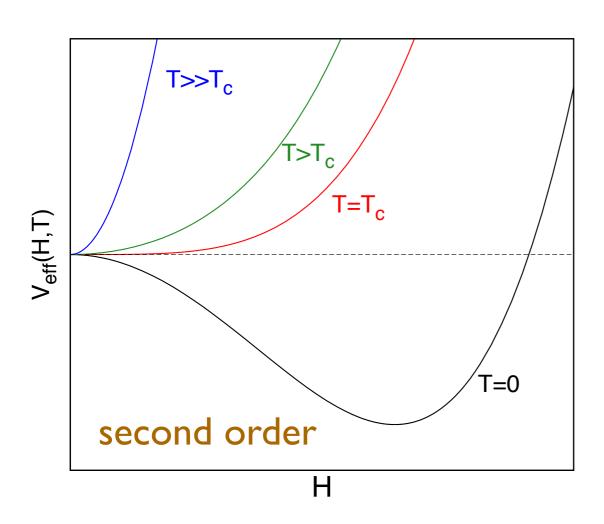
=>> EW symmetry breaking

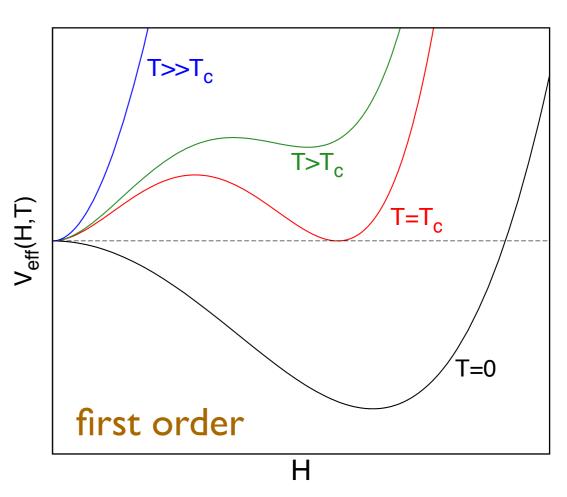
Phase transition in the (very) early Universe



SM: no phase transition (crossover)

Kajantie et al. (1996); Csikor et al. (1998)







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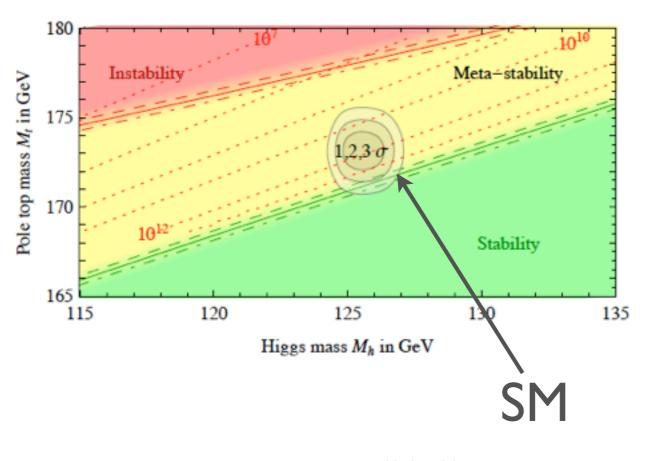
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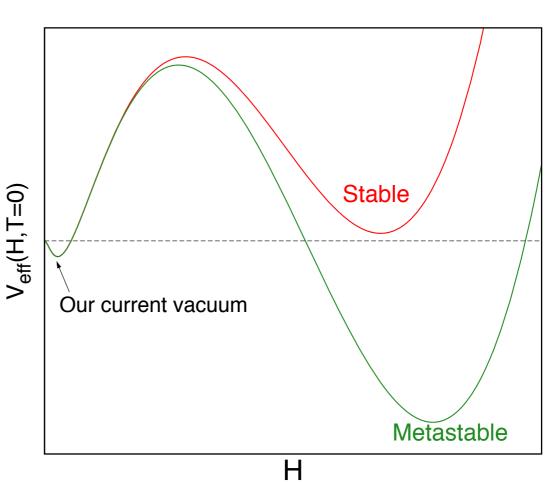


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• The fate of our Universe: stable vs metastable vacuum!





Degrassi et al. (2012)



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Raman Sundrun BSM Wishlist (Snowmass21 EF meeting)

Strong first order PT

General EW PT distinguish

SM crossover

$$\delta \lambda_{hhh} \gtrsim 10\%$$

$$\delta \lambda_{hhh} \ll 1\%$$

$$\delta \lambda_{hhh} \sim 0\%$$



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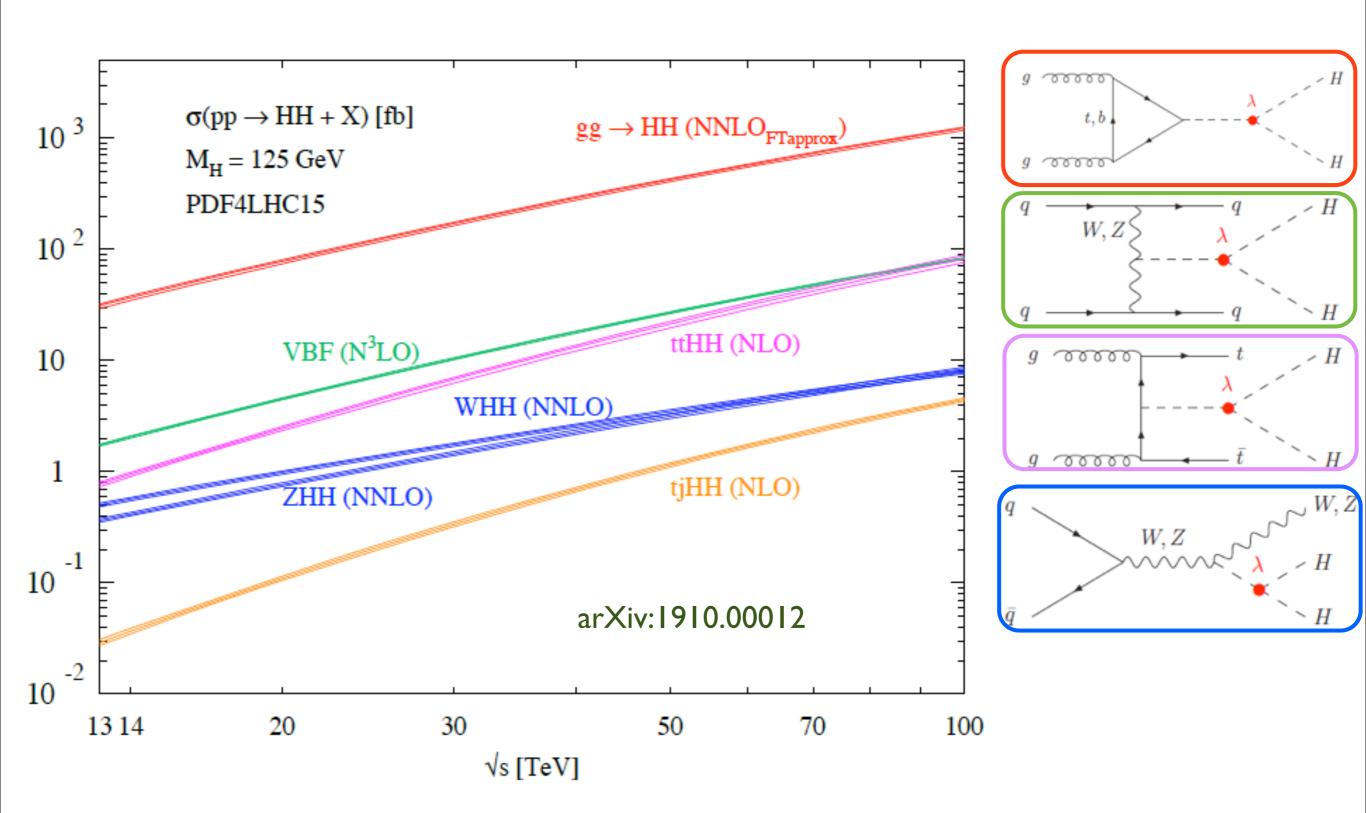
Perturbation theory breaks at scale

$$\frac{13 \text{ TeV}}{|\delta \lambda_{hhh}|}$$

Chang et al. (2019)

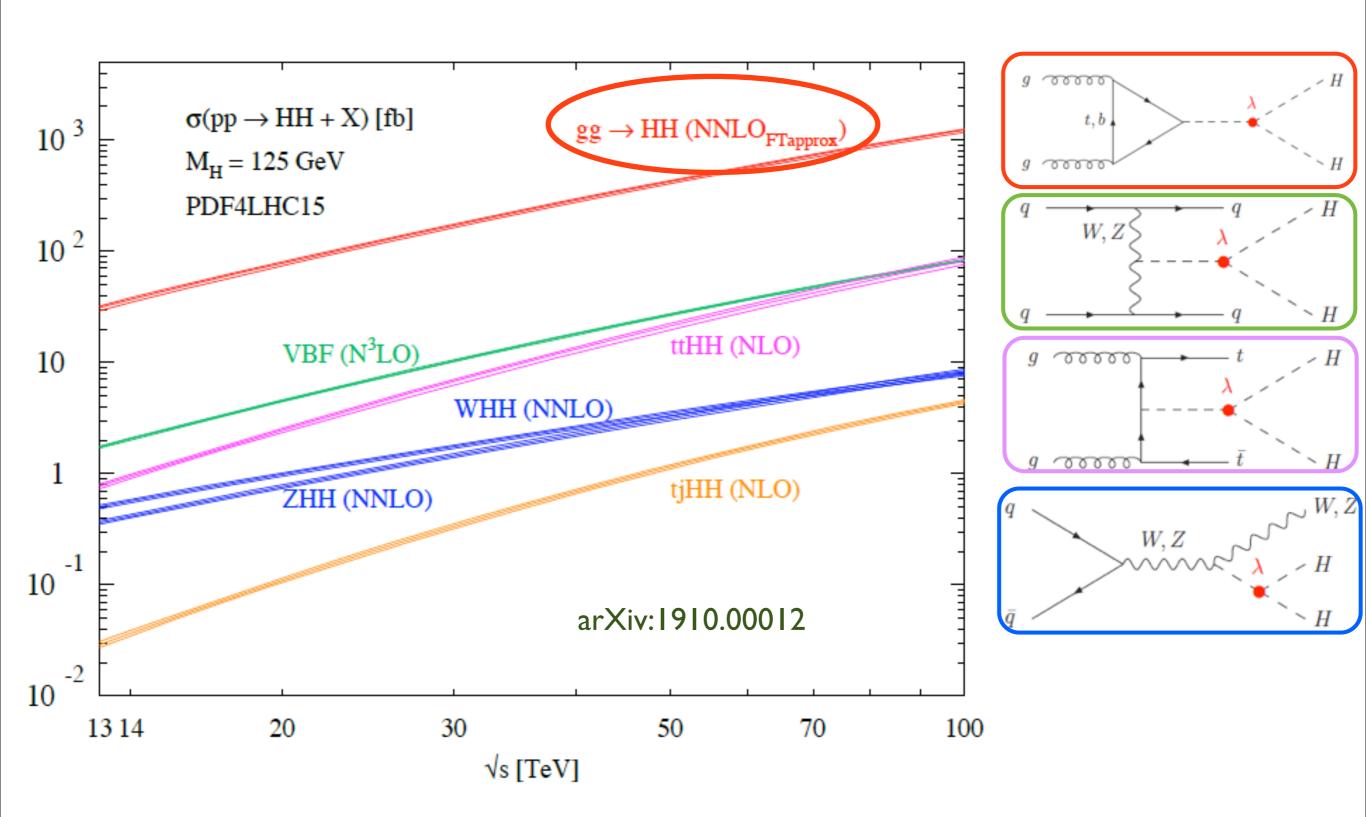
HIGGS BOSON PAIR PRODUCTION





HIGGS BOSON PAIR PRODUCTION

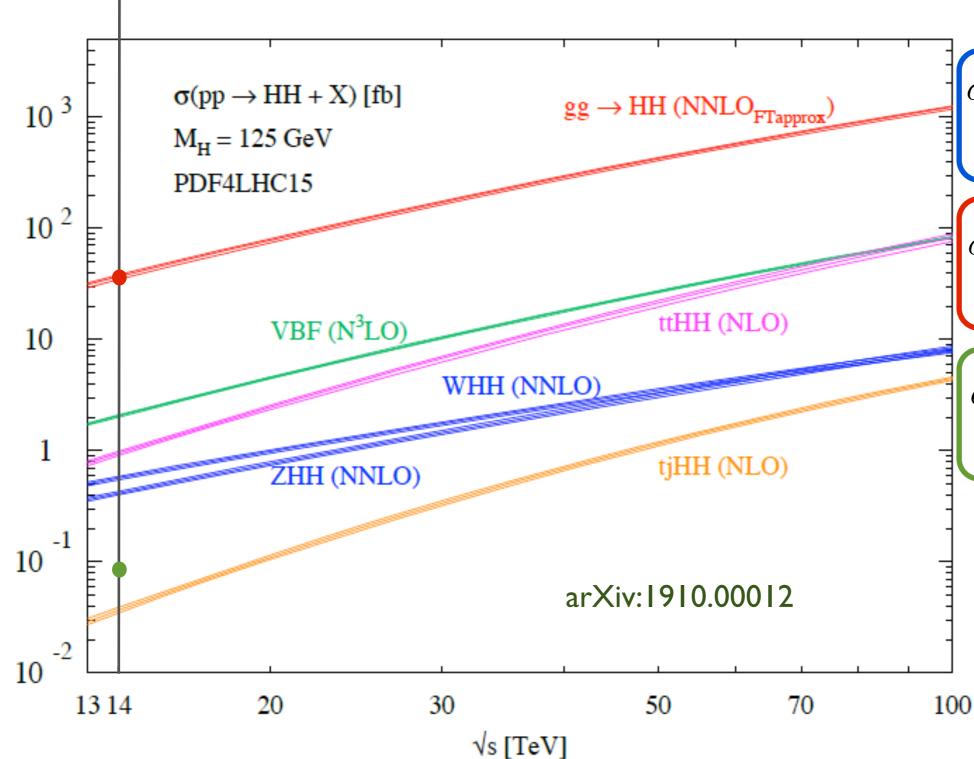




GLUON FUSION CROSS SECTIONS







$$\sigma_h^{\rm N^3LO} = 54.72 \text{ pb}$$

arXiv:1902.00134

$$\sigma_{hh}^{\mathrm{NNLO_{FT_{approx}}}} = 36.69 \text{ fb}$$

arXiv:1910.00012

$$\sigma_{hhh}^{\rm NLO_{FT_{approx}}} = 89.4 \text{ ab}$$

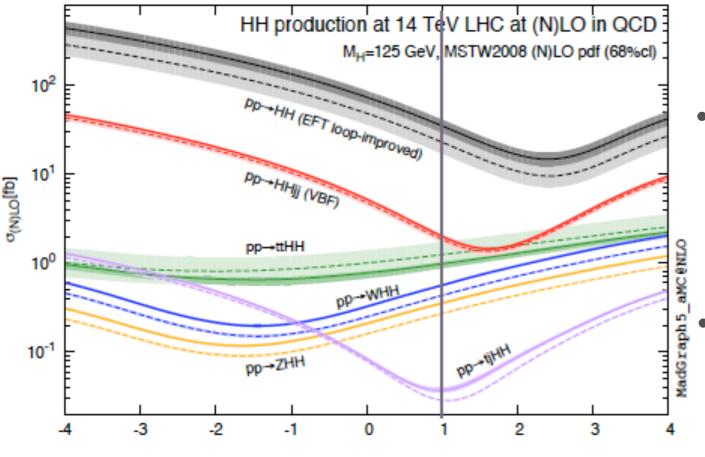
arXiv:1408.6542

Cross sections for hh(h) increase by a factor of 20 (60) at 100 TeV

PROBING HIGGS SELF COUPLING



arXiv:1401.7340



 The self-coupling value can be extracted by measuring the cross sections.

However:

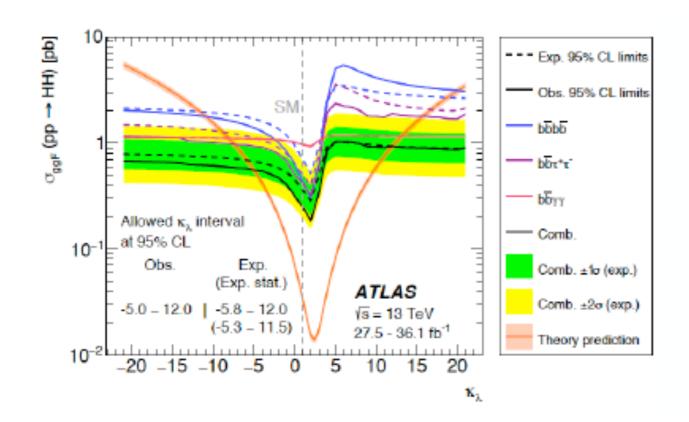
- Interpretations of these bounds in terms of BSM always need additional assumptions on how the SM has been deformed.
- The most commonly assumption is only changing the value of λ_{hhh} , which leads to (differential) cross section variations

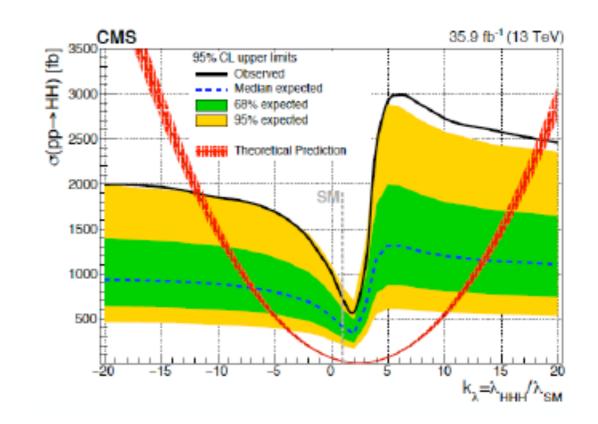
$$\sigma = \sigma_{SM}[1 + (\kappa_{\lambda} - 1)A_1 + (\kappa_{\lambda}^2 - 1)A_2]$$

THE MEASUREMENTS



$$\kappa_{\lambda} = rac{\lambda_{hhh}}{\lambda_{hhh}^{ ext{SM}}}$$





$$-5.0 < \kappa_{\lambda} < 12$$

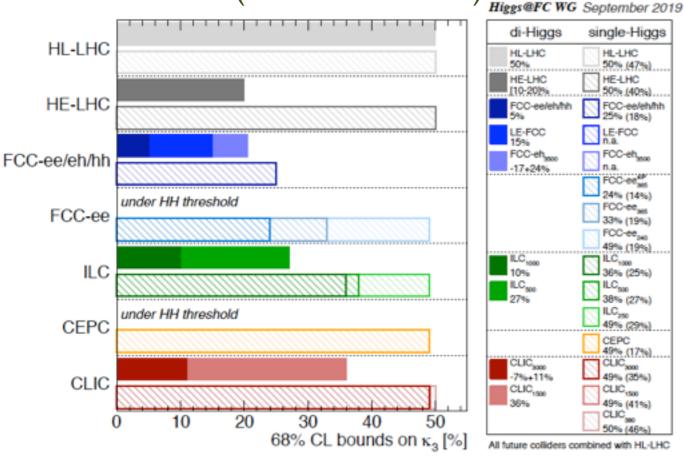
$$-11.8 < \kappa_{\lambda} < 18.8$$

- Strong shape effects with κλ variations
- Soft spectra for κ_λ ≈ 5 → difficult to constrain anomalous positive values

FUTURE PROSPECTS



de Blas et al. (arXiv:1905.03764)



Caterina Vernieri (Snowmass21EF meeting)

collider	single-H	HH	combined
HL-LHC	100-200%	50%	50%
CEPC ₂₄₀	49%	-	49%
ILC_{250}	49%	_	49%
ILC_{500}	38%	27%	22%
ILC_{1000}	36%	10%	10%
$CLIC_{380}$	50%	_	50%
CLIC_{1500}	49%	36%	29%
$CLIC_{3000}$	49%	9%	9%
FCC-ee	33%	_	33%
FCC-ee (4 IPs)	24%	_	24%
HE-LHC	-	15%	15%
FCC-hh	-	5%	5%

50% accuracy (HL-LHC): sensitive to BSM with the largest new physics effects

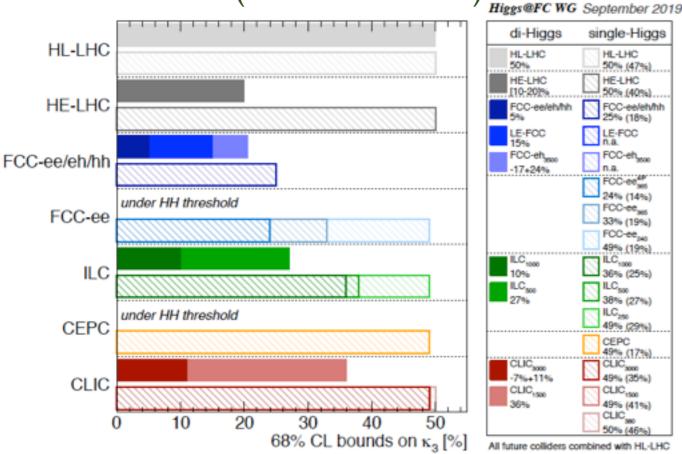
20% accuracy (future e+e-): discovery of SM-like λ_{hhh}

5% accuracy (FCC-hh): sensitive to BSM loop corrections

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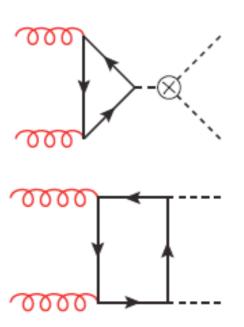
5% accuracy (FCC-hh): sensitive to BSM loop corrections

Ultimate precision machine!

Mangano et al. (arXiv:2004.03505)



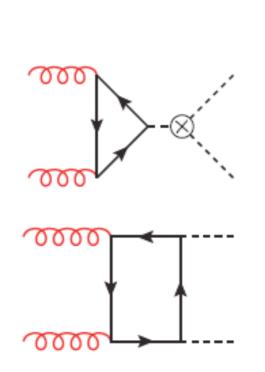
- Full top-quark mass dependence
 - Leading order (LO) is a loop-induced process

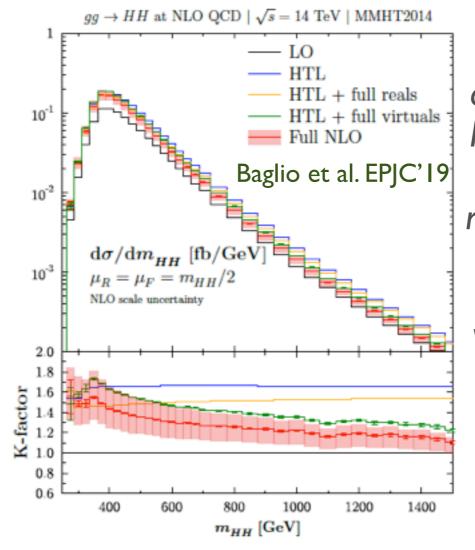




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 - Next-to-leading order (NLO) was computed numerically

Borowka et al. PRL'16, JHEP'16; Baglio et al. EPJC'19, JHEP'20





Reasonable
approximations to extend
I/m_t result (rescaled exact
Born, include exact real
radiation) can fail the true
K factor significantly.

virtual is so crucial, which is remaining to be understood

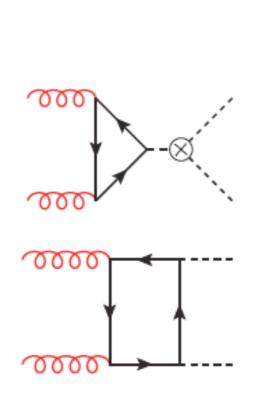


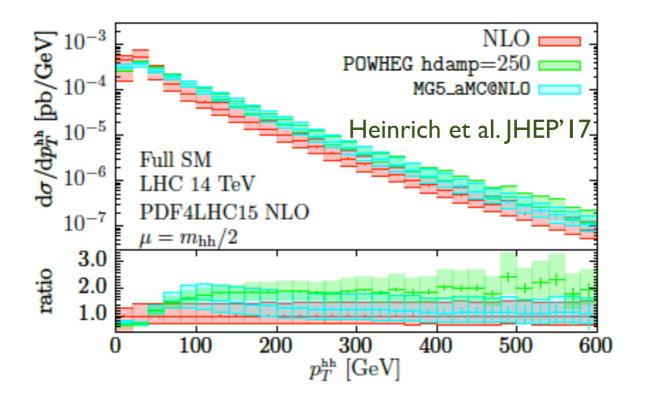
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... even after matching to parton showers (i.e. NLO+PS)

Heinrich et al. JHEP'17, JHEP'19; Jones, Kuttimalai JHEP'18





Matching scheme dependence starts to be significant at large p_T^{hh}



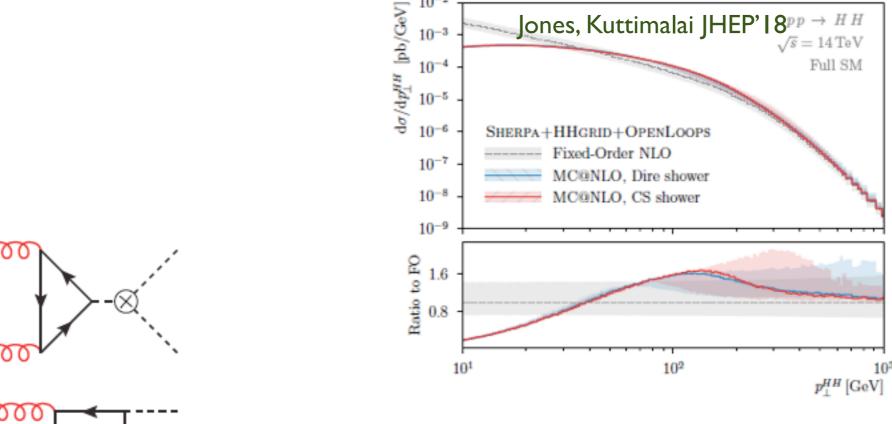
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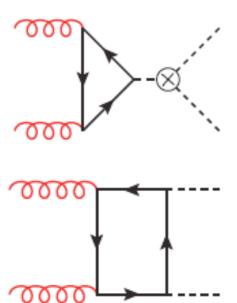
Jones, Kuttimalai JHEP' 18pp

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Shower scale uncertainty is also significant at large





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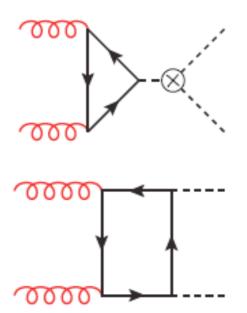
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Heinrich et al. JHEP'17, JHEP'19; Jones, Kuttimalai JHEP'18

Scale unc. (>10%)

Energy	13 TeV	14 TeV	27 TeV	100 TeV
NLO	27.78 ^{+13.8%} _{-12.8%} fb	32.88 ^{+13.5%} _{-12.5%} fb	127.7 ^{+11.5%} _{-10.4%} fb	1147 ^{+10.7%} _{-9.9%} fb





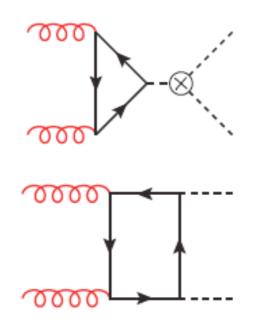
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- Scale unc. (>10%)
- ... and large top-quark mass scheme dependence



$$\frac{d\sigma(gg \to HH)}{dQ} \Big|_{Q=300 \text{ GeV}} = 0.02978(7)_{-34\%}^{+6\%} \text{ fb/GeV},$$

$$\frac{d\sigma(gg \to HH)}{dQ} \Big|_{Q=400 \text{ GeV}} = 0.1609(4)_{-13\%}^{+0\%} \text{ fb/GeV},$$

$$\frac{d\sigma(gg \to HH)}{dQ} \Big|_{Q=600 \text{ GeV}} = 0.03204(9)_{-30\%}^{+0\%} \text{ fb/GeV},$$

$$\frac{d\sigma(gg \to HH)}{dQ} \Big|_{Q=1200 \text{ GeV}} = 0.000435(4)_{-35\%}^{+0\%} \text{ fb/GeV},$$



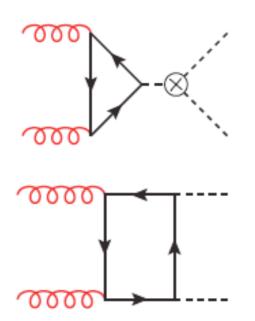
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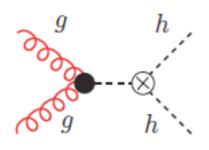
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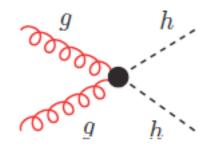
- Scale unc. (>10%)
- ... and large top-quark mass scheme dependence
- A lot of analytical approximations (well-motivated to deepen understanding)
 Grigo et al. NPB'13, NPB'15; Degrassi EPJC'16;, Davies et al. JHEP'18, JHEP'19;
 Bonciani et al. PRL'18; Xu and Yang JHEP'19; Davies and Steinhauser (1909.01361)





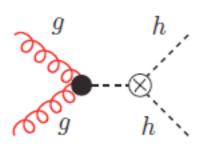
• Infinite top-quark mass limit
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\ \mu\nu} \left(C_h \frac{h}{v} - C_{hh} \frac{h^2}{2v^2} \right)$$

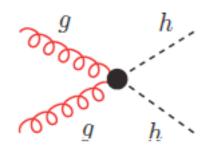






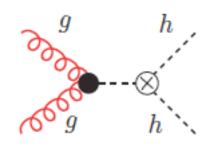
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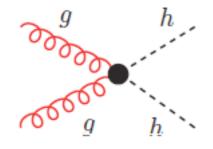






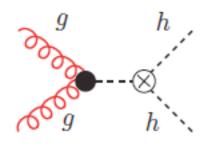
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 - · Technically, it is much easier to achieve high precision
 - NLO was 20 years old Dawson PRD'98
 - NNLO was known as well Florian and Mazzitelli PLB'13,PRL'13; Grigo et al. NPB'14; Florian et al. JHEP'16
 - Threshold resummation Shao et al. JHEP'13; Florian and Mazzitelli JHEP'15, JHEP'18
 - NLO_{FTapprox}: NLO plus full top quark mass in Born and real Frederix et al. PLB'14; Maltoni et al. JHEP'14
 - Combine NNLO with full top-quark mass NLO Grazzini et al. JHEP'18

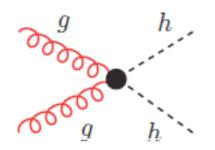






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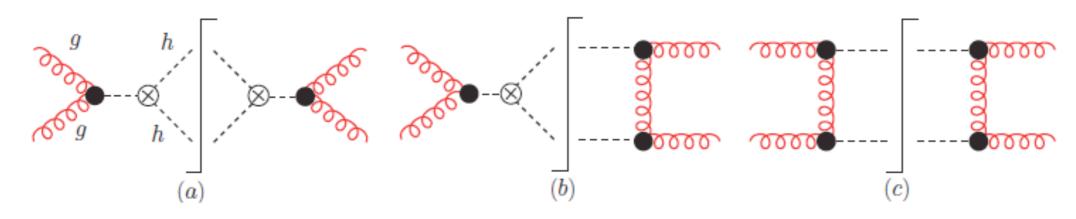




Our aim is to push the calculation to N^3LO !



Infinite top-quark mass limit
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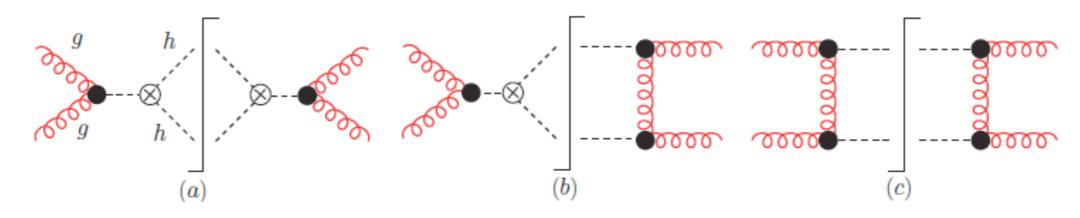
	LO	NLO	NNLO	N^3LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
c	0	0	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$

$$d\sigma_{hh} = d\sigma_{hh}^a + d\sigma_{hh}^b + d\sigma_{hh}^c$$

HIGGS PAIR GLUON FUSION PRO



Infinite top-quark mass limit
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right)$$



	LO	NLO	NNLO	N_3 LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
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Chen, Li, HSS, Wang (1909.06808)

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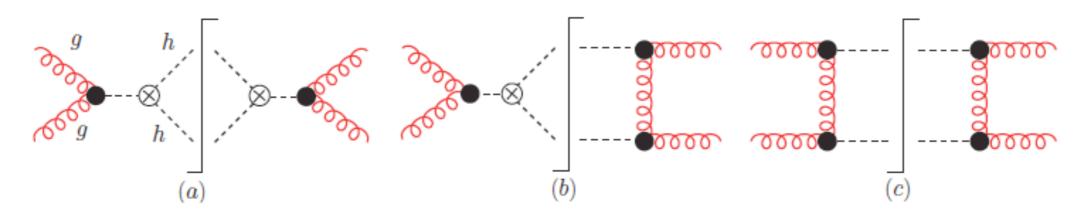
class-a: same topology as ggH

$$\frac{d\sigma_{hh}^{a}}{dm_{hh}} = f_{h\to hh} \left(\frac{C_{hh}}{C_{h}} - \frac{6\lambda v^{2}}{m_{hh}^{2} - m_{h}^{2}} \right)^{2} \times \sigma_{h}(m_{h} \to m_{hh})$$

$$f_{h\to hh} = \frac{\sqrt{m_{hh}^{2} - 4m_{h}^{2}}}{16\pi^{2}v^{2}}$$



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			NNLO	
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
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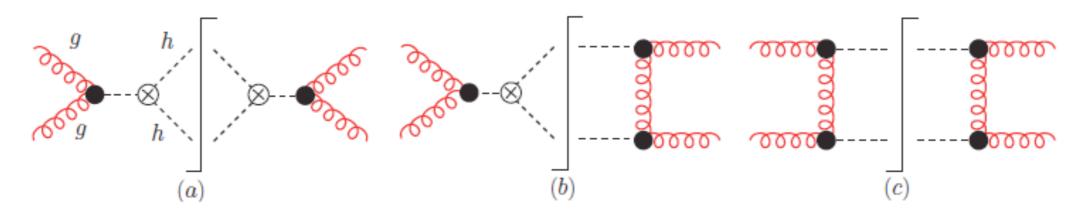
$$f_{h\to hh} = \frac{\sqrt{m_{hh}^2 - 4m_h^2}}{m_{hh}^2 - 4m_h^2}$$

From iHixs2

Dulat et al. CPC'18



Infinite top-quark mass limit
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right)$$

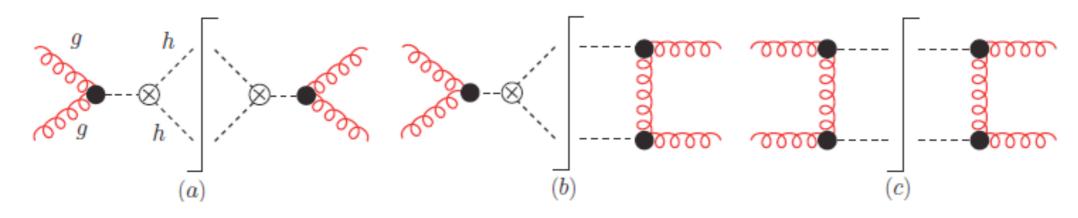


	LO	NLO	NNLO	N^3LO
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Infinite top-quark mass limit
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right)$$



	LO	NLO	NNLO	N_3 LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
c	0	0	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$

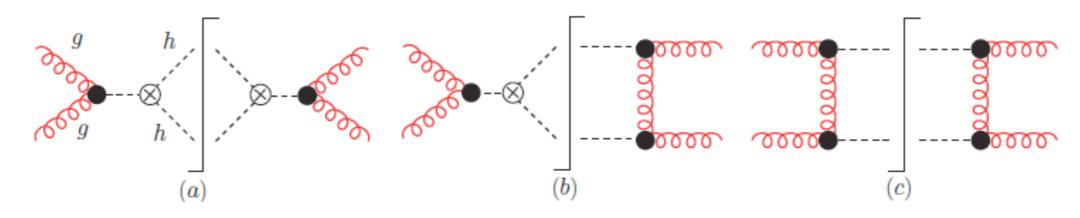
$$d\sigma_{hh} = d\sigma_{hh}^a + d\sigma_{hh}^b + d\sigma_{hh}^c$$

• class-b: need NNLO as its as is zero (qT subtraction, Catani & Grazzini PRL'07)

$$d\sigma_{hh}^b = d\sigma_{hh}^b \Big|_{p_T^{hh} < p_T^{\text{veto}}} + d\sigma_{hh}^b \Big|_{p_T^{hh} > p_T^{\text{veto}}}$$



Infinite top-quark mass limit
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right)$$



	LO	NLO	NNLO	N_3 LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
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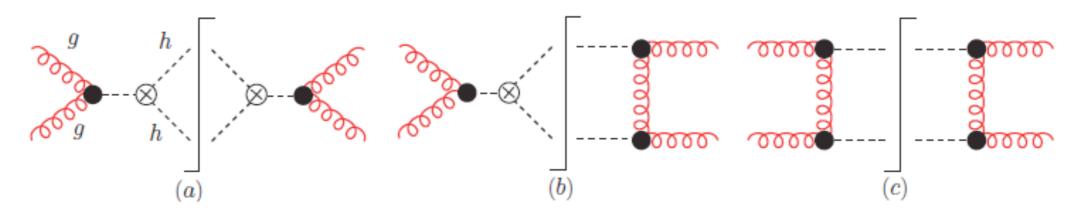
$$\left. d\sigma_{hh}^b = \left. d\sigma_{hh}^b \right|_{p_T^{hh} < p_T^{\text{veto}}} + \left. d\sigma_{hh}^b \right|_{p_T^{hh} > p_T^{\text{veto}}}$$

SCET:
$$d\sigma_{hh}^b\big|_{p_T^{hh} < p_T^{\text{veto}}} = \mathcal{H} \otimes \mathcal{B}_1 \otimes \mathcal{B}_2 + \mathcal{O}\left(\left(\frac{p_T^{\text{veto}}}{Q}\right)^2\right)$$



Infinite top-quark mass limit

$$\mathcal{L}_{eff} = -\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu}\left(C_{h}\frac{h}{v} - C_{hh}\frac{h^{2}}{2v^{2}}\right)$$



	LO	NLO	NNLO	N_3 LO
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Chen, Li, HSS, Wang (1909.06808)

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• class-b: need NNLO as its as is zero (qT subtraction, Catani & Grazzini PRL'07)

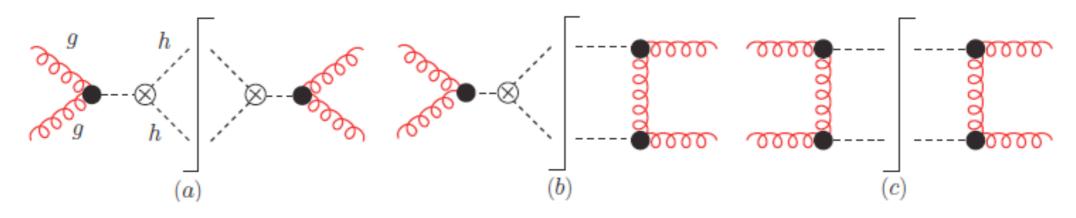
$$\left. d\sigma_{hh}^b = \left. \frac{d\sigma_{hh}^b}{\left|_{p_T^{hh} < p_T^{\text{veto}}}\right|} + d\sigma_{hh}^b \right|_{p_T^{hh} > p_T^{\text{veto}}}$$
 \mathcal{H} hard function two-loop amplitude Banerjee et al., JHEP' I8 new one-loop amplitude

 ${\cal H}$ hard function two-loop amplitude



Infinite top-quark mass limit

$$\mathcal{L}_{eff} = -\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu}\left(C_{h}\frac{h}{v} - C_{hh}\frac{h^{2}}{2v^{2}}\right)$$



	LO	NLO	NNLO	N_3 LO
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b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
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Chen, Li, HSS, Wang (1909.06808)

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$$d\sigma_{hh}^{b} = \left. d\sigma_{hh}^{b} \right|_{p_{T}^{hh} < p_{T}^{\text{veto}}} + \left. d\sigma_{hh}^{b} \right|_{p_{T}^{hh} > p_{T}^{\text{veto}}}$$

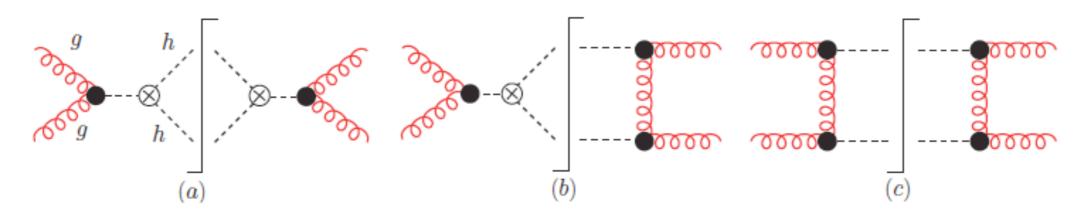
$$\text{SCET: } \left. d\sigma_{hh}^{b} \right|_{p_{T}^{hh} < p_{T}^{\text{veto}}} = \mathcal{H} \otimes \mathcal{B}_{\mathcal{I}} \otimes \mathcal{B}_{\mathcal{I}} + \mathcal{O} \left(\left(\frac{p_{T}^{\text{veto}}}{Q} \right)^{2} \right)$$

 \mathcal{B}_i TMD beam function two-loop exp. known Gehrmann et al.PRL'12,

JHEP'14; Luebbert et al., JHEP'16; Echevarria et al., JHEP'16; Luo et al., '19



• Infinite top-quark mass limit
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right)$$



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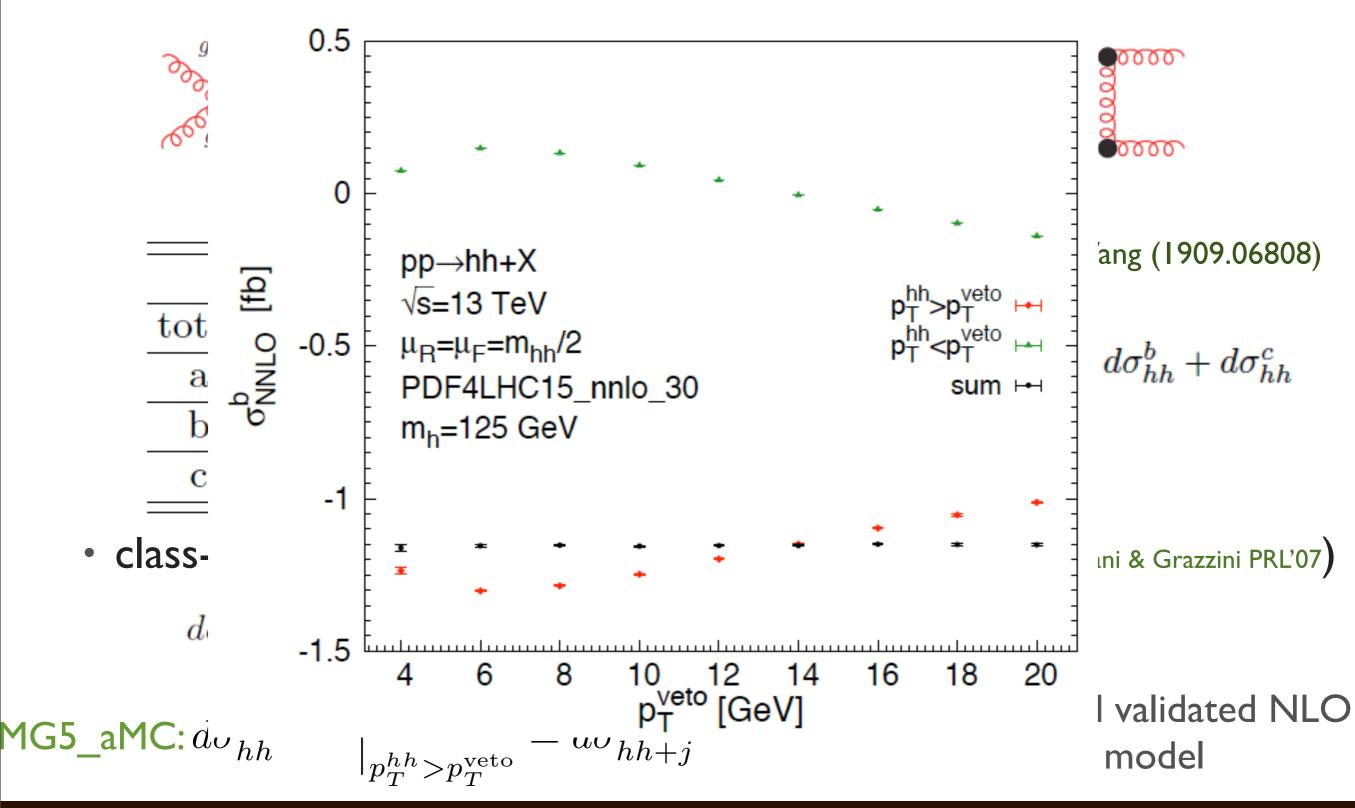
$$\left. d\sigma_{hh}^b = \left. d\sigma_{hh}^b \right|_{p_T^{hh} < p_T^{\text{veto}}} + \left. d\sigma_{hh}^b \right|_{p_T^{hh} > p_T^{\text{veto}}}$$

$$exttt{MG5_aMC:} d\sigma_{hh}^{b, ext{NNLO}} \Big|_{p_T^{hh} > p_T^{ ext{veto}}} = d\sigma_{hh+j}^{b, ext{NLO}}$$

New and validated NLO model

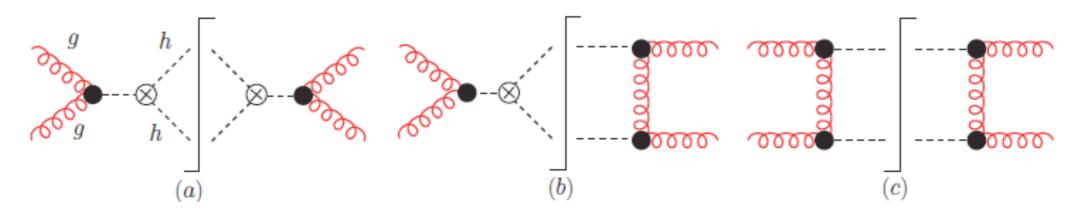


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$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right)$$





Infinite top-quark mass limit
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right)$$



	LO	NLO	NNLO	N_3 LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
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Chen, Li, HSS, Wang (1909.06808)

$$d\sigma_{hh} = d\sigma_{hh}^a + d\sigma_{hh}^b + d\sigma_{hh}^c$$

class-c: need NLO (full fledged)



• Infinite top-quark mass limit
$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\ \mu\nu}\left(C_h\frac{h}{v} - C_{hh}\frac{h^2}{2v^2}\right)$$

A lot of cross checks

Chen, Li, HSS, Wang (1909.06808)

			NNLO	
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
\mathbf{a}	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
c	0	0	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$



• Infinite top-quark mass limit

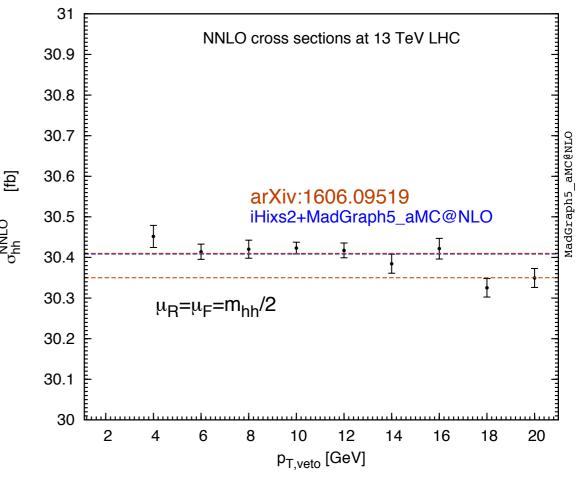
$$\mathcal{L}_{eff} = -\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu}\left(C_{h}\frac{h}{v} - C_{hh}\frac{h^{2}}{2v^{2}}\right)$$

A lot of cross checks

Chen, Li, HSS, Wang (1909.06808)

	LO	NLO	NNLO	N_3 LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(lpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
b		$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
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√ At least two independent calculations





• Infinite top-quark mass limit

$$\mathcal{L}_{eff} = -\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu}\left(C_{h}\frac{h}{v} - C_{hh}\frac{h^{2}}{2v^{2}}\right)$$

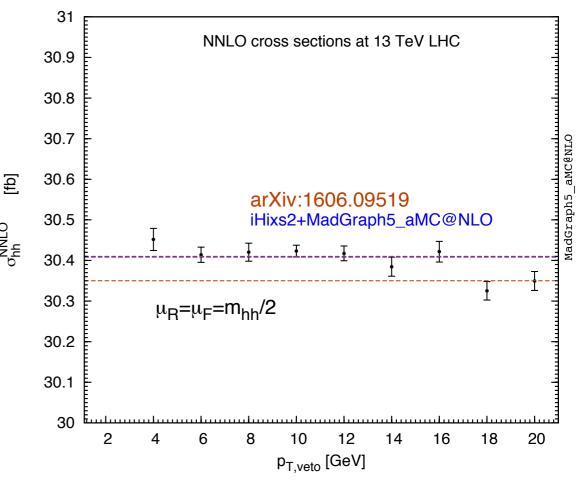
A lot of cross checks

Chen, Li, HSS, Wang (1909.06808)

	LO	NLO	NNLO	N_3 LO
total	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
a	$\mathcal{O}(\alpha_s^2)$	\ _ /	$\mathcal{O}(\alpha_s^4)$	\ _ /
b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
c	0	0	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$

√ At least two independent calculations

 $\sqrt{}$ Orthogonal check with NNLO ggHH





Infinite top-quark mass limit

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} G^{a}_{\mu\nu} G^{a \mu\nu} \left(C_{h} \frac{h}{v} - C_{hh} \frac{h^{2}}{2v^{2}} \right)$$

A lot of cross checks

Chen, Li, HSS, Wang (1909.06808)

	LO	NLO	NNLO	N_3 LO
total	\ _ /	\ - /	$\mathcal{O}(\alpha_s^4)$	\ _ /
a			$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
b	0	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$
c	0	0	$\mathcal{O}(\alpha_s^4)$	$\mathcal{O}(\alpha_s^5)$

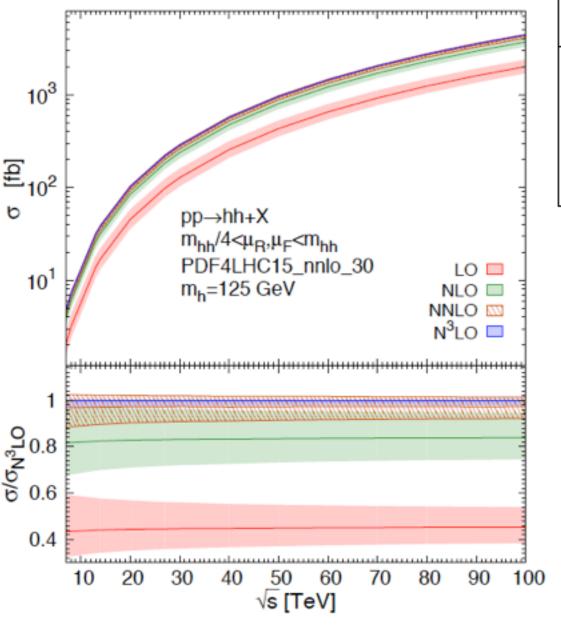
√ At least two independent calculations

 $\sqrt{\text{Orthogonal check with NNLO ggHH}}$

√ Check piece-by-piece



- Infinite top-quark mass limit
 - N³LO cross sections



in unit of fb

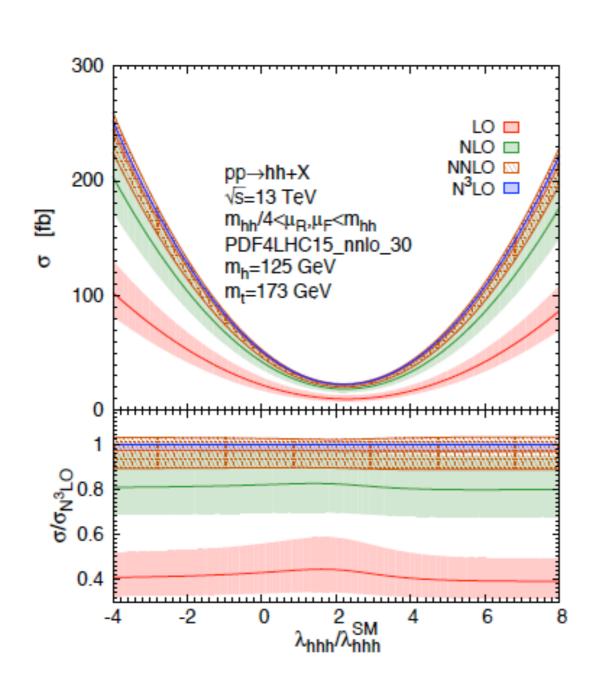
Chen, Li, HSS, Wang (1909.06808)

$\frac{\sqrt{s}}{\text{order}}$	13 TeV	$14~{ m TeV}$	$27~{ m TeV}$	$100~{ m TeV}$
LO	$13.80^{+31\%}_{-22\%}$	$17.06^{+31\%}_{-22\%}$	$98.22^{+26\%}_{-19\%}$	$2015^{+19\%}_{-15\%}$
NLO	$25.81^{+18\%}_{-15\%}$	$31.89^{+18\%}_{-15\%}$	$183.0^{+16\%}_{-14\%}$	$3724_{-11\%}^{+13\%}$
NNLO	$30.41^{+5.3\%}_{-7.8\%}$	$37.55^{+5.2\%}_{-7.6\%}$	$214.2^{+4.8\%}_{-6.7\%}$	$4322^{+4.2\%}_{-5.3\%}$
N^3LO	$31.31^{+0.66\%}_{-2.8\%}$	$38.65^{+0.65\%}_{-2.7\%}$	$220.2^{+0.53\%}_{-2.4\%}$	$4438^{+0.51\%}_{-1.8\%}$

- Scale unc. is significantly reduced!
- PDF unc. > Scale unc. now!
- Very good perturbative convergence!



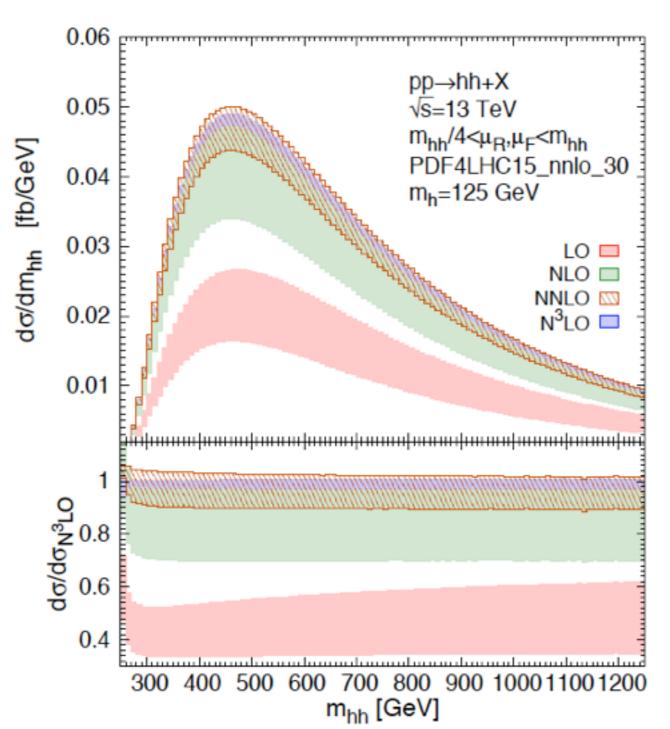
- Infinite top-quark mass limit
 - N³LO cross sections



- Shapes change from LO to NLO and from NLO to NNLO
- The shape variation from NNLO to N³LO is quite invisible



- Infinite top-quark mass limit
 - a N³LO differential distribution



Chen, Li, HSS, Wang (1909.06808)

- Scale unc. is significantly reduced!
- Very good perturbative convergence!
- N³LO/NNLO is quite flat

TOP QUARK MASS APPROXIMATIONS



k > l

Several approximations:

assuming we have $\begin{cases} N^k LO \\ N^l LO \end{cases}$

Chen, Li, HSS, Wang (1912.13001)

infinite top-quark mass limit

full top-quark mass dependence

• $N^kLO \oplus N^lLO_{m_t}$

$$d\sigma^{\mathbf{N^kLO}\oplus\mathbf{N^lLO}_{\mathbf{m_t}}} = d\sigma^{\mathbf{N^lLO}}_{m_t} + \left(d\sigma^{\mathbf{N^kLO}}_{m_t=\infty} - d\sigma^{\mathbf{N^lLO}}_{m_t=\infty}\right)$$
 missing top mass in correction

• $N^kLO\otimes N^lLO_{m_t}$

$$d\sigma^{\mathbf{N^{k}LO}\otimes\mathbf{N^{l}LO_{m_{t}}}} = d\sigma_{m_{t}}^{\mathbf{N^{l}LO}} \frac{d\sigma_{m_{t}=\infty}^{\mathbf{N^{k}LO}}}{d\sigma_{m_{t}=\infty}^{\mathbf{N^{l}LO}}}$$

Same K factor for mass correction

• $N^k LO_{B-i} \oplus N^l LO_{m_t}$

$$d\sigma^{\mathbf{N^{k}LO_{B-i}}\oplus\mathbf{N^{l}LO_{m_{t}}}} = d\sigma^{\mathbf{N^{l}LO}}_{m_{t}} + \Delta\sigma^{k,l}_{m_{t}=\infty} \frac{d\sigma^{\mathbf{LO}}_{m_{t}}}{d\sigma^{\mathbf{LO}}_{m_{t}=\infty}}$$
 Born mass improved for correction

TOP QUARK MASS APPROXIMATIONS



Several approximations:

assuming we have $\begin{cases} N^k LO \\ N^l LO \end{cases}$

infinite top-quark mass limit

k > l

full top-quark mass dependence

Chen, Li, HSS, Wang (1912.13001)

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$$d\sigma^{\mathbf{N^kLO}\oplus\mathbf{N^lLO}_{\mathbf{m_t}}} = d\sigma^{\mathbf{N^lLO}}_{m_t} + \left(d\sigma^{\mathbf{N^kLO}}_{m_t=\infty} - d\sigma^{\mathbf{N^lLO}}_{m_t=\infty}\right)$$
 missing top mass in correction

• $N^k LO \otimes N^l LO_{m_s}$

$$d\sigma^{\mathbf{N^{k}LO}\otimes\mathbf{N^{l}LO_{m_{t}}}} = d\sigma_{m_{t}}^{\mathbf{N^{l}LO}} \frac{d\sigma_{m_{t}=\infty}^{\mathbf{N^{k}LO}}}{d\sigma_{m_{t}=\infty}^{\mathbf{N^{l}LO}}}$$

Same K factor for mass correction

• $N^k LO_{B-i} \oplus N^l LO_{m_t}$

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 Born mass improved for correction

TOP QUARK MASS RESULTS



- Top-quark mass dependent results NLO_{mt} from Powheg, arXiv:1903.08137
 - N³LO cross sections

Chen, Li, HSS, Wang (1912.13001)

in unit of fb

\sqrt{s}	13 TeV	14 TeV	27 TeV	100 TeV
NLO_{m_t}	$27.56^{+14\%}_{-13\%}$	$32.64^{+14\%}_{-12\%}$	$126.2^{+12\%}_{-10\%}$	$1119^{+13\%}_{-13\%}$
$\mathrm{NNLO} \oplus \mathrm{NLO}_{m_t}$	$32.16^{+5.9\%}_{-5.9\%}$	$38.29^{+5.6\%}_{-5.5\%}$	$157.3^{+3.0\%}_{-4.7\%}$	$1717^{+5.8\%}_{-12\%}$
$\text{NNLO}_{\mathrm{B-i}} \oplus \text{NLO}_{m_t}$	$33.08^{+5.0\%}_{-4.9\%}$	$39.16^{+4.9\%}_{-5.0\%}$	$150.8^{+4.6\%}_{-5.7\%}$	$1330^{+4.0\%}_{-7.2\%}$
$\mathrm{NNLO} \otimes \mathrm{NLO}_{m_t}$	$32.47^{+5.3\%}_{-7.8\%}$	$38.42^{+5.2\%}_{-7.6\%}$	$147.6^{+4.8\%}_{-6.7\%}$	$1298^{+4.2\%}_{-5.3\%}$
$N^3LO \oplus NLO_{m_t}$	$33.06^{+2.1\%}_{-2.9\%}$	$39.40^{+1.7\%}_{-2.8\%}$	$163.3^{+4.0\%}_{-8.3\%}$	$1833^{+14\%}_{-20\%}$
$N^3LO_{B-i} \oplus NLO_{m_t}$	$34.17^{+1.9\%}_{-4.6\%}$	$40.44^{+1.9\%}_{-4.7\%}$	$155.5^{+2.3\%}_{-5.0\%}$	$1372^{+2.8\%}_{-5.0\%}$
$N^3LO\otimes NLO_{m_t}$	$33.43^{+0.66\%}_{-2.8\%}$	$39.56^{+0.64\%}_{-2.7\%}$	$151.7^{+0.53\%}_{-2.4\%}$	$1333^{+0.51\%}_{-1.8\%}$

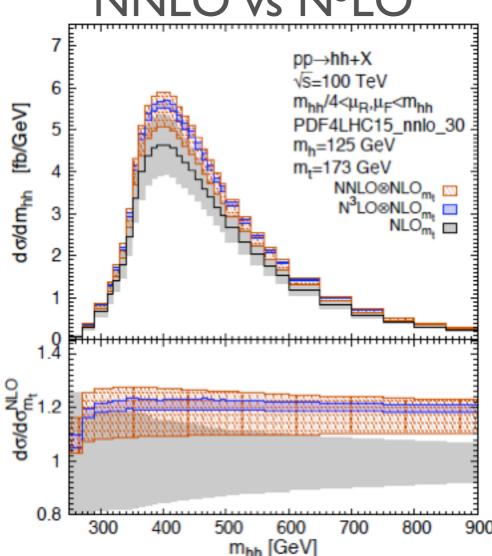
- N³LO enhances NNLO by 3% but enhances NLO by 20%
- N³LO reduces scale unc. to 3%
- The missing top quark mass uncer. at N³LO is around 5%
- The top mass scheme uncer. is unknown (not expected to be improved)

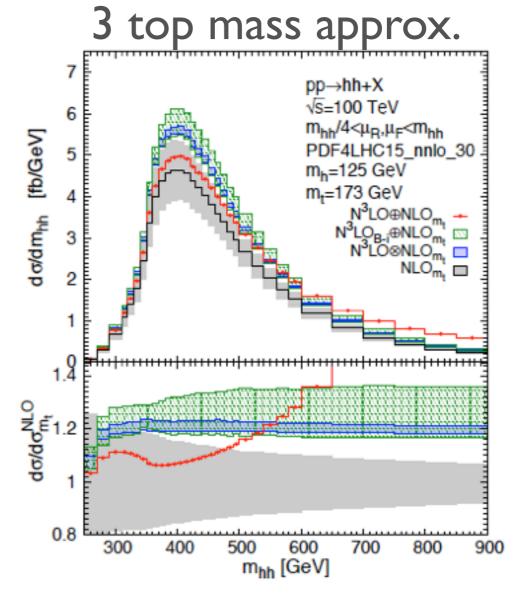
TOP QUARK MASS RESULTS



- Top-quark mass dependent results NLO_{mt} from Powheg, arXiv:1903.08137
 - N³LO distributions

NNLO vs N3LO $pp \rightarrow hh + X$





- Scale is again significantly reduced from NNLO to N³LO
- Missing top-quark mass effect at large mhh is very bad (red)

CONCLUSIONS



- We have carried out N³LO calculations for Higgs pair production in the gluon fusion channel with the infinite top-quark mass limit.
- The scale uncertainty is significantly reduced to be below 3% (2%) at 13 (100) TeV. PDF uncertainty is bigger than scale uncertainty.
- The perturbative convergence in the process shows pretty good at N³LO.

CONCLUSIONS



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- Remaining (theory) challenges:
 - How to improve the big top-quark mass scheme dependence seen at NLO?
 - How to further improve the finite top quark mass corrections?
 - Other theoretical uncertainties (e.g. EW corr., parameterical errors)?

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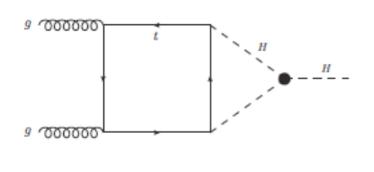
Thank you for your attention!

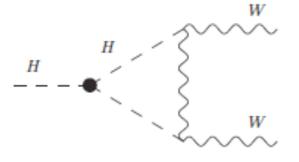
BACKUP

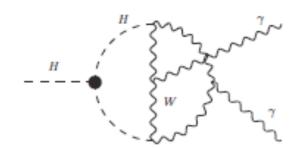
THE INDIRECT PROBE











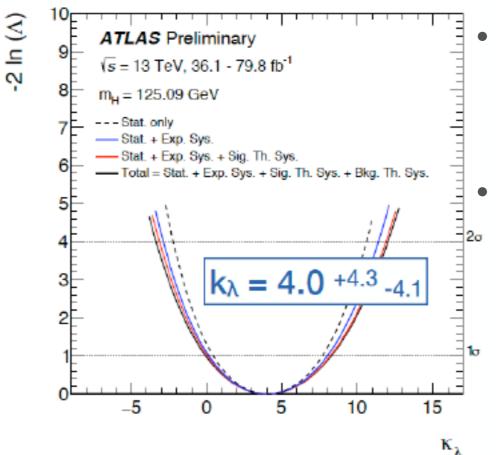
• Exploit the dependence of single-Higgs production and decay rates on λ_{hhh} entering via loops

 One can perform a one-parameter fit assuming other couplings being SM like.

$$\mu_i^f = \frac{\sigma_i \cdot Br^f}{\sigma_{SM,i} \cdot Br_{SM}^f} = \mu_i \cdot \mu^f$$

$$\mu_i = 1 + \delta \sigma_i (\lambda_{hhh})$$

$$\mu^f = 1 + \delta Br^f (\lambda_{hhh})$$

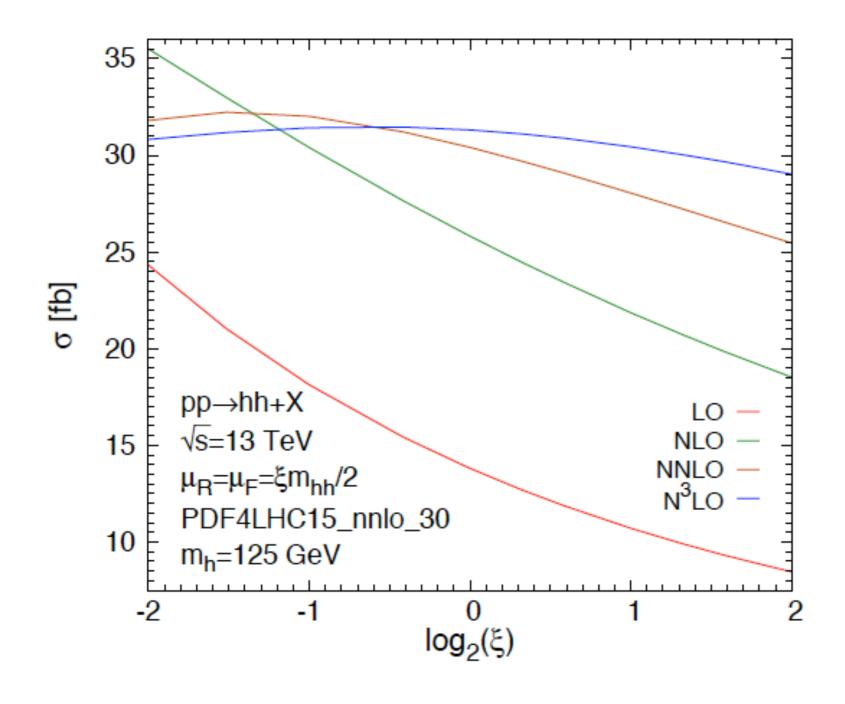


- Similar constraints than from hh measurements.
- However, it is
 limited by
 systematics. Then,
 less room to be
 improved at HL LHC than hh.



- Infinite top-quark mass limit
 - N³LO cross sections

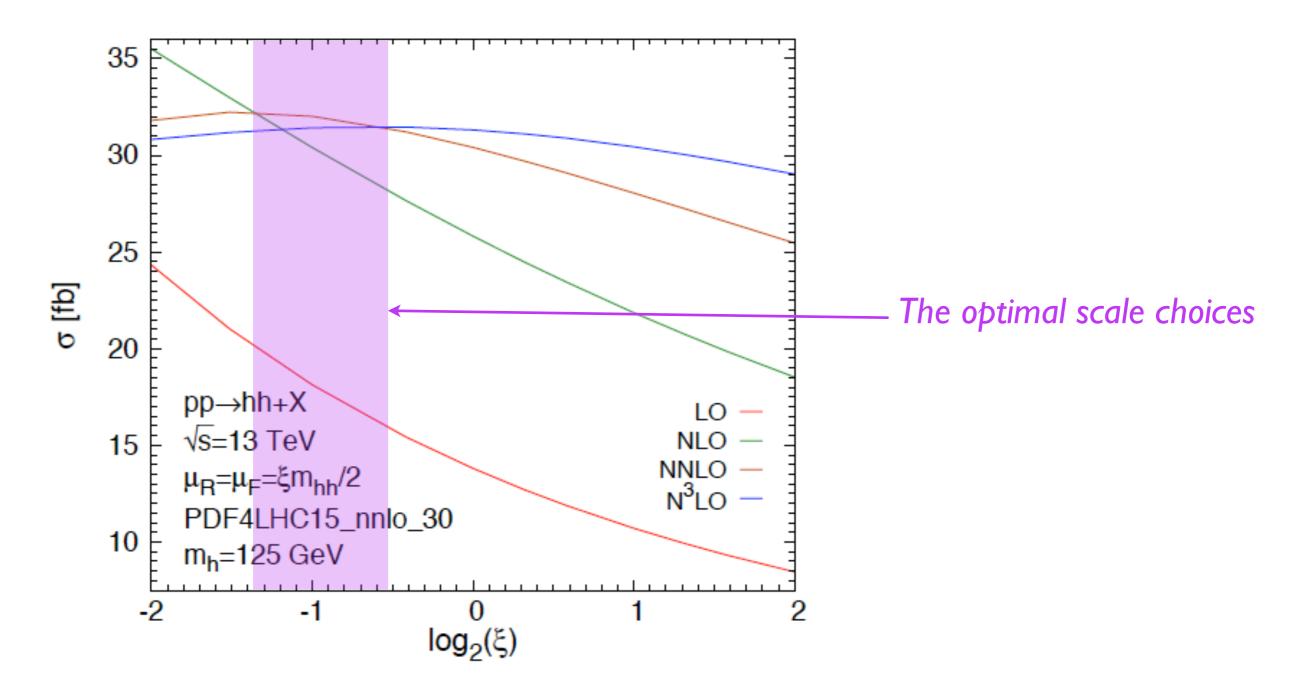
Chen, Li, HSS, Wang (1909.06808)





- Infinite top-quark mass limit
 - N³LO cross sections

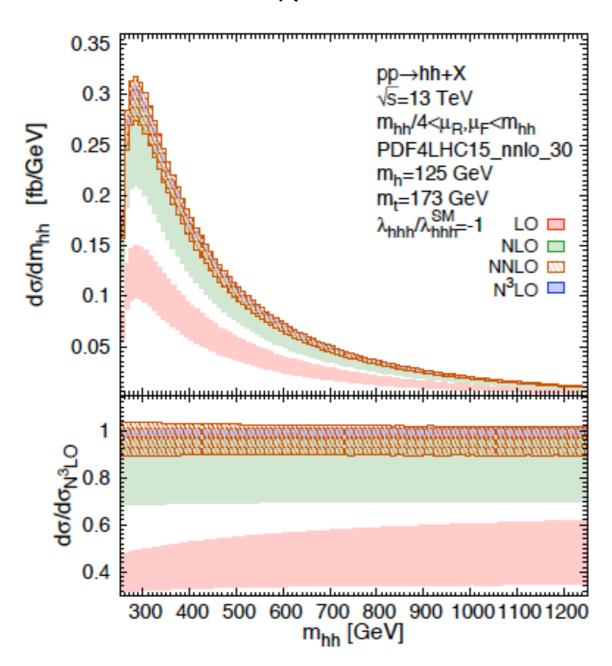
Chen, Li, HSS, Wang (1909.06808)





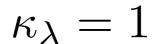
- Infinite top-quark mass limit
 - a N³LO differential distribution

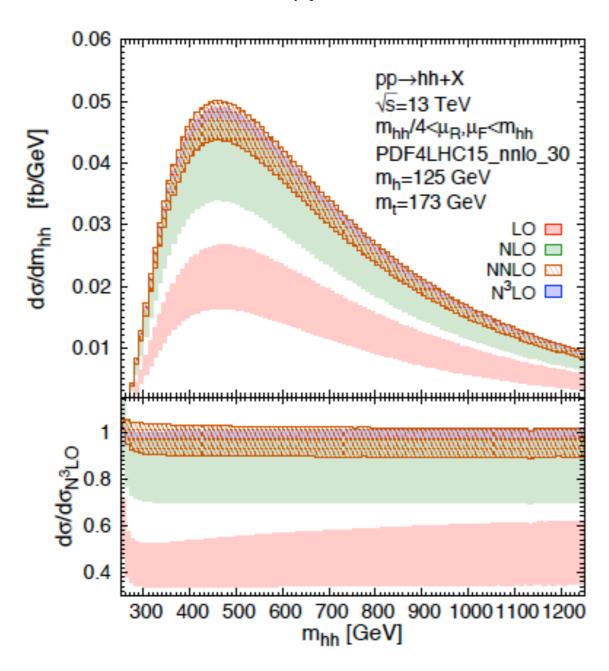
$$\kappa_{\lambda} = -1$$





- Infinite top-quark mass limit
 - a N³LO differential distribution

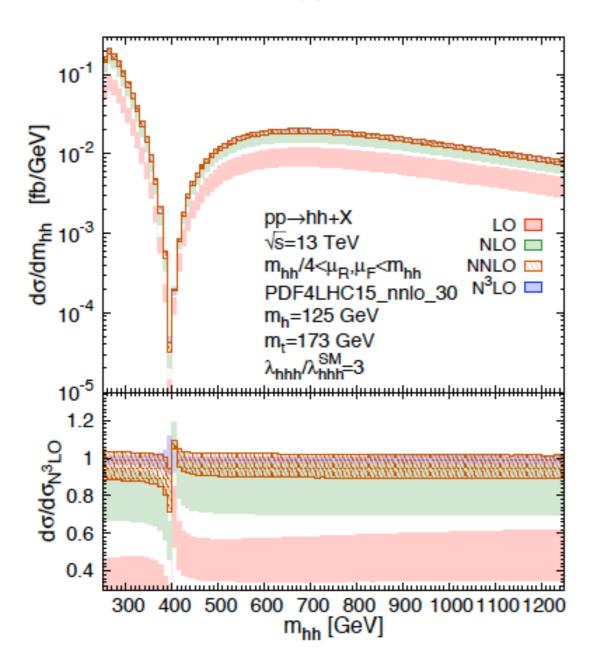






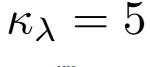
- Infinite top-quark mass limit
 - a N³LO differential distribution

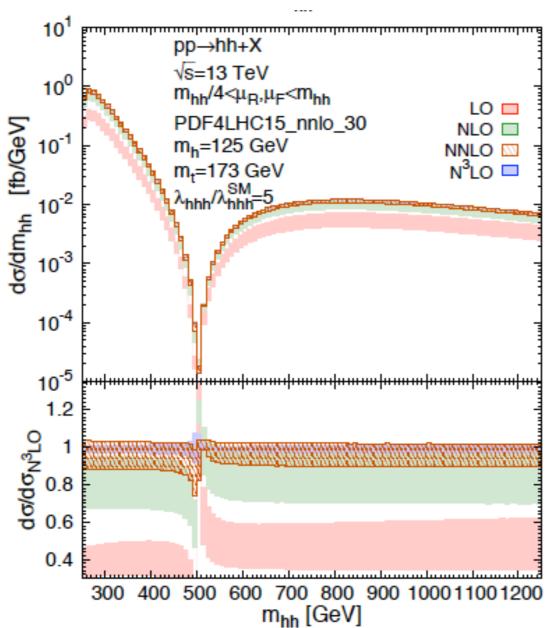
$$\kappa_{\lambda} = 3$$





- Infinite top-quark mass limit
 - a N³LO differential distribution

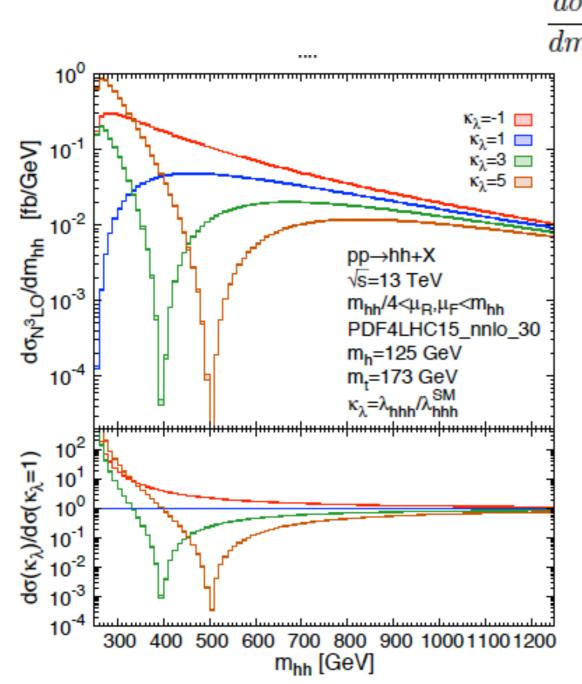






- Infinite top-quark mass limit
 - a N³LO differential distribution

Chen, Li, HSS, Wang (1912.13001)



$$\frac{d\sigma_{hh}^a}{dm_{hh}} = f_{h\to hh} \left(\frac{C_{hh}}{C_h} - \frac{6\lambda v^2}{m_{hh}^2 - m_h^2}\right)^2 \times \sigma_h(m_h \to m_{hh})$$

$$f_{h\to hh} \propto \sqrt{m_{hh}^2 - 4m_h^2} \qquad m_{hh} > 2m_h$$

$$\left(\frac{C_{hh}}{C_h} - \frac{6\lambda_{hhh}v^2}{m_{hh}^2 - m_h^2}\right)^2 \simeq \left(1 - \kappa_\lambda \frac{3m_h^2}{m_{hh}^2 - m_h^2}\right)^2$$

$$\kappa_\lambda < 0 \qquad \qquad \text{w} \quad m_{hh} \uparrow$$

$$0 < \kappa_\lambda \le 1 \qquad \text{w} \quad m_{hh} \uparrow$$

$$\kappa_\lambda > 1 \quad \text{zero when}$$

$$m_{hh} = \sqrt{1 + 3\kappa_\lambda} m_h$$

Potential useful in BSM searches



- Infinite top-quark mass limit
 - N³LO other differential distribution



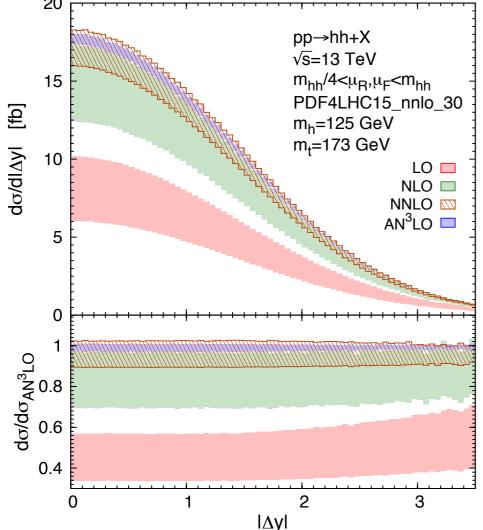
- Infinite top-quark mass limit
 - N³LO other differential distribution Chen, Li, HSS, Wang (1912.13001)
 - Impossible as we are even lacking of fully-differential ggH



- Infinite top-quark mass limit
 - N³LO other differential distribution

- Impossible as we are even lacking of fully-differential ggH
- but ... possible with some approximations

e.g. rapidity difference
$$\frac{d\sigma_{hh}^{\text{AN}^{3}\text{LO}}}{dO} = \frac{d\sigma_{hh}^{(\text{a},1),\text{NNLO}}}{dO} + \frac{\sigma_{hh}^{(\text{a},1),\text{NNLO}}}{\sigma_{hh}^{(\text{a},1),\text{NNLO}}} + \frac{d\sigma_{hh}^{(\text{a},2),\text{N}^{3}\text{LO}}}{dO} + \frac{d\sigma_{hh}^{\text{b},\text{NNLO}}}{dO} + \frac{d\sigma_{hh}^{\text{b},\text{NNLO}}}{dO} + \frac{d\sigma_{hh}^{\text{b},\text{NNLO}}}{dO} + \frac{d\sigma_{hh}^{\text{c},\text{NLO}}}{dO}$$



$$d\sigma_{hh}^{(a,1)} = d\sigma_{hh}^a|_{C_{hh}=C_h}$$
$$d\sigma_{hh}^{(a,2)} = d\sigma_{hh}^a - d\sigma_{hh}^{(a,1)}$$

$$d\sigma_{hh}^{(a,2)} = d\sigma_{hh}^a - d\sigma_{hh}^{(a,1)}$$