Two-loop corrections to the Higgs trilinear coupling in models with extended scalar sectors

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based on
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with **Shinya Kanemura**

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Probing the shape of the Higgs potential

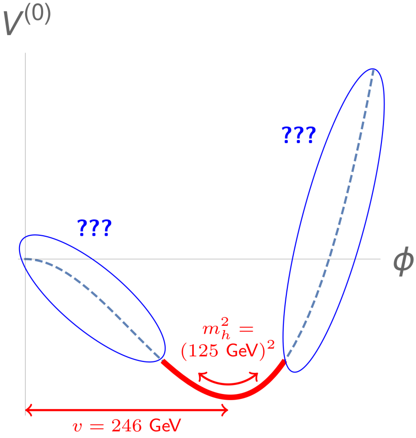
- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:

  → the location of the EW minimum: $v \simeq 246$ GeV
  → the curvature of the potential around the EW minimum: $m_h \simeq 125$ GeV

  However what we still don't know is the **shape** of the Higgs potential, which **depends on** λ_{hhh}

- λ_{hhh} determines the nature of the EWPT!

  $\Rightarrow \mathcal{O}(30\%)$ deviation of λ_{hhh} from its SM prediction needed to have a strongly first-order EWPT
  → necessary for EWBG scenario
  [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]



Investigating the Higgs trilinear coupling λ_{hhh}

Alignment with or without decoupling

- Aligned scenarios – for which Higgs couplings are SM-like at **tree-level** – seem to be strongly favoured by experimental results $\rightarrow$ *why*?

  $\rightarrow$ Alignment **through decoupling** ($i.e.$ BSM states out of reach)?
  or
  $\rightarrow$ Alignment **without** decoupling?

- If alignment without decoupling, Higgs couplings like λ_{hhh} can still exhibit **large deviations** from SM predictions because of large **non-decoupling effects** in BSM loops!

- Current best limit on λ_{hhh}: $-3.7 < \lambda_{hhh}/\lambda_{hhh}^{\text{SM}} < 11.5$ (at 95% CL) [ATLAS-CONF-2019-049]

- Improvement at future colliders: **HL-LHC**: $\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ within $\sim 50-100\%$; **lepton colliders** (ILC, CLIC) within some tens of $\%$; even down to $5-7\%$ at **100-TeV hadron collider** (*details in backup*)

c.f. [de Blas et al., 1905.03764], [Cepeda et al., 1902.00134], [Di Vita et al. 1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130, 1908.00753], etc.

Non-decoupling effects in λ_{hhh}

The Two-Higgs-Doublet Model (2HDM)

- CP-conserving 2HDM, with softly-broken $\mathbb{Z}_2$ symmetry $(\Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2)$ to avoid tree-level FCNCs
- 2 $SU(2)_L$ doublets $\Phi_{1,2} = \begin{pmatrix} \Phi_{1,2}^+ \\ \Phi_{1,2}^0 \end{pmatrix}$ of hypercharge $1/2$

$$V_{\text{2HDM}}^{(0)} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)\,,$$

and $\tan\beta \equiv \langle\Phi_2^0\rangle/\langle\Phi_1^0\rangle$.

- Doublets expanded in terms of mass eigenstates:
  h, H: CP-even, A: CP-odd, $H^\pm$: charged Higgs bosons
- Additional BSM scalars $(\Phi = H, A, H^\pm)$ have 2 sources of mass:

(1) Higgs VEV v
(2) $\mathbb{Z}_2$-symmetry soft-breaking mass scale $M^2 = 2m_3^2/s_{2\beta}$

$\Rightarrow$ $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$

($\tilde{\lambda}_\Phi$: some combination of λ_i)

Non-decoupling effects in λ_{hhh} at one loop

First studies of the one-loop corrections to λ_{hhh} in the 2HDM in [Kanemura, Kiyoura, Okada, Senaha, Yuan '02] and [Kanemura, Okada, Senaha, Yuan '04]

- Up to leading one-loop corrections (for $s_{\beta-\alpha}=1$, and with $[M_h^2]_{V_{\text{eff}}}$: Higgs effective potential mass)

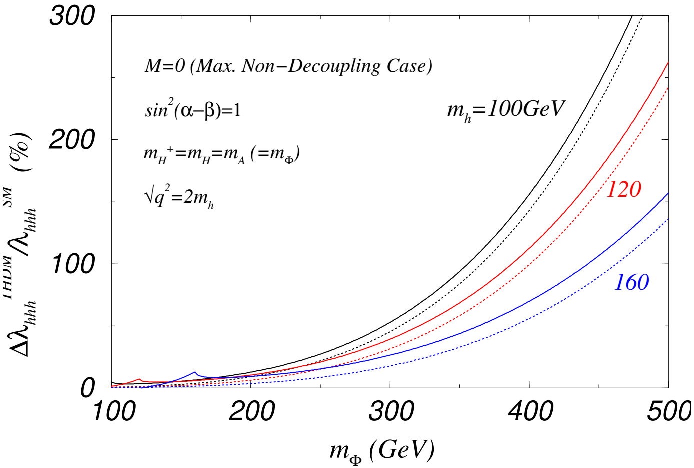
$$\lambda_{hhh} = \lambda_{hhh}^{(0)} + \frac{1}{16\pi^2}\delta^{(1)}\lambda_{hhh} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \frac{1}{16\pi^2}\left[\underbrace{-\frac{48m_t^4}{v^3}}_{\text{SM-like}} + \sum_{\Phi=H,A,H^\pm}\underbrace{\frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3}_{\text{BSM}}\right]$$

- BSM terms have
  (1) a power-like dependence of $\propto m_\Phi^4$
  (2) a suppression factor that depends on how the BSM scalars acquire their mass (recall that $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$)

$$\left(1-\frac{M^2}{m_\Phi^2}\right)^3 \to \begin{cases} 0, & \text{for } M^2 \gg \tilde{\lambda}_\Phi v^2 \\ 1, & \text{for } M^2 \ll \tilde{\lambda}_\Phi v^2 \end{cases}$$

$\Rightarrow$ for $M \to 0$, **large BSM effects are possible at one loop!** $\to$ see figure in next slide
(Note that this doesn't pose a problem for perturbativity, as the large effects in $\delta^{(1)}\lambda_{hhh}$ involve new couplings that are not present at tree level)

Non-decoupling effects in λ_{hhh} at one loop



(figure from [Kanemura, Okada, Senaha, Yuan '04])

- **Huge deviations possible, without violating unitarity!**
  → **non-decoupling effects**

⇒ What happens at **two loops**? Can new huge corrections occur?

⇒ We derive the **dominant** two-loop corrections to λ_{hhh} in several BSM models [JB, Kanemura '19]

Two-loop calculation of λ_{hhh}

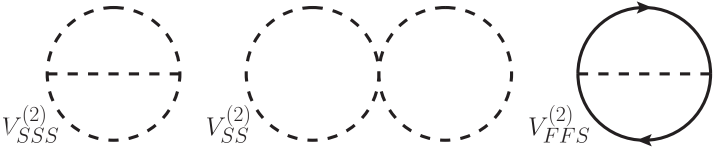
Setup of our effective-potential calculation

Step 1: calculate $\underbrace{V_{\text{eff}}}_{\overline{\text{MS}}}$ $\rightarrow$ **Step 2:** $\underbrace{\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\Big|_{\text{min.}}}_{\overline{\text{MS}}}$ $\rightarrow$ **Step 3:** convert from $\overline{\text{MS}}$ to OS scheme

- $\overline{\text{MS}}$-renormalised two-loop effective potential is

$$V_{\text{eff}} = V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)} \qquad \left(\kappa \equiv \frac{1}{16\pi^2}\right)$$

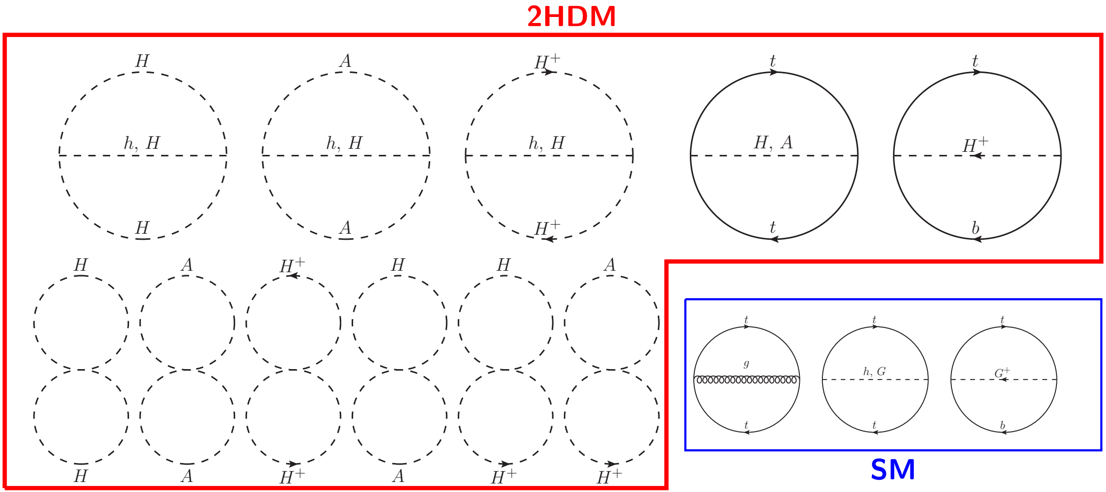
- $V^{(2)}$: 1PI vacuum bubble diags., and we want to study the leading two-loop BSM corrections from **additional scalars** and **top quark**, so we only need



- Also, we **neglect subleading contributions** from h, $G, G^{\pm}$, gauge bosons, and light fermions
- **Scenarios without mixing**: aligned 2HDM ($s_{\beta-\alpha} = 1$) $\Rightarrow$ evade exp. constrains!
(loop-induced deviations from alignment also neglected)

λ_{hhh} at two loops in the 2HDM

2HDM $\to$ 15 new BSM diagrams appearing in $V^{(2)}$ w.r.t. the SM case

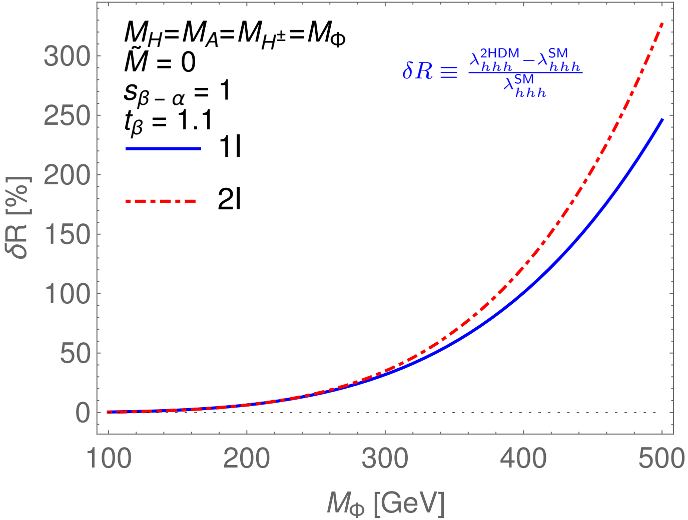


Numerical results

In the following: some results for the BSM deviation

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{BSM}} - \lambda_{hhh}^{\text{SM}}}{\lambda_{hhh}^{\text{SM}}}$$

Non-decoupling effects in the 2HDM



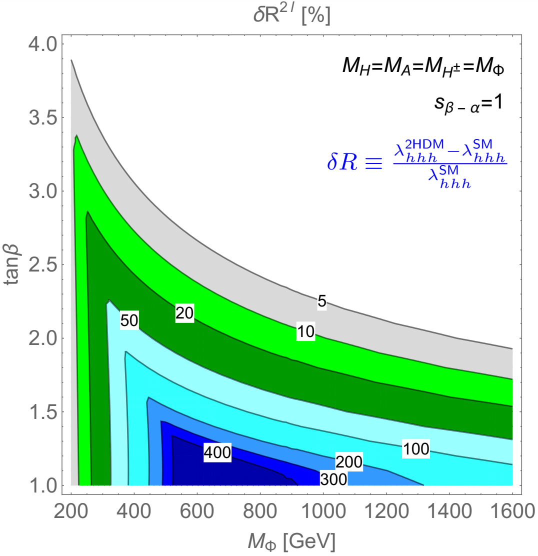
- $\tilde{M} = 0 \to$ maximal non-decoupling effects

  [$\tilde{M}$: "OS" version of M, defined to ensure proper decoupling for $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$ (c.f. backup)]

- For $\tilde{M} = 0$, $\tan\beta = 1.1$, tree-level perturbative unitarity is lost around $M_\Phi \approx 600$ GeV [Kanemura, Kubota, Takasugi '93]

- $\delta^{(2)}\hat{\lambda}_{hhh}$ typically 10-20% of $\delta^{(1)}\hat{\lambda}_{hhh}$ for most of the range of M_Φ (at most ∼30%)

Maximal possible BSM deviations in the 2HDM



- Maximal deviation δR $(1\ell+2\ell)$ allowed while fulfilling perturbative unitarity shown in $(\tan\beta, M_\Phi)$ plane
- Max. deviations for low $\tan\beta$ and $M_\Phi \sim 600\text{-}800$ GeV $\rightarrow$ BSM scalars acquire all their mass from the Higgs VEV and become heavy
  - 1 loop: up to $\sim 300\%$ deviation at most
  - 2 loops: additional 100% (for same points)
- For increasing $\tan\beta$, unitarity constraints become more stringent $\rightarrow$ smaller effects
- Blue region: probed at the **HL-LHC** (50% accuracy on λ_{hhh})
- Green region: probed at lepton colliders, *e.g.* **ILC** (50% accuracy at 250 GeV; 27% at 500 GeV; 10% at 1 TeV)

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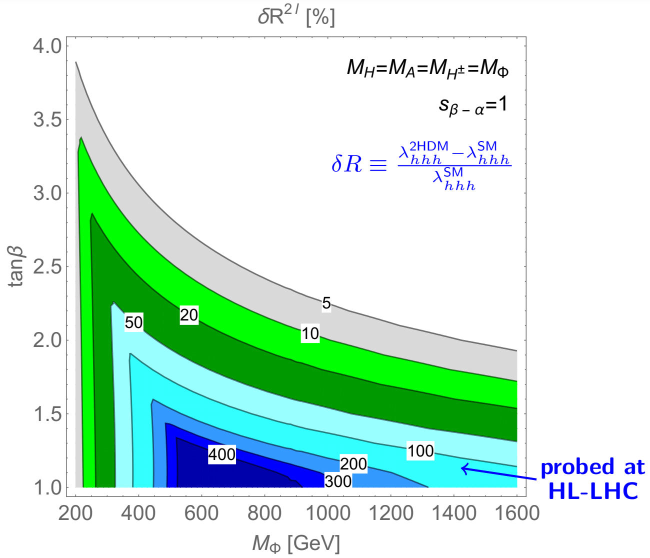

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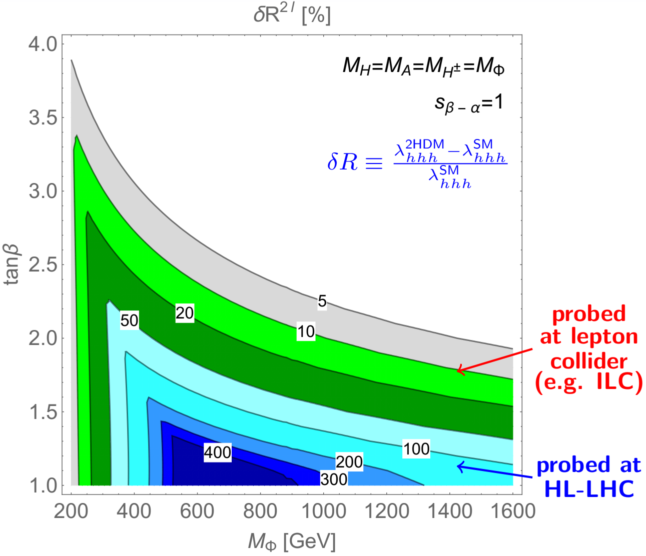

- Maximal deviation δR (1ℓ+2ℓ) allowed while fulfilling perturbative unitarity shown in $(\tan\beta, M_\Phi)$ plane
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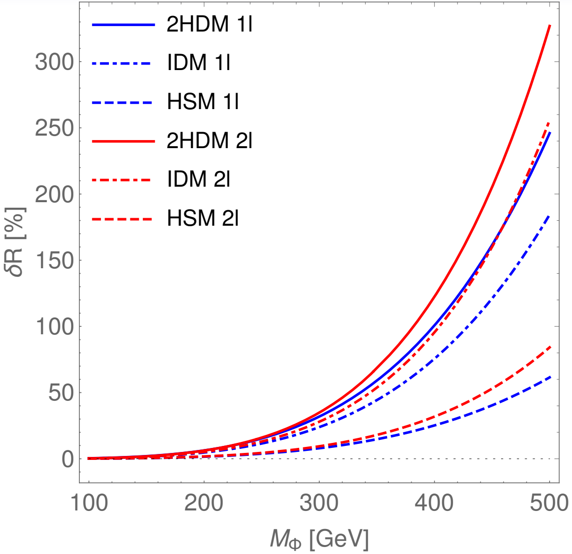
Two-loop calculations for more models



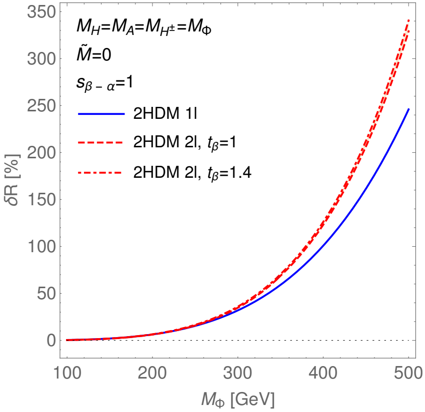
- We considered several more models
  [JB, Kanemura 1911.11507]
  - 2HDM → previously presented
  - Inert Doublet Model (IDM), in DM-inspired scenario (H light; A, $H^\pm$ heavy)
  - Real-singlet extension of the SM (HSM)

- Size of BSM deviation ∝ # heavy d.o.f.
  - 2HDM → 4 (H, A, $H^\pm$)
  - IDM → 3 (A, $H^\pm$)
  - HSM → 1 (S)

Two-loop calculation for more models

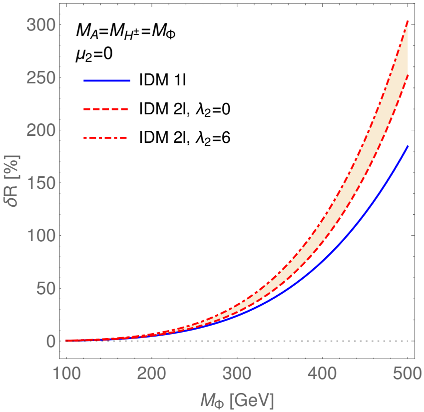
Each model contains a new parameter appearing from two loops:

Aligned 2HDM $\rightarrow \tan\beta$



$\tan\beta$ constrained by perturbative unitarity
$\rightarrow$ only small effects

IDM $\rightarrow \lambda_2$ (inert doublet quartic coupling)



λ_2 is less contrained $\rightarrow$ enhancement is possible
(but 2ℓ effects remains well smaller than 1ℓ ones)

Summary

- **First two-loop calculation of λ_{hhh} in 2HDM**, in a scenario with alignment
  $+$ also **IDM** and **HSM**

- Two-loop corrections amount typically to $10\text{-}20\%$ of one-loop contributions (max. $\sim 30\%$)

$\Rightarrow$ Non-decoupling effects found at one loop are **not drastically changed**

$\Rightarrow$ Computations beyond one loop will be **necessary** given the expected accuracy of the measurement of λ_{hhh} at future colliders – **HL-LHC** (50% acc.), **ILC** (27% at 500 GeV, down to 10% at 1 TeV)

- Precise calculation of Higgs couplings (λ_{hhh}, etc.) can allow distinguishing aligned scenarios **with or without decoupling**, by accessing **non-decoupling effects**!

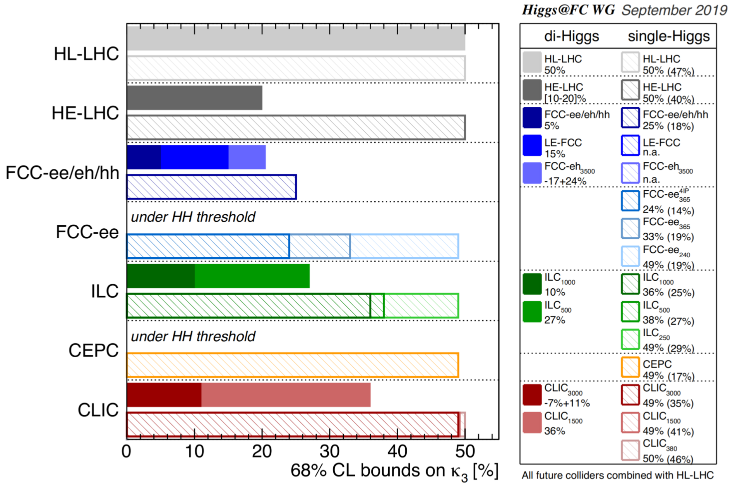
See also JB and Kanemura, *PLB 796 (2019) 38-46 & EPJC 80 (2020) 3, 227* for details, and please feel free to write to me at braathen@het.phys.sci.osaka-u.ac.jp

Thank you for your attention!

Backup

Future measurements prospects for the Higgs trilinear coupling λ_{hhh}

$$\left(\kappa_3 \equiv \frac{\lambda_{hhh}}{\lambda_{hhh}^{\mathrm{SM}}}\right)$$



[Higgs@FC report, 1905.03764]

Setup of our effective-potential calculation (detailed)

- OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\text{MS}}\text{ parameters} \\ \text{translated to OS ones}}}$$

- Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\text{MS}}}) + \kappa f^{(1)}(x^{\overline{\text{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\text{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

and

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa\left[f^{(1)}(X^{\text{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x\right] + \kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2\right]$$

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$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

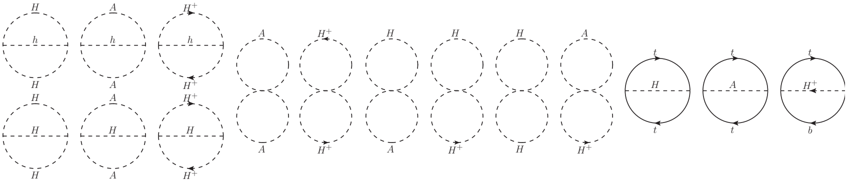
then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa\left[f^{(1)}(X^{\text{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x\right]$$

$$+\kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2\right]$$

because we neglect m_h in the loop corrections and $\lambda_{hhh}^{(0)} = 3m_h^2/v$ (in absence of mixing)

λ_{hhh} at two loops in the 2HDM



- In the $\overline{\text{MS}}$ scheme

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4 + 9\cot^2 2\beta\right)\left(1 - \frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2)\,\overline{\log}\, m_\Phi^2\right]$$
$$+ \frac{192 m_\Phi^6 \cot^2 2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^4\left[1 + 2\,\overline{\log}\, m_\Phi^2\right]$$
$$+ \frac{96 m_\Phi^4 m_t^2 \cot^2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3\left[-1 + 2\,\overline{\log}\, m_\Phi^2\right] + \mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

(Recall: aligned scenario, degenerate masses, dominant corrections only)

Decoupling behaviour of the $\overline{\mathrm{MS}}$ expressions

- Seeing whether corrections from additional BSM states decouple if said state is taken to be very massive is a good way to check the consistency of the calculation

$$\delta^{(1)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3$$

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4+9\cot^2 2\beta\right)\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2-m_\Phi^2+(M^2+2m_\Phi^2)\,\overline{\log}\,m_\Phi^2\right]$$
$$+\frac{192m_\Phi^6\cot^2 2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[1+2\,\overline{\log}\,m_\Phi^2\right]$$
$$+\frac{96m_\Phi^4 m_t^2\cot^2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^3\left[-1+2\,\overline{\log}\,m_\Phi^2\right]+\mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

where $m_\Phi^2 = M^2+\tilde{\lambda}_\Phi v^2$

- To have $m_\Phi\to\infty$, then we must take $M\to\infty$, otherwise the quartic couplings grow out of control
- Fortunately all of these terms go like

$$(m_\Phi^2)^{n-1}\left(1-\frac{M^2}{m_\Phi^2}\right)^n \underset{m_\Phi^2=M^2+\tilde{\lambda}_\Phi v^2}{=} \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2+\tilde{\lambda}_\Phi v^2} \xrightarrow[\tilde{\lambda}_\Phi v^2\ \text{fixed}]{M\to\infty} 0$$

Decoupling behaviour and $\overline{\text{MS}}$ to OS scheme conversion

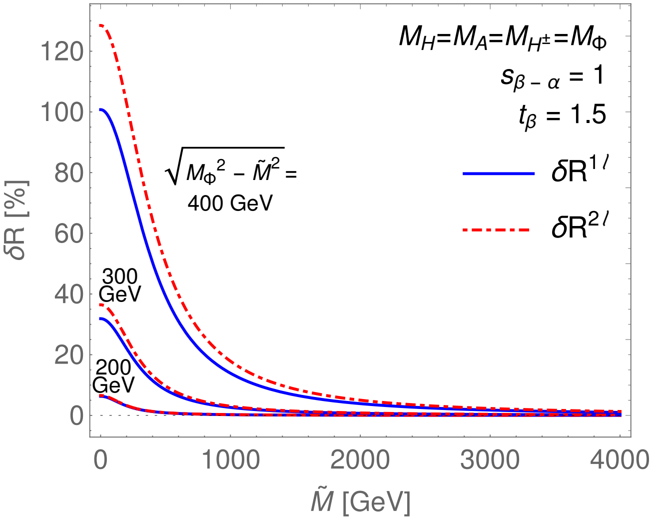
- To obtain $\hat{\lambda}_{hhh} = -\Gamma_{hhh}(0,0,0)$, we must express our results in terms of physical parameters

$$\overline{\text{MS}}\text{ scheme:}\Big\{\underbrace{m_H, m_A, m_{H\pm}}_{m_\Phi}, m_t, v\Big\} \longrightarrow \text{OS scheme:}\Big\{\underbrace{M_H, M_A, M_{H\pm}}_{M_\Phi}, M_t, v_{\text{phys}} = (\sqrt{2}G_F)^{-1/2}\Big\}$$

- A priori, M is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable
  ... but then, two-loop expressions do not decouple for $M_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ and $M \to \infty$!
- This is because we should relate M_Φ, renormalised in OS scheme, and M, renormalised in $\overline{\text{MS}}$ scheme, with a **one-loop relation** $\to$ then the two-loop corrections decouple properly
- We give a new "OS" prescription for the finite part of the counterterm for M be requiring that the decoupling of $\delta^{(2)}\hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$

$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{48 M_\Phi^6}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^4 \left\{4 + 3\cot^2 2\beta\left[3 - \frac{\pi}{\sqrt{3}}\left(\frac{\tilde{M}^2}{M_\Phi^2} + 2\right)\right]\right\} + \frac{576 M_\Phi^6 \cot^2 2\beta}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^4$$
$$+ \frac{288 M_\Phi^4 M_t^2 \cot^2\beta}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^3 + \frac{168 M_\Phi^4 M_t^2}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^3 - \frac{48 M_\Phi^6}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^5 + \mathcal{O}\left(\frac{M_\Phi^2 M_t^4}{v_{\text{phys}}^5}\right)$$

Decoupling behaviour



- δR size of BSM contributions to λ_{hhh}:

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

- $\tilde{M}$: "OS" version of M, defined so as to ensure proper decoupling for $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

- Radiative corrections from additional scalars + top quark indeed decouple properly for $\tilde{M} \to \infty$